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*Research Article*

# The Analytical Solutions of the Stochastic Fractional RKL Equation via Jacobi Elliptic Function Method

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This article considers the stochastic fractional Radhakrishnan-Kundu-Lakshmanan equation (SFRKLE), which is a higher order nonlinear Schrödinger equation with cubic nonlinear terms in Kerr law. To find novel elliptic, trigonometric, rational, and stochastic fractional solutions, the Jacobi elliptic function technique is applied. Due to the Radhakrishnan-Kundu-Lakshmanan equation's importance in modeling the propagation of solitons along an optical fiber, the derived solutions are vital for characterizing a number of key physical processes. Additionally, to show the impact of multiplicative noise on these solutions, we employ MATLAB tools to present some of the collected solutions in 2D and 3D graphs. Finally, we demonstrate that multiplicative noise stabilizes the analytical solutions of SFRKLE at zero.

## 1. Introduction

Deterministic partial differential equations (DPDEs) are utilized to explain the dynamic behavior of the phenomena in physics and other scientific areas including nonlinear optics, biology, elastic media, fluid dynamics, molecular biology, hydrodynamics, surface of water waves, quantum mechanics, and plasma physics. As a result, solving nonlinear problems is crucial in nonlinear sciences. Some of these methods, such as Darboux transformation [1], sine-cosine [2, 3], exp $(-\phi(\varsigma))$-expansion [4], $(G'/G)$-expansion [5, 6], Hirota's function [7], perturbation [8, 9], Jacobi elliptic function [10, 11], trial function [12], tanh-sech [13], fractal semi-inverse method [14, 15], $F$-expansion method [16], and homotopy perturbation method [17], have been recently developed. However, it is completely obvious that the phenomena that happen in the environment are not always deterministic. Recently, fluctuations/noise has been demonstrated to play an important role in a wide range in describing different phenomena that appear in oceanography, environmental sciences, finance, meteorology, information systems, biology, physics, and other fields [18–24]. Therefore, partial differential equations with noise or random effects are ideal mathematical problems for modeling complex systems.

On the other hand, fractional partial differential equations (FPDEs) have been used to explain many physical phenomena in biology, physics, finance, engineering applications, electromagnetic theory, mathematical, signal processing, and different scientific studies; see, for example, [25–35]. These new fractional-order models are better equipped than the previously utilized integer-order models because fractional-order integrals and derivatives allow for the representation of memory and hereditary qualities of different substances [36]. Compared to integer-order models, where such effects are ignored, fractional-order models have the most significant advantage.

It appears that studying stochastic equations with fractional derivative is more essential. As a result, the next stochastic fractional Radhakrishnan-Kundu-Lakshmanan equation (SFRKLE) [37–39] perturbed by multiplicative noise in the Stratonovich sense is treated:

$$\begin{aligned} &id\Psi + \left[\ell_1 \mathbb{D}_{xx}^{\alpha}\Psi_{xx} - i\ell_2 \mathbb{D}_{x}^{\alpha}\Psi + \ell_3 |\Psi|^2\Psi - i\ell_4 \Psi \mathbb{D}_x^{\alpha}\left(|\Psi|^2\right) - \right. \\ &\left. -i\ell_5 \mathbb{D}_x^{\alpha}\left(|\Psi|^2\Psi\right) + i\ell_6 \mathbb{D}_{xxx}^{\alpha}\Psi\right] dt + i\sigma\Psi \circ d\mathscr{W} = 0, \end{aligned} \tag{1}$$

where $\Psi \in \mathbb{C}$, $\mathbb{D}_x^{\alpha}$ is the conformable fractional derivative (CFD) [40], $\ell_1$ is the group-velocity dispersion, $\ell_2$ is the intermodal dispersion, $\ell_3$ is the coefficient of nonlinearity, $\ell_4$ is the higher-order dispersion coefficient, $\ell_5$ is the coefficient of self-steepening for short pulses, and $\ell_6$ is the third-order dispersion term. While $\sigma$ denotes the noise intensity, $\mathscr{W}(t)$ is a standard Wiener process (SWP).

Many researchers have recently developed exact solutions of SFRKLE (1), with $\sigma = 0$, using a variety of methods including trial equation method [41], Lie group analysis [42], sine-cosine method [43], first integral method [44], extended simple equation method [45], the modified Khater method [46], and improved tan $(\phi(\varsigma)/2)$-expansion method [47], while the analytical solutions of SFRKLE (1) have not yet been investigated.

