Research Article
Image Enhancement of Cross-Border E-Commerce Logistics Video Surveillance Based on Partial Differential Equations

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Abstract

Although the development time of cross-border e-commerce in China is very short, the scale of its transactions and the speed of development are amazing, and as a supporting foundation for promoting economic and trade globalization, cross-border e-commerce has an extremely important strategy for this guiding role. This not only brings new opportunities to cross-border e-commerce companies but also excavates a huge potential market for the logistics industry. Cross-border e-commerce not only breaks through the trade barriers between countries; it makes trade move towards borderlessness and at the same time triggers major changes in international trade. This paper introduces partial differential equations into the video surveillance image enhancement system of cross-border e-commerce logistics. Aiming at the shortcomings of the contrast enhancement method based on gradient field equalization, this paper proposes a partial differential enhancement method based on histogram equalization. By proposing a gradient transformation function, the edges and textures with relatively small gradient values are enhanced to make the original weaker texture details clearer. In order to better adjust the brightness and contrast of the image, combined with histogram equalization, we propose an inverse equalization transformation. When the histogram equalization and the inverse equalization transform are combined reasonably, the brightness and contrast of the image can be adjusted very well. In this paper, the finite difference method is used for discretization when solving partial differential equations, and Euler's equation is obtained by applying the principle of least squares. By introducing the heat equation, the direct solution of Euler's equation is converted into an iterative form, which greatly reduces the amount of calculation. This article uses statistical methods to obtain the empirical formula of the fractional differential order. This empirical formula makes the calculation of the order of the fractional derivative easy and can be extended to other fractional image enhancement models and overcomes the shortcomings of the traditional fractional derivative order obtained through experience or a large number of experiments. Experiments show that the proposed algorithm not only enhances detailed texture information but also improves image clarity, overall brightness, and contrast without color distortion. The objective evaluation indicators also show the superiority of the algorithm.

1. Introduction

Driven by the development of international trade, because commodities need to be transported from the country where the seller is the subject of the transaction to the country and at the level of space transportation, it needs to cross different national borders or customs borders, resulting in cross-border logistics. It shows that bulk commodities flow from the seller to the buyer through land transportation, sea transportation, air transportation, pipeline, or international multimodal transportation [1, 2]. Cross-border logistics in the context of international trade belongs to a broad concept. The broad cross-border logistics refers to the logistics service activities carried out between two or more countries. Cross-border logistics is a kind of advanced stage of logistics service development. In the context of cross-border e-commerce, cross-border logistics has obvious characteristics of e-commerce. Commodity transportation is no longer manifested as the cross-border spatial displacement of bulk commodities. The cross-border logistics model realizes the
cross-border spatial displacement of small batches and multiple frequencies [3]. So this belongs to the category of cross-border logistics in a narrow sense. Cross-border logistics in a narrow sense refers specifically to the context of cross-border e-commerce, because the transaction entities belong to different countries or regions, and the goods that flow from the seller to the buyer need to cross different countries, so as to realize the spatial displacement of the goods from the seller to the buyer and realize the final logistics and distribution activities in the buyer’s country, as well as a series of related activities [4].

There is a coexistence relationship between cross-border e-commerce and cross-border e-commerce logistics, but it is not a relationship of prosperity and loss, because cross-border e-commerce logistics has a greater impact on cross-border e-commerce than cross-border e-commerce logistics [5]. The development of international trade under the new form of cross-border e-commerce determines the future development of cross-border e-commerce logistics and also puts forward new standards for cross-border e-commerce logistics, such as the quality of logistics and transportation operations, the timeliness of transportation, and the safety of goods. Of course, the development of cross-border e-commerce logistics also affects the development of international trade, because in the development of e-commerce, information flow and capital flow can be completed through the uniqueness of the Internet, but only logistics must be specific and practical [6]. The transportation tool completes the operation. Therefore, in the actual operation of enterprises, the superior development of cross-border e-commerce logistics does not necessarily mean the superior development of cross-border e-commerce, but the backward development of cross-border e-commerce logistics will restrict the development of cross-border e-commerce to a greater extent [7]. Development shows the importance of cross-border e-commerce logistics development. Therefore, a stable cross-border e-commerce logistics environment will promote the better development of international trade, while a disorderly and inefficient cross-border logistics environment will become a bottleneck for the development of international trade [8].

