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Research Article

Diverse Soliton Structures of the $(2+1)$-Dimensional Nonlinear Electrical Transmission Line Equation

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In this work, the $(2+1)$-dimensional nonlinear electrical transmission line equation (NETLE) is investigated by applying three recent technologies, namely, the variational approach, Hamiltonian approach, and energy balance approach. Diverse exact soliton solutions such as the bright, bright-like, kinky bright, bright-dark soliton, and periodic soliton solutions are successfully constructed. The outlines of the different solutions are shown in the form of the 3-D plot with the help of the Wolfram Mathematica. It reveals that the used methods are concise and effective and are expected to provide some inspiration for the study of travelling wave solutions of the PDEs in physics.

1. Introduction

Nonlinear partial differential equations (NLPDEs) appear in mathematics, physics, engineering, and other fields. Many complex phenomena occurring in nature can be described by the NLPDEs. The study of their soliton solutions is of great significance since they can make us more deeply understand the natural phenomena and their internal relations. So far, there are many effective methods available for constructing the soliton solutions such as the exp-function method [1–4], tanh-function method [5–9], (G'/G)-expansion method [10–12], F-expansion method [13, 14], extended rational sine-cosine and sinh-cosh methods [15–18], Sardar-subequation method [19–21], and Sine-Gordon expansion method [22, 23] [24–31]. In the current work, we aim to study the $(2+1)$-dimensional NETLE, which is expressed by [32]

$$\frac{\partial^2}{\partial t^2}\left(u-\alpha u^2+\beta u^3\right)-\mu_0^2\left(\delta_1^2\frac{\partial^2 u}{\partial x^2}+\frac{\delta_1^4}{12}\frac{\partial^4 u}{\partial x^4}\right)-\varpi_0^2\left(\delta_2^2\frac{\partial^2 u}{\partial y^2}+\frac{\delta_2^4}{12}\frac{\partial^4 u}{\partial y^4}\right)=0. \tag{1}$$

In Eq. (1), α, β, u_0, and ϖ_0 are nonzero constants, δ_1 and δ_2 represent the longitudinal and transverse distance between two adjacent sections, respectively. Eq. (1) plays an important role in the field of telecommunication and network engineering. Up to now, some effective approaches have been adopted to solve Eq. (1) such as the modified simple equation method [33], Jacobi elliptic function expansion method [34], sine-Gordon expansion method [35], and the Kudryashov method [36]. In recent years, the variational theory-based methods such as the variational approach (VA), Hamiltonian approach (HA), and energy balance approach (EBA) have caught a wide attention for solving the PDEs since they all probe the problem in view of the energy conservation and obtain the solutions by the stationary conditions. Additionally, these methods can help us insight the problem from a physical perspective. Thus, in this work, we aim to seek for the various soliton solutions by means of the variational theory-based methods, which are the VA, HA, and EBA. The rest content of this paper is arranged as follows. In Section 2, the variational principle and Hamiltonian of the studied problem are presented. In Section 3, the various soliton solutions are derived by applying the three methods. The behaviors of the different solutions are presented through the 3-D plot in Section 4. Finally, we get a conclusion in Section 5.

2. Variational Principle and the Hamiltonian

For solving Eq. (1), we introduce the following transformation [36]:

$$u(x, y, t) = U(\chi), \chi = \sqrt{\wp}(x + y - \theta_0 t). \tag{2}$$

In Eq. (2), $\wp$ and θ_0 represent the wave number and wave speed, respectively. Applying Eq. (2) to Eq. (1), integrating the results twice with respect to χ and setting the integrating constants to zero, we get

$$\begin{aligned} &\left[M - \left(\mu_0^2 \Pi_1 + \omega_0^2 \Pi_2\right)\right] U + M\left(\beta U^3 - \alpha U^2\right) \\ &- \frac{1}{12}\left(\mu_0^2 \Pi_1^2 + \omega_0^2 \Pi_2^2\right) \frac{d^2 U}{d\chi^2} = 0, \end{aligned} \tag{3}$$

where $M = \wp\theta_0^2$, $\Pi_1 = \delta_1^2 \wp$, and $\Pi_2 = \delta_2^2 \wp$.

