Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 3828446, 6 pages
https://doi.org/10.1155/2022/3828446

Research Article

Asymmetric Bidirectional Controlled Quantum Teleportation of Three- and Four-Qubit States

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Received 4 July 2022; Revised 25 July 2022; Accepted 28 July 2022; Published 31 August 2022

Academic Editor: Meraj Ali Khan

In this paper, we theoretically realize bidirectional controlled quantum teleportation by using ten-qubit entangled state method. This paper uses a case to introduce the specific process of realizing quantum teleportation: Alice sends an unknown four-qubit GHZ state to Bob, and Bob sends an arbitrary three-qubit GHZ state to Alice. In addition, Charlie controls the transfer to ensure the integrity of the protocol. A ten-qubit quantum channel is constructed and used in this paper. Then, the unitary matrix transformation is used to complete the communication protocol. The research results show that the communication protocol constructed in this paper is more efficient than most communication protocols.

1. Introduction

Quantum information has become increasingly popular in recent years. Quantum communication is a new communication method which uses quantum superposition state and quantum entanglement effect to transmit information. Quantum communication is based on three principles, along with uncertainty, measurement collapse, and no-cloning theorem in quantum mechanics. Quantum communication is an absolutely secure means of communication that cannot be eavesdropped or cracked. Quantum communication is mainly divided into quantum teleportation and quantum key distribution. This paper studies the communication mode of quantum teleportation.

In this paper, BQCT by using ten-qubit entangled state is devised. Alice has unknown qubit state A, B, C, D, a, b, c, d; Bob has unknown qubit state E, F, G, e, f, g, h, i; and Charlie has unknown qubit state e. Alice sends arbitrary four-qubit GHZ state to Bob, Bob transmits unknown three-qubit GHZ state to Alice, and ten-qubit entangled state is used as quantum channel. Alice performs a five-qubit GHZ-state measurement on qubits A, B, C, D, a; and Bob operates a four-qubit GHZ-state measurement on qubits E, F, G, f. Both Alice and Bob tells Charlie to the basis of measurement, and Charlie controls the process of the protocol. If Charlie believes the protocol is safety, Charlie measures the remaining quantum state using single-qubit basis and tells Alice and Bob about information of the used basis. Alice and Bob can obtain the initial state by appropriate unitary operations. In contrast, this protocol efficiency is relatively high.

2. Literature Review

In 1935, Einstein et al. proposed a paradox to prove the incompleteness of quantum mechanics, which is referred to as “EPR paradox” [1]. In 1964, Bell presented Bell inequality to support localized realism and can prove the completeness of quantum mechanics in mathematics [2].

In the field of quantum information, quantum teleportation is very important. In 1993, quantum teleportation was first proposed [3]. In 2013, Zha et al. present the first bidirectional quantum controlled teleportation (BQCT) protocol [4]. In 2016, the scheme which has three controllers was proposed for BCQT via seven-qubit entangled state to convey one-qubit each other [5]. In 2017, Zadeh et al. presented bidirectional quantum teleportation (BQT) without controller to teleport an arbitrary two-qubit state to each other simultaneously via an eight-qubit entangled state [6]. In 2018, Sarvaghad-Moghaddam et al. used five-qubit entangled states as a quantum channel to teleport one-qubit each other under permission of controller [7]. In 2019, Zhou et al. used six-qubit cluster state to send single-qubit and three-qubit GHZ state to each other [8]. In 2020, Zhou et al. proposed BQCT of two-qubit states through seven-qubit entangled state [9]. Protocol which transmits two-qubit each other and two-qubit and three-qubit each other about six-qubit quantum channel was reported as well [10]. In 2021, Jiang et al. presented BQCT of three-qubit GHZ state through an entangled eleven-qubit quantum channel [11] and Huo et al. presented asymmetric BCQT of two- and three-qubit states via an entangled eleven-qubit quantum channel [12]. In 2022, Kazemikhah et al. present asymmetric bidirectional controlled quantum teleportation protocol of two-qubit and three-qubit unknown states using eight-qubit cluster state [13].

