Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 2815298, 10 pages
https://doi.org/10.1155/2022/2815298

Research Article

Different Wave Structures for the (2+1)-Dimensional Korteweg-de Vries Equation

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Received 21 August 2021; Accepted 24 March 2022; Published 12 April 2022

Academic Editor: Wen-Xiu Ma

In this article, a (2+1)-dimensional Korteweg-de Vries equation is investigated. Abundant periodic wave solutions are obtained based on the Hirota's bilinear form and a direct test function. Meanwhile, the interaction solutions between lump and periodic waves are presented. What is more, we derive the interaction solutions among lump, periodic, and solitary waves. Based on the extended homoclinic test technique, some new double periodic-soliton solutions are presented. Finally, some 3D and density plots are simulated and displayed to respond the dynamic behavior of these obtained solutions.

1. Introduction

Korteweg-de Vries (KdV) equation [1–11]

$$u_t + 6uu_x + u_{xxx} = 0, \quad (1)$$

has been used to depict the shallow-water waves, stratified internal waves, lattice dynamics, and so on, where $u = u(x, t)$. Its extensions, namely, the KdV-type models, have been presented in fields such as fluid flows, plasma physics, and solid-state physics [12–15]. Solitary wave solutions have wide applications in many fields of natural science such as plasmas, hydrodynamics, nonlinear optics, fiber optics, and solid state physics, and that the interaction of solitons plays an important role [16, 17], which can keep their velocities and shapes after the elastic collisions [18–21]. Periodic waves, as solitary waves, have amusing applications in nature. For the ultrashort pulse-train generation from the beating of two-mode signals, for instance, one must research periodic wave solutions of nonlinear equations governing the fiber system [22]. However, the interaction properties between periodic waves are rarely discussed because the mathematics is more involved.

In this paper, based on symbolic computation [23–30], we will investigate the following (2+1)-dimensional KdV equation for nonlinear waves such as the shallow-water waves and surface and internal waves [31]

$$u_t + 3(uv)_x + u_{xxx} = 0, \quad u_x = v_y, \quad (2)$$

where $u = u(x, y, t)$, $v = v(x, y, t)$. Equation (1) was obtained by Boiti et al. in Ref. [31] by using the weak Lax pair, also named as Boiti-Leon-Manna-Pempinelli equation [32] and read as the ubiquitous KdV equation when $v = u$ and $y = x$ [33]. The rich dromion structures and localized structures [34, 35], exact periodic solitary wave and Jacobi elliptic function double periodic solutions [33], periodic type of three-wave solutions [36], lump solutions [37], a new Bäklund transformation and new representation of the N-soliton solution [38], invariant solutions [39], M-lump solutions [40], and breathers and interaction solutions [41, 42] for Equation (1) have been studied. Ma [43] obtained N-soliton solutions and given the Hirota N-soliton conditions of Equation (1) by using the Hirota bilinear formulation. However, the interaction solutions between lump and periodic waves and interaction solutions among lump, periodic, and solitary waves have not been seen in literature, which will become our main work.

The organization of this article is as follows. In Section 2, abundant periodic wave solutions are obtained based on the Hirota's bilinear form and a direct test function. In Section 3, the interaction solutions between lump and periodic waves are obtained. In Section 4, we present the interaction solutions among lump, periodic, and solitary waves. Dynamic behavior is analyzed by some 3D and density plots. In Section 5, we present new double periodic-soliton solutions for the (2+1)-dimensional KdV equation by using the extended homoclinic. In Section 6, the conclusions are made.

2. Periodic Wave Solutions

Under two logarithmic transformations [36]

$$u = 2(\ln f)_{xy}, \quad v = 2(\ln f)_{xx}, \tag{3}$$

Equation (2) has the following bilinear form:

$$(D_yD_t + D_x^3D_y)f\cdot f = f_{yt}f - f_yf_t + ff_{xxxy} + 3f_{xy}f_{xx} - 3f_xf_{xxy} - f_yf_{xxx} = 0, \tag{4}$$

where $f = f(x, y, t)$. To study the periodic solitary wave solutions of Equation (1), suppose that

$$f = k_1e^{\theta_1} + e^{-\theta_1} + k_2 \tan\theta_2 + k_3 \tanh\theta_3, \tag{5}$$

where $\theta_i = \alpha_i x + \beta_i y + \delta_i t + \sigma_i (i = 1, 2, 3)$ and α_i, β_i, δ_i, and σ_i are undetermined constants. Substituting Equation (5) into Equation (4) and equating all the coefficients of different powers of e^{θ_1}, $e^{-\theta_1}$, $\tan\theta_2$, and $\tanh\theta_3$ and constant term to zero, we have

Case 1.

$$k_3 = \beta_1 = \alpha_2 = \delta_2 = 0, \delta_1 = -\alpha_1^3. \tag{6}$$

Substituting Equation (6) into Equation (5), we have

$$f = e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_2 \tan(y\beta_2 + \sigma_2). \tag{7}$$

Thus, the first new periodic wave solution is obtained as

$$u_1 = -\frac{2\sec^2(y\beta_2 + \sigma_2)k_2\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1\alpha_1 - e^{t\alpha_1^3 - x\alpha_1 - \sigma_1}\alpha_1\right)\beta_2}{\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_2\tan(y\beta_2 + \sigma_2)\right)^2}, \tag{8}$$

$$v_1 = \frac{2\left(e^{t\alpha_1^3 - x\alpha_1 - \sigma_1}\alpha_1^2 + e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1\alpha_1^2\right)}{e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_2\tan(y\beta_2 + \sigma_2)} - \frac{2\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1\alpha_1 - e^{t\alpha_1^3 - x\alpha_1 - \sigma_1}\alpha_1\right)^2}{\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_2\tan(y\beta_2 + \sigma_2)\right)^2}. \tag{9}$$

Dynamic behavior of Equation (8) is shown in Figure 1 in $x - y$.