With the aid of the semi-inverse method [37–43], we establish the variational principle of Eq. (3) as

$$\begin{aligned} J(U) = \int &\left\{ \frac{1}{2}\left(U'\right)^2 + \frac{3M\beta}{\mu_0^2 \Pi_1^2 + \varpi_0^2 \Pi_2^2} U^4 - \frac{4M\alpha}{\mu_0^2 \Pi_1^2 + \varpi_0^2 \Pi_2^2} U^3 \right. \\ &\left. + \frac{6\left[M - \left(\mu_0^2 \Pi_1 + \varpi_0^2 \Pi_2\right)\right]}{\mu_0^2 \Pi_1^2 + \varpi_0^2 \Pi_2^2} U^2 \right\} d\chi. \end{aligned} \tag{4}$$

For the convenience of calculation, we reexpress Eq. (4) as

$$\begin{aligned} J(U) &= \int \left\{ \frac{1}{2}\left(U'\right)^2 + \lambda_1 U^4 - \lambda_2 U^3 + \lambda_3 U^2 \right\} d\chi \\ &= \int \{\mathfrak{R} - \mathfrak{I}\} d\chi, \end{aligned} \tag{5}$$

where $\lambda_1 = 3M\beta/\mu_0^2 \Pi_1^2 + \varpi_0^2 \Pi_2^2$, $\lambda_2 = 4M\alpha/\mu_0^2 \Pi_1^2 + \varpi_0^2 \Pi_2^2$, and $\lambda_3 = 6[M - (\mu_0^2 \Pi_1 + \varpi_0^2 \Pi_2)]/\mu_0^2 \Pi_1^2 + \varpi_0^2 \Pi_2^2$. And there are

$$\begin{aligned} \mathfrak{R} &= \frac{1}{2}\left(U'\right)^2, \\ \mathfrak{I} &= -\lambda_1 U^4 + \lambda_2 U^3 - \lambda_3 U^2. \end{aligned} \tag{6}$$

Here, $\mathfrak{R}$ is the kinetic energy, and $\mathfrak{I}$ indicates the potential energy. Then, we can get the Hamiltonian as [44, 45]

$$H_m = \mathfrak{R} + \mathfrak{I} = \frac{1}{2}\left(U'\right)^2 - \lambda_1 U^4 + \lambda_2 U^3 - \lambda_3 U^2. \tag{7}$$

3. The Solutions

3.1. The VA. The main target of this subsection is to apply the VA to search for the abundant exact solutions of Eq. (1).

3.1.1. The Bright Soliton Solution. Here, we suppose the solution of Eq. (3) is [46]

$$U(\chi) = \Xi \operatorname{sech}(\chi). \tag{8}$$

Taking it into Eq. (5) yields

$$\begin{aligned} J(\Xi) &= \int_0^\infty \left\{ \frac{1}{2}\left(U'\right)^2 + \lambda_1 U^4 - \lambda_2 U^3 + \lambda_3 U^2 \right\} d\chi \\ &= \int_0^\infty \left\{ \frac{1}{2}[\Xi \sec h(\chi) \tanh(\chi)]^2 + \lambda_1 [\Xi \sec h(\chi)]^4 \right. \\ &\quad \left. - \lambda_2 [\Xi \sec h(\chi)]^3 + \lambda_3 [\Xi \sec h(\chi)]^2 \right\} d\chi \\ &= \frac{\Xi^2\left(8\lambda_1 \Xi^2 - 3\Xi\lambda_2 \pi + 12\lambda_3 + 2\right)}{12}. \end{aligned} \tag{9}$$