Our motivation of this article is to achieve exact stochastic-fractional solutions for SFRKLE (1). This is the first study to attain the exact solutions of SFRKLE (1) in the existence of a stochastic term and fractional derivative. To get a wide variety of solutions such as trigonometric, hyperbolic, elliptic, and rational functions, we apply the Jacobi elliptic function method. Due to the significance of the RKL in modeling the propagation of solitons through an optical fiber, the solutions obtained are useful for describing some important physical phenomena. In addition, we investigate the impact of BM on the acquired solutions of SFRKLE (1) by generating 3D and 2D diagrams for these solutions.

The outline of this article is as follows. In Section 2, we use a proper wave transformation to deduce the SFRKLE's wave equation (1). While in Section 3, we utilize Jacobi elliptic function method to create the analytic solutions of SFRKLE (1). In Section 4, the influence of the SWP on the obtained solutions is investigated. The conclusion of the document is displayed last.

## 2. Wave Equation for SFRKLE

The next wave transformation is used to get the wave equation of SFRKLE (1):

$$\begin{aligned} &\Psi(x,t) = \Phi(\eta) e^{\left(i\theta(x,t) - \sigma\mathscr{W}(t) - \sigma^2 t\right)}, \quad \eta = \frac{x^{\alpha}}{\alpha} - \nu t, \\ &\theta(x,t) = -\frac{k}{\alpha}x^{\alpha} + \omega t, \end{aligned} \tag{2}$$

where $\Phi$ is deterministic function that describes the profile of the pulse, $\theta(x,t)$ is the phase component of the soliton, and $\nu$,$k$, and $\omega$ are nonzero constants. Plugging equation (2) into equation (1) and using

$$\begin{aligned} d\Psi &= \left[\left(-\nu\Phi' + i\omega\Phi + \frac{1}{2}\sigma^2\Phi - \sigma^2\Phi\right)dt - \sigma\Phi d\mathscr{W}\right] e^{\left(i\theta(x,t) - \sigma\mathscr{W}(t) - \sigma^2 t\right)} \\ &= \left[\left(-\nu\Phi' + i\omega\Phi\right)dt - \sigma\Phi \circ d\mathscr{W}\right] e^{\left(i\theta(x,t) - \sigma\mathscr{W}(t) - \sigma^2 t\right)}, \end{aligned} \tag{3}$$

where $(1/2)\sigma^2\Phi$ is the Itô correction term, and

$$\begin{aligned} \mathbb{D}_x^{\alpha}\Psi &= \left(\Phi' - ik\Phi\right) e^{\left(i\theta(x,t) - \sigma\mathscr{W}(t) - \sigma^2 t\right)}, \\ \mathbb{D}_{xx}^{\alpha}\Psi &= \left[\Phi'' - 2ik\Phi' - k^2\Phi\right] e^{\left(i\theta(x,t) - \sigma\mathscr{W}(t) - \sigma^2 t\right)}, \\ \mathbb{D}_{xxx}^{\alpha}\Psi &= \left[\Phi''' - 3ik\Phi'' - 3k^2\Phi' + ik^3\Phi\right] e^{\left(i\theta(x,t) - \sigma\mathscr{W}(t) - \sigma^2 t\right)}, \\ \Psi\mathbb{D}_x^{\alpha}\left(|\Psi|^2\right) &= 2\Phi^2\Phi' e^{\left(i\theta(x,t) - 3\sigma\mathscr{W}(t) - 3\sigma^2 t\right)}, \\ \mathbb{D}_x^{\alpha}\left(|\Psi|^2\Psi\right) &= \left(3\Phi^2\Phi' - ik\Phi^3\right) e^{\left(i\theta(x,t) - 3\sigma\mathscr{W}(t) - 3\sigma^2 t\right)}, \end{aligned} \tag{4}$$

we get for imaginary part

$$\begin{aligned} &\ell_6 k^3 \Phi''' - \left(3\ell_6 k^2 + \ell_2 + 2k\ell_1 + \nu\right)\Phi' \\ &\quad - (3\ell_5 + 2\ell_4)\Phi^2\Phi' e^{\left(-2\sigma\mathscr{W}(t) - 2\sigma^2 t\right)} = 0, \end{aligned} \tag{5}$$

and for real part,

$$\begin{aligned} &(\ell_1 + 3k\ell_6)\Phi'' - \left(k^2\ell_1 + k\ell_2 - k^3\ell_6\right)\Phi \\ &\quad + (\ell_3 - k\ell_5)\Phi^3 e^{\left(-2\sigma\mathscr{W}(t) - 2\sigma^2 t\right)} = 0. \end{aligned} \tag{6}$$