Case 2.

$$k_3 = k_1 = \alpha_2 = \delta_2 = 0, \delta_1 = -\alpha_1^3. \tag{10}$$

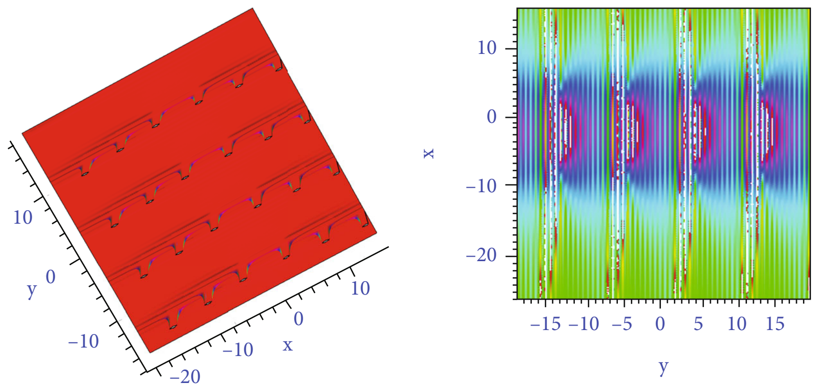
Substituting Equation (10) into Equation (5), we have

$$f = k_2\tan(y\beta_2 + \sigma_2) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}. \tag{11}$$

Thus, we derive the second new periodic wave solution as follows:

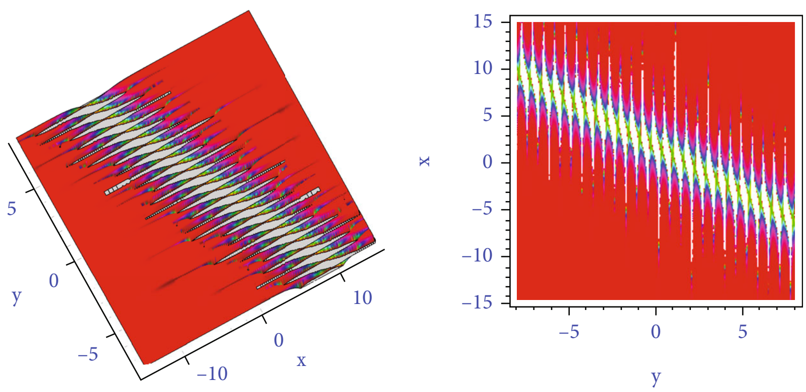
$$u_2 = \frac{2e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\alpha_1\beta_1}{k_2\tan(y\beta_2 + \sigma_2) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}} + \frac{2e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\alpha_1\left(\sec^2(y\beta_2 + \sigma_2)k_2\beta_2 - e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\beta_1\right)}{\left(k_2\tan(y\beta_2 + \sigma_2) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\right)^2}, \tag{12}$$

$$v_2 = \frac{2e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\alpha_1^2}{k_2\tan(y\beta_2 + \sigma_2) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}} - \frac{2e^{2t\alpha_1^3 - 2x\alpha_1 - 2y\beta_1 - 2\sigma_1}\alpha_1^2}{\left(k_2\tan(y\beta_2 + \sigma_2) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\right)^2}. \tag{13}$$



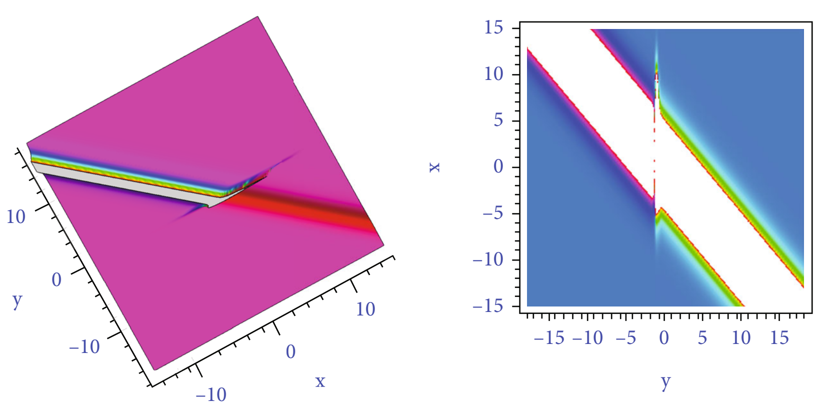
(a) (b)

Figure 1: $\alpha_1 = \sigma_2 = -1$, $k_1 = 5$, $k_2 = \sigma_1 = -2$, $\beta_2 = 1$, and $t = 0$. (a) 3D plot and (b) density plot.



(a) (b)

Figure 2: $\alpha_1 = \sigma_2 = -1$, $\beta_1 = -1$, $k_2 = 2$, $\sigma_1 = 1$, $\beta_2 = 5$, and $t = 0$. (a) 3D plot and (b) density plot.



(a) (b)

Figure 3: $\alpha_1 = \beta_1 = 1$, $\sigma_1 = -1$, $k_3 = -2$, $\sigma_3 = \beta_3 = 5$, and $t = 0$. (a) 3D plot and (b) density plot.