According to the Ritz-like method (RLM), we take its stationary condition as

$$\frac{dJ}{d\Xi} = 0, \tag{10}$$

which leads to

$$\frac{\Xi\left(32\lambda_1 \Xi^2 - 9\Xi\lambda_2 \pi + 24\lambda_3 + 4\right)}{12} = 0. \tag{11}$$

On solving it by the Wolfram Mathematica, we have

$$\Xi = \frac{9\lambda_2 \pi + \sqrt{81\lambda_2^2 \pi^2 - 3072\lambda_1 \lambda_3 - 512\lambda_1}}{64\lambda_1}, \tag{12}$$

Or

$$\Xi = \frac{9\lambda_2 \pi - \sqrt{81\lambda_2^2 \pi^2 - 3072\lambda_1 \lambda_3 - 512\lambda_1}}{64\lambda_1}. \tag{13}$$

So, the solution of Eq. (3) can be obtained as

$$U(\chi) = \frac{9\lambda_2 \pi + \sqrt{81\lambda_2^2 \pi^2 - 3072\lambda_1 \lambda_3 - 512\lambda_1}}{64\lambda_1} \operatorname{sech}(\chi), \tag{14}$$

or

$$U(\chi) = \frac{9\lambda_2 \pi - \sqrt{81\lambda_2^2 \pi^2 - 3072\lambda_1 \lambda_3 - 512\lambda_1}}{64\lambda_1} \operatorname{sech}(\chi). \tag{15}$$

In the view of Eq. (2), we have

$$u(x,y,t)=\frac{9\lambda_2\pi+\sqrt{81\lambda_2^2\pi^2-3072\lambda_1\lambda_3-512\lambda_1}}{64\lambda_1}\cdot \operatorname{sech}\left[\sqrt{\wp}(x+y-\theta_0 t)\right], \tag{16}$$

or

$$u(x,y,t)=\frac{9\lambda_2\pi-\sqrt{81\lambda_2^2\pi^2-3072\lambda_1\lambda_3-512\lambda_1}}{64\lambda_1}\cdot \operatorname{sech}\left[\sqrt{\wp}(x+y-\theta_0 t)\right]. \tag{17}$$

3.1.2. The Bright-Like Soliton Solution. The solution of Eq. (3) is assumed as the following form [47]:

$$U(\chi)=\frac{\Xi_2}{1+\cosh(\chi)}. \tag{18}$$

Putting above equation into Eq. (5), it gives

$$\begin{aligned} J(\Xi_2)&=\int_0^\infty\left\{\frac{1}{2}\left(U'\right)^2+\lambda_1U^4-\lambda_2U^3+\lambda_3U^2\right\}d\chi\\ &=\int_0^\infty\left\{\frac{1}{2}\left[-\frac{\Xi_2\sinh(\chi)}{(1+\cosh(\chi))^2}\right]^2+\lambda_1\left[\frac{\Xi_2}{1+\cosh(\chi)}\right]^4\right.\\ &\quad\left.-\lambda_2\left[\frac{\Xi_2}{1+\cosh(\chi)}\right]^3+\lambda_3\left[\frac{\Xi_2}{1+\cosh(\chi)}\right]^2\right\}d\chi\\ &=\frac{\Xi_2^2[4\Xi_2(3\lambda_1\Xi_2-7\lambda_2)+70\lambda_3+7]}{210}. \end{aligned} \tag{19}$$

Based on the RLM, there is

$$\frac{dJ}{d\Xi_2}=0. \tag{20}$$

It results into

$$\frac{\Xi_2\left(24\lambda_1\Xi_2^2-42\lambda_2\Xi_2+70\lambda_3+7\right)}{150}=0. \tag{21}$$

Solving above equation, we have

$$\Xi_2=\frac{21\lambda_2+\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-8\lambda_1\right)}}{24\lambda_1}, \tag{22}$$

or

$$\Xi_2=\frac{21\lambda_2-\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-8\lambda_1\right)}}{24\lambda_1}. \tag{23}$$