Taking expectation $\mathbb{E}(\cdot)$ on both sides for equations (5) and (6) and using

$$\mathbb{E}\left(e^{\sigma\mathscr{W}(t)}\right) = e^{\left(\sigma^2/2\right)t}, \tag{7}$$

we have

$$\ell_6 k^3 \Phi''' - \left(3\ell_6 k^2 + \ell_2 + 2k\ell_1 + \nu\right)\Phi' - (3\ell_5 + 2\ell_4)\Phi^2\Phi' = 0, \tag{8}$$

$$(\ell_1 + 3k\ell_6)\Phi'' - \left(\omega + k^2\ell_1 + k\ell_2 + k^3\ell_6\right)\Phi - (k\ell_5 - \ell_3)\Phi^3 = 0, \tag{9}$$

where $\Phi$ is deterministic functions. Integrating equation (8) and setting the integration constant to zero, we get

$$\ell_6 k^3 \Phi'' - \left(3\ell_6 k^2 + \ell_2 + 2k\ell_1 + \nu\right)\Phi - \left(\ell_5 + \frac{2}{3}\ell_4\right)\Phi^3 = 0. \tag{10}$$

Since the same function $\Phi$ fulfills both equations (9) and (10), we get the next constraint conditions:

$$\frac{\ell_1 + 3k\ell_6}{\ell_6 k^3} = \frac{\omega + k^2\ell_1 + k\ell_2 + k^3\ell_6}{3\ell_6 k^2 + \ell_2 + 2k\ell_1 + \nu} = \frac{3(k\ell_5 - \ell_3)}{3\ell_5 + 2\ell_4}, \quad (11)$$

whenever

$$\ell_3 = -\frac{3\ell_5\ell_1 + \ell_1\ell_4 + 6k\ell_6\ell_5 + 3k\ell_6\ell_4}{3\ell_6}, \quad (12)$$

$$\omega = \frac{8k^3\ell_6^2 + 8k^2\ell_1\ell_6 + 2k\ell_1^2 + 2k\ell_2\ell_6 + \ell_1\ell_2 + \nu(3k\ell_6 + \ell_1)}{\ell_6}. \quad (13)$$

Plugging equation (13) into equation (9), we have the wave equation as follows:

$$\Phi'' - \hbar_1\Phi^3 - \hbar_2\Phi = 0, \quad (14)$$

where

$$\hbar_1 = \frac{3\ell_5\ell_1 + \ell_1\ell_4 + 9k\ell_6\ell_5 + 3k\ell_6\ell_4}{3\ell_6(\ell_1 + 3k\ell_6)},$$
$$\hbar_2 = \frac{9k^3\ell_6^2 + 9k^2\ell_1\ell_6 + 2k\ell_1^2 + 3k\ell_2\ell_6 + \ell_1\ell_2 + \nu(3k\ell_6 + \ell_1)}{\ell_6(\ell_1 + 3k\ell_6)}. \quad (15)$$

## 3. Analytical Solutions of SFRKLE

To determine the solutions to equation (14), we employ the Jacobi elliptic function method [48]. As a result, we are able to acquire the exact solutions of SFRKLE (1).

*3.1. Jacobi Elliptic Function Method.* Initially, let the solutions of equation (14) have the form

$$\Phi(\eta) = \sum_{i=1}^{M} a_i \varphi^i(\eta), \quad (16)$$

where $\varphi$ solves

$$\varphi' = \sqrt{\frac{1}{2}p\varphi^4 + q\varphi^2 + r}, \quad (17)$$

where $p$, $q$, and $r$ are real parameters and $M$ is a positive integer number and will be defined later in (19).

We note that equation (17) has different kind of solutions depending on $p$, $q$, and $r$.