Dynamic behavior of Equation (12) is shown in Figure 2 in $x-y$.

Case 3.

$$k_2 = k_1 = \alpha_3 = \delta_3 = 0, \delta_1 = -\alpha_1^3. \tag{14}$$

Substituting Equation (14) into Equation (5), we have

$$f = k_3 \tanh(y\beta_3 + \sigma_3) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}. \tag{15}$$

Thus, the third new periodic wave solution is

$$u_3 = \frac{2e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\alpha_1\beta_1}{k_3 \tanh(y\beta_3 + \sigma_3) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}} + \frac{2e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\alpha_1\left(\sec h^2(y\beta_3+\sigma_3)k_3\beta_3 - e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\beta_1\right)}{\left(k_3 \tanh(y\beta_3 + \sigma_3) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\right)^2}, \tag{16}$$

$$v_3 = \frac{2e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\alpha_1^2}{k_3 \tanh(y\beta_3 + \sigma_3) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}} - \frac{2e^{2t\alpha_1^3 - 2x\alpha_1 - 2y\beta_1 - 2\sigma_1}\alpha_1^2}{\left(k_3 \tanh(y\beta_3 + \sigma_3) + e^{t\alpha_1^3 - x\alpha_1 - y\beta_1 - \sigma_1}\right)^2}. \tag{17}$$

Dynamic behavior of Equation (16) is shown in Figure 3 in $x-y$.

Case 4.

$$k_2 = \beta_1 = \alpha_3 = \delta_3 = 0, \delta_1 = -\alpha_1^3. \tag{18}$$

Substituting Equation (20) into Equation (5), we have

$$f = e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_3 \tanh(y\beta_3 + \sigma_3). \tag{19}$$

Then, the fourth new periodic wave solution is presented as follows:

$$u_4 = -\frac{2\sec h^2(y\beta_3+\sigma_3)k_3\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1\alpha_1 - e^{t\alpha_1^3 - x\alpha_1 - \sigma_1}\alpha_1\right)\beta_3}{\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_3 \tanh(y\beta_3 + \sigma_3)\right)^2}, \tag{20}$$

$$v_4 = \frac{2\left(e^{t\alpha_1^3 - x\alpha_1 - \sigma_1}\alpha_1^2 + e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1\alpha_1^2\right)}{e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_3 \tanh(y\beta_3 + \sigma_3)} - \frac{2\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1\alpha_1 - e^{t\alpha_1^3 - x\alpha_1 - \sigma_1}\alpha_1\right)^2}{\left(e^{-t\alpha_1^3 + x\alpha_1 + \sigma_1}k_1 + e^{t\alpha_1^3 - x\alpha_1 - \sigma_1} + k_3 \tanh(y\beta_3 + \sigma_3)\right)^2}. \tag{21}$$

Dynamic behavior of Equation (20) is shown in Figure 4 in $x-y$.

3. Lump-Periodic Waves

Under the transformations [41]

$$u = 2(\ln f)_{xy}, \quad v = \gamma + 2(\ln f)_{xx}, \tag{22}$$

Equation (2) has the more general bilinear form

$$\left(D_yD_t + D_x^3D_y + 3\gamma D_xD_y\right)f\cdot f = f_{yt}f - f_yf_t + 3\gamma\left(f_{yx}f - f_yf_x\right) + ff_{xxxy} + 3f_{xy}f_{xx} - 3f_xf_{xxy} - f_yf_{xxx} = 0. \tag{23}$$

To discuss the interaction between lump and periodic waves, assume

$$\begin{aligned} f = {} & \alpha_9 + k_1 \sin(\alpha_{14} + \alpha_{13}t + \alpha_{11}x + \alpha_{12}y) \\ & + (\alpha_8 + \alpha_7t + \alpha_5x + \alpha_6y)^2 + (\alpha_4 + \alpha_3t + \alpha_1x + \alpha_2y)^2 \\ & + k_2 \cos(\alpha_{24} + \alpha_{23}t + \alpha_{21}x + \alpha_{22}y), \end{aligned} \tag{24}$$

where $\alpha_i (i = 1,\cdots,9)$, α_{j1}, α_{j2}, α_{j3}, and $\alpha_{j4} (j = 1, 2)$ are undetermined constants. Substituting Equation (24) into Equation (23), we have

$$\begin{aligned} & \alpha_{13} = \alpha_{11}^3 - 3\alpha_{11}\gamma, \alpha_{23} = \alpha_{21}^3 - 3\alpha_{21}\gamma, \alpha_{12} = \alpha_{22} = 0, \\ & \alpha_6 = -\frac{\alpha_1\alpha_2}{\alpha_5}, \alpha_7 = -3\alpha_5\gamma, \alpha_3 = -3\alpha_1\gamma. \end{aligned} \tag{25}$$

