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Research Article

Analytical Solutions to Two-Dimensional Nonlinear Telegraph Equations Using the Conformable Triple Laplace Transform Iterative Method

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The conformable fractional triple Laplace transform approach, in conjunction with the new Iterative method, is used to examine the exact analytical solutions of the $(2+1)$-dimensional nonlinear conformable fractional Telegraph equation. All the fractional derivatives are in a conformable sense. Some basic properties and theorems for conformable triple Laplace transform are presented and proved. The linear part of the considered problem is solved using the conformable fractional triple Laplace transform method, while the noise terms of the nonlinear part of the equation are removed using the novel Iterative method's consecutive iteration procedure, and a single iteration yields the exact solution. As a result, the proposed method has the benefit of giving an exact solution that can be applied analytically to the presented issues. To confirm the performance, correctness, and efficiency of the provided technique, two test modeling problems from mathematical physics, nonlinear conformable fractional Telegraph equations, are used. According to the findings, the proposed method is being used to solve additional forms of nonlinear fractional partial differential equation systems. Moreover, the conformable fractional triple Laplace transform iterative method has a small computational size as compared to other methods.

1. Introduction

The fractional formulation of differential equations is a development of the fractional calculus, which was initially introduced in 1695 when L'Hôpital and Leibniz addressed the expansion of the integer-order derivative to the order 1/2 derivative. Both Euler and Lacroix researched the fractional-order derivative and defined it using the power function's n^{th} derivative formulation [1]. Fractional partial differential equations (FPDEs) have become increasingly important in recent years for modeling a wide range of applications in real-world sciences and engineering, including fluid dynamics, mathematical biology, electrical circuits, optics, and quantum mechanics [2]. As a result, many researchers have focused on solving FPDEs in recent decades [3, 4]. Since many physical and mechanical systems contain internal damping, which makes it impossible to derive equations describing the physical behavior of a non-conservative system using the traditional energy-based approach, fractional derivative formulations can be used to model them more accurately. In non-conservative systems, fractional derivative formulations can be constructed by minimizing specific functionals containing fractional derivative terms using techniques from the calculus of variations [5]. Many definitions of fractional derivatives and integrals have been published in the literature, including Riemann-Liouville fractional definitions [6], Caputo fractional definitions [7], Grünwald-Letnikov fractional derivatives [8], and Hadamard fractional integral [9]. All known fractional derivatives satisfy one of the well-known properties of classical derivatives, namely, the linear property. However, the other properties of classical derivatives, such as the derivatives of a constant are zero, the product rule, quotient rule, and the chain rule either do not hold or are too complicated for many fractional derivatives. For instance, $D_a^{\alpha}(1)=0$ does not fulfill the Riemann-Liouville definition. In Caputo's definition, $f(x)$ is assumed to be differentiable; otherwise, one cannot use such a definition. Moreover, Liouville's theorem in the fractional setting does not hold. Therefore, it is clear that all definitions of fractional derivatives seem deficient regarding certain mathematical properties, such as Rolle's theorem and the mean value theorem [10].

To resolve these issues, Khalil et al. [11] recently introduced a novel fractional derivative called conformable fractional derivative (CFD) in 2014. This definition is formulated as follows:

Definition 1 (see [12–14]). For the initial real value a, the conformable fractional derivative $D_a^\alpha f(x)$ of a real function f : [a,∞) ⟶ ℝ, $\alpha \in (0, 1]$ is defined as

$$D_a^\alpha f(x) = \lim_{h \longrightarrow 0} \frac{f\left(x + h(x-a)^{1-\alpha}\right) - f(x)}{h}, \text{for all}\, x > a, \alpha \in (0, 1]. \tag{1}$$

The initial value a can be zero, and if the limit exists, f(x) is said to be partially α – differentiable at t > 0.

The CFD's Definition 1 is very similar to the classical derivative. It depends upon the basic limit definition and consequently allows the easier extension of some typical theorems in calculus that the existing definitions of fractional derivatives did not allow, due to its simple nature. Along with the CFD's Definition 1, various classical properties, such as the mean value theorem and the product, quotient, and chain rules, are fulfilled. Moreover, this definition is provided by the Leibniz rule, which other fractional derivatives cannot achieve (see [15]). Another study [16] conducted by Abdeljawad presented the left and right conformable fractional derivatives and fractional integrals of higher order concepts. In addition, the authors also defined the fractional chain rule, fractional integration by parts formulae, Gronwall inequality, fractional power series expansion, and fractional Laplace transform. Following this definition, a new approach for finding fractional operators was introduced by Antagan and Baleanu [17] with a nonsingular Mittag-Leffler kernel with a memory effect. Growing attention has been paid to exploring the conformable fractional derivative due to the enormous number of its meaningful applications in many fields of science. Recently, in [18], Rabha et al. introduced different vitalization of the growth of COVID-19 by using controller terms based on the concept of conformable calculus. Ghanbari et al. [19] studied the dynamic behavior of allelopathic stimulator phytoplankton species with Mittag-Leffler (ML) law by using the Atangana-Baleanu fractional derivative (ABC). The interested reader might consult the monograph [20–22] for more information.