This article mainly applies partial differential equations to the image enhancement field of cross-border e-commerce logistics video surveillance. Using the idea of partial differential equations, the image is transformed into the gradient domain, and the purpose of enhancing the image is achieved through the transformation of the gradient field. Since there are relatively few studies on partial differential enhancement methods, a typical method is an image contrast enhancement method based on gradient field equalization. This paper proposes a partial differential enhancement method based on histogram equalization, using a new gradient transformation function. By changing the distribution of the image gradient field, the feature information of the image with small gradient values and unclear texture edges can be enhanced. At the same time, in order to enhance the brightness and contrast of the image, the method in this paper also combines histogram equalization and proposes an inverse equalization transformation, which can well enhance the detail information of the dark and bright areas in the image and enhance the brightness and contrast of the entire image. It will not produce overenhancement and saturation, so as to achieve better visual effects. The algorithm in this paper enhances detailed texture information; improves image clarity, overall brightness, and contrast; and does not display color distortion. From the perspective of objective evaluation indicators, the algorithm in this paper also has significant advantages. This paper also compares the proposed algorithms in terms of enhancement effect and running time. Through analysis, the three image enhancement algorithms for cross-border e-commerce logistics video surveillance proposed in this paper can enhance the brightness and contrast of the image from a visual point of view and achieve the enhanced effect. From the perspective of objective evaluation indicators, the algorithm proposed in this paper also has significant advantages. In terms of running time, the histogram equalization algorithm of partial differential equations proposed in this paper has obvious advantages in time.

2. Related Work

Related scholars have discussed the logistics problems of cross-border e-commerce development, such as unreasonable logistics system construction, imperfect infrastructure construction, low logistics information level, low comprehensive level, and weak information processing capabilities of third-party logistics companies, which restrict foreign business development [9]. Because scholars believe that while strengthening government policy support and strengthening enterprise informatization and standardization efforts, it is necessary to further explore the new cross-border third-party logistics enterprise model [10]. Relevant scholars believe that the laws and regulations related to cross-border e-commerce logistics are relatively lacking, so they explain the importance of establishing a sound cross-border e-commerce logistics legal mechanism [11]. In addition, starting from e-commerce companies, in order to reduce the logistics costs of self-operated logistics companies, it is proposed to establish a logistics strategic alliance between cross-border e-commerce companies. Relevant scholars believe that international parcels and international express delivery are the selection criteria for small- and medium-sized cross-border e-commerce companies in my country, which are relatively small in scale but account for the main body of cross-border e-commerce [12]. Companies are monopolized, and their fees are often high.

The isotropic heat conduction model is the original form of image noise reduction and smoothness. It is a process of transforming a physical model based on the heat conduction method of physics into a mathematical equation. The image that has been preprocessed with noise reduction is analogous to an object with different temperatures everywhere and uneven heat distribution, affected by diffusion, and the internal heat will follow the principle of gradual diffusion from high temperature to low temperature. In the process of image noise reduction, the unevenly distributed heat in the object when heat conduction occurs is the different gray
values of image pixels. They will eventually become a stable temperature field with heat balance, that is, a smoothed image. This smoothing principle is similar to linear Gaussian filtering. Isotropic diffusion determines that it cannot identify different areas in the image during noise reduction. The smoothness of each part is the same. The signal has removed the high-frequency signal containing image noise and details. Although the noise is effectively filtered out, it also causes the loss of the image texture, edges, and other details, making the overall look and feel of the processed image blurred.