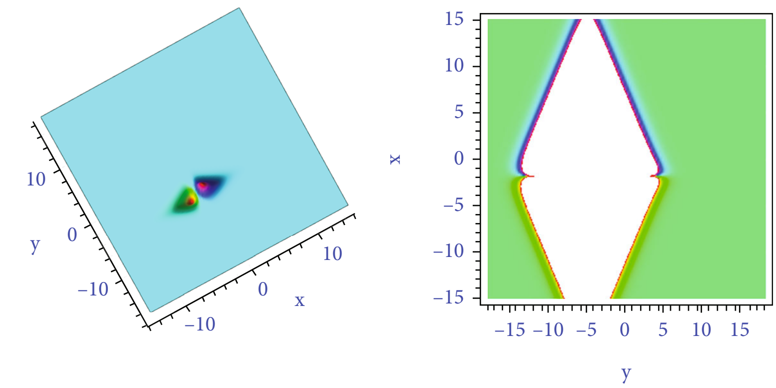
Equation (24) will become

$$\begin{aligned} f = {} & \alpha_9 + k_1 \sin\left[\alpha_{14} + t\left(\alpha_{11}^3 - 3\alpha_{11}\gamma\right) + \alpha_{11}x\right] \\ & + (\alpha_4 - 3\alpha_1\gamma t + \alpha_1x + \alpha_2y)^2 \\ & + \left(\alpha_8 - 3\alpha_5\gamma t + \alpha_5x - \frac{\alpha_1\alpha_2y}{\alpha_5}\right)^2 \\ & + k_2 \cos\left[\alpha_{24} + t\left(\alpha_{21}^3 - 3\alpha_{21}\gamma\right) + \alpha_{21}x\right]. \end{aligned} \tag{26}$$

Combining Equations (22) and (26), we can obtain the following interaction solution:

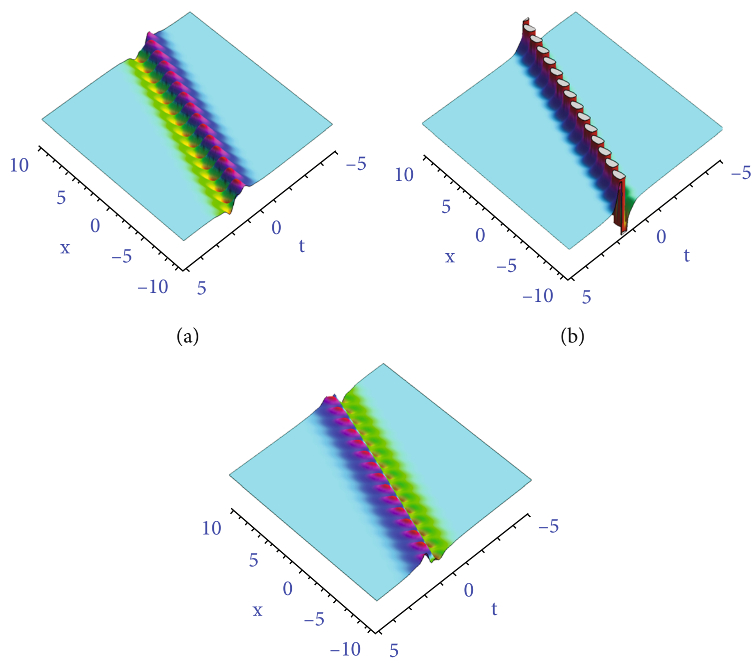
$$u = 2(\ln f)_{xy}, \quad v = \gamma + 2(\ln f)_{xx}. \tag{27}$$

Dynamic behavior of Equation (27) is shown in Figure 5. Lump and periodic waves can be seen in Figure 5. With the



(a) (b)

Figure 4: $\alpha_1=\beta_3=\sigma_1=1$, $k_3=-2$, $\delta_1=-5$, $k_1=\sigma_3=5$, and $t=0$. (a) 3D plot and (b) density plot.



(c)

Figure 5: $\alpha_1=\alpha_4=\alpha_{11}=\alpha_9=k_1=1$, $k_2=\alpha_2=\alpha_{21}=3$, $\gamma=\alpha_8=\alpha_{14}=2$, $\alpha_{24}=4$, $\alpha_5=-2$. (a) $y=-3$, (b) $y=0$, and (c) $y=2$.

change of y value, the amplitude of wave changes correspondingly and reaches the maximum at a certain moment.

In addition, we can also derive another three sets of solutions for the parameters $\alpha_i(i=1,\cdots,9)$, α_{j1}, α_{j2}, α_{j3}, and $\alpha_{j4}(j=1,2)$.

$$\begin{gathered}\alpha_{13}=\alpha_{11}^3-3\alpha_{11}\gamma,\ \alpha_{23}=\alpha_{12}=\alpha_{21}=0,\\ \alpha_6=-\frac{\alpha_1\alpha_2}{\alpha_5},\ \alpha_7=-3\alpha_5\gamma,\ \alpha_3=-3\alpha_1\gamma,\\ \alpha_{13}=\alpha_{11}=\alpha_{22}=0,\ \alpha_{23}=\alpha_{21}^3-3\alpha_{21}\gamma,\\ \alpha_6=-\frac{\alpha_1\alpha_2}{\alpha_5},\ \alpha_7=-3\alpha_5\gamma,\ \alpha_3=-3\alpha_1\gamma,\\ \alpha_{13}=\alpha_{23}=\alpha_{11}=\alpha_{21}=0,\\ \alpha_6=-\frac{\alpha_1\alpha_2}{\alpha_5},\ \alpha_7=-3\alpha_5\gamma,\ \alpha_3=-3\alpha_1\gamma.\end{gathered}\tag{28}$$

Substituting these sets of solutions for the parameters into Equations (22) and (24), the corresponding interaction solutions can be obtained.