The conformable telegraph equations have a wide range of applications in science and engineering, with the most common application being in optimizing propagation-oriented and propagating electrical communication systems [23, 24]. Therefore, as one of the crucial equations in different fields of sciences, many scholars have recently focused their efforts on investigating the solutions of conformable fractional telegraph equations using various methodologies. Using a double conformable Sumudu matching transformation approach, [25] discovered accurate and convergent numerical solutions of linear space-time matching telegraph fractional equations in 2021. Using the cosine family of linear operators, Bouaouid et al. [26] established the existence, uniqueness, and stability of the integral solution of a nonlocal telegraph equation in the conformable time-fractional derivative (see [12, 27–30] for more related work on the solution of conformable telegraph equations).

Because the Laplace transform method (LTM) [13, 14, 31] is an integral transform method for getting the approximate and precise solutions of FDEs, many authors are still working hard to develop and generalize this transform so that it can be used with the newly created fractional derivatives and integrals. For instance, the authors of the paper [32] present a fractional Laplace transform in terms of conformable fractional-order Bessel functions (CFBFs). They also established several important formulas of the fractional Laplace integral operator acting on the CFBFs and give the solutions of a generalized class of fractional kinetic equations associated with the CFBFs in view of the fractional Laplace transform method. Ozan özkan and Ali Kurt in 2018 proposed a new generalization of the double Laplace transform called the conformable double Laplace transform (CDLT), which they used to solve the conformable fractional partial heat equation and the conformable fractional partial Telegraph equation [33]. This method was later used by many authors to handle a variety of real-world challenges resulting from various occurrences such as conformable fractional partial differential equations, Singular conformable pseudoparabolic equations, and system of conformable fractional differential equations [34–36].

Several researchers have recently extended the conformable double Laplace transform method to the conformable triple Laplace transform method (CTLTM) to obtain the exact/approximate solution of two-dimensional nonlinear CFDEs that occur in a variety of natural events. The conformable triple Laplace transform reduces a linear differential equation to an algebraic equation, which can then be solved by the formal rules of algebra. The original differential equation can then be solved by applying the inverse triple conformable Laplace transform. In comparison to other known approaches, the conformable triple Laplace transform method provides rapid convergence of the exact solution without any restrictive assumptions about the answer. Unfortunately, some complex nonlinear partial differential equations that arise in mathematical physics, engineering, and other relevant branches of research that involve nonlinear phenomena are not solved by this technique. In such cases, the conformable triple Laplace transform method is frequently used with other efficient approaches, such as the Adomian decomposition method and homotopy analysis method to tackle a similar problem. For example, the conformable triple Laplace transform decomposition was employed by the authors in [37] to discover the solution of linear and nonlinear homogeneous and nonhomogeneous partial fractional differential equations. This method's important result and theorems are also discussed. In 2022, [38] gives some key discoveries on conformable fractional partial derivatives and shows how to solve nonlinear partial differential equations in two dimensions using the conformable triple Laplace and Sumudu transform method in conjunction with the Adomian decomposition approach. In paper [39], the authors present the solution of the incompressible second-grade fluid models by using the generalized ρ-Laplace transform method in conjunction with the homotopy analysis method in the sense of the Liouville-Caputo fractional derivative.

The main objective of this paper is to introduce the new method called the conformable triple Laplace transform iterative method (CTLTIM) to investigate an accurate solution to the two-dimensional nonlinear conformable telegraph equation under the given initial and boundary conditions. This method is the combination of the two powerful techniques, the conformable triple Laplace transform method (CTLTM) and the new iterative method (NIM) introduced by Daftardar-Gejji and Jafari [40]. In practical scientific areas, solving integer and fractional-order nonlinear differential equations with linear and nonlinear ordinary and partial differential equations utilizing the NIM is a fascinating problem [41]. The iterative strategy employed in this method produces a series that can be summed to obtain an analytical formula or utilized to construct an appropriate approximation with a faster convergent series solution [42, 43]. The approximation error can be reduced by properly truncating the series [44]. Recently, the NIM is combined with other known methods like the Sumudu transform method and Laplace transform method to obtain the approximate or exact solution of the nonlinear partial differential equation. The authors of the paper [45] successfully implemented the combined double Sumudu transform with the iterative method to get the approximate analytical solution of the one-dimensional coupled nonlinear sine-Gordon equation (NLSGE) subject to the appropriate initial and boundary conditions which cannot be solved by applying double Sumudu transform only. Through this approach, the solution of the linear part was solved by the double Sumudu transform method, and the nonlinear part of the problem was solved by a successive iterative method. Deresse et al. [46] present the triple Laplace transform coupled with an iterative method to obtain the exact solution of the two-dimensional nonlinear sine-Gordon equation (NLSGE) subject to the appropriate initial and boundary conditions. The noise term in this equation vanished by a successive iterative method. As a result, the proposed technique has the advantage of producing an exact solution, and it is easily applied to the given problems analytically. However, the amalgamation of the conformable triple Laplace transform method and the new iterative method that is CTLTIM has not previously been studied to solve the two-dimensional nonlinear fractional telegraph equations; this is the main motivation of the current research work.

The proposed CTLTIM has been utilized to solve the problems as follows. First, the source term $f((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))$ of the considered problem (2) is decomposed into two functions namely $f_1((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))$ and $f_2((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))$. The importance of this decomposition is that the part $f_1((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))$ with the terms in Equation (2) always leads to the simple algebraic expression while applying the conformable triple Laplace transform and the part $f_2((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))$ is combined with the nonlinear term of Equation (2) to avoid noise terms in the iteration process. Next, the conformable triple Laplace transform method is applied to the linear part of the problem. Finally, the successive iterative method is applied to the nonlinear part of the problem as it introduced in Section 4. While applying this iterative method, the noise terms in the iteration process are avoided, and a single iteration gives the exact solution. Therefore, using the described method one can obtain the exact solution to nonlinear partial fractional derivatives with less computational size. Moreover, the proposed approach allows the user with analytical approximation, and it is applied directly to the problems without requiring any discretization, linearization, or perturbation parameters like Adomian polynomials and SOS polynomials ([42, 47] see the references therein). This is the main advantage of the proposed method CFTLTIM over the other existing approaches in the literature.