So, there are

$$u(x,y,t)=\frac{21\lambda_2+\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-8\lambda_1\right)}}{24\lambda_1}\cdot\frac{1}{1+\cosh\left[\sqrt{\wp}(x+y-\theta_0 t)\right]}, \tag{24}$$

or

$$u(x,y,t)=\frac{21\lambda_2-\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-8\lambda_1\right)}}{24\lambda_1}\cdot\frac{1}{1+\cosh\left[\sqrt{\wp}(x+y-\theta_0 t)\right]}. \tag{25}$$

3.1.3. The Kinky-Bright Soliton. Inspired by the research results obtained in [47], we can also assume that Eq. (3) has the following solution to construct the kinky-bright soliton:

$$U(\chi)=\Xi_3\operatorname{sech}^2(\chi). \tag{26}$$

By the same way, we have

$$J(\Xi_3)=\frac{2\Xi_3^2\left(24\lambda_1\Xi_3^2-28\lambda_2\Xi_3+35\lambda_3+14\right)}{105}. \tag{27}$$

Computing its stationary condition with respect to Ξ_3 yields

$$\frac{4\Xi_3\left(48\lambda_1\Xi_3^2-42\lambda_2\Xi_3+35\lambda_3+14\right)}{105}=0, \tag{28}$$

from which, we obtain

$$\Xi_3=\frac{21\lambda_2+\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-32\lambda_1\right)}}{48\lambda_1}, \tag{29}$$

or

$$\Xi_3=\frac{21\lambda_2-\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-32\lambda_1\right)}}{48\lambda_1}. \tag{30}$$

With this, we have

$$u(x,y,t)=\frac{21\lambda_2+\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-32\lambda_1\right)}}{48\lambda_1}\cdot\operatorname{sech}^2\left[\sqrt{\wp}(x+y-\theta_0 t)\right], \tag{31}$$

or

$$u(x,y,t)=\frac{21\lambda_2-\sqrt{21\left(21\lambda_2^2-80\lambda_1\lambda_3-32\lambda_1\right)}}{48\lambda_1}\cdot\operatorname{sech}^2\left[\sqrt{\wp}(x+y-\theta_0 t)\right]. \tag{32}$$

3.1.4. The Bright-Dark Soliton. Here, we search for a solution of Eq. (3) in the following form [47]:

$$U(\chi) = \Xi_4 \operatorname{sech}(\chi) \tanh(\chi). \tag{33}$$

Substituting Eq.(33) into Eq. (5), it produces

$$J = \frac{\Xi_4^2}{210}\left(12\lambda_1\Xi_4^2 - 28\lambda_2\Xi_4 + 70\lambda_3 + 49\right). \tag{34}$$

Making J stationary with Ξ_4 results in

$$\frac{\Xi_4}{105}\left(24\lambda_1\Xi_4^2 - 42\lambda_2\Xi_4 + 70\lambda_3 + 49\right) = 0. \tag{35}$$

From Eq. (35), we have

$$\Xi_4 = \frac{21\lambda_2 + \sqrt{21\left(21\lambda_2^2 - 80\lambda_1\lambda_3 - 56\lambda_1\right)}}{24\lambda_1}, \tag{36}$$

or

$$\Xi_4 = \frac{21\lambda_2 - \sqrt{21\left(21\lambda_2^2 - 80\lambda_1\lambda_3 - 56\lambda_1\right)}}{24\lambda_1}. \tag{37}$$

So, we have

$$u(x, y, t) = \frac{21\lambda_2 + \sqrt{21\left(21\lambda_2^2 - 80\lambda_1\lambda_3 - 56\lambda_1\right)}}{24\lambda_1} \cdot \operatorname{sech}\left[\sqrt{k}(x + y - v_0 t)\right] \tanh\left[\sqrt{\wp}(x + y - \theta_0 t)\right], \tag{38}$$

or

$$u(x, y, t) = \frac{21\lambda_2 - \sqrt{21\left(21\lambda_2^2 - 80\lambda_1\lambda_3 - 56\lambda_1\right)}}{24\lambda_1} \cdot \operatorname{sech}\left[\sqrt{k}(x + y - v_0 t)\right] \tanh\left[\sqrt{\wp}(x + y - \theta_0 t)\right]. \tag{39}$$