3. Construction of Quantum Channel

Quantum communication is a new communication method which uses quantum superposition state and quantum entanglement effect to transmit information. Quantum communication is an absolutely secure means of communication that cannot be eavesdropped or cracked. Therefore, in this paper, the quantum channel adopted is

$$|\Psi\rangle_{abccdef jhij} = \frac{1}{2}\Big(|0000000000\rangle_{abcdef jhij} + |0000011111\rangle_{abcdef jhij} \cdot + |1111100000\rangle_{abcdef jhij} + |1111111111\rangle_{abcdef jhij}\Big). \tag{1}$$

This quantum channel can not only be theoretically proposed but also constructed. The step method is as follows.

Step 1. The ten-qubit initial state is prepared like

$$|\Psi_0\rangle_{abccdef jhij} = |0\rangle_a \otimes |0\rangle_b \otimes |0\rangle_c \otimes |0\rangle_d \otimes |0\rangle_e \otimes |0\rangle_f \otimes |0\rangle_g \otimes |0\rangle_h \otimes |0\rangle_i \otimes |0\rangle_j. \tag{2}$$

Step 2. Two Hadamard gates are implemented to qubits a and f. Then, the state $|\psi\rangle_{abcdef ghij}$ changes into

$$|\Psi_1\rangle_{abccdef jhij} = \frac{\left(|0\rangle_a + |1\rangle_a\right)}{\sqrt{2}} \otimes |0\rangle_b \otimes |0\rangle_c \otimes |0\rangle_d \otimes |0\rangle_e \otimes \frac{\left(|0\rangle_f + |1\rangle_f\right)}{\sqrt{2}} \otimes |0\rangle_g \otimes |0\rangle_h \otimes |0\rangle_i \otimes |0\rangle_j. \tag{3}$$

Step 3. When qubit a can be control qubits and qubits b, c, d, e are target qubits, CNOT gates operate on $|\Psi_1\rangle_{abccdef jhij}$. In the same way, CNOT gates operate on $|\Psi_1\rangle_{abccdef jhij}$ when qubits f can be control qubits and qubits g, h, i, j are target qubits. We can obtain the quantum channel $|\Psi_1\rangle_{abccdef jhij}$.

4. Bidirectional Quantum Controlled Teleportation

4.1. Quantum Teleportation. Suppose Alice has an arbitrary four-qubit GHZ state

$$|\Psi\rangle_{ABCD} = \alpha|0000\rangle_{ABCD} + \beta|1111\rangle_{ABCD}. \tag{4}$$

And Bob has an arbitrary three-qubit GHZ state

$$|\Psi\rangle_{EFG} = \upsilon|000\rangle_{EFG} + \mu|111\rangle_{EFG}, \tag{5}$$

where $|\alpha|^2 + |\beta|^2 = 1, |\nu|^2 + |\mu|^2 = 1$. Alice and Bob do not know what α, β, ν, and μ are. Alice wants to transmit A, B, C, D to Bob who wants to transmit E, F, G to Alice through ten-qubit quantum channel. Supervisor Charlie who has qubit e controls whether or not the protocol continues. We have ten-qubit state quantum channel

$$|\Psi\rangle_{abccdef jhij} = \frac{1}{2}\Big[|0000000000\rangle_{abcdef jhij} + |0000011111\rangle_{abcdef jhij} + |1111100000\rangle_{abcdef jhij} + |1111111111\rangle_{abcdef jhij}\Big]. \tag{6}$$

Here, qubits a, b, c, d belong to Alice, qubits f, g, h, i belong to Bob, and qubit j belongs to Charlie, respectively. The initial state of the total system is

$$|\Psi\rangle_{ABCDEFGabcdefghij} = |\Psi\rangle_{ABCD} \otimes |\Psi\rangle_{EFG} \otimes |\Psi\rangle_{abcdefghij}. \quad (7)$$

Four-qubit GHZ states which form a set of basis can be described as

$$\begin{aligned}
\left|\xi_1^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|0000\rangle \pm |1111\rangle), \left|\xi_2^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|0001\rangle \pm |1110\rangle),\\
\left|\xi_3^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|0011\rangle \pm |1100\rangle), \left|\xi_4^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|0111\rangle \pm |1000\rangle),\\
\left|\xi_5^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|0101\rangle \pm |1010\rangle), \left|\xi_6^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|0110\rangle \pm |1001\rangle),\\
\left|\xi_7^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|0100\rangle \pm |0100\rangle), \left|\xi_8^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|0010\rangle \pm |1111\rangle).
\end{aligned} \quad (8)$$