$sn(\eta) = sn(\eta, m)$, $cn(\eta) = cn(\eta, m)$, $dn(\eta, m) = dn(\eta, m)$ are the Jacobi elliptic functions (JEFs) for $0 < m < 1$. When $m \longrightarrow 1$, the JEFs are transformed into the following hyperbolic functions:

$$cn(\eta) \longrightarrow \operatorname{sech}(\eta), sn(\eta) \longrightarrow \tanh(\eta), cs(\eta) \longrightarrow \operatorname{csch}(\eta),$$
$$ds \longrightarrow \operatorname{csch}(\eta), dn(\eta) \longrightarrow \operatorname{sech}(\eta). \quad (18)$$

*3.2. Solutions of SFRKLE.* Now, let us determine the parameter $M$ by balancing $\Phi''$ with $\Phi^3$ in equation (14) as

$$M + 2 = 3M \Rightarrow M = 1. \quad (19)$$

Rewriting equation (16) with $M = 1$ as

$$\Phi = a_0 + a_1\varphi. \quad (20)$$

Differentiating equation (20) twice, we have, by using (17),

$$\Phi'' = a_1 q\varphi + a_1 p\varphi^3. \quad (21)$$

Substituting equations (20) and (21) into equation (14), we obtain

$$\left(a_1 p - \hbar_1 a_1^3\right)\varphi^3 - 3a_0 a_1^2\hbar_1\varphi^2 + \left(a_1 q - 3\hbar_1 a_0^2 a_1 - \hbar_2 a_1\right)\varphi - \left(\hbar_1 a_0^3 + \hbar_2 a_0\right) = 0. \quad (22)$$

Putting each coefficient of $\varphi^k$ equal zero, we get for $k = 0, 1, 2, 3$

$$\begin{aligned} a_1 p - \hbar_1 a_1^3 &= 0, \\ 3a_0 a_1^2 \hbar_1 &= 0, \\ a_1 q - 3\hbar_1 a_0^2 a_1 - \hbar_2 a_1 &= 0, \\ \hbar_1 a_0^3 + \hbar_2 a_0 &= 0. \end{aligned} \quad (23)$$

Solving these equations, we obtain

$$a_0 = 0, a_1 = \pm\sqrt{\frac{p}{\hbar_1}}, q = \hbar_2. \quad (24)$$

Hence, the solution of equation (14) is

$$\Phi(\eta) = \pm\sqrt{\frac{p}{\hbar_1}}\varphi(\eta), \quad (25)$$

for $p/\hbar_1 > 0$. There are two sets depending only on $p$ and $\hbar_1$ as follows.

*First set*: if $p > 0$ and $\hbar_1 > 0$, then the solutions $\varphi(\eta)$ of equation (17) corresponding to Table 1 are as follows.

If $m \longrightarrow 1$, then the above table degenerates to the following.

Table 1: All solutions of equation (17).

| Case | $p$ | $q$ | $r$ | $\varphi$ |
|---|---|---|---|---|
| 1 | $2m^2$ | $-(1+m^2)$ | $1$ | $sn(\eta)$ |
| 2 | $2$ | $2m^2-1$ | $-m^2(1-m^2)$ | $ds(\eta)$ |
| 3 | $2$ | $2-m^2$ | $(1-m^2)$ | $cs(\eta)$ |
| 4 | $-2m^2$ | $2m^2-1$ | $(1-m^2)$ | $cn(\eta)$ |
| 5 | $-2$ | $2-m^2$ | $(m^2-1)$ | $dn(\eta)$ |
| 6 | $\frac{m^2}{2}$ | $\frac{(m^2-2)}{2}$ | $\frac{1}{4}$ | $\frac{sn(\eta)}{1\pm dn(\eta)}$ |
| 7 | $\frac{m^2}{2}$ | $\frac{(m^2-2)}{2}$ | $\frac{m^2}{4}$ | $\frac{sn(\eta)}{1\pm dn(\eta)}$ |
| 8 | $\frac{-1}{2}$ | $\frac{(m^2+1)}{2}$ | $\frac{-(1-m^2)^2}{4}$ | $mcn(\eta)\pm dn(\eta)$ |
| 9 | $\frac{m^2-1}{2}$ | $\frac{(m^2+1)}{2}$ | $\frac{(m^2-1)}{4}$ | $\frac{dn(\eta)}{1\pm sn(\eta)}$ |
| 10 | $\frac{1-m^2}{2}$ | $\frac{(1-m^2)}{2}$ | $\frac{(1-m^2)}{4}$ | $\frac{cn(\eta)}{1\pm sn(\eta)}$ |
| 11 | $\frac{(1-m^2)^2}{2}$ | $\frac{(1-m^2)^2}{2}$ | $\frac{1}{4}$ | $\frac{sn(\eta)}{dn\pm cn(\eta)}$ |
| 12 | $2$ | $0$ | $0$ | $\frac{c}{\eta}$ |
| 13 | $0$ | $1$ | $0$ | $ce^{\eta}$ |

