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Research Article

White Noise Functional Solutions for Wick-Type Stochastic Fractional Mixed KdV-mKdV Equation Using Extended (G'/G)-Expansion Method

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In this paper, white noise functional solutions of Wick-type stochastic fractional mixed KdV-mKdV equations have been obtained by using the extended (G'/G)-expansion method and the Hermite transform. Firstly, the Hermite transform is used to transform Wick-type stochastic fractional mixed KdV-mKdV equations into deterministic fractional mixed KdV-mKdV equations. Secondly, the exact traveling wave solutions of deterministic fractional mixed KdV-mKdV equations are constructed by applying the extended (G'/G)-expansion method. Finally, a series of white noise functional solutions are obtained by the inverse Hermite transform.

1. Introduction

The study of stochastic differential equation (SDE) can be traced back to Einstein's classical paper in 1905, which proposed the microscopic random motion of particles and macrodiffusion equations. In Einstein's research, the exact dynamics of the system is quite uncertain and which can be modeled by SDE. In 1949, Itô [1], a Japanese mathematician, first defined a random integral named Itô stochastic integral, which laid the theoretical foundation for the research of SDEs. In fact, the physical quantities of the objective world generally change with time and space, which are usually simulated by partial differential equation. Thus, stochastic partial differential equation (SPDE) [2] is usually used to simulate mathematical problems in the fields of science and engineering.

Fractional calculus was proposed before the birth of SPDE. In 1695, Leibniz and L'Hospital have discussed the definition and significance of derivative when the order of derivative is 1/2. In recent years, with the combination of fractional calculus theory and SPDE, it is gradually found that fractional SPDE can describe some nonlinear phenomena in the fields of natural science and engineering applications [3, 4]. In recent years, white noise functional solution is a very important topic in the research of fractional SPDEs. Many researchers have proposed many methods to construct the white noise functional solutions of fractional SPDEs, such as the Exp-function method [5], the Kudryashov method [6], improved computational method [7], and computerized symbolic [8]. The biggest obstacle in finding the white noise functional solution of SPDE is that the nonlinear ordinary differential equation obtained by the Hermite transform and random traveling wave transform is a nonlinear ordinary differential equation with variable coefficients. Therefore, many methods of constructing the white noise functional solutions of fractional partial differential equations are not applicable. So, it is particularly important to find a new method to construct the solution of nonlinear differential equation with variable coefficients. As early as 2008, Professor Wang [9] proposed a method named (G'/G)-expansion method to construct the exact traveling wave solution of partial differential equation. Later, many experts and scholars [10, 11] further expanded the method to enrich the solutions of partial differential equations. In this paper, we intend to find the solutions of fractional SPDES by the extended (G'/G)-expansion method.

In recent years, with the development of fractional derivative, the Wick-type stochastic fractional mixed KdV-mKdV equation (see [5, 6, 8, 12, 13]), a very important class of fractional SPDE, has been widely concerned by many researchers. This model can be described as follows:

$$D_t^\alpha U + \Theta_1(t) \diamond U \diamond D_x^\alpha U + \Theta_2(t) \diamond U^{\diamond 2} \diamond D_x^\alpha U + D_x^{3\alpha} U = 0, \quad (1)$$

where $U = U(t, x)$, $(t, x) \in \mathbb{R}_+ \times \mathbb{R}$, and $0 < \alpha \le 1$. $D_t^\alpha U$, $D_x^\alpha U$, and $D_x^{3\alpha} U$ are the conformable fractional derivative. $\Theta_1(t)$ and $\Theta_2(t)$ are integrable white noise functionals from $\mathbb{R}_+$ to the Kondrative distribution space $\mathscr{D}_{-1}$. The operator $\diamond$ represents the Wick product on $\mathscr{D}_{-1}$.