The following two-dimensional nonlinear conformable telegraph equation was the subject of the current study (for $\alpha=\beta=1$ see [30]):

$$\begin{aligned}&\frac{\partial^{2\alpha}u}{\partial t^{2\alpha}}+a\frac{\partial^{\alpha}u}{\partial t^{\alpha}}+bN\left(u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\\&\quad=c\left(\frac{\partial^{2\beta}u}{\partial x^{2\beta}}+\frac{\partial^{2\gamma}u}{\partial y^{2\gamma}}\right)+f\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right),\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\\&\quad>0\&\alpha,\beta,\gamma\in(0,1],\end{aligned}\tag{2}$$

depending on the starting conditions

$$\begin{aligned}u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},0\right)&=\varphi_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma}\right),\frac{\partial^\alpha}{\partial t^\alpha}u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},0\right)\\&=\varphi_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma}\right),\end{aligned}\tag{3}$$

and boundary conditions (Cauchy type BCs)

$$\begin{aligned}u\left(0,\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)&=g_1\left(\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right),\frac{\partial^\beta}{\partial x^\beta}u\left(0,\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\\&=g_2\left(\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right),\end{aligned}\tag{4}$$

$$\begin{aligned}u\left(\frac{x^\beta}{\beta},0,\frac{t^\alpha}{\alpha}\right)&=g_3\left(\frac{x^\beta}{\beta},\frac{t^\alpha}{\alpha}\right),\frac{\partial^\gamma}{\partial y^\gamma}u\left(\frac{x^\beta}{\beta},0,\frac{t^\alpha}{\alpha}\right)\\&=g_4\left(\frac{x^\beta}{\beta},\frac{t^\alpha}{\alpha}\right),\end{aligned}\tag{5}$$

where a, b and c are known real constants; $u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))$ denotes either the voltage or current through the two-dimensional conductor at position (x, y) at the time t; N is the general continuous nonlinear resorting term; and $f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ is the source term which is assumed to be analytic in time t. Equation (2) reduces to the undamped telegraph equation in two space variables when $\alpha = 0$, and to the damped one when $\alpha > 0$.

The rest of the paper is organized as follows: Section 2 covers the definitions, properties, and theorems of conformable fractional derivatives. Section 3 contains some basic CFTLTM definitions, properties, and theorem proofs. In Section 4, the details of the new iterative method and its convergence are discussed. Section 5 displays the model's description and how CTLTIM is used to obtain the exact analytical solutions to the specified conformable fractional telegraph equations. In Section 6, we demonstrate the proposed method's reliability, convergence, and efficiency using two exemplary instances. Finally, Section 7 outlines concluding observations.

2. Conformable Fractional Derivative

This section introduces the essential definitions and features of conformable fractional partial derivatives, which are then applied to the current topic.

Definition 2 (see [48]). The fractional derivative of a suitable mapping f by Riemann-Liouville is given as

$$D_x^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^n \int_0^x (x-t)^{n-\alpha-1} f(t)dt, \quad n-1 < \alpha \le n. \tag{6}$$

Caputo fractional derivative of a suitable mapping f is given as

$$D_x^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} f^{(n)}(t)dt, \quad n-1 < \alpha \le n, \tag{7}$$

where α is the order of fractional derivative and $\alpha \in Z^+$.

If this limit exists, f is said to be partially α – differentiable at $t > 0$.

Definition 3 (see [37, 38]). Given a function $f : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$, then the conformable partial fractional derivatives (CPFDs) of $f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ having order α, β and γ are defined by.

$$\begin{cases} \partial_x^\beta f = \dfrac{\partial^\beta}{\partial x^\beta} f\left(\dfrac{x^\beta}{\beta}, \dfrac{y^\gamma}{\gamma}, \dfrac{t^\alpha}{\alpha}\right) = \lim\limits_{h \longrightarrow 0} \dfrac{f\left((x^\beta/\beta) + hx^{1-\beta}, (y^\gamma/\gamma), (t^\alpha/\alpha)\right) - f\left((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)\right)}{h}, \\ \partial_y^\gamma f = \dfrac{\partial^\gamma}{\partial y^\gamma} f\left(\dfrac{x^\beta}{\beta}, \dfrac{y^\gamma}{\gamma}, \dfrac{t^\alpha}{\alpha}\right) = \lim\limits_{k \longrightarrow 0} \dfrac{f\left((x^\beta/\beta), (y^\gamma/\gamma) + ky^{1-\gamma}, (t^\alpha/\alpha)\right) - f\left((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)\right)}{k}, \\ \partial_t^\alpha f = \dfrac{\partial^\alpha}{\partial t^\alpha} f\left(\dfrac{x^\beta}{\beta}, \dfrac{y^\gamma}{\gamma}, \dfrac{t^\alpha}{\alpha}\right) = \lim\limits_{\varepsilon \longrightarrow 0} \dfrac{f\left((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha) + \varepsilon t^{1-\alpha}\right) - f\left((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)\right)}{\varepsilon}, \end{cases} \tag{8}$$

where $0 < \alpha, \beta, \gamma \le 1, x^\beta/\beta, y^\gamma/\gamma, t^\alpha/\alpha > 0$, and $\partial_x^\beta f = (\partial^\beta/\partial x^\beta)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)), \partial_y^\gamma f = (\partial^\gamma/\partial y^\gamma)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$, and $\partial_t^\alpha f = (\partial^\alpha/\partial t^\alpha)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ are called the fractional partial derivatives of orders α, β and γ, respectively.