3.2. The HA. In this section, we aim to use the HA to search for the periodic solution of Eq. (3) as [48–50]

$$U(\chi) = \Lambda \cos(\Omega\chi), \tag{40}$$

where Λ is the amplitude and Ω is the frequency.

Form the obtained Hamiltonian provided by Eq. (7), we construct a new function $\breve{h}(U)$ as

$$\breve{h}(U) = \int_0^{T/4}\left\{\frac{1}{2}\left(U'\right)^2 - \lambda_1 U^4 + \lambda_2 U^3 - \lambda_3 U^2\right\} d\chi. \tag{41}$$

Taking Eq. (40) into above equation, it yields

$$\begin{aligned}\breve{h}(U) &= \int_0^{T/4}\left\{\frac{1}{2}[-\Lambda\Omega \sin(\Omega\chi)]^2 - \lambda_1[\Lambda\cos(\Omega\chi)]^4 + \lambda_2[\Lambda\cos(\Omega\chi)]^3 - \lambda_3[\Lambda\cos(\Omega\chi)]^2\right\} d\chi \\ &= \frac{\Lambda^2}{48\Omega}\left[32\lambda_2\Lambda - 9\lambda_1\Lambda^2\pi + 6\left(\Omega^2 - 2\lambda_3\right)\pi\right].\end{aligned} \tag{42}$$

By the Hamiltonian approach, we set

$$\frac{\partial}{\partial\Lambda}\left(\frac{\partial\breve{h}}{\partial(1/\Omega)}\right) = 0, \tag{43}$$

which leads to

$$\Omega = \sqrt{\frac{8\Lambda\lambda_2}{\pi} - 2\lambda_3 - 3\lambda_1\Lambda^2}. \tag{44}$$

So, the periodic soliton solution of Eq. (1) is found as

$$u(x, y, t) = \Lambda\cos\left(\sqrt{\frac{8\Lambda\lambda_2}{\pi} - 2\lambda_3 - 3\lambda_1\Lambda^2}\left[\sqrt{\wp}(x + y - \theta_0 t)\right]\right). \tag{45}$$

3.3. The EBA. To use the EBA [50, 51], we first assume the solution of Eq.(3) is

$$U(\chi) = \Delta\cos(\Theta\chi). \tag{46}$$

And we have

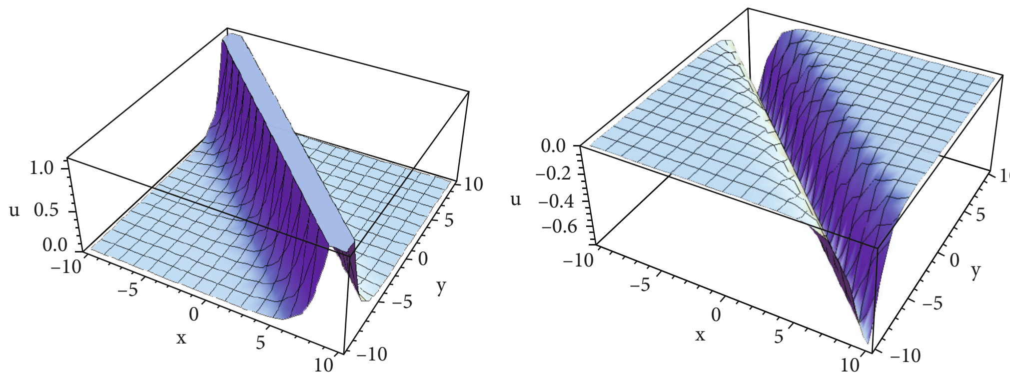
$$U(0) = \Delta, U'(0) = 0. \tag{47}$$

The energy conservation reveals that the Hamiltonian invariant keep unchanged for the system; so, inserting Eq. (47) into Eq. (7) yields