Related scholars have proposed an improved anisotropic diffusion model PM model, which is the first time that partial differential equations have been used in real sense for image noise reduction [13]. The PM model uses the diffusion function to formulate the diffusion coefficient according to the calculation results of the gradient information at different positions of the image. The texture details and the edge part of the image change obviously, and the gradient value is large. For the flat areas of the image such as the background, the gray value changes are not obvious, and the gradient value is small. The diffusion coefficient obtained by the diffusion function is inversely proportional to the image gradient value. The diffusion intensity of the flat area of the image is large, and the diffusion intensity of the detail edge part is small, which protects the image structure information while filtering image noise. The appearance of the PM model has significantly improved the quality of image noise reduction, but as its applications increase, its shortcomings have begun to appear. Some scholars have found that when large noises are located at the edge of the image or are distributed in areas with large gray values, these noises are difficult to filter out, and the noise reduction effect is not ideal [14]. At the same time, for two input images with small differences, the smoothed images after PM model noise reduction are very different from each other. Related studies have confirmed that the PM model is an ill-conditioned equation [15]. For this reason, the researchers proposed a regularized PM model of selective diffusion, added Gaussian smoothing regularization to the original model, and proved the uniqueness of its partial differential equation solution [16]. Related scholars have also proposed the MCM diffusion model of mean curvature flow, which improves the function of the PM model and improves the stability [17].

Researchers introduced the theory of scale space to digital image processing, using multiple scale functions to represent the same image, which is the basis for the application of partial differential equations in the field of image processing [18]. The earliest multiscale image is obtained by Gaussian filtering of the original image. In the partial differential equation, it can be considered that the thermal diffusion equation is used to transform the original image to obtain an isotropic diffusion model. Later research found that the thermal diffusion equation is not the only partial differential equation that can be used to define multiscale images [19], as long as the transformation equation that satisfies the principle of maximum value can create a multiscale space. Research has found that adding threshold operation in the Gaussian filtering process can obtain geometric partial differential equations, among which the most famous type of partial differential equations is curvature motion [20].

3. Construction of Cross-Border e-Commerce Logistics System

3.1. The Components of a Cross-Border e-Commerce Logistics System

The logistics information subsystem mainly includes four elements: order processing, document transmission, logistics information tracking, and customer data sorting. By constructing a complete logistics information subsystem, the various entities involved in cross-border e-commerce logistics can be integrated through the construction of an information network. It speeds up the circulation of information among various entities, reduces the duplication of documents, greatly improves the efficiency of cross-border e-commerce logistics, and reduces the cost of cross-border e-commerce logistics. The structure of the cross-border e-commerce ecosystem is shown in Figure 1.

Order processing refers to the transmission speed of orders between relevant subjects in cross-border e-commerce platforms, customs, cargo stations, logistics companies, and other cross-border e-commerce systems and the processing speed of each subject to orders. The transmission of documents refers to whether there is a unified document entry port between various entities, and the unified document entry port can effectively avoid multiple entry of documents. Entering documents once will effectively improve the operational efficiency of cross-border e-commerce logistics. A good logistics information tracking system can track the transportation of cross-border goods in real time, improving the safety of transported goods in transit. In addition, customers can query the transportation status of the goods in real time by entering the order number, which improves the level of customer satisfaction. Customer data collation mainly refers to the statistics of the types, quantities, and origins of imported goods, the types, quantities, and destinations of exported goods through the information system; and the use of big data technology for data analysis and processing.

3.1.1. Infrastructure Subsystem. The infrastructure subsystem is composed of the necessary facilities for cross-border e-commerce logistics operations in the airport area, including the area of customs supervision warehouses operating at the airport, storage facilities, special customs supervision areas, (cross-border) e-commerce industrial parks, and other facilities. The infrastructure subsystem is the basic guarantee for the operation of the cross-border e-commerce logistics system.

Warehousing facilities mainly refer to whether the types of storage facilities in the airport area can fully meet the development needs of cross-border e-commerce logistics and whether there are cold storage and special warehouses in the area to ensure the diversified development of cross-border e-commerce logistics. The special customs supervision zone is mainly to ensure that the airport area can carry out the cross-border e-commerce bonded import and export model, and the development of the bonded model will
increase the volume of cross-border e-commerce logistics in the airport area. The construction of e-commerce industrial parks can effectively drive the chain and systematic development of the cross-border e-commerce industry in the airport area and provide integrated and comprehensive supporting services for cross-border e-commerce and logistics express companies, thereby ensuring the sustained and stable development of the country.