Theorem 4 (see [37, 38]). *Let $\alpha, \beta, \gamma \in (0, 1]$ and $f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ be differentiable at a point for $x^\beta/\beta, y^\gamma/\gamma, t^\alpha/\alpha > 0$. Then,*

(1) $\partial_x^\beta f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = (\partial^\beta/\partial x^\beta)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = x^{1-\beta}(\partial/\partial x)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$.

(2) $\partial_y^\gamma f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = (\partial^\gamma/\partial y^\gamma)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = y^{1-\gamma}(\partial/\partial y)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$.

(3) $\partial_t^\alpha f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = (\partial^\alpha/\partial t^\alpha)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = t^{1-\alpha}(\partial/\partial t)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$.

Proof (1). With the help of CFPD definition, we have

$$\partial_x^\beta f = \frac{\partial^\beta}{\partial x^\beta} f\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) = \lim_{h \longrightarrow 0} \frac{f\left((x^\beta/\beta) + hx^{1-\beta}, (y^\gamma/\gamma), (t^\alpha/\alpha)\right) - f\left((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)\right)}{h}. \tag{9}$$

Using $\lambda = hx^{1-\beta}$ in the above equation, we get

$$\begin{aligned}\partial_x^\beta f = \frac{\partial^\beta}{\partial x^\beta} f\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) &= \lim_{\lambda \longrightarrow 0} \frac{f\left(\left(x^\beta/\beta\right) + \lambda, \left(y^\gamma/\gamma\right), \left(t^\alpha/\alpha\right)\right) - f\left(\left(x^\beta/\beta\right), \left(y^\gamma/\gamma\right), \left(t^\alpha/\alpha\right)\right)}{\lambda x^{\beta-1}}, \\ &= x^{1-\beta} \lim_{\lambda \longrightarrow 0} \frac{f\left(\left(x^\beta/\beta\right) + \lambda, \left(y^\gamma/\gamma\right), \left(t^\alpha/\alpha\right)\right) - f\left(\left(x^\beta/\beta\right), \left(y^\gamma/\gamma\right), \left(t^\alpha/\alpha\right)\right)}{\lambda} \\ &= x^{1-\beta} f\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right).\end{aligned} \tag{10}$$

We can prove the results of (2) and (3) in the same way. □

Proposition 5. *Let $\alpha, \beta, \gamma \in (0, 1]$ and $a, b, c \in \mathbb{R}$, $l, m, n \in \mathbb{N}$. Then, we have the following:*

(i) $(\partial^\beta/\partial x^\beta)(af((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) + bg((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))) = a(\partial^\beta/\partial x^\beta)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) + b(\partial^\beta/\partial x^\beta)g((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$

(ii) $(\partial^\beta/\partial x^\beta)(fg)((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = g((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))(\partial^\beta/\partial x^\beta)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) + f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))(\partial^\beta/\partial x^\beta)g((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$

(iii) $(\partial^\beta/\partial x^\beta)(f/g)((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = g((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))(\partial^\beta/\partial x^\beta)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) - f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))(\partial^\beta/\partial x^\beta)g((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))/(g((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)))^2$

provided that $g((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) \neq 0$.

(iv) $(\partial^\alpha/\partial t^\alpha)f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha)) = 0$, *if* $((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ *was a function depending only on* $(x^\beta/\beta)\&(y^\gamma/\gamma)$

(v) $(\partial^{\beta+\gamma+\alpha}/\partial x^\beta \partial y^\gamma \partial t^\alpha)((x^\beta/\beta)^m (y^\gamma/\gamma)^n (t^\alpha/\alpha)^l) = mnl (x^\beta/\beta)^{m-\beta}(y^\gamma/\gamma)^{n-\gamma}(t^\alpha/\alpha)^{l-\alpha}$

(vi) $(\partial^\beta/\partial x^\beta)((x^\beta/\beta)^m (y^\gamma/\gamma)^n (t^\alpha/\alpha)^l) = m(x^\beta/\beta)^{m-1}(y^\gamma/\gamma)^n (t^\alpha/\alpha)^l$, $(\partial^\gamma/\partial y^\gamma)((x^\beta/\beta)^m (y^\gamma/\gamma)^n (t^\alpha/\alpha)^l) = n(x^\beta/\beta)^m (y^\gamma/\gamma)^{n-1}(t^\alpha/\alpha)^l$ *and* $(\partial^\alpha/\partial t^\alpha)((x^\beta/\beta)^m (y^\gamma/\gamma)^n (t^\alpha/\alpha)^l) = l(x^\beta/\beta)^m (y^\gamma/\gamma)^n (t^\alpha/\alpha)^{l-1}$