Five-qubit GHZ states which form a set of basis can be described as

$$\begin{aligned}
\left|\gamma_1^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|00000\rangle \pm |11111\rangle), \left|\gamma_2^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|00001\rangle \pm |11110\rangle),\\
\left|\gamma_3^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|00010\rangle \pm |11101\rangle), \left|\gamma_4^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|00100\rangle \pm |11011\rangle),\\
\left|\gamma_5^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|01000\rangle \pm |10111\rangle), \left|\gamma_6^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|00011\rangle \pm |11100\rangle),\\
\left|\gamma_7^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|00110\rangle \pm |11001\rangle), \left|\gamma_8^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|01100\rangle \pm |10011\rangle),\\
\left|\gamma_9^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|00111\rangle \pm |11000\rangle), \left|\gamma_{10}^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|01110\rangle \pm |10001\rangle),\\
\left|\gamma_{11}^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|01101\rangle \pm |10010\rangle), \left|\gamma_{12}^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|01011\rangle \pm |10100\rangle),\\
\left|\gamma_{13}^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|01110\rangle \pm |10001\rangle), \left|\gamma_{14}^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|01111\rangle \pm |10000\rangle),\\
\left|\gamma_{15}^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}(|01010\rangle \pm |10101\rangle), \left|\gamma_{16}^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|00101\rangle \pm |11010\rangle).
\end{aligned} \quad (9)$$

Alice can carry out a five-qubit GHZ-state measurement on qubits A, B, C, D, a, and Bob can carry out a four-qubit GHZ-state measurement on qubits E, F, G, f. Then, quantum state $|\Psi\rangle_{ABCDEFGabcdefghij}$ can be expressed as

$$\begin{aligned}
|\Psi\rangle_{ABCDEFGabcdefghij} = \alpha\nu \Big[&(|\gamma_1^+\rangle + |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_1^+\right\rangle + \left|\xi_1^-\right\rangle\right)_{EFGf}\left|00000000\right\rangle_{bcdeghij} + (|\gamma_2^+\rangle + |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_1^+\right\rangle + \left|\xi_1^-\right\rangle\right)_{EFGf}\left|11100000\right\rangle_{bcdeghij}\\
&+ (|\gamma_1^+\rangle \quad + |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle + \left|\xi_2^-\right\rangle\right)_{EFGf}\left|00001111\right\rangle_{bcdeghij} + (|\gamma_2^+\rangle + |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle + \left|\xi_2^-\right\rangle\right)_{EFGf}\left|11111111\right\rangle_{bcdeghij}\Big]\\
&+ \beta\nu\Big[(|\gamma_2^+\rangle - |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_1^+\right\rangle + \left|\xi_1^-\right\rangle\right)_{EFGf}\left|00000000\right\rangle_{bcdeghij} + (|\gamma_1^+\rangle - |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_1^+\right\rangle + \left|\xi_1^-\right\rangle\right)_{EFGf}\left|11100000\right\rangle_{bcdeghij}\\
&+ (|\gamma_2^+\rangle - |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle + \left|\xi_2^-\right\rangle\right)_{EFGf}\left|00001111\right\rangle_{bcdeghij} + (|\gamma_1^+\rangle - |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle + \left|\xi_2^-\right\rangle\right)_{EFGf}\left|11111111\right\rangle_{bcdeghij}\Big]\\
&+ \alpha\mu\Big[(|\gamma_1^+\rangle + |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle - \left|\xi_2^-\right\rangle\right)_{EFGf}\left|00000000\right\rangle_{bcdeghij} + (|\gamma_2^+\rangle - |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle + \left|\xi_2^-\right\rangle\right)_{EFGf}\left|11100000\right\rangle_{bcdeghij}\\
&+ (|\gamma_1^+\rangle + |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_1^+\right\rangle + \left|\xi_1^-\right\rangle\right)_{EFGf}\left|00001111\right\rangle_{bcdeghij} + (|\gamma_2^+\rangle + |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle - \left|\xi_2^-\right\rangle\right)_{EFGf}\left|11111111\right\rangle_{bcdeghij}\Big]\\
&+ \beta\mu\Big[(|\gamma_2^+\rangle - |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle - \left|\xi_2^-\right\rangle\right)_{EFGf}\left|00000000\right\rangle_{bcdeghij} + (|\gamma_1^+\rangle - |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle - \left|\xi_2^-\right\rangle\right)_{EFGf}\left|11100000\right\rangle_{bcdeghij}\\
&+ (|\gamma_2^+\rangle - |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_1^+\right\rangle - \left|\xi_1^-\right\rangle\right)_{EFGf}\left|00001111\right\rangle_{bcdeghij} + (|\gamma_1^+\rangle - |\gamma_1^-\rangle)_{ABCDa}\left(\left|\xi_1^+\right\rangle - \left|\xi_1^-\right\rangle\right)_{EFGf}\left|11111111\right\rangle_{bcdeghij}\Big]\\
&\cdot \beta\mu\Big[(|\gamma_2^+\rangle - |\gamma_2^-\rangle)_{ABCDa}\left(\left|\xi_2^+\right\rangle - \left|\xi_2^-\right\rangle\right)_{EFGf}\left|00000000\right\rangle_{bcdeghij}\Big] + .
\end{aligned} \quad (10)$$