Now, using Table 2 (or Table 3) and equations (2) and (25), we get the solutions of SFRKLE (1) as follows:

$$\Psi(x,t)=\sqrt{\frac{p}{\hbar_1}}\varphi(\eta)e^{\left(i\theta(x,t)-\sigma\mathscr{W}(t)-\sigma^2 t\right)}\ \text{for}\ \frac{p}{\hbar_1}>0, \tag{26}$$

where $\eta=(x^{\alpha}/\alpha)-\nu t$.

*Second set*: if $p<0$ and $\hbar_1<0$, then the solutions $\varphi(\eta)$ of equation (17) corresponding to Table 1 are as follows.

If $m\longrightarrow 1$, then Table 3 degenerates to the following.

In this case, using Table 4 (or Table 5), we obtain the analytical solutions of SFRKLE (1) as stated in equation (26).

## 4. The Impact of SWP on the Solutions of SFRKLE

The effect of SWP on the analytical solutions of SFRKLE (1) is discussed here. Fix the parameters $\ell_1=\ell_2=\ell_4=\ell_5=\ell_6=1$, $k=-1$, $\nu=-11/3$, and $m=\sqrt{2/3}$. Hence, $\ell_3=5/3$, $\hbar_1=4/3$, and $\hbar_2=-5/3$. Now, we offer some graphs for distinct value of $\sigma$ (noise strength) and $\alpha$ (fractional order) for $t,x\in[0,6]$. We utilize the MATLAB tools to create some graphs for the following solutions:

$$\Psi(x,t)=sn\left(\frac{x^{\alpha}}{\alpha}+\frac{11}{3}t\right)e^{\left(i\theta(x,t)-\sigma\mathscr{W}(t)-\sigma^2 t\right)}, \tag{27}$$

Table 2: All solutions $\varphi(\eta)$ of equation (17) for $p>0$ and $\hbar_1>0$.

| Case | $p$ | $q$ | $r$ | $\varphi(\eta)$ |
|---|---|---|---|---|
| 1 | $2m^2$ | $-(1+m^2)$ | $1$ | $sn(\eta)$ |
| 2 | $2$ | $2m^2-1$ | $-m^2(1-m^2)$ | $ds(\eta)$ |
| 3 | $2$ | $2-m^2$ | $(1-m^2)$ | $cs(\eta)$ |
| 4 | $\frac{m^2}{2}$ | $\frac{(m^2-2)}{2}$ | $1/4$ or $m^2/4$ | $\frac{sn(\eta)}{1\pm dn(\eta)}$ |
| 5 | $\frac{1-m^2}{2}$ | $\frac{(1-m^2)}{2}$ | $\frac{(1-m^2)}{4}$ | $\frac{cn(\eta)}{1\pm sn(\eta)}$ |
| 6 | $\frac{(1-m^2)^2}{2}$ | $\frac{(1-m^2)^2}{2}$ | $\frac{1}{4}$ | $\frac{sn(\eta)}{dn\pm cn(\eta)}$ |
| 7 | $2$ | $0$ | $0$ | $\frac{c}{\eta}$ |

Table 3: All solutions of equation (17) when $m\longrightarrow 1$.

| Case | $p$ | $q$ | $r$ | $\varphi(\eta)$ |
|---|---|---|---|---|
| 1 | $2$ | $-2$ | $1$ | $\tanh(\eta)$ |
| 2 | $2$ | $1$ | $0$ | $\operatorname{csch}(\eta)$ |
| 3 | $\frac{1}{2}$ | $\frac{-1}{2}$ | $\frac{1}{4}$ | $\frac{\tanh(\eta)}{1\pm \operatorname{sech}(\eta)}$ |
| 4 | $2$ | $0$ | $0$ | $\frac{c}{\eta}$ |