(vii) $(\partial^\beta/\partial x^\beta)(e^{a(x^\beta/\beta)+b(y^\gamma/\gamma)+c(t^\alpha/\alpha)}) = ae^{a(x^\beta/\beta)+b(y^\gamma/\gamma)+c(t^\alpha/\alpha)}$, $(\partial^\gamma/\partial y^\gamma)(e^{a(x^\beta/\beta)+b(y^\gamma/\gamma)+c(t^\alpha/\alpha)}) = be^{a(x^\beta/\beta)+b(y^\gamma/\gamma)+c(t^\alpha/\alpha)}$ *and* $(\partial^\alpha/\partial t^\alpha)(e^{a(x^\beta/\beta)+b(y^\gamma/\gamma)+c(t^\alpha/\alpha)}) = ce^{a(x^\beta/\beta)+b(y^\gamma/\gamma)+c(t^\alpha/\alpha)}$

(viii) $(\partial^\beta/\partial x^\beta)\sin(x^\beta/\beta)\cos(t^\alpha/\alpha) = \cos(x^\beta/\beta)\cos(t^\alpha/\alpha)$ *and* $(\partial^\gamma/\partial y^\gamma)\sin(x^\beta/\beta)\cos(y^\gamma/\gamma)\cos(t^\alpha/\alpha) = -\sin(x^\beta/\beta)\sin(y^\gamma/\gamma)\cos(t^\alpha/\alpha)$.

3. Some Results and Theorems of the Conformable Triple Laplace Transform

In this section, we will go over the fundamental concepts of fractional conformable Laplace transforms as well as certain results that will be useful later. The conformable triple Laplace transform is also defined (see [36–39, 46] for more information).

Definition 6. Let the function u : $(0,\infty) \longrightarrow \mathbb{R}$ and $0 < \alpha \leq 1$. The conformable Laplace transform (CLT) of the exponential of order α function $u(t^\alpha/\alpha)$ is thus defined and denoted by

$$L_t^\alpha\left(u\left(\frac{t^\alpha}{\alpha}\right)\right) = U_\alpha(s) = \int_0^\infty e^{-s\left(\frac{t^\alpha}{\alpha}\right)} u\left(\frac{t^\alpha}{\alpha}\right) t^{\alpha-1} dt, t > 0. \tag{11}$$

Definition 7. Let $u((x^\beta/\beta), (y^\gamma/\gamma))$ be a piece-wise continuous function of exponential order on the domain D of $\mathbb{R}^+ \times \mathbb{R}^+$. After that, the conformable double Laplace transform (FCDLT) of $u((x^\beta/\beta), (y^\gamma/\gamma))$ is defined and denoted by

$$\begin{aligned}L_x^\beta L_y^\gamma\left(u\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}\right)\right) = U_{\beta,\gamma}(k, p) = \int_0^\infty \int_0^\infty e^{-\beta\left(\frac{x^\beta}{\beta}\right) - \gamma\left(\frac{y^\gamma}{\gamma}\right)} u \\ \cdot \left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}\right) x^{\beta-1} y^{\gamma-1} dx dy,\end{aligned} \tag{12}$$

where $x^\beta/\beta, y^\gamma/\gamma > 0$, $k, p \in \mathbb{C}$, $\beta, \gamma \in (0, 1]$.

Now, we define fractional conformable triple Laplace transform, for $\alpha, \beta, \gamma \in (0, 1]$ and $k, p, s \in \mathbb{C}$ are the Laplace variables.

Definition 8. Let $u((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ be a piece-wise continuous function of exponential order on the domain D of $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$. After that, the conformable triple Laplace transform (FCDLT) of $u((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ is defined and denoted by

$$L_x^{\beta}L_y^{\gamma}L_t^{\alpha}\left(u\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)\right)=U_{\alpha,\beta,\gamma}(k,p,s)=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-k\left(\frac{x^{\beta}}{\beta}\right)-p\left(\frac{y^{\gamma}}{\gamma}\right)-s\left(\frac{t^{\alpha}}{\alpha}\right)}u\cdot\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}dxdydt, \tag{13}$$

where $k, p, s \in \mathbb{C}$ are Laplace variables of $x^{\beta}/\beta, (y^{\gamma}/\gamma)$ and $(t^{\alpha}/\alpha) > 0$, respectively, and $\alpha, \beta, \gamma \in (0, 1]$.

The conformable inverse triple Laplace transform, abbreviated by $u((x^{\beta}/\beta), (y^{\gamma}/\gamma), (t^{\alpha}/\alpha))$, is defined as follows:

$$u\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)=L_k^{-1}L_p^{-1}L_s^{-1}\left(U_{\alpha,\beta,\gamma}(k,p,s)\right)=\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty}e^{k\left(\frac{x^{\beta}}{\beta}\right)}\left[\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}e^{p\left(\frac{y^{\gamma}}{\gamma}\right)}\left[\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}e^{s\left(\frac{t^{\alpha}}{\alpha}\right)}U_{\alpha,\beta,\gamma}(k,p,s)ds\right]dp\right]dk. \tag{14}$$

Definition 9. The following is the definition of a unit step or Heaviside unit step function:

$$H\left(\left(\frac{x^{\beta}}{\beta}\right)-a,\left(\frac{y^{\gamma}}{\gamma}\right)-b,\left(\frac{t^{\alpha}}{\alpha}\right)-c\right)=\begin{Bmatrix}1;\ \dfrac{x^{\beta}}{\beta}>a,\ \dfrac{y^{\gamma}}{\gamma}>b,\ \dfrac{t^{\alpha}}{\alpha}>c\\ 0;\ \dfrac{x^{\beta}}{\beta}<a,\ \dfrac{y^{\gamma}}{\gamma}<b,\ \dfrac{t^{\alpha}}{\alpha}<c\end{Bmatrix}. \tag{15}$$