3.1.2. Collection and Distribution Subsystem. The collection and distribution subsystem includes the number of routes at the airport, navigable cities, the number of flights, and the construction of a comprehensive transportation system. The collection and distribution subsystem directly affects the operational efficiency and cost of the airport’s cross-border e-commerce logistics system.

The internationalization, small batch, and multibatch characteristics of cross-border e-commerce require that the airport must have a well-connected international and domestic route network. The construction of a comprehensive transportation system mainly includes the connection between the airport and other modes of transportation such as shipping, railway, and highway; the construction of internal road traffic in the airport area; and the construction of road traffic connecting the airport area and the outside. The construction of a comprehensive transportation system determines whether cross-border goods can be transported in and out efficiently.

3.1.3. Commodity Inspection Subsystem. The commodity inspection subsystem mainly includes customs and inspection and quarantine. The supervision of customs and inspection and quarantine is one of the most important characteristics that distinguish cross-border e-commerce from traditional e-commerce. The speed and release rate of customs and inspection and quarantine departments will directly affect the operational efficiency of cross-border e-commerce logistics.

Customs supervision is a necessary link in the import and export of cross-border e-commerce goods. Therefore, the customs supervision mode and the speed of customs clearance will directly determine the operational efficiency of cross-border e-commerce logistics. At present, in order to promote the rapid development of cross-border e-commerce logistics, in most cross-border e-commerce pilot cities, innovative customs supervision models such as “distribution and collective reporting” and “declaration on the way” have been actively explored.

3.1.4. Management Subsystem. The management subsystem mainly includes the management ability of the management department, the support of the government, and the training of relevant talents. Good management capabilities are the guarantee for the healthy development of cross-border e-commerce logistics, and the strong support of the government can promote the rapid development of cross-border e-commerce logistics. The training of cross-border e-commerce logistics-related talents is the health.

The management ability of the management department will affect the development direction of regional cross-border e-commerce logistics from the decision-making level. The management ability of the management department is divided into comprehensive management ability and internal management ability. The comprehensive management
ability is mainly reflected in the effective coordination of cross-border e-commerce. The relationship between the relevant entities of customs, cargo stations, and express companies in the logistics system determines whether the relevant entities can develop in coordination. Internal management capabilities refer to the management capabilities of the customs, cargo stations, and other internal management departments. As an emerging industry development field, cross-border e-commerce involves trade negotiations and taxation issues with other countries. Therefore, government support is a strong guarantee for cross-border e-commerce logistics to participate in international competition.

3.2. **Structural Design of Cross-Border e-Commerce Logistics System.** The basic structure of the system means that the subsystems contained in the system and the constituent elements of the subsystems are interconnected in some form to make them a complete organic whole. As a complete organic operation entity, the cross-border e-commerce logistics system has a mutual restriction and influence relationship between various subsystems and their constituent elements. The basic structure of the cross-border e-commerce logistics system is shown in Figure 2.

The cross-border e-commerce logistics system takes the logistics information subsystem as the core, through the construction of a cross-border e-commerce logistics integrated information service platform, unifies the information entry ports of each subsystem, and realizes the one-time entry of logistics information. The platform can more comprehensively collect the data information of the various subsystems in the cross-border e-commerce logistics system. Through the comprehensive processing and analysis of the data collected by the information service platform, the results of the processing and analysis are fed back to the various subsystems, thereby promoting the development of the various subsystems.

The expansion of cross-border e-commerce logistics service functions and the extension of service scope have promoted the development of peripheral industrial subsystems. At the same time, the development of the surrounding industrial subsystems has promoted the improvement of the service level of the cross-border e-commerce logistics system, provided a stable supply of goods for the development of cross-border e-commerce logistics, and promoted the development of the collection and distribution subsystem and the infrastructure subsystem. In addition, the peripheral industry subsystem is the link between the cross-border e-commerce logistics system and the regional economy, which can drive the cross-border e-commerce logistics system to better serve the regional social and economic development. At the same time, the regional social economy can provide better external economic conditions for the development of cross-border e-commerce logistics system through the connection of surrounding industrial subsystems.