Based on Eq. (1), we consider the fractional mixed KdV-mKdV equation which is a very important fractional partial differential equation and usually used to simulate shallow water surface waves phenomena [14–19].

$$D_t^\alpha u + \theta_1(t) u D_x^\alpha u + \theta_2(t) u^2 D_x^\alpha u + D_x^{3\alpha} u = 0, 0 < \alpha < 1, \quad (2)$$

where $u(t, x)$ stands for the wave profile. $\theta_1(t)$ and $\theta_2(t)$ are integrable function on $\mathbb{R}_+$. The fractional derivatives are considered in the sense of conformable fractional derivatives [20–23].

Our work is as follows. In Section 2, we review the white noise theory and briefly introduce the extend (G'/G)-expansion method of Wick-type fractional SPDE. In Section 3, white noise functional solutions of Eq. (1) are constructed by apply Hermite transform, fractional traveling wave transformation, and the extend (G'/G)-expansion method. In Section 4, we give a summary.

2. Preliminaries

2.1. White Noise Theory. Assume that the rigging $\mathscr{D}(\mathbb{R}^N) \subset L^2(\mathbb{R}^N) \subset \mathscr{D}^*(\mathbb{R}^N)$, where $\mathscr{D}(\mathbb{R}^N)$ represents the Schwartz test functions space. $\mathscr{D}^*(\mathbb{R}^N)$ stands for the tempered distributions space. In Ref. [24], Holden et al. have proved that there is a unique measure of white noise, that is measure μ on $(\mathscr{D}^*(\mathbb{R}^N), \mathscr{B}(\mathscr{D}^*(\mathbb{R}^N)))$, where $\mathscr{B}(\mathscr{D}^*(\mathbb{R}^N))$ is the family of all Borel sets in $\mathscr{D}^*(\mathbb{R}^N)$. The Hermite function $e_n(x)$ is defined by $e_n(x) = e^{-1/2x^2} h_n(\sqrt{2}x)/(\pi(n-1)!)^{1/2}$, where $n \ge 1$, $h_n(x)$, is the Hermite polynomial. Define

$$\mathbf{H}_m(\omega) = \prod_{i=1}^{\infty} h_{m_i}(\langle \omega, e_i \rangle), \omega \in \mathscr{D}^*(\mathbb{R}^N), \quad (3)$$

where $(h_{m_i})_{i \ge 1}$ represents the Hermite polynomials. $(e_i)_{i \ge 1}$ denotes the orthonormal basis in $L^2(\mathbb{R}^N)$. Let $(\mathscr{D})_1^n$ be the Kondrative space of stochastic test functions space, and then $(\mathscr{D})_{-1}^n$ stands for the Kondrative space of stochastic distributions. Let $F = \sum_m a_m \mathbf{H}_m$, $G = \sum_{\bar{m}} a_{\bar{m}} \mathbf{H}_{\bar{m}} \in (\mathscr{D})_{-1}^n$, where $a_m, b_{\bar{m}} \in \mathbb{R}^n$. The wick product is defined by

$$F \diamond G = \sum_{m, \bar{m}} (a_m, b_{\bar{m}}) \mathbf{H}_{m+\bar{m}}. \quad (4)$$

Then, the Hermite transform of F is given by

$$\mathscr{H} F(z) = \tilde{F}(z) = \sum_m a_m z^m \in \mathbb{C}^n, \quad (5)$$

where $z = (z_1, z_2, \cdots, z_n) \in \mathbb{C}^n$, $F = \sum_m a_m \mathbf{H}_m \in (\mathscr{D})_{-1}^n$. Next, we define the Hermite transform

$$\widetilde{F \diamond G}(z) = \tilde{F}(z) \tilde{G}(z), \quad (6)$$

where $\tilde{F}(z)$ and $\tilde{G}(z)$ exist.