$$\begin{aligned}H_m &= \mathfrak{R} + \mathfrak{I} \\ &= \frac{1}{2}\left(U'\right)^2 - \lambda_1 U^4 + \lambda_2 U^3 - \lambda_3 U^2 \\ &= -\lambda_1\Delta^4 + \lambda_2\Delta^3 - \lambda_3\Delta^2.\end{aligned} \tag{48}$$

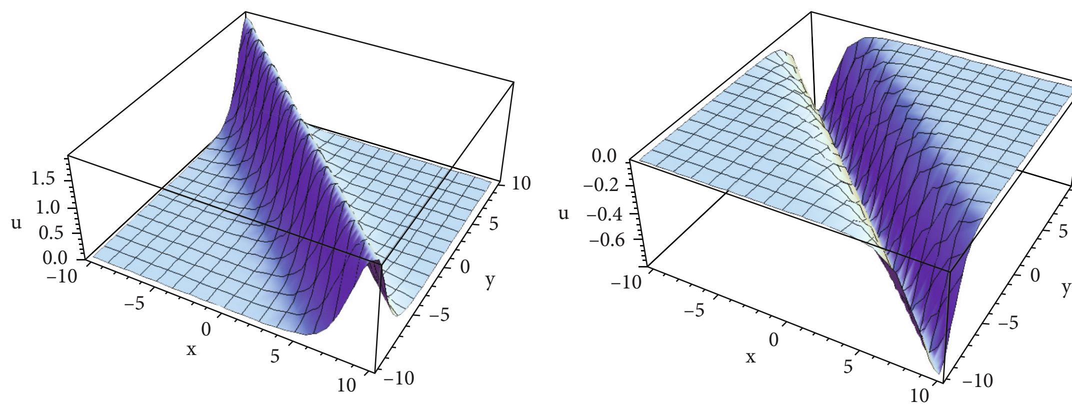
Then, we substitute Eq. (46) into Eq. (48) and set $\Theta\chi = \pi/4$, and there is

$$\begin{aligned}&\frac{1}{2}\left(-\Delta\Theta\sin\left(\frac{\pi}{4}\right)\right)^2 - \lambda_1\left(\Delta\cos\left(\frac{\pi}{4}\right)\right)^4 + \lambda_2\left(\Delta\cos\left(\frac{\pi}{4}\right)\right)^3 - \lambda_3\left(\Delta\cos\left(\frac{\pi}{4}\right)\right)^2 \\ &= -\lambda_1\Delta^4 + \lambda_2\Delta^3 - \lambda_3\Delta^2.\end{aligned} \tag{49}$$



(a) (b)

FIGURE 1: The outline of the bright soliton. (a) for Eq. (16) and (b) for Eq. (17).



(a) (b)

FIGURE 2: The behaviors of the bright-like soliton. (a) for Eq. (24) and (b) for Eq. (25).

Solving it gives

$$\Theta = \sqrt{\left(4-\sqrt{2}\right)\lambda_2 \Delta - 2\lambda_3 - 3\lambda_1 \Delta^2}, \tag{50}$$

which has a well agreement with Eq. (44). It strongly proves the correctness of the two different methods. Thus, we get the periodic soliton solution of Eq. (1) as

$$u(x, y, t) = \Delta \cos\left(\sqrt{\left(4-\sqrt{2}\right)\lambda_2 \Delta - 2\lambda_3 - 3\lambda_1 \Delta^2}\left[\sqrt{\wp}(x + y - \theta_0 t)\right]\right). \tag{51}$$

4. Results and Discussion

This section will give the graphical representations and physical interpretation of the obtained solutions in Section 3 by using proper parameters.