4. Lump-Periodic-Solitary Waves

In order to investigate the interaction among lump, periodic, and solitary waves, suppose

$$\begin{aligned}f=&\ \alpha_9+k_1\exp\left(\alpha_{14}+\alpha_{13}t+\alpha_{11}x+\alpha_{12}y\right)\\ &+\left(\alpha_8+\alpha_7t+\alpha_5x+\alpha_6y\right)^2+\left(\alpha_4+\alpha_3t+\alpha_1x+\alpha_2y\right)^2\\ &+k_2\cos\left(\alpha_{24}+\alpha_{23}t+\alpha_{21}x+\alpha_{22}y\right).\end{aligned}\tag{29}$$

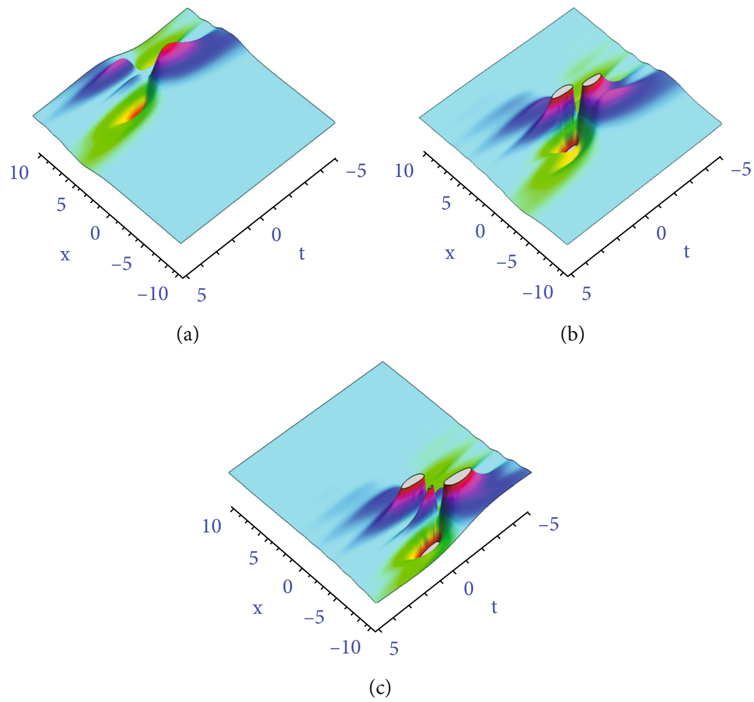


Figure 6: $\alpha_1 = \alpha_4 = \alpha_{11} = \alpha_9 = k_1 = 1$, $k_2 = \alpha_2 = \alpha_{21} = 3$, $\gamma = \alpha_8 = \alpha_{14} = 2$, $\alpha_{24} = 4$, $\alpha_5 = -2$. (a) $t = -1$, (b) $t = 0$, and (c) $t = 1$.

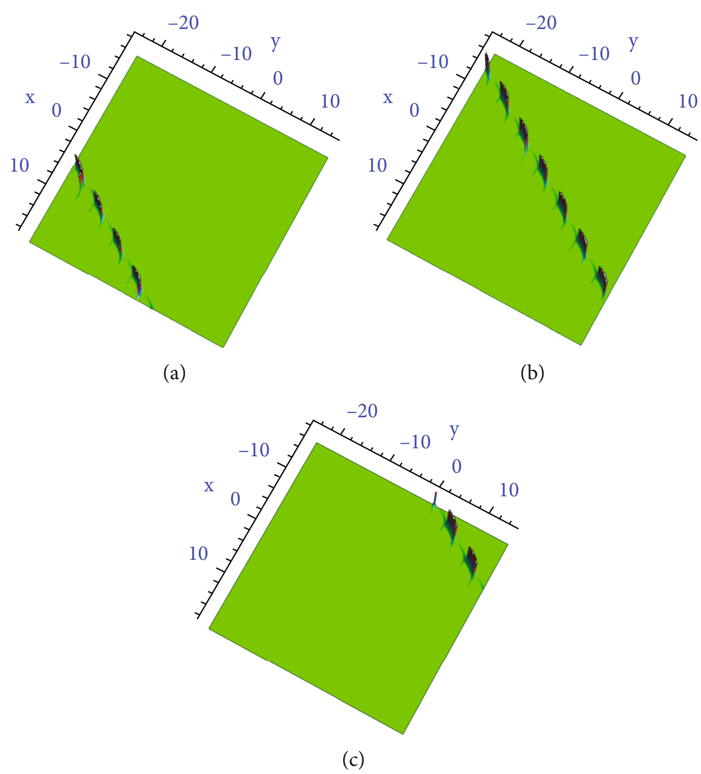


Figure 7: Evolution of the periodic wave for solution (49) at $\alpha_1 = \alpha_3 = \gamma_2 = -1$, $\alpha_2 = \alpha_4 = \gamma_3 = 1$, $\beta_1 = \gamma_1 = 5$, $k_2 = -2$, and $\gamma_4 = 2$. (a) $t = -5$, (b) $t = 0$, and (c) $t = 5$.