4.2. Quantum Teleportation Results. As mentioned above, both Alice and Bob tell each other the measurement basis by the classical channel and different basis vectors which Alice and Bob choose and the corresponding collapse state is as Table 1. Then, Charlie is told the measurement results by the classical communication channel. And Charlie can perform single-qubit Von Neumann measurement on $|+\rangle$ or $|-\rangle$ and

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \quad (11)$$

Then, if Charlie wants to continue the protocol, he needs to deliver his result to both Alice and Bob. Finally, Alice and Bob use correct unitary operations on their state to obtain the state teleported by the other party. The different collapse states and unitary operations in $|+\rangle$ or $|-\rangle$ are as Tables 2 and 3. In

Table 1: The collapsed states of qubits b, c, d, e, g, h, i, j under Alice's and Bob's GHZ-state measurement.

Alice's results	Bob's results	Collapsed state of qubits b, c, d, e, g, h, i, j
$\|\gamma_1^+\rangle$	$\|\xi_1^+\rangle$	$\alpha\nu\|00000000\rangle + \beta\nu\|11110000\rangle + \alpha\mu\|00001111\rangle + \beta\mu\|11111111\rangle$
$\|\gamma_1^+\rangle$	$\|\xi_1^-\rangle$	$\alpha\nu\|00000000\rangle + \beta\nu\|11110000\rangle - \alpha\mu\|00001111\rangle - \beta\mu\|11111111\rangle$
$\|\gamma_1^+\rangle$	$\|\xi_2^+\rangle$	$\alpha\nu\|00001111\rangle + \beta\nu\|11111111\rangle + \alpha\mu\|00000000\rangle + \beta\mu\|11110000\rangle$
$\|\gamma_1^+\rangle$	$\|\xi_2^-\rangle$	$\alpha\nu\|00001111\rangle + \beta\nu\|11111111\rangle - \alpha\mu\|00000000\rangle - \beta\mu\|00000000\rangle$
$\|\gamma_1^-\rangle$	$\|\xi_1^+\rangle$	$\alpha\nu\|00000000\rangle - \beta\nu\|11110000\rangle + \alpha\mu\|00001111\rangle - \beta\mu\|11111111\rangle$
$\|\gamma_1^-\rangle$	$\|\xi_1^-\rangle$	$\alpha\nu\|00000000\rangle - \beta\nu\|11110000\rangle - \alpha\mu\|00001111\rangle + \beta\mu\|00000000\rangle$
$\|\gamma_1^-\rangle$	$\|\xi_1^-\rangle$	$\alpha\nu\|00001111\rangle - \beta\nu\|11111111\rangle + \alpha\mu\|00000000\rangle - \beta\mu\|11110000\rangle$
$\|\gamma_1^-\rangle$	$\|\xi_1^-\rangle$	$\alpha\nu\|00001111\rangle - \beta\nu\|11111111\rangle - \alpha\mu\|00000000\rangle + \beta\mu\|11110000\rangle$
$\|\gamma_2^+\rangle$	$\|\xi_1^+\rangle$	$\alpha\nu\|11110000\rangle + \beta\nu\|00000000\rangle + \alpha\mu\|11111111\rangle + \beta\mu\|00001111\rangle$
$\|\gamma_2^+\rangle$	$\|\xi_1^-\rangle$	$\alpha\nu\|11110000\rangle + \beta\nu\|00000000\rangle - \alpha\mu\|11111111\rangle - \beta\mu\|00001111\rangle$
$\|\gamma_2^+\rangle$	$\|\xi_2^+\rangle$	$\alpha\nu\|11111111\rangle + \beta\nu\|00001111\rangle + \alpha\mu\|11110000\rangle + \beta\mu\|00000000\rangle$
$\|\gamma_2^+\rangle$	$\|\xi_2^-\rangle$	$\alpha\nu\|11111111\rangle + \beta\nu\|00001111\rangle - \alpha\mu\|11110000\rangle - \beta\mu\|00000000\rangle$
$\|\gamma_2^-\rangle$	$\|\xi_1^+\rangle$	$\alpha\nu\|11110000\rangle - \beta\nu\|00000000\rangle + \alpha\mu\|11111111\rangle - \beta\mu\|00001111\rangle$
$\|\gamma_2^-\rangle$	$\|\xi_1^-\rangle$	$\alpha\nu\|11110000\rangle - \beta\nu\|11110000\rangle - \alpha\mu\|00001111\rangle + \beta\mu\|00000000\rangle$
$\|\gamma_2^-\rangle$	$\|\xi_2^-\rangle$	$\alpha\nu\|11111111\rangle - \beta\nu\|00001111\rangle - \alpha\mu\|11110000\rangle - \beta\mu\|00000000\rangle$
$\|\gamma_2^-\rangle$	$\|\xi_2^-\rangle$	$\alpha\nu\|11111111\rangle - \beta\nu\|00001111\rangle - \alpha\mu\|11110000\rangle + \beta\mu\|00000000\rangle$