For $q < \infty$, $\rho > 0$, then we define the neighborhoods $N_q(\rho)$

$$N_q(\rho) = \left\{ (z_1, z_2, \cdots) \in \mathbb{C}^N : \sum_{m \ne 0} |z^\alpha|^2 (2\mathbb{N})^{q\alpha} < \rho^2 \right\}. \quad (7)$$

Theorem 1 (see [24]). *Suppose $u : \mathbf{D} \times N_q(\rho) \longrightarrow \mathbb{R}$ be a strong solution of the following equation*

$$P\left(t, x, D_t^\alpha, D_{x_1}^\alpha, D_{x_2}^\alpha, \cdots, D_{x_n}^\alpha, u, z\right) = 0, \quad (8)$$

where (t, x) in open bounded set in $\mathbf{D} \subset \mathbb{R}_+ \times \mathbb{R}^n$, $z \in N_q(r)$. Then, there exists $U(t, x) \in (\mathscr{D})_{-1}^n$ such that $u(t, x, z) = \tilde{U}(t, x)(z)$ and solves the stochastic equation

$$P^\diamond\left(t, x, D_t^\alpha, D_{x_1}^\alpha, D_{x_2}^\alpha, \cdots, D_{x_n}^\alpha, U, \omega\right) = 0, \text{ in } (\mathscr{D})_{-1}^n. \quad (9)$$

2.2. Extended (G'/G)-Expansion Method. Now, we consider a wick-type stochastic fractional partial differential equation

$$\mathbf{P}^\diamond\left(t, x, U, D_t^{\diamond\alpha} U, D_x^{\diamond\alpha} U, D_x^{\diamond 2\alpha} U, \cdots, \omega\right) = 0, \quad (10)$$

where $D_t^{\diamond\alpha} U, D_x^{\diamond\alpha} U, D_x^{\diamond 2\alpha} U, \cdots$ are the conformable fractional derivatives of u in the wick-type sense. Applying the Hermite transform, we can get a fractional partial differential equation as follows

$$\tilde{\mathbf{P}}\left(t, x, \tilde{U}, D_t^\alpha \tilde{U}, D_x^\alpha \tilde{U}, D_x^{2\alpha} \tilde{U}, \cdots, z\right) = 0, \quad (11)$$

where $\tilde{U} = \mathscr{H}(U)$ is the Hermite transform of U. $z = (z_1, z_2, \cdots, z_n) \in \mathbb{C}^n$. Then, we introduced the transformation:

$$\tilde{U}(t, x, z) = u(\xi), \xi(t, x, z) = k\left(\frac{x^\alpha}{\alpha}\right) + c\int_a^t \frac{\theta(\eta, z)}{\eta^{1-\alpha}} d\eta, \quad (12)$$

where $a > 0$. k and c are constants. θ is a nonzero function.

Next, substituting Eq. (12) into Eq. (11), we can obtain the following of the ordinary differential equation

$$\mathbf{Q}\left(u, \frac{du}{d\xi}, \frac{d^2u}{d\xi^2}, \frac{d^3u}{d\xi^3}, \cdots\right) = 0. \tag{13}$$

Next, we briefly introduce the extended (G'/G)-expansion method. Firstly, we assume that the traveling wave solution of Eq. (13) can be described as follows:

$$u(\xi) = \sum_{i=0}^{N} a_i \left(\frac{G'}{G}\right)^i + \sum_{i=1}^{N} b_i \left(\frac{G'}{G}\right)^{-i}, \tag{14}$$

where $G = G(\xi)$ satisfies

$$GG'' = AGG' + B\left(G'\right)^2 + CG^2, \tag{15}$$

where A, B, and C are real number. Secondly, substituting Eq. (14) into Eq. (13), and balancing the highest order derivative term and the highest order nonlinear term, then, we can determine the positive integer N. Next, substituting Eq. (14) together with (15) into Eq. (13) with the value of N determined in the previous step, we can get the polynomials about $(G'/G)^{-i}$ $(i = 0, 1, 2, \cdots)$ and $(G'/G)^i$ $(i = 0, 1, 2, \cdots)$, then we set all coefficients of the polynomial to zero, and we have a set of algebraic equation. Finally, we solve the algebraic equations, and we can get the values of a_i and b_i. The readers can refer to [25] for details of this method.