Theorem 10. *If $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(u((x^{\beta}/\beta),(y^{\gamma}/\gamma),(t^{\alpha}/\alpha)))=U_{\alpha,\beta,\gamma}(k, p,s)$, $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(v((x^{\beta}/\beta),(y^{\gamma}/\gamma),(t^{\alpha}/\alpha)))=V_{\alpha,\beta,\gamma}s(k,p,s)$, and a, b and c are constants then the followings hold:*

(1) Linearity property:

$$L_x^{\beta}L_y^{\gamma}L_t^{\alpha}\left(au\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)+bv\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)\right)=aL_x^{\beta}L_y^{\gamma}L_t^{\alpha}u\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)+bL_x^{\beta}L_y^{\gamma}L_t^{\alpha}v\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)=aU_{\alpha,\beta,\gamma}(k,p,s)+bV_{\alpha,\beta,\gamma}(k,p,s) \tag{16}$$

(2) $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(c)=c/kps$, where c is the constant

(3) $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}((x^{\beta}/\beta)^m(y^{\gamma}/\gamma)^n(t^{\alpha}/\alpha)^l)=\Gamma(m+1)\Gamma(n+1)\Gamma(l+1)/k^{m+1}p^{n+1}s^{l+1}$, where $\Gamma(.)$ is the gamma function. Note that $\Gamma(n+1)=n!$, for $n=0,1,2,3,\cdots$.

(4) $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(e^{a(x^{\beta}/\beta)+b(y^{\gamma}/\gamma)+c(t^{\alpha}/\alpha)})=L_xL_yL_t(e^{ax+by+ct})=1/(k-a)(p-b)(s-c)$ and $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(e^{a(x^{\beta}/\beta)-b(y^{\gamma}/\gamma)-c(t^{\alpha}/\alpha)})=L_xL_yL_t(e^{ax-by-ct})=1/(k+a)(p+b)(s+c)$

(5) The conformable triple Laplace transform's first shifting theorem:

If $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(u((x^{\beta}/\beta),(y^{\gamma}/\gamma),(t^{\alpha}/\alpha)))=U_{\alpha,\beta,\gamma}(k,p,s)$, then $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(e^{a(x^{\beta}/\beta)+b(y^{\gamma}/\gamma)+c(t^{\alpha}/\alpha)}u((x^{\beta}/\beta),(y^{\gamma}/\gamma),(t^{\alpha}/\alpha)))=U_{\alpha,\beta,\gamma}(k-a,p-b,s-c)$.

(6) $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(\sin(a(x^{\beta}/\beta))\sin(b(y^{\gamma}/\gamma))\sin(c(t^{\alpha}/\alpha)))=a\,bc/(k^2+a^2)(p^2+b^2)(s^2+c^2)$,

$L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(\cos(a(x^{\beta}/\beta))\cos(b(y^{\gamma}/\gamma))\cos(c(t^{\alpha}/\alpha)))=kps/(k^2+a^2)(p^2+b^2)(s^2+c^2)$

(7) If $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}(u((x^{\beta}/\beta),(y^{\gamma}/\gamma),(t^{\alpha}/\alpha)))=U_{\alpha,\beta,\gamma}(k,p,s)$, then $L_x^{\beta}L_y^{\gamma}L_t^{\alpha}((x^{\beta}/\beta)^m(y^{\gamma}/\gamma)^n(t^{\alpha}/\alpha)^l u((x^{\beta}/\beta),(y^{\gamma}/\gamma),(t^{\alpha}/\alpha)))=(-1)^{m+n+l}(d^{m+n+l}/dk^m dp^n ds^l)U_{\alpha,\beta,\gamma}(k,p,s)$.

Proof (7). Using the definition of the conformable triple Laplace transform method, the demonstration of outcomes 1–6 is simple. As a result, we will show how to prove result 7 using the conformable triple Laplace transform definition.

$$L_x^{\beta}L_y^{\gamma}L_t^{\alpha}\left(u\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)\right)=U_{\alpha,\beta,\gamma}(k,p,s)=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-k\left(\frac{x^{\beta}}{\beta}\right)-p\left(\frac{y^{\gamma}}{\gamma}\right)-s\left(\frac{t^{\alpha}}{\alpha}\right)}u\ \text{Än}\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}dxdydt. \tag{17}$$

Differentiating with respect to k, m – times, we get

$$\frac{d^m}{dk^m}U_{\alpha,\beta,\gamma}(k,p,s)=\frac{d^m}{dk^m}\left\{\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-k\left(\frac{x^{\beta}}{\beta}\right)-p\left(\frac{y^{\gamma}}{\gamma}\right)-s\left(\frac{t^{\alpha}}{\alpha}\right)}u\cdot\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)\times x^{\beta-1}y^{\gamma-1}t^{\alpha-1}dxdydt\right\},=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\frac{d^m}{dk^m}\left[e^{-k\left(\frac{x^{\beta}}{\beta}\right)-p\left(\frac{y^{\gamma}}{\gamma}\right)-s\left(\frac{t^{\alpha}}{\alpha}\right)}\times u\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}\right]dxdydt. \tag{18}$$