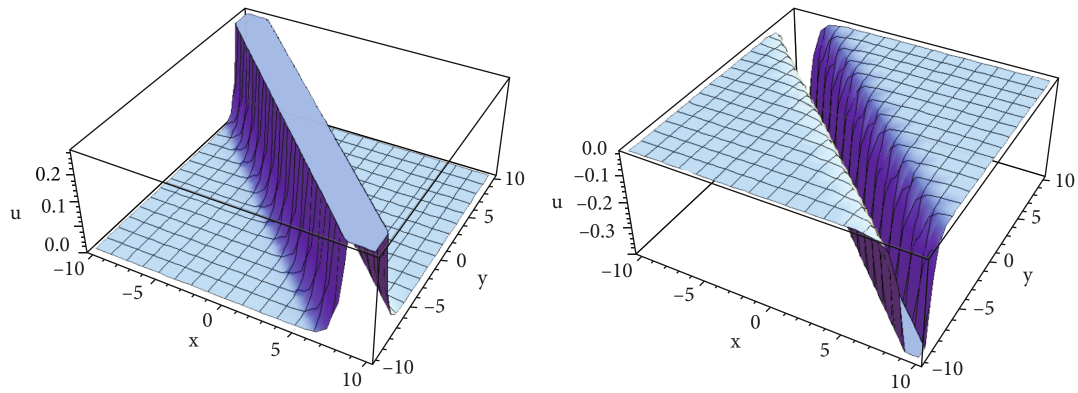
For $\wp = 1$, $\theta_0 = 1$, $\delta_1^2 = \delta_2^2 = 1$, $\varpi_0 = 1$,, $\mu_0 = 1$, $\alpha = 1$, and $\beta = 1$, Figure 1 plots the performance of the bright solitons obtained by Eqs. (16) and (17) within the interval $-10 < x < 10$ and $-10 < y < 10$. Obviously, they have the bright soliton characteristics. $M = 1$.

We plot the behaviors of the bright-like soliton solutions given by Eqs. (24) and (25) in Figure 2 by choosing $\wp = 1$, $\theta_0 = 1$, $\delta_1^2 = \delta_2^2 = 1$, $\varpi_0 = 1$, $M = 1$, $\mu_0 = 1$,$\alpha = 1$, and $\beta = 1$, where the performance are like bright soliton.

Solutions of Eqs. (31) and (32) are the kinky-bright solitons. We plot their behaviors in Figure 3 within the interval on $-10 < x < 10$ and $-10 < y < 10$ with for $\wp = 1$, $\theta_0 = 1$, $\delta_1^2 = \delta_2^2 = 1$, $\varpi_0 = 1$, $M = 1$, $\mu_0 = 1$, $\alpha = 1$, and $\beta = 1$. As expected, they all have the bright soliton characteristics.

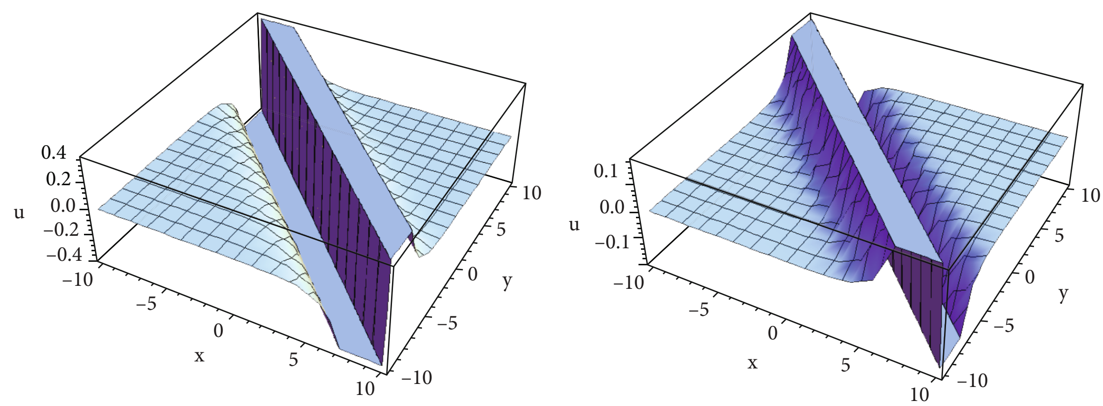
Selecting $\wp = 1$, $\theta_0 = 1$, $\delta_1^2 = \delta_2^2 = 1$, $\varpi_0 = 1$, $M = 1$, $\mu_0 = 1$, $\alpha = 1$, and $\beta = 1$, the 3-D plots of the bright-dark solitons given by Eqs. (38) and (39) are presented in Figure 4 within the interval $-10 < x < 10$ and $-10 < y < 10$. It can be observed that the performances are bright-dark soliton.