3. Explicit Solutions of System (1)

Applying Hermite transform, Eq. (1) can be transformed into fractional partial differential equation in the sense of conformable fractional derivatives

$$\begin{aligned} &D_t^\alpha \tilde{U}(t,x,z) + \tilde{\Theta}_1(t,z)\tilde{U}(t,x,z)D_x^\alpha \tilde{U}(t,x,z) \\ &\quad + \tilde{\Theta}_2(t,z)\tilde{U}^2(t,x,z)D_x^\alpha \tilde{U}(t,x,z) \\ &\quad + D_x^{3\alpha}\tilde{U}(t,x,z) = 0, \end{aligned} \tag{16}$$

where $z = (z_1, z_2, \cdots) \in \mathbb{C}^N$. Next, we introduce the transformation $\tilde{\Theta}_1(t,z) = \theta_1(t,z)$, $\tilde{\Theta}_2(t,z) = \theta_2(t,z)$, and $\tilde{U}(t,x,z) = u(\xi(t,x,z))$ with

$$\xi(t,x,z) = k\left(\frac{x^\alpha}{\alpha}\right) + c\int_a^t \frac{\theta(\eta,z)}{\eta^{1-\alpha}}d\eta, \tag{17}$$

where $a > 0$. k and c are nonzero constants. θ is a nonzero function.

Substituting (16) into (3.1), we obtain

$$c\theta\frac{du}{d\xi} + k\theta_1 u\frac{du}{d\xi} + k\theta_2 u^2\frac{du}{d\xi} + k^3\frac{d^3u}{d\xi^3} = 0. \tag{18}$$

Integrating Eq. (18) with respect to ξ and assuming the integral constant to zero, we obtain

$$c\theta u + \frac{k\theta_1}{2}u^2 + \frac{k\theta_2}{3}u^3 + k^3\frac{d^2u}{d\xi^2} = 0. \tag{19}$$

By making the homogeneous balance between u^3 and $d^2u/d\xi^2$ in Eq. (19), we have $N = 1$. Thus, the solution of Eq. (19) can be written as follows:

$$u(\xi) = a_0 + a_1\frac{G'}{G} + b_1\left(\frac{G'}{G}\right)^{-1}, \tag{20}$$

where a_0, a_1, and b_1 are undetermined parameters.

Substituting Eq. (20) together with Eq. (15) into Eq. (19), Eq. (19) is transformed into polynomials in $(G'/G)^{-i}$ $(i = 1, 2, 3)$ and $(G'/G)^i (i = 0, 1, 2, 3)$. Collecting each coefficient of the polynomials yields a set of algebraic equations. By solving the algebraic equations, we can get following.

Set 1: $a_1 = \pm\sqrt{-6k^2(A-1)^2/\theta_2}$, $a_0 = \pm B(A-1)\sqrt{-3k^2/2(A-1)^2\theta_2} - \theta_1/2\theta_2$, $b_1 = 0$, and $c = -(k\theta_1/4\theta)a_0 - (3k^3BCa_1/2\theta a_0) + (k^3/2\theta a_0)[B^2 + 2C(A-1)]$.

Set 2: $a_1 = 0$, $a_0 = \pm BC\sqrt{-3k^2/2C^2\theta_2} - \theta_1/2\theta_2$, $b_1 = \pm\sqrt{-6k^2C^2/\theta_2}$, and $c = -(k\theta_1/4\theta)a_0 - (3k^3B(A-1)b_1/2\theta a_0) + (k^3/2\theta a_0)[B^2 + 2C(A-1)]$.