Table 4: All solutions $\varphi(\eta)$ of equation (17) when $p<0$.

| Case | $p$ | $q$ | $r$ | $\varphi$ |
|---|---|---|---|---|
| 1 | $-2m^2$ | $2m^2-1$ | $(1-m^2)$ | $cn(\eta)$ |
| 2 | $-2$ | $2-m^2$ | $(m^2-1)$ | $dn(\eta)$ |
| 3 | $\frac{-1}{2}$ | $\frac{(m^2+1)}{2}$ | $\frac{-(1-m^2)^2}{4}$ | $mcn(\eta)\pm dn(\eta)$ |
| 4 | $\frac{m^2-1}{2}$ | $\frac{(m^2+1)}{2}$ | $\frac{(m^2-1)}{4}$ | $\frac{dn(\eta)}{1\pm sn(\eta)}$ |

Table 5: All solutions $\varphi(\eta)$ of equation (17) when $m\longrightarrow 1$ and $p<0$.

| Case | $p$ | $q$ | $r$ | $\varphi$ |
|---|---|---|---|---|
| 1 | $-2$ | $1$ | $0$ | $\operatorname{sech}(\eta)$ |
| 2 | $\frac{-1}{2}$ | $2$ | $0$ | $2\operatorname{sech}(\eta)$ |

$$\Psi(x,t)=\sqrt{\frac{3}{2}}ds\left(\frac{x^{\alpha}}{\alpha}+\frac{11}{3}t\right)e^{\left(i\theta(x,t)-\sigma\mathscr{W}(t)-\sigma^2 t\right)}. \tag{28}$$

If $\sigma=0$, we can see how the surface oscillates (periodic solutions) in Figure 1 and the surface expands as the fractional order increases $\alpha=0.3,0.5,0.7,1$.

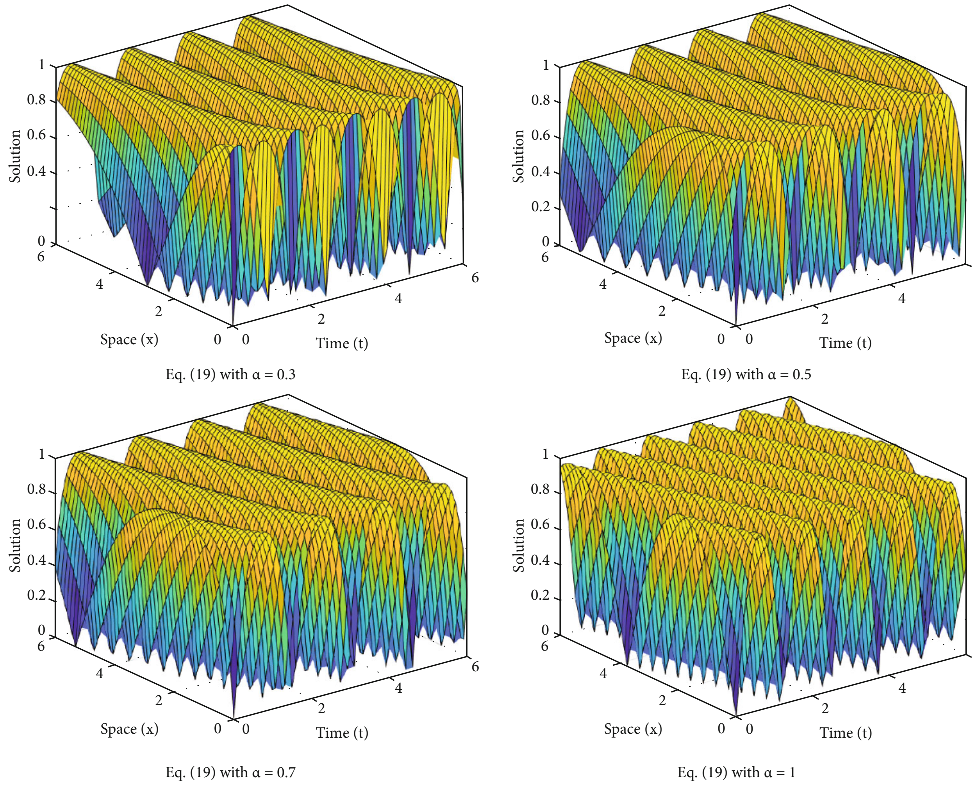


FIGURE 1: 3D graphs of equation (27) with $\sigma = 0$ and for different values of $\alpha$.