Table 2: The specific unitary transformation and collapsed states correspond to Alice's, Bob's, and Charlie's measurement results.

Alice's results	Bob's results	Charlie's results	Collapsed state of qubits b, c, d, g, h, i, j	Alice's unitary operator	Bob's unitary operator
$\|\gamma_1^+\rangle$	$\|\xi_1^+\rangle$	$\|+\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (\nu\|0000\rangle + \mu\|1111\rangle)$	$I \otimes I \otimes I$	$I \otimes I \otimes I \otimes I$
$\|\gamma_1^+\rangle$	$\|\xi_1^+\rangle$	$\|-\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (\nu\|0000\rangle + \mu\|1111\rangle)$	$Z \otimes I \otimes I$	$I \otimes I \otimes I \otimes I$
$\|\gamma_1^+\rangle$	$\|\xi_1^-\rangle$	$\|+\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (\nu\|0000\rangle - \mu\|1111\rangle)$	$I \otimes I \otimes I$	$Z \otimes I \otimes I \otimes I$
$\|\gamma_1^+\rangle$	$\|\xi_1^-\rangle$	$\|-\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (\nu\|0000\rangle - \mu\|1111\rangle)$	$Z \otimes I \otimes I$	$Z \otimes I \otimes I \otimes I$
$\|\gamma_1^+\rangle$	$\|\xi_2^+\rangle$	$\|+\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (\nu\|0000\rangle + \mu\|1111\rangle)$	$I \otimes I \otimes I$	$I \otimes I \otimes I \otimes I$
$\|\gamma_1^+\rangle$	$\|\xi_2^+\rangle$	$\|-\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (\mu\|0000\rangle + \nu\|1111\rangle)$	$Z \otimes I \otimes I$	$X \otimes X \otimes X \otimes X$
$\|\gamma_1^+\rangle$	$\|\xi_2^-\rangle$	$\|+\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (\mu\|0000\rangle + \nu\|1111\rangle)$	$I \otimes I \otimes I$	$X \otimes X \otimes X \otimes X$
$\|\gamma_1^+\rangle$	$\|\xi_2^-\rangle$	$\|-\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (-\mu\|0000\rangle + \nu\|1111\rangle)$	$Z \otimes I \otimes I$	$iY \otimes I \otimes I \otimes I$
$\|\gamma_1^-\rangle$	$\|\xi_1^+\rangle$	$\|+\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (\nu\|0000\rangle + \mu\|1111\rangle)$	$Z \otimes I \otimes I$	$I \otimes I \otimes I \otimes I$
$\|\gamma_1^-\rangle$	$\|\xi_1^+\rangle$	$\|-\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (\nu\|0000\rangle + \mu\|1111\rangle)$	$I \otimes I \otimes I$	$I \otimes I \otimes I \otimes I$
$\|\gamma_1^-\rangle$	$\|\xi_1^-\rangle$	$\|+\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (\nu\|0000\rangle - \mu\|1111\rangle)$	$Z \otimes I \otimes I$	$Z \otimes I \otimes I \otimes I$
$\|\gamma_1^-\rangle$	$\|\xi_1^-\rangle$	$\|-\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (\nu\|0000\rangle - \mu\|1111\rangle)$	$I \otimes I \otimes I$	$Z \otimes I \otimes I \otimes I$
$\|\gamma_1^-\rangle$	$\|\xi_2^+\rangle$	$\|+\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (\mu\|0000\rangle + \nu\|1111\rangle)$	$I \otimes I \otimes I$	$X \otimes X \otimes X \otimes X$
$\|\gamma_1^-\rangle$	$\|\xi_2^+\rangle$	$\|-\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (\mu\|0000\rangle + \nu\|1111\rangle)$	$I \otimes I \otimes I$	$X \otimes X \otimes X \otimes X$
$\|\gamma_1^-\rangle$	$\|\xi_2^-\rangle$	$\|+\rangle$	$(\alpha\|000\rangle - \beta\|111\rangle) \otimes (-\mu\|0000\rangle + \nu\|1111\rangle)$	$Z \otimes I \otimes I$	$iY \otimes X \otimes X \otimes X$
$\|\gamma_1^-\rangle$	$\|\xi_2^-\rangle$	$\|-\rangle$	$(\alpha\|000\rangle + \beta\|111\rangle) \otimes (-\mu\|0000\rangle + \nu\|1111\rangle)$	$I \otimes I \otimes I$	$iY \otimes I \otimes I \otimes I$