4. **Image Enhancement Algorithm Based on Partial Differential Equations**

4.1. **Gradient Domain Image Enhancement Algorithm.** There are usually two types of image enhancement methods, one is time domain enhancement, and the other is frequency domain enhancement. The image enhancement method based on partial differential equations transforms the image into the gradient domain for processing, which is also called
Gradient domain enhancement. Strictly speaking, this kind of enhancement method should belong to the scope of time domain enhancement, but this kind of method has its own uniqueness, so this kind of method is discussed separately. Gradient domain enhancement uses the theory of partial differentiation to convert the intuitive texture information and brightness and contrast information of the image into quantized gradient values for processing, so as to achieve the purpose of enhancing the image.

The objective function of using the idea of gradient domain enhancement to enhance image contrast is

\[
E(\Omega) = \frac{\Omega}{2} \prod_{p \in \Omega} \prod_{q \in N_q(p)} \frac{u'(p) - u'(q)}{u'(p) + u'(q)}.
\]

Among them, \( u \) represents the original image, \( u' \) represents the enhanced image, \( \Omega \) represents the domain of the image, and \( N_q(p) \) represents the four areas of point \( p \). The original image \( u \) is restricted by the following formula:

\[
-1 < \frac{u(p) - u(q)}{u'(p) - u'(q)} < \delta - 1,
\]

\[
L < 1 - u'(p) < U.
\]

\( \delta \) represents the value of image contrast enhancement; \( L \) and \( U \) represent the grayscale range of the image, for 8-bit grayscale images, \( L = 0 \), \( U = 255 \).

\( u(p) - u(q) \) represents the change of the domain gray value of a certain point in the original image, \( u'(p) - u'(q) \) represents the change of the domain gray value of the point in the enhanced image. If you want to make the image enhancement effect better, that is, the contrast of the image is higher, it is required that the gray value of a certain point of the image after enhancement and the gray value of the field change more; that is, \( E(\Omega) \) is required to be greater. When \( E(\Omega) \) takes the maximum value, the enhancement effect of the image can be maximized.

The direct gradient domain enhancement method also improves the contrast of the image by changing the magnitude of the gradient value, and in this method, the information in the low-contrast area of the image is enhanced by reducing the transformation range of the gradient domain. The specific process of this method is as follows:

1. Perform a nonlinear logarithmic transformation on the original image, that is, \( H = \log(u) \), where \( u \) is the original image.
2. Take the inverse of the transformed image \( H \) to get \( \Delta H \).
3. Construct a function \( \Phi \) to realize the enhancement of \( \Delta H \), and while the function \( \Phi \) enhances \( \Delta H \), the sign of \( \Delta H \) is kept unchanged; that is, the transformation direction of the gradient is kept unchanged, and only the transformation range of the gradient is changed. The function \( \Phi \) reduces the large gradient of the image and increases the small gradient, so that the detailed information in the shadow or highlight area is enhanced. Suppose the enhanced gradient domain is defined as \( G \), then

\[
G(x, y) = \Phi(-x, y) \cdot \nabla H(x, y)
\]

(3)

4. In order to reconstruct the image from the enhanced gradient field \( G \), an objective function is constructed using the least square method:

\[
E(u) = \int_\Omega [(\nabla u - G)^2 - u^2] dxdy
\]

(4)

Using the principle of extreme value, when the above formula takes the minimum value, the required Euler-Lagrange equation can be obtained:

\[
\Delta u = \text{div}(G) \cdot \Phi(x, y).
\]

(5)

(5) After the restored image \( u \) undergoes inverse transformation-exponential transformation, the final desired enhanced image \( u' \): \( u' = \exp(u) \) can be obtained.

The enhancement effect of this method is closely related to the construction function \( \Phi \), and the function \( \Phi \) is different, and the enhancement effect of the enhanced image obtained is different, so this method is not universal.

4.2. Image Contrast Enhancement Algorithm Based on Gradient Field Equalization. Through the idea of nonlinear diffusion partial differential equations, we can understand that in order to achieve image enhancement based on partial differential equations, it is necessary to improve the contrast of the image by changing the distribution of the image gradient field. The image is transformed into the gradient domain. Because the visual effect of the image is good, the image gradient is strong, and the texture is clear, so the edge texture of the image can be enhanced by enhancing the gradient field of the image.