This suggests that

$$\frac{d^m}{dk^m}U_{\alpha,\beta,\gamma}(k,p,s)=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\left(-\left(\frac{x^{\beta}}{\beta}\right)\right)^m\cdot\left[e^{-k\left(\frac{x^{\beta}}{\beta}\right)-p\left(\frac{y^{\gamma}}{\gamma}\right)-s\left(\frac{t^{\alpha}}{\alpha}\right)}\times u\left(\frac{x^{\beta}}{\beta},\frac{y^{\gamma}}{\gamma},\frac{t^{\alpha}}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}\right]dxdydt. \tag{19}$$

Differentiating (19) with respect to p, n – times, produces

$$\frac{d^n}{dp^n}U_{\alpha,\beta,\gamma}(k,p,s)=\int_0^\infty\int_0^\infty\int_0^\infty\left(-\left(\frac{y^\gamma}{\gamma}\right)\right)^n \cdot\left[e^{-k\left(\frac{x^\beta}{\beta}\right)-p\left(\frac{y^\gamma}{\gamma}\right)-s\left(\frac{t^\alpha}{\alpha}\right)} \times u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}\right]dxdydt. \tag{20}$$

Again, differentiating (20) with respect to s, l – times, we obtain

$$\frac{d^l}{ds^l}U_{\alpha,\beta,\gamma}(k,p,s)=\int_0^\infty\int_0^\infty\int_0^\infty\left(-\left(\frac{t^\alpha}{\alpha}\right)\right)^l \cdot\left[e^{-k\left(\frac{x^\beta}{\beta}\right)-p\left(\frac{y^\gamma}{\gamma}\right)-s\left(\frac{t^\alpha}{\alpha}\right)} \times u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}\right]dxdydt. \tag{21}$$

Using Equations (19), (20), and (21), we get

$$\frac{d^{m+n+l}}{dk^m dp^n ds^l}nU_{\alpha,\beta,\gamma}(k,p,s)=\int_0^\infty\int_0^\infty\int_0^\infty\left(-\left(\frac{x^\beta}{\beta}\right)\right)^m\left(-\left(\frac{y^\gamma}{\gamma}\right)\right)^n\left(-\left(\frac{t^\alpha}{\alpha}\right)\right)^l \cdot\begin{bmatrix} e^{-k\left(\frac{x^\beta}{\beta}\right)-p\left(\frac{y^\gamma}{\gamma}\right)-s\left(\frac{t^\alpha}{\alpha}\right)} \\ \times u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}\end{bmatrix}dxdydt,$$

$$=(-1)^{m+n+l}\int_0^\infty\int_0^\infty\int_0^\infty\begin{bmatrix} e^{-k\left(\frac{x^\beta}{\beta}\right)-p\left(\frac{y^\gamma}{\gamma}\right)-s\left(\frac{t^\alpha}{\alpha}\right)}\times\left(\frac{x^\beta}{\beta}\right)^m\left(\frac{y^\gamma}{\gamma}\right)^n\left(\frac{t^\alpha}{\alpha}\right)^l \\ \times u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}\end{bmatrix}dxdydt. \tag{22}$$

This suggests that

$$\frac{d^{m+n+l}}{dk^m dp^n ds^l}U_{\alpha,\beta,\gamma}(k,p,s)=(-1)^{m+n+l}L_x^\beta L_y^\gamma L_t^\alpha \cdot\left(\left(\frac{x^\beta}{\beta}\right)^m\left(\frac{y^\gamma}{\gamma}\right)^n\left(\frac{t^\alpha}{\alpha}\right)^l u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right). \tag{23}$$

Multiplying both sides of the Equation (23) by $(-1)^{m+n+l}$, we obtain

$$L_x^\beta L_y^\gamma L_t^\alpha\left(\left(\frac{x^\beta}{\beta}\right)^m\left(\frac{y^\gamma}{\gamma}\right)^n\left(\frac{t^\alpha}{\alpha}\right)^l u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right) =(-1)^{m+n+l}\frac{d^{m+n+l}}{dk^m dp^n ds^l}U_{\alpha,\beta,\gamma}(k,p,s). \tag{24}$$

□

Theorem 11. *If* $L_x^\beta L_y^\gamma L_t^\alpha(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha)))=U_{\alpha,\beta,\gamma}(k,p,s)$, *then*

$$L_x^\beta L_y^\gamma L_t^\alpha\left(u\left(\frac{x^\beta}{\beta}-\frac{\xi^\beta}{\beta},\frac{y^\gamma}{\gamma}-\frac{\eta^\gamma}{\gamma},\frac{t^\alpha}{\alpha}-\frac{\mu^\alpha}{\alpha}\right)\times H\left(\frac{x^\beta}{\beta}-\frac{\xi^\beta}{\beta},\frac{y^\gamma}{\gamma}-\frac{\eta^\gamma}{\gamma},\frac{t^\alpha}{\alpha}-\frac{\mu^\alpha}{\alpha}\right)\right) =e^{-k\left(\frac{\xi^\beta}{\beta}\right)-p\left(\frac{\eta^\gamma}{\gamma}\right)-s\left(\frac{\mu^\alpha}{\alpha}\right)}U_{\alpha,\beta,\gamma}(k,p,s), \tag{25}$$

where the Heaviside unit step function $H(x,y,t)$*is defined as in Equation (15)*
(see [37, 38] for the proof).