Tables 2 and 3, i is an imaginary unit, X, Y, and Z are Pauli matrices, and I is the identity matrix. These matrices have the form

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (12)$$

Table 3: Following the above table.

Alice's results	Bob's results	Charlie's results	Collapsed state of qubits b, c, d, g, h, i, j	Alice's unitary operator	Bob's unitary operator
$\vert\gamma_2^+\rangle$	$\vert\xi_1^+\rangle$	$\vert+\rangle$	$(\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (\nu\vert 0000\rangle + \mu\vert 1111\rangle)$	$X \otimes X \otimes X$	$I \otimes I \otimes I \otimes I$
$\vert\gamma_2^+\rangle$	$\vert\xi_1^+\rangle$	$\vert-\rangle$	$(\beta\vert 000\rangle - \alpha\vert 111\rangle) \otimes (\nu\vert 0000\rangle + \mu\vert 1111\rangle)$	$-iY \otimes X \otimes X$	$I \otimes I \otimes I \otimes I$
$\vert\gamma_2^+\rangle$	$\vert\xi_1^-\rangle$	$\vert+\rangle$	$(\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (\nu\vert 0000\rangle - \mu\vert 1111\rangle)$	$X \otimes X \otimes X$	$Z \otimes I \otimes I \otimes I$
$\vert\gamma_2^+\rangle$	$\vert\xi_1^-\rangle$	$\vert-\rangle$	$(\beta\vert 000\rangle - \alpha\vert 111\rangle) \otimes (-\nu\vert 0000\rangle + \mu\vert 1111\rangle)$	$iY \otimes X \otimes X$	$-iY \otimes I \otimes I \otimes I$
$\vert\gamma_2^+\rangle$	$\vert\xi_2^+\rangle$	$\vert+\rangle$	$(\alpha\vert 000\rangle + \beta\vert 111\rangle) \otimes (\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$X \otimes X \otimes X$	$X \otimes X \otimes X \otimes X$
$\vert\gamma_2^+\rangle$	$\vert\xi_2^+\rangle$	$\vert-\rangle$	$(-\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$-iY \otimes X \otimes X$	$X \otimes X \otimes X \otimes X$
$\vert\gamma_2^+\rangle$	$\vert\xi_2^-\rangle$	$\vert+\rangle$	$(\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (-\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$X \otimes X \otimes X$	$-iY \otimes X \otimes X \otimes X$
$\vert\gamma_2^+\rangle$	$\vert\xi_2^-\rangle$	$\vert-\rangle$	$(-\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (-\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$-iY \otimes X \otimes X$	$iY \otimes X \otimes X \otimes X$
$\vert\gamma_2^-\rangle$	$\vert\xi_1^+\rangle$	$\vert+\rangle$	$(-\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (\nu\vert 0000\rangle + \mu\vert 1111\rangle)$	$-iY \otimes X \otimes X$	$I \otimes I \otimes I \otimes I$
$\vert\gamma_2^-\rangle$	$\vert\xi_1^+\rangle$	$\vert-\rangle$	$(-\beta\vert 000\rangle - \alpha\vert 111\rangle) \otimes (\nu\vert 0000\rangle + \mu\vert 1111\rangle)$	$-X \otimes X \otimes X$	$I \otimes I \otimes I \otimes I$
$\vert\gamma_2^-\rangle$	$\vert\xi_1^-\rangle$	$\vert+\rangle$	$(-\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (\nu\vert 0000\rangle - \mu\vert 1111\rangle)$	$iY \otimes X \otimes X$	$Z \otimes I \otimes I \otimes I$