Substituting the solutions into (20), we have

$$\begin{aligned} u(\xi) &= \pm B(A-1)\sqrt{-\frac{3k^2}{2(A-1)^2\theta_2}} - \frac{\theta_1}{2\theta_2} \pm \sqrt{-\frac{6k^2(A-1)^2}{\theta_2}}\left(\frac{G'}{G}\right), \\ u(\xi) &= \pm BC\sqrt{-\frac{3k^2}{2C^2\theta_2}} - \frac{\theta_1}{2\theta_2} \pm \sqrt{-\frac{6k^2C^2}{\theta_2}}\left(\frac{G'}{G}\right)^{-1}. \end{aligned} \tag{21}$$

3.1. The Solutions of Eq. (19). Next, we can obtain the solutions of Eq. (19) as follows.

Family 1. When $B^2 - 4(A-1)C > 0$ and $A \neq 1$, we obtain

$$u_1(\xi) = -\frac{\theta_1}{2\theta_2} \pm \frac{1}{1-A}\sqrt{-\frac{3k^2(A-1)^2(B^2+4C-4AC)}{2\theta_2}}H_1, \tag{22}$$

$$\begin{aligned} u_2(\xi) &= \pm BC\sqrt{-\frac{3k^2}{2C^2\theta_2}} - \frac{\theta_1}{2\theta_2} \pm \sqrt{-\frac{6k^2C^2}{\theta_2}} \\ &\quad \cdot \left[\frac{\sqrt{B^2+4C-4AC}}{2(1-A)}H_1 + \frac{B}{2(1-A)}\right]^{-1}, \end{aligned} \tag{23}$$

where $\xi(t,x,z) = k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta,z)/\eta^{1-\alpha})d\eta$, $H_1 = C_1 \sinh((\sqrt{B^2+4C-4AC}/2)\xi) + C_2\cosh((\sqrt{B^2+4C-4AC}/2)\xi)/C_1\cosh((\sqrt{B^2+4C-4AC}/2)\xi) + C_2\sinh((\sqrt{B^2+4C-4AC}/2)\xi)$. C_1 and C_2 are constants.

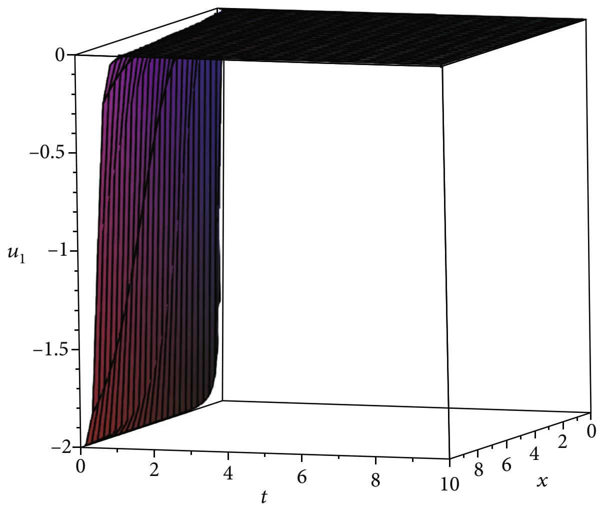


FIGURE 1: The solution $u_1(t,x)$ of Equation (22) for differential parameter.

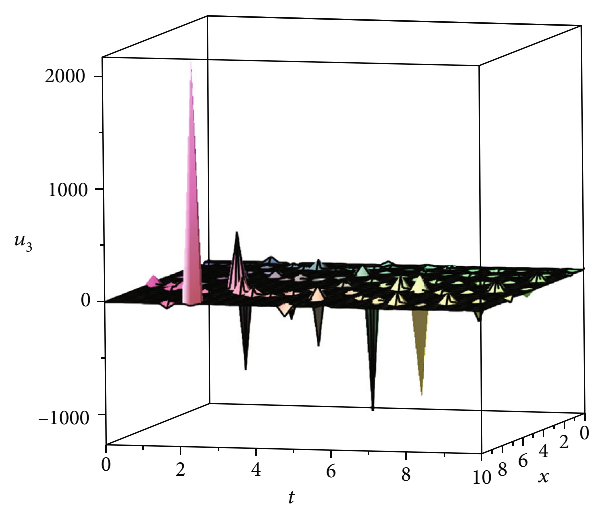


FIGURE 2: The solution $u_3(t,x)$ of Equation (24) for differential parameter.