Substituting Equation (29) into Equation (23), we have

$$\begin{gathered}\alpha_{13}=-\alpha_{11}^3-3\alpha_{11}\gamma, \alpha_{23}=\alpha_{21}^3-3\alpha_{21}\gamma, \alpha_{12}=\alpha_{22}=0,\\ \alpha_6=-\frac{\alpha_1\alpha_2}{\alpha_5}, \alpha_7=-3\alpha_5\gamma, \alpha_3=-3\alpha_1\gamma.\end{gathered} \tag{30}$$

Equation (29) will become

$$\begin{aligned}f=&\alpha_9+k_1e^{\alpha_{14}+t\left(-3\alpha_{11}\gamma-\alpha_{11}^3\right)+\alpha_{11}x}+(\alpha_4-3\alpha_1\gamma t+\alpha_1x+\alpha_2y)^2\\ &+\left(\alpha_8-3\alpha_5\gamma t+\alpha_5x-\frac{\alpha_1\alpha_2y}{\alpha_5}\right)^2\\ &+k_2\cos\left[\alpha_{24}+t\left(\alpha_{21}^3-3\alpha_{21}\gamma\right)+\alpha_{21}x\right].\end{aligned} \tag{31}$$

Combining Equations (22) and (31), we can obtain the following interaction solution:

$$u=2(\ln f)_{xy},\quad v=\gamma+2(\ln f)_{xx}. \tag{32}$$

Dynamic behavior of Equation (32) is shown in Figure 6. Lump, periodic, and solitary waves can be found in Figure 6.

In addition, we can also derive another three sets of solutions for the parameters $\alpha_i(i=1,\cdots,9)$, α_{j1}, α_{j2}, α_{j3}, and $\alpha_{j4}(j=1,2)$.

$$\begin{gathered}\alpha_{13}=-\alpha_{11}^3-3\alpha_{11}\gamma, \alpha_{23}=\alpha_{12}=\alpha_{21}=0,\\ \alpha_6=-\frac{\alpha_1\alpha_2}{\alpha_5}, \alpha_7=-3\alpha_5\gamma, \alpha_3=-3\alpha_1\gamma,\\ \alpha_{13}=\alpha_{11}=\alpha_{22}=0, \alpha_{23}=\alpha_{21}^3-3\alpha_{21}\gamma,\\ \alpha_6=-\frac{\alpha_1\alpha_2}{\alpha_5}, \alpha_7=-3\alpha_5\gamma, \alpha_3=-3\alpha_1\gamma,\\ \alpha_{13}=\alpha_{23}=\alpha_{11}=\alpha_{21}=0,\\ \alpha_6=-\frac{\alpha_1\alpha_2}{\alpha_5}, \alpha_7=-3\alpha_5\gamma, \alpha_3=-3\alpha_1\gamma.\end{gathered} \tag{33}$$

Substituting these sets of solutions for the parameters into Equations (22) and (29), the corresponding interaction solutions can be obtained.

5. Double Periodic-Soliton Solutions

Supposing the function f in Equation (4) has the following double-periodic soliton structures:

$$f=e^{\theta_1}[\gamma_1\cos(\theta_2)+\gamma_2\sin(\theta_2)]+k_1e^{2\theta_1}+e^{\theta_3}[\gamma_3\cos(\theta_4)+\gamma_4\sin(\theta_4)]+k_2e^{\theta_4}, \tag{34}$$

where $\theta_i=\alpha_i x+\beta_i y+\delta_i t, i=1,2,3,4$ and α_i, β_i, and δ_i are constants to be determined later. Substituting Equation (34) into Equation (4), we can obtain a set of algebraic equations for α_i, β_i, and δ_i yields a set of algebraic equations. Solving these algebraic equations with the aid of symbolic computation, we obtain the following:

Case 5.

$$k_1=\beta_2=\beta_4=0, \beta_3=\beta_1, \delta_2=\alpha_2\left(\alpha_2^2-3(\alpha_1-2\alpha_4)^2\right), \tag{35}$$

$$\delta_1=-\alpha_1^3+3\alpha_2^2\alpha_1-14\alpha_4^3-12(\alpha_1-2\alpha_3)\alpha_4^2+6\left(\alpha_1^2-\alpha_2^2-\alpha_3^2\right)\alpha_4, \tag{36}$$

$$\delta_3=-\alpha_3^3+15\alpha_4^2\alpha_3-20\alpha_4^3, \delta_4=\alpha_4\left(-3\alpha_3^2+12\alpha_4\alpha_3-11\alpha_4^2\right). \tag{37}$$

Case 6.

$$\begin{gathered}k_1=\beta_2=\delta_4=\alpha_4=0, \delta_1=-\alpha_1^3+3\alpha_3\alpha_1^2+3\left(\alpha_2^2-\alpha_3^2\right)\alpha_1-3\alpha_2^2\alpha_3,\\ \gamma_4=-\frac{\beta_4\gamma_3}{2\beta_4-\beta_3}, \gamma_2=\frac{\left(\alpha_2^2+\alpha_1(\alpha_3-\alpha_1)\right)\gamma_1}{\alpha_2(2\alpha_1-\alpha_3)}, \delta_3=-\alpha_3^3,\\ \beta_1=2\beta_4, \delta_2=\alpha_2\left(\alpha_2^2-3(\alpha_1-\alpha_3)^2\right).\end{gathered} \tag{38}$$

Case 7.

$$\begin{gathered}k_1=\alpha_3=\alpha_4=\beta_2=\delta_3=\delta_4=0, \gamma_4=-\frac{\beta_4\gamma_3}{\beta_1-\beta_3}, \gamma_2=\frac{2\alpha_1\alpha_2\gamma_1}{\alpha_1^2-\alpha_2^2},\\ \delta_1=3\alpha_1\alpha_2^2-\alpha_1^3, \delta_2=\alpha_2^3-3\alpha_1^2\alpha_2.\end{gathered} \tag{39}$$

Case 8.