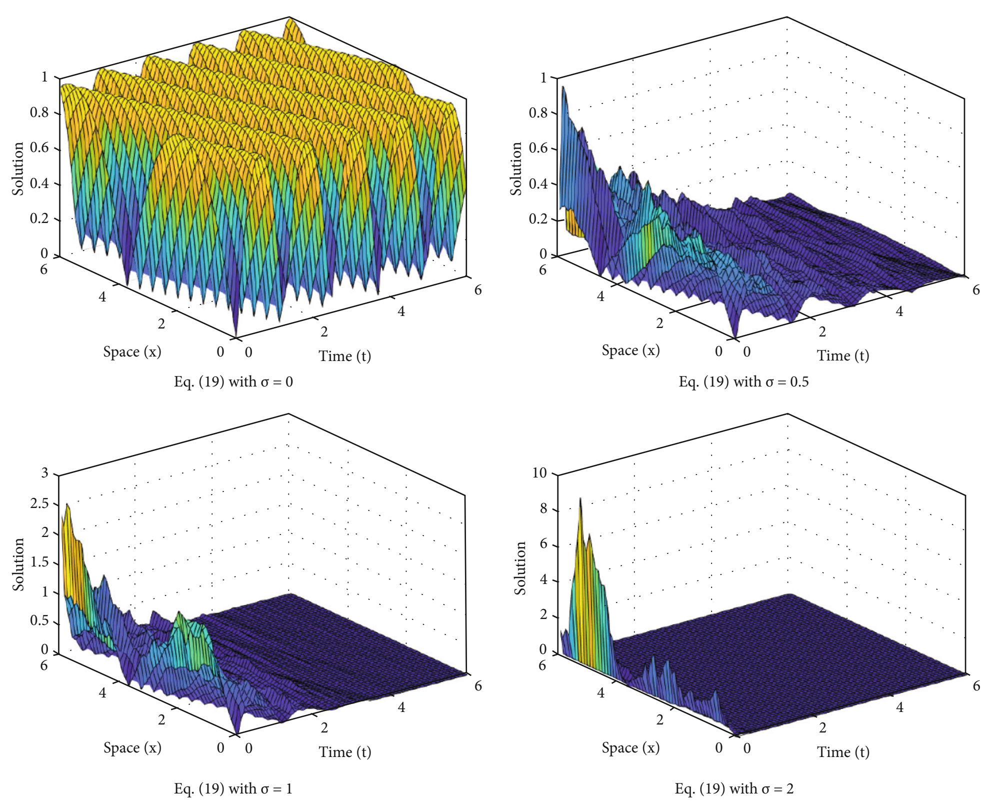


FIGURE 2: 3D shape of equation (27) with $\alpha = 1$ and for different values of $\sigma$.