Fixed parameters are as follows: $A=2$, $B=4$, $C=3$, $C_1\neq 0$, $C_2=0$, $\theta_1=-12$, $\theta_2=-6$, $\theta=2$, $k=1$, and $\alpha=1/2$; then, three-dimensional portrait can be drawn in Figure 1.

Family 2. When $4(A-1)C-B^2>0$ and $A\neq 1$, we have

$$u_3(\xi)=-\frac{\theta_1}{2\theta_2}\pm\frac{1}{1-A}\sqrt{-\frac{3k^2(A-1)^2(4(A-1)C-B^2)}{2\theta_2}}H_2, \tag{24}$$

$$u_4(\xi)=\pm \mathrm{BC}\sqrt{-\frac{3k^2}{2C^2\theta_2}-\frac{\theta_1}{2\theta_2}\pm\sqrt{-\frac{6k^2C^2}{\theta_2}}}\cdot\left[\frac{\sqrt{4(A-1)C-B^2}}{2(1-A)}H_2+\frac{B}{2(1-A)}\right]^{-1}, \tag{25}$$

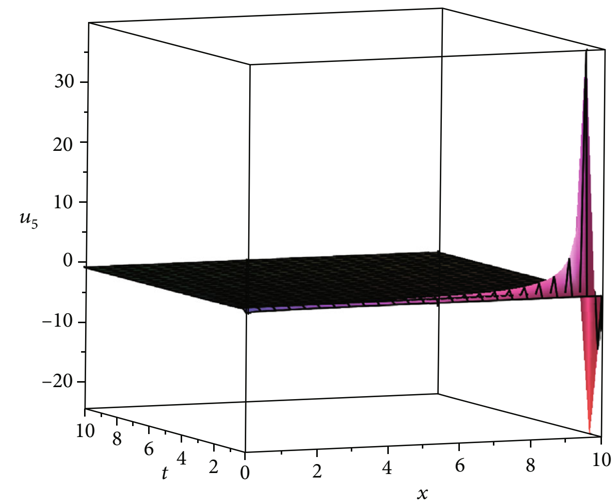


FIGURE 3: The solution $u_5(t,x)$ of Equation (26) for differential parameter.

where $\xi(t,x,z)=k(x^\alpha/\alpha)+c\int_a^t(\theta(\eta,z)/\eta^{1-\alpha})d\eta$, $H_2=-C_1\sin((\sqrt{4AC-4C-B^2}/2)\xi)+C_2\cos((\sqrt{4AC-4C-B^2}/2)\xi)/C_1\cos((\sqrt{4AC-4C-B^2}/2)\xi)+C_2\sin((\sqrt{4AC-4C-B^2}/2)\xi)$. C_1 and C_2 are constants.

Fixed parameters are as follows: $A=2$, $B=4$, $C=5$, $C_1\neq 0$, $C_2=0$, $\theta_1=-12$, $\theta_2=-6$, $\theta=2$, $k=1$, and $\alpha=1/2$; then, three-dimensional portrait can be drawn in Figure 2.

Family 3. When $B^2-4(A-1)C=0$ and $A\neq 1$, we obtain

$$u_5(\xi)=-\frac{\theta_1}{2\theta_2}\pm\frac{1}{1-A}\sqrt{-\frac{6k^2(A-1)^2}{\theta_2}}\cdot\frac{C_1}{C_1\xi+C_2}, \tag{26}$$

$$u_6(\xi)=\pm \mathrm{BC}\sqrt{-\frac{3k^2}{2C^2\theta_2}-\frac{\theta_1}{2\theta_2}\pm(1-A)\sqrt{-\frac{6k^2C^2}{\theta_2}}}\left[\frac{C_1}{C_1\xi+C_2}+\frac{B}{2}\right]^{-1}, \tag{27}$$