The partial differential enhancement method based on gradient field equalization mainly considers that for uneven illumination images, in the gradient domain, the number of occurrences of small gradient values is more, and the number of occurrences of large gradient values is less. The field is equalized to enhance the less frequent details in the gradient field and suppress the more frequent details, so that the partial dark or bright details in the image caused by uneven illumination can be enhanced. The specific process is as follows.

We define the image gradient after transformation as \( G \). In order to enhance the image gradient field while
maintaining the change direction of the image gradient, we use

\[ G = \nabla u_0 \cdot \frac{L(\nabla u_0)}{|\nabla u_0|}. \]  

Among them, \( u_0 \) is the original image. After the gradient field is enhanced, the objective function is obtained using the principle of least squares:

\[ E(u) = \int_{\Omega} [(\nabla u + 2G)^2 - 2u^2]dxdy. \]

Minimizing the above formula, the Euler-Lagrange equation is

\[ \Delta u = \Phi(x, y) \cdot \text{div}(-2G). \]

Although this method can enhance the dark or bright part of the image caused by uneven illumination, this method has certain limitations: (1) when there is noise in the image, the histogram equalizes the noise at the same time. The texture is also enhanced more obviously, so the restored image noise is enlarged. (2) This method is not obvious for images with unclear texture details and low-contrast images.

### 4.3. Partial Differential Enhancement Algorithm Based on Histogram Equalization

This paper proposes a partial differential enhancement method based on histogram equalization. In this method, it is mainly divided into two parts. The first part first transforms the image into the gradient domain, proposes a new gradient transformation function in the gradient domain, and then applies the principle of least squares to reconstruct the image from the transformed gradient field. This part mainly enhances the texture details of the image, highlighting the original unclear details. The second part is to improve the histogram equalization for uneven illumination and low contrast, so that the brightness and contrast of the image are adjusted more suitable for human eyes.

#### 4.3.1. Image Texture Enhancement Based on Partial Differential Equations

Applying the theory of partial differential equations, first, we transform the image to the gradient domain. In the gradient domain, the smooth area of the image with a relatively small gradient value is still kept smooth after the transformation, while noise can be suppressed, and the texture edge area with a relatively large gradient value is transformed. Then, we make the gradient value larger and the texture edge more prominent. So a new gradient transformation function is proposed. We define the gradient function of the original image as \( \nabla u \). In order to ensure that the transformed gradient field maintains the change direction of the original gradient field and the algorithm meets the requirement of contrast invariance, the transformed gradient function \( S \) is

\[ S = \left[ 1 - \cos \left( \frac{\nabla u - \min \nabla u}{\max \nabla u - \min \nabla u} \pi \right) \right] \cdot \min \nabla u \cdot \frac{\pi}{2\nabla u}. \]

How to reconstruct an enhanced image from the transformed gradient field is also an important issue, and the least square method is now used to achieve the purpose of reconstruction.

\[ E(u) = \int_{\Omega} (S - 2\nabla u)^2 dxdy. \]

#### 4.3.2. Image Brightness and Contrast Enhancement Based on Histogram Equalization

Histogram equalization is one of the most basic and simplest and most widely used algorithms in the field of image enhancement. It can expand the histogram of the image to the entire gray level and can
enhance the brightness and contrast of the image. In theory, it can tend to maximize the entropy; that is, it can enhance the darker area in the image, while suppressing the brighter area in the image, so that the probability of all the information in the image tends to be consistent.

Now, the histogram equalization is improved, and an inverse equalization transformation is proposed, which brightens the originally dark areas while darkening the highlight areas and then combines with the histogram equalization, so that the brightness and contrast of the entire image can be adjusted very well. So all the information can be displayed well. The specific algorithm is as follows.