Figure 5 plots the profile of Eqs. (45) and (51) within the interval $-5 < x < 5$ and $-5 < y < 5$ by using the parameters as $\wp = 1$, $\theta_0 = 1$, $\delta_1^2 = \delta_2^2 = 1$, $\varpi_0 = 1$, $M = 1$, $\mu_0 = 1$, $\Lambda = 1$ $(\Delta = 1)$, $\alpha = 1$, and $\beta = 1$. It can be seen that the two profiles are all



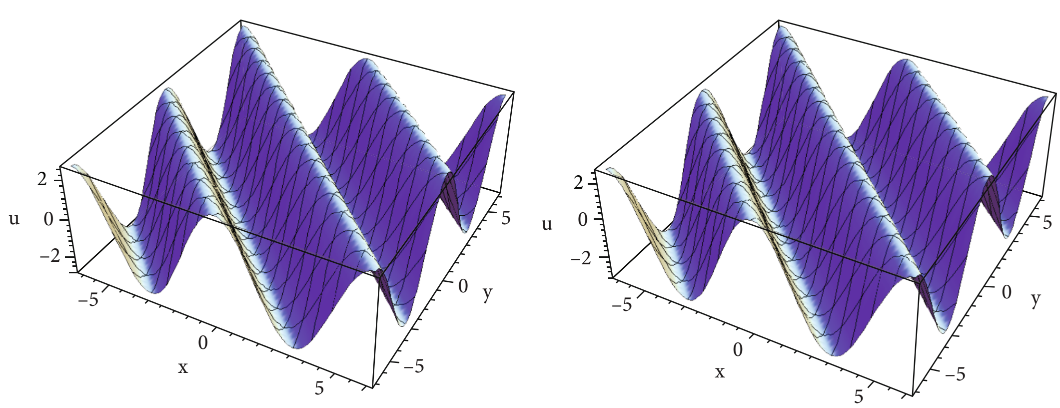
(a) (b)

Figure 3: The outline of the kinky-bright soliton. (a) for Eq. (31) and (b) for Eq. (32).



(a) (b)

Figure 4: The outline of the bright-dark soliton. (a) for Eq.(38) and (b) for Eq.(39).



(a) (b)

Figure 5: The outline of the periodic soliton. (a) for Eq. (45) and (b) for Eq. (51).

perfect periodic waves. In addition, the two contours are basically the same.

5. Conclusion

In this work, diverse soliton solutions of $(2+1)$-dimensional nonlinear electrical transmission line equation like the bright soliton, bright-like soliton, kinky-bright soliton, bright-dark soliton, and periodic soliton solutions were constructed by using the variational approach, Hamiltonian approach, and energy balance approach. The profiles of the solutions were presented through the 3-D plots via selecting the appropriate parameters by means of the Wolfram Mathematica. The results revealed that the proposed methods were straightforward, simple, and effective, which can be adopted to study the traveling wave theory of the PDEs arising in physics.

Data Availability

The data generated and/or analyzed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

Conflicts of Interest

This work does not have any conflicts of interest.

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