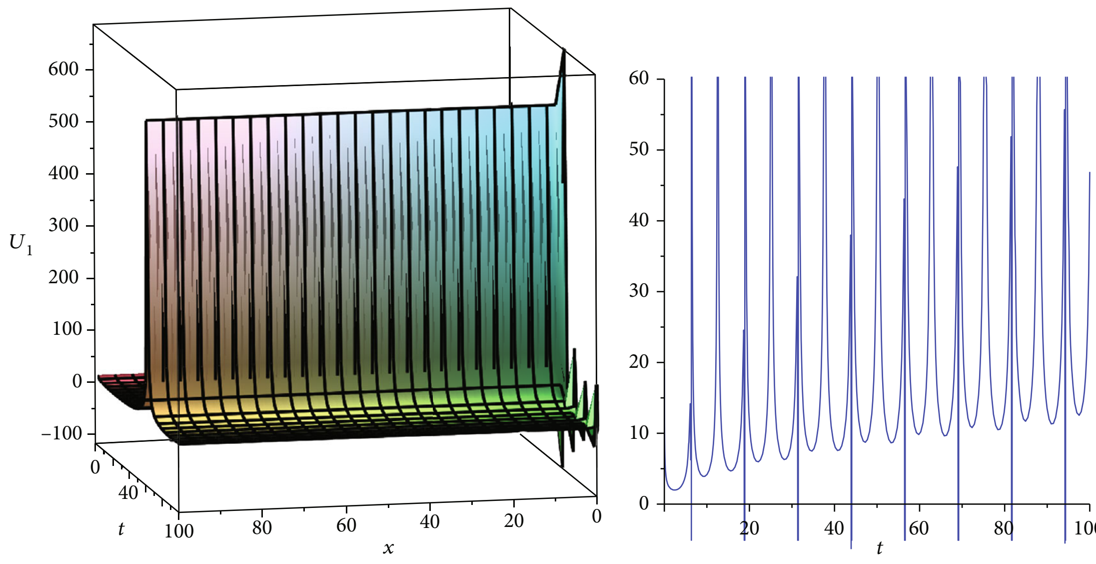
where $\xi(t,x,z)=k(x^\alpha/\alpha)+c\int_a^t(\theta(\eta,z)/\eta^{1-\alpha})d\eta$. C_1 and C_2 are constants.

Fixed parameters are as follows: $A=2$, $B=4$, $C=4$, $C_1=1$, $C_2=2$, $\theta_1=-12$, $\theta_2=-6$, $\theta=2$, $k=1$, and $\alpha=1/2$; then, three-dimensional portrait can be drawn in Figure 3.

3.2. White Noise Functional Solutions of Eq. (1). In order to construct the white noise functional solutions of Eq. (1), we apply the inverse Hermite transform and Theorem 1 to the solutions $u_1(\xi), u_2(\xi), \cdots, u_6(\xi)$. Then, we obtain six white noise functional solution.

Family 1. When $B^2-4(A-1)C>0$ and $A\neq 1$, we obtain

$$U_1(t,x)=-\frac{\Theta_1(t)}{2\Theta_2(t)}\pm\frac{1}{1-A}\left(-\frac{3k^2(A-1)^2(B^2+4C-4AC)}{2\Theta_2(t)}\right)^{\diamond\frac{1}{2}}\diamond H_1,$$



(a) Perspective view of the wave

(b) The wave along t

FIGURE 4: Wave profiles of exact solution $U_1(t,x)$ with $A=2, B=1, C=0, k=1, c=1, \theta_1(t)=0, \theta_2(t)=-t/\sin^2 0.5t$.

$$U_2(t,x) = \pm BC\left(-\frac{3k^2}{2C^2\Theta_2(t)}\right)^{\diamond\frac{1}{2}} - \frac{\Theta_1(t)}{2\Theta_2(t)} \pm \left(-\frac{6k^2C^2}{\Theta_2(t)}\right)^{\diamond\frac{1}{2}} \diamond \cdot \left[\frac{\sqrt{B^2+4C-4AC}}{2(1-A)}H_1 + \frac{B}{2(1-A)}\right]^{-1}, \tag{28}$$

where $H_1 = C_1 \sinh^{\diamond}((\sqrt{B^2+4C-4AC}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta)) + C_2 \cosh^{\diamond}((\sqrt{B^2+4C-4AC}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta))/C_1 \cosh^{\diamond}((\sqrt{B^2+4C-4AC}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta)) + C_2 \sinh^{\diamond}((\sqrt{B^2+4C-4AC}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta))$. C_1 and C_2 are constants.