Assuming that the total number of pixels in the image is \( n \), the number of times that the first gray level appears in the image is \( n_{k_0} \), and the total number of gray levels is \( l \); the probability \( P_x(r_k) \) of the \( k \)th gray level value is taken as

\[
P_x(r_k) = \frac{n_k}{n-l}, \quad k = 0, 1, \ldots, l-1, -1 < r_k < 1.
\]  

Then, the transformation function of the inverse equalization transformation is

\[
s_k = \prod_{j=k-1}^{l-1} P_x(r_j).
\]

The image after histogram equalization is denoted as \( J(u) \), and the image after antiequalization is denoted as \( J'(u) \); then, the part of the enhanced image \( G(u) \) can be written as

\[
G(u) = \lambda_1 J(-u) - \lambda_2 J'(u).
\]

In the formula, \( \lambda_1 \) and \( \lambda_2 \) are adjustment coefficients. The solution process of the image enhancement method based on partial differential equations still adopts the wired difference method. The gradient is approximated by forward difference:

![Figure 4: Experimental results of different image enhancement algorithms: (a) original image; (b) gradient domain image enhancement algorithm processing; (c) image contrast enhancement algorithm processing based on gradient field equalization; (d) partial differential enhancement based on histogram equalization algorithm processing.](image-url)
The two-dimensional Laplacian is decomposed into two one-dimensional Laplacian implementations in orthogonal directions.

In order to realize the simple and rapid solution of the algorithm, direct calculation is not possible, and the heat equation is now introduced. Assuming that the image area is $\Omega$ and its boundary is $\Gamma$, the heat equation on this image area is

$$\frac{\partial u}{\partial t}(t,x,y) - \Delta u(t,x,y) = 0,$$

$$u(-1,x) = u_1(x), \quad x \in \Omega, t > -1,$$

$$\left.\frac{\partial u}{\partial n}\right|_t = -1.$$

In the formula, $n$ is the direction of the outer normal of $\Gamma$, and $\Omega$ is divided by straight lines parallel to the coordinate axis with an interval of 1, and the interval of the time axis $t$ is set to $\Delta t$. The Laplace transform of the enhanced image is transformed into the derivative of the enhanced image with respect to time $t$, so that the entire Poisson equation can be transformed into an iterative form. The equation for solving the image enhancement method based on partial differential equations in this article can be written in the following form:

$$\nabla u = [u(x,y+1) - u(x,y)] - [u(x+1,y) - u(x,y)].$$

The divergence is approximated by backward difference:

$$\text{div}(S) = 2S(x,y) - S(x+1,y) - S(x-1,y).$$

The contrast enhancement method based on gradient field equalization adopts the method of directly solving the Poisson equation in the process of numerical solution. The two-dimensional Laplacian is decomposed into two one-dimensional Laplacian implementations in orthogonal directions.

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$$u(-1,x) = u_1(x), \quad x \in \Omega, t > -1,$$

$$\left.\frac{\partial u}{\partial n}\right|_t = -1.$$
order of the image. Because image brightness and texture complexity have an impact on the optimal order, the image’s brightness characteristics are reflected by statistical image mean, variance, skewness, kurtosis, and second norm, and image texture is reflected by statistical image contrast. We analyze the relationship between these features and the preliminary optimal order and finally use the regression analysis method to obtain the functional relationship between the corresponding fractional differential order and these features when the enhancement effect is optimal, so as to determine the optimal fractional derivative order.

In this paper, \( M = 100 \), \( h_1 = 0.1 \), and 100 cross-border e-commerce logistics video surveillance images with different brightness, texture, and scenes are selected. We calculate the preliminary optimal order and image characteristics (mean, variance, skewness, kurtosis, second norm, and contrast). In order to determine the relationship between the order of the fractional derivative and each statistical feature, we take the order as the horizontal axis and each statistic as the vertical axis and draw a scatter plot as shown in Figure 3.