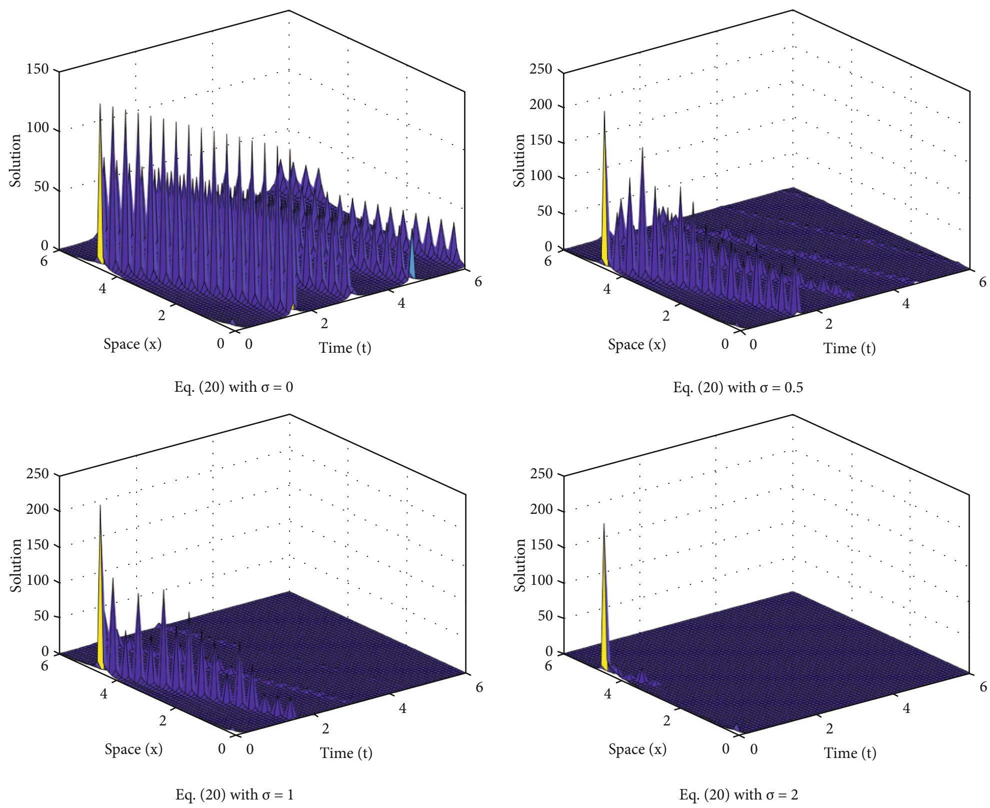


Figure 3: 3D shape of equation (28) with $\sigma = 0{,}0.5{,}1, 2$.

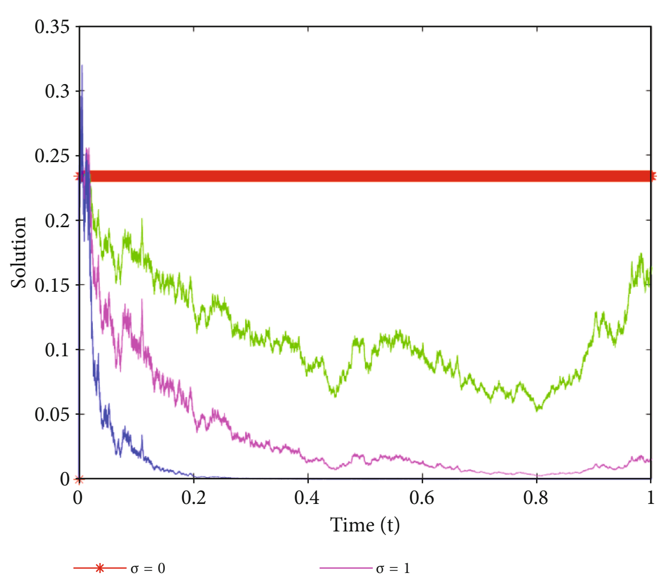


Figure 4: 2D shape of equation (27) with $\sigma = 0{,}0.5{,}1, 2$.

In Figures 2 and 3, we can see that when noise is introduced after small transit patterns, the surface starts to be flat as the noise intensity increases $\sigma = 0.5{,}1, 2$.

Figure 4 shows the 2D shape of equation (27) with $\sigma = 0{,}0.5{,}1, 2$ which highlight the above results.

We can deduce from Figures 1–4 that

(1) the solutions of SFRKLE (1) are stabilized around zero by the SWP

(2) as the fractional order $\alpha$ decreases, the surface shrinks

## 5. Conclusions

We considered here the stochastic fractional Radhakrishnan-Kundu-Lakshmanan equation (1) which has never been considered before with fractional derivative and stochastic term. To get hyperbolic, rational, and elliptic stochastic fractional solutions, we used the Jacobi elliptic function method. Because of the importance of SFRKLE in representing the propagation of solitons via an optical fiber, the derived solutions may be utilized to represent a wide range of exciting physical phenomena. Finally, we achieved by plotting the derived solutions to show how multiplicative noise and fractional derivative influence these solutions. We deduced that the SWP stabilizes the solutions around zero when the noise strength increases. In future work, we can try to get the exact solutions of SFRKLE (1) with additive noise or multiplicative color noise.

## Data Availability

All data are available in this paper.

## Conflicts of Interest

The authors declare that they have no competing interests.

## Authors' Contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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