Fixed parameters are as follows: $A=2, B=1, C=0, k=1, c=1, \theta_1(t)=0, \theta_2(t)=-t/\sin^2 0.5t$; then, three-dimensional portrait and two-dimensional portrait can be drawn in Figure 4.

Family 2. When $4(A-1)C-B^2>0$ and $A\neq 1$, we have

$$U_3(t,x) = -\frac{\Theta_1(t)}{2\Theta_2(t)} \pm \frac{1}{1-A}\left(-\frac{3k^2(A-1)^2(4(A-1)C-B^2)}{2\Theta_2(t)}\right)^{\diamond\frac{1}{2}} \diamond H_2,$$

$$U_4(t,x) = \pm BC\left(-\frac{3k^2}{2C^2\Theta_2(t)}\right)^{\diamond\frac{1}{2}} - \frac{\Theta_1(t)}{2\Theta_2(t)} \pm \left(-\frac{6k^2C^2}{\Theta_2(t)}\right)^{\diamond\frac{1}{2}} \diamond \cdot \left[\frac{\sqrt{4(A-1)C-B^2}}{2(1-A)}H_2 + \frac{B}{2(1-A)}\right]^{\diamond(-1)}, \tag{29}$$

where $H_2 = -C_1 \sin^{\diamond}((\sqrt{4AC-4C-B^2}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta)) + C_2 \cos^{\diamond}((\sqrt{4AC-4C-B^2}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta))/C_1 \cos^{\diamond}((\sqrt{4AC-4C-B^2}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta)) + C_2 \sin^{\diamond}((\sqrt{4AC-4C-B^2}/2)(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta))$. C_1 and C_2 are constants.

Family 3. When $B^2-4(A-1)C=0$ and $A\neq 1$, we obtain

$$U_5(t,x) = -\frac{\Theta_1(t)}{2\Theta_2(t)} \pm \frac{1}{1-A} \cdot \left(-\frac{6k^2(A-1)^2}{\Theta_2(t)}\right)^{\diamond\frac{1}{2}} \diamond \frac{C_1}{C_1\left(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta\right) + C_2},$$

$$U_6(t,x) = \pm BC\left(-\frac{3k^2}{2C^2\Theta_2(t)}\right)^{\diamond\frac{1}{2}} - \frac{\Theta_1(t)}{2\Theta_2(t)} \pm (1-A)\left(-\frac{6k^2C^2}{\Theta_2(t)}\right)^{\diamond\frac{1}{2}} \diamond \cdot \left[\frac{C_1}{C_1\left(k(x^\alpha/\alpha) + c\int_a^t (\theta(\eta)/\eta^{1-\alpha})d\eta\right) + C_2} + \frac{B}{2}\right)\Bigg]^{\diamond(-1)}, \tag{30}$$

where C_1 and C_2 are constants.

4. Conclusion

In this paper, we constructed the white noise functional solutions of Wick-type stochastic fractional mixed KdV-mKdV equation by using the extended (G'/G)-expansion method and the Hermite transform. Compared with the existing literature [5, 6, 8, 12, 13], the negative power solutions $U_2(t,x)$, $U_4(t,x)$, and $U_6(t,x)$ obtained in the paper are not reported. The method discussed in the paper is not only applicable to Eq. (1), but also can help mathematicians and physicists find the white noise functional solutions of Wick-type fractional SPDEs. In the future, our work will mainly focus on the white noise functional solutions of SPDEs.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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