From the selected 100 images, we select the 2 images shown in Figure 4 to display the effect of algorithm processing. It can be seen from Figure 4 that the processing effect of the partial differential enhancement algorithm based on histogram equalization proposed in this paper is the best, and the visual dehazing and enhancement effects are better than the other two algorithms.
Figures 5 and 6, respectively, show the average gradient and information entropy of the experimental results of the two methods. It can be seen that the average gradient and information entropy of the partial differential enhancement algorithm based on histogram equalization are higher than those of the other two algorithms. Therefore, the partial differential enhancement algorithm based on histogram equalization proposed in this paper is not only easy to calculate but also effective. This also confirms the effectiveness of the partial differential enhancement algorithm based on histogram equalization proposed in this paper.

5.2. Effectiveness Verification of Partial Differential Image Enhancement Algorithm Based on Histogram Equalization

In order to objectively evaluate the effectiveness of the algorithm in this paper, Figures 7 and 8 show the average gradient and information entropy value of the image after the enhancement of each algorithm under different fractional differential orders. It can be seen that the average gradient and information entropy value of the image processed by the algorithm in this paper are both the highest. That is to say, the enhanced image has more advantages in the richness of information, which further verifies the effectiveness and efficiency of the algorithm in this paper.

In the Matlab 2019 environment, the running times of the three models in the above experiments are counted, as shown in Table 1. It can be seen from Table 1 that compared with the gradient domain image enhancement algorithm and the image contrast enhancement algorithm based on gradient field equalization, the partial differential enhancement algorithm based on histogram equalization has a shorter running time.

### Table 1: Comparison of enhanced algorithm running time (unit: s).

<table>
<thead>
<tr>
<th>Testing frequency</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gradient domain image enhancement algorithm</td>
<td>21.11</td>
<td>24.02</td>
<td>25.09</td>
<td>20.11</td>
<td>26.10</td>
</tr>
<tr>
<td>Image contrast enhancement algorithm based on gradient field equalization</td>
<td>18.39</td>
<td>17.22</td>
<td>19.03</td>
<td>15.01</td>
<td>16.08</td>
</tr>
<tr>
<td>Partial differential enhancement algorithm based on histogram equalization</td>
<td>4.01</td>
<td>3.33</td>
<td>4.21</td>
<td>1.03</td>
<td>2.08</td>
</tr>
</tbody>
</table>

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### 6. Conclusion

Inspired by the powerful role of partial differential equations in the field of image denoising, we consider applying partial differential equations to image enhancement for cross-border e-commerce logistics video surveillance. A typical method of gradient field enhancement—contrast enhancement method based on gradient field equalization is introduced. The advantages and disadvantages of this method are analyzed, and a partial differential enhancement method based on histogram equalization is proposed. In this new method, a gradient transformation function is proposed, which can effectively enhance the weak texture in the image. At the same time, combined with histogram equalization, an inverse equalization transformation is proposed, which can well enhance the brightness and contrast of the image. In the numerical solution process, this paper adopts a simple solution method and introduces a heat equation, which greatly reduces the amount of calculation and shortens the running time, which provides a guarantee for the real-time image enhancement processing in the future. Taking the image mean, variance, skewness, kurtosis, second norm, and contrast as the statistics, the relationship between the
order of the fractional derivative and these statistical features is obtained through regression analysis. Linear relationship gives an empirical formula to determine the order of fractional derivatives. Experimental results show that the adaptive fractional differential order obtained by this method is close to the optimal order manually selected, and the enhancement effect is optimal. Compared with other image enhancement algorithms, this algorithm can improve the brightness and contrast of the image and obtain good visual effects. The two objective evaluation indicators of information entropy and average gradient also show the effectiveness of this method. The image enhancement method based on partial differential equations proposed in this paper has obvious advantages. It can not only enhance the unclear image edge texture caused by camera defocusing but also enhance the impact of uneven lighting or foggy environment. But when the image is affected by very large noise, this method shows its shortcomings. When this method enhances the useful information in the image, it cannot effectively suppress the influence of noise, so that the image effect after enhancement is not very good. Therefore, the noise suppression algorithm of the image to be processed is a research direction in the future. In addition, from the perspective of running time, although the running time of the algorithm in this paper is faster than the comparison algorithm, the time required to find the preliminary optimal order is not counted, and the algorithm adopts an iterative method that cannot meet the real-time requirements. How to find more efficient adaptive algorithm is the problem to be solved in our follow-up research.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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