$\vert\gamma_2^-\rangle$	$\vert\xi_1^-\rangle$	$\vert-\rangle$	$(\beta\vert 000\rangle - \alpha\vert 111\rangle) \otimes (\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$iY \otimes X \otimes X$	$X \otimes X \otimes X \otimes X$
$\vert\gamma_2^-\rangle$	$\vert\xi_2^+\rangle$	$\vert+\rangle$	$(\alpha\vert 000\rangle - \beta\vert 111\rangle) \otimes (\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$-X \otimes X \otimes X$	$X \otimes X \otimes X \otimes X$
$\vert\gamma_2^-\rangle$	$\vert\xi_2^+\rangle$	$\vert-\rangle$	$(-\beta\vert 000\rangle - \alpha\vert 111\rangle) \otimes (\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$-X \otimes X \otimes X$	$X \otimes X \otimes X \otimes X$
$\vert\gamma_2^-\rangle$	$\vert\xi_2^-\rangle$	$\vert+\rangle$	$(\beta\vert 000\rangle - \alpha\vert 111\rangle) \otimes (-\mu\vert 0000\rangle + \nu\vert 1111\rangle)$	$iY \otimes X \otimes X$	$iY \otimes X \otimes X \otimes X$
$\vert\gamma_2^-\rangle$	$\vert\xi_2^-\rangle$	$\vert-\rangle$	$(\beta\vert 000\rangle + \alpha\vert 111\rangle) \otimes (\mu\vert 0000\rangle - \nu\vert 1111\rangle)$	$X \otimes X \otimes X$	$-iY \otimes X \otimes X \otimes X$

Table 4: Comparing the efficiency of different protocols.

Year and reference	The number of Alice's transmitted qubits	The number of Bob's transmitted qubits	The number of quantum channel	The efficiency of protocol
2019 [14]	3	3	6	54.6%
2020 [10]	2	2	6	40%
2020 [10]	2	3	6	45.5%
2021 [11]	3	3	11	30%
2022 [13]	2	3	8	38.5%
This paper	4	3	10	46.7%

5. Comparison of Efficiency

The protocol efficiency of bidirectional quantum controlled teleportation can be defined as

$$\eta = \frac{c}{q + p}. \tag{13}$$

Here, c represent the total number of qubits to be transmitted by both parties and q is the total number of quantum channel in the protocol. In this paper, the total number of qubits to be transmitted is seven and the total number of quantum channel is ten. The efficiency of this bidirectional quantum controlled teleportation η is equal to 46.7%. The other protocols are as Table 4, and the efficiency of this scheme is relatively high.

6. Conclusion

In conclusion, this paper proves that the implementation of BQCT protocol using quantum channel constructed by entanglement of ten-qubit is more efficient than traditional methods. In addition, quantum communication is an absolutely safe means of communication because it cannot be eavesdropped or cracked. Therefore, the quantum channel constructed in this paper can be used for communication with better security and confidentiality than the existing communication means. However, at present, the research results of this paper only verify its feasibility in theory, and future empirical research is needed to verify its feasibility in practice.

Data Availability

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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