$$\begin{gathered}k_1=\alpha_2=\beta_4=\delta_2=0, \delta_1=-2\alpha_4\left(3\alpha_3^2-12\alpha_4\alpha_3+11\alpha_4^2\right),\\ \delta_3=-\alpha_3^3+15\alpha_4^2\alpha_3-20\alpha_4^3, \delta_4=-\alpha_4\left(3\alpha_3^2-12\alpha_4\alpha_3+11\alpha_4^2\right),\\ \gamma_4=-\frac{(\alpha_3-3\alpha_4)(\alpha_3-\alpha_4)\gamma_3}{2(\alpha_3-2\alpha_4)\alpha_4}, \alpha_1=2\alpha_4.\end{gathered} \tag{40}$$

Case 9.

$$\begin{aligned}k_1=\alpha_2=\alpha_4=\delta_2=\delta_4=0, \alpha_1=\alpha_3, \gamma_4\\ =-\frac{\beta_4\gamma_1\gamma_3}{(\beta_1-\beta_3)\gamma_1+\beta_2\gamma_2}, \delta_1=\delta_3=-\alpha_3^3.\end{aligned} \tag{41}$$

Case 10.

$$\begin{gathered}k_2=\alpha_2=\beta_4=\beta_2=\delta_2=0, \beta_3=2\beta_1, \delta_1=-\alpha_1^3, \gamma_3=\frac{\left(-\alpha_3^2+2\alpha_1\alpha_3+\alpha_4^2\right)\gamma_4}{2(\alpha_1-\alpha_3)\alpha_4},\\ \delta_3=3(\alpha_3-\alpha_1)\alpha_4^2-\alpha_3\left(3\alpha_1^2-3\alpha_3\alpha_1+\alpha_3^2\right), \delta_4=\alpha_4^3-3(\alpha_1-\alpha_3)^2\alpha_4.\end{gathered} \tag{42}$$

Case 11.

$$\begin{gathered}k_2=\alpha_4=\beta_2=\delta_4=0, \gamma_4=\frac{(\beta_1-\beta_3)\gamma_3}{\beta_4}, \alpha_3=2\alpha_1, \delta_1=3\alpha_1\alpha_2^2-\alpha_1^3,\\ \delta_2=\alpha_2^3-3\alpha_1^2\alpha_2, \delta_3=6\alpha_1\alpha_2^2-2\alpha_1^3.\end{gathered} \tag{43}$$

Case 12.

$$k_2=\alpha_2=\delta_2=\alpha_4=\delta_4=0,\ \alpha_3=2\alpha_1,\ \delta_3=-2\alpha_1^3,\ \delta_1=-\alpha_1^3,\\ \gamma_4=-\frac{(\beta_2\gamma_1+(\beta_3-\beta_1)\gamma_2)\gamma_3}{\beta_4\gamma_2}. \tag{44}$$

Case 13.

$$\alpha_1=\alpha_2=\delta_1=\delta_2=\alpha_4=\delta_4=0,\ \beta_4=\beta_1,\\ \delta_3=-\alpha_3^3,\ \gamma_4=-\frac{(\beta_2\gamma_1+(\beta_3-\beta_1)\gamma_2)\gamma_3}{\beta_1\gamma_2}. \tag{45}$$

Case 14.

$$\alpha_4=\beta_2=\delta_4=0,\ \beta_4=\beta_1,\ \alpha_3=2i\epsilon\alpha_2,\ \alpha_1=i\epsilon\alpha_2,\ \delta_3=8i\epsilon\alpha_2^3, \tag{46}$$

$$\delta_2=4\alpha_2^3,\ \delta_1=4i\epsilon\alpha_2^3,\ \gamma_4=\left(1-\frac{\beta_3}{\beta_1}\right)\gamma_3. \tag{47}$$

Case 15.

$$\delta_1=\alpha_1=\alpha_4=\delta_4=\beta_2=0,\ \delta_3=-\alpha_3^3,\ \alpha_2=i\epsilon\alpha_3,\ \gamma_2=-i\epsilon\gamma_1,\\ \gamma_3=-\frac{(\beta_1-\beta_3)\gamma_4}{\beta_4},\ \delta_2=-i\epsilon\alpha_3^3, \tag{48}$$

where $\varepsilon=\pm1$. Substituting Equations (35)–(47) into Equations (3) and (34), respectively, we can obtain abundant double-periodic soliton solutions of Equation (1). As an example, substituting Equation (47) into Equation (34), we have

$$\begin{aligned} f={}&e^{2\left[x\alpha_4+t\left(-3\alpha_3^2+12\alpha_4\alpha_3-11\alpha_4^2\right)\alpha_4\right]}k_2\\ &+\left[\cos\left[x\alpha_2+t\left[\alpha_2^2-3(\alpha_1-2\alpha_4)^2\right]\alpha_2\right]\gamma_1+\sin\left[x\alpha_2+t\left[\alpha_2^2-3(\alpha_1-2\alpha_4)^2\right]\alpha_2\right]\gamma_2\right]\\ &\quad Exp\left[x\alpha_1+t\left[-\alpha_1^3+3\alpha_2^2\alpha_1-14\alpha_4^3-12(\alpha_1-2\alpha_3)\alpha_4^2+6\left(\alpha_1^2-\alpha_2^2-\alpha_3^2\right)\alpha_4\right]+y\beta_1\right]\\ &+e^{x\alpha_3+t\left(-\alpha_3^3+15\alpha_4^2\alpha_3-20\alpha_4^3\right)+y\beta_1}\left[\cos\left[x\alpha_4+t\left(-3\alpha_3^2+12\alpha_4\alpha_3-11\alpha_4^2\right)\alpha_4\right]\gamma_3+\sin\left[x\alpha_4+t\left(-3\alpha_3^2+12\alpha_4\alpha_3-11\alpha_4^2\right)\alpha_4\right]\gamma_4\right]. \end{aligned} \tag{49}$$

Therefore, the corresponding double-periodic soliton solutions can be presented as follows:

$$\begin{aligned} u_1={}&\left[2\left[e^{x\alpha_1+y\beta_1+t\delta_1}\alpha_1\beta_1(\cos(x\alpha_2+t\delta_2)\gamma_1+\sin(x\alpha_2+t\delta_2)\gamma_2)+e^{x\alpha_1+y\beta_1+t\delta_1}\beta_1[\cos(x\alpha_2+t\delta_2)\alpha_2\gamma_2-\sin(x\alpha_2+t\delta_2)\alpha_2\gamma_1]\right.\right.\\ &\left.\left.+e^{x\alpha_3+y\beta_1+t\delta_3}\alpha_3\beta_1[\cos(x\alpha_4+t\delta_4)\gamma_3+\sin(x\alpha_4+t\delta_4)\gamma_4]+e^{x\alpha_3+y\beta_1+t\delta_3}\beta_1[\cos(x\alpha_4+t\delta_4)\alpha_4\gamma_4-\sin(x\alpha_4+t\delta_4)\alpha_4\gamma_3]\right]\right]\\ &/\left[e^{2(x\alpha_4+t\delta_4)}k_2+e^{x\alpha_1+y\beta_1+t\delta_1}[\cos(x\alpha_2+t\delta_2(\gamma_1+\sin(x\alpha_2+t\delta_2)\gamma_2]+e^{x\alpha_3+y\beta_1+t\delta_3}[\cos(x\alpha_4+t\delta_4)\gamma_3+\sin(x\alpha_4+t\delta_4)\gamma_4]\right]\\ &-\left[2\left[e^{x\alpha_1+y\beta_1+t\delta_1}\beta_1[\cos(x\alpha_2+t\delta_2)\gamma_1+\sin(x\alpha_2+t\delta_2)\gamma_2]+e^{x\alpha_3+y\beta_1+t\delta_3}\beta_1[\cos(x\alpha_4+t\delta_4)\gamma_3+\sin(x\alpha_4+t\delta_4)\gamma_4]\right]\right.\\ &\quad\left[2e^{2(x\alpha_4+t\delta_4)}k_2\alpha_4+e^{x\alpha_1+y\beta_1+t\delta_1}\alpha_1[\cos(x\alpha_2+t\delta_2)\gamma_1+\sin(x\alpha_2+t\delta_2)\gamma_2]+e^{x\alpha_1+y\beta_1+t\delta_1}[\cos(x\alpha_2+t\delta_2)\alpha_2\gamma_2-\sin(x\alpha_2+t\delta_2)\alpha_2\gamma_1]\right.\\ &\quad\quad\left.\left.+e^{x\alpha_3+y\beta_1+t\delta_3}\alpha_3[\cos(x\alpha_4+t\delta_4)\gamma_3+\sin(x\alpha_4+t\delta_4)\gamma_4]+e^{x\alpha_3+y\beta_1+t\delta_3}[\cos(x\alpha_4+t\delta_4)\alpha_4\gamma_4-\sin(x\alpha_4+t\delta_4)\alpha_4\gamma_3]\right]\right]\\ &/\left[\left[e^{2(x\alpha_4+t\delta_4)}k_2+e^{x\alpha_1+y\beta_1+t\delta_1}[\cos(x\alpha_2+t\delta_2)\gamma_1+\sin(x\alpha_2+t\delta_2)\gamma_2]+e^{x\alpha_3+y\beta_1+t\delta_3}[\cos(x\alpha_4+t\delta_4)\gamma_3+\sin(x\alpha_4+t\delta_4)\gamma_4]\right]^2\right]. \end{aligned} \tag{50}$$

Dynamic behavior of expression (49) is shown in Figure 7.

6. Conclusion

In this paper, we study a (2+1)-dimensional KdV equation. Abundant periodic wave solutions are obtained based on the Hirota's bilinear form and a direct test function. Corresponding dynamic behavior is shown in Figures 1–4. Meanwhile, the interaction solutions between lump and periodic waves are obtained. Corresponding dynamic behavior is seen in Figure 5. From Figure 5, we can observe the interaction between lump and periodic waves. With the change of y value, the amplitude of wave changes correspondingly and reaches the maximum at a certain moment. We present the interaction solutions among lump, periodic, and solitary waves. Corresponding dynamic behavior is seen in Figure 6. From Figure 6, we can observe the lump wave, periodic wave, and solitary wave at the same time. Finally, with the aid of the extended homoclinic test technique and an ansätz functions, double periodic-soliton solutions of the (2+1)-dimensional Korteweg-de Vries equation are obtained. Corresponding dynamic behavior is shown in Figure 7.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Ethical Approval

The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this article.

Acknowledgments

This project was supported by the National Natural Science Foundation of China (Grant No. 12161048), Doctoral Research Foundation of Jiangxi University of Chinese Medicine (Grant No 2021WBZR007), and Development Plan of University Level Scientific and Technological Innovation Team of Jiangxi University of Chinese Medicine.

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