Theorem 12. *For* $\alpha,\beta,\gamma\in(0,1]$. *Let* $u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))$ *be the real-valued piece-wise continuous function defined on the domain* $(0,\infty)\times(0,\infty)\times(0,\infty)$. *The CFTLT of the conformable partial fractional derivatives of order* $\alpha,\beta,$ *and*γ *is given bySS*

(1) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^\beta/\partial x^\beta)(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=kU(k,p,s)-U(0,p,s)$

(2) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^\gamma/\partial y^\gamma)(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=pU(k,p,s)-U(k,0,s)$

(3) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^\alpha/\partial t^\alpha)(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=sU(k,p,s)-U(k,p,0)$

(4) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^{2\beta}/\partial x^{2\beta})(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=k^2U(k,p,s)-kU(0,p,s)-(\partial/\partial x)U(0,p,s)$

(5) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^{2\gamma}/\partial y^{2\gamma})(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=p^2U(k,p,s)-pU(k,0,s)-(\partial/\partial y)U(k,0,s)$

(6) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^{2\alpha}/\partial t^{2\alpha})(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=s^2U(k,p,s)-sU(k,p,0)-(\partial/\partial t)U(k,p,0)$

(7) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^{3\beta}/\partial x^{3\beta})(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=k^3U(k,p,s)-k^2U(0,p,s)-(\partial/\partial x)U(0,p,s)-(\partial^2/\partial x^2)U(0,p,s)$

(8) $L_x^\beta L_y^\gamma L_t^\alpha((\partial^{\beta+\gamma+\alpha}/\partial x^\beta\partial y^\gamma\partial^\alpha)(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))))=kpsU(k,p,s)-kU(k,0,0)-pU(0,p,0)-sU(0,0,s)-kpU(k,p,0)-ksU(k,0,s)-psU(0,p,s)-U(0,0,0)$

Proof (1). Using the CFTLT definition (6), we have

$$L_x^\beta L_y^\gamma L_t^\alpha\left(\frac{\partial^\beta}{\partial x^\beta}\left(u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right) = \int_0^\infty\int_0^\infty\int_0^\infty e^{-k\left(\frac{x^\beta}{\beta}\right)-p\left(\frac{y^\gamma}{\gamma}\right)-s\left(\frac{t^\alpha}{\alpha}\right)}\frac{\partial^\beta}{\partial x^\beta}u \ddot{A}n\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)x^{\beta-1}y^{\gamma-1}t^{\alpha-1}dxdydt. \quad (26)$$

By using Theorem 4, we have

$$\frac{\partial^\beta}{\partial x^\beta}u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = x^{1-\beta}\frac{\partial}{\partial x}u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right). \quad (27)$$

Then, Equation (26) reduced to

$$L_x^\beta L_y^\gamma L_t^\alpha\left(\frac{\partial^\beta}{\partial x^\beta}\left(u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right) = \int_0^\infty\int_0^\infty e^{-p\left(\frac{y^\gamma}{\gamma}\right)-s\left(\frac{t^\alpha}{\alpha}\right)}\left(\int_0^\infty e^{-k\left(\frac{x^\beta}{\beta}\right)}\frac{\partial}{\partial x}u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)dx\right)\cdot y^{\gamma-1}t^{\alpha-1}dydt. \quad (28)$$

Taking integration by parts and Theorem 4 to the integrals inside the bracket produces

$$\int_0^\infty e^{-k\left(\frac{x^\beta}{\beta}\right)}\frac{\partial}{\partial x}u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)dx = kU(k,y,t)-U(0,y,t). \quad (29)$$

We get the needed outcome by substituting Equation (29) into Equation (28), then simplifying $L_x^\beta L_y^\gamma L_t^\alpha((\partial^\beta/\partial x^\beta)(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha)))) = kU(k,p,s)-U(0,p,s)$, and this result can be generalized to

$$L_x^\beta L_y^\gamma L_t^\alpha\left(\frac{\partial^{m\beta}}{\partial x^{m\beta}}\left(u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right) = k^mU(k,p,s)-\sum_{n=0}^{m-1}k^{m-1-n}\frac{\partial^n}{\partial x}U(0,p,s). \quad (30)$$

The process outlined above can be used to receive verification of the remaining results. □

4. Basic Idea of the New Iterative Method(NIM)

Consider the following general functional equation [40] for the main principle of the new iterative method:

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = N\left(u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)+f\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right), \quad (31)$$

where N is a nonlinear operator in a Banach space such that $N : B \longrightarrow B$ and f is a known function.

We are looking for a solution u of the Equation (31) having the series form:

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = \sum_{i=0}^\infty u_i\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right). \quad (32)$$

The nonlinear operation N can then be decomposed as

$$N\left(\sum_{i=0}^\infty u_i\right) = N(u_0)+\sum_{i=1}^\infty u_i\left\{N\left(\sum_{r=0}^i u_r\right)-N\left(\sum_{r=0}^{i-1}u_r\right)\right\}. \quad (33)$$

From Equations (33) and (32), Equation (31) is equivalent to

$$\sum_{i=0}^\infty u_i = f+N(u_0)+\sum_{i=1}^\infty u_i\left\{N\left(\sum_{r=0}^i u_r\right)-N\left(\sum_{r=0}^{i-1}u_r\right)\right\}. \quad (34)$$

Equation (34) yields the following recurrence relation:

$$G_0 = u_0 = f_0, \quad (35)$$

$$G_1 = u_1 = N(u_0), \quad (36)$$

$$G_m = u_{m+1} = N(u_0+\cdots+u_m) - N(u_0+\cdots+u_{m-1}),\ m=1,2,\cdots \quad (37)$$

Then,

$$u_1+\cdots+u_{m+1} = N(u_0+\cdots+u_m),\ m=1,2,\cdots, \quad (38)$$

and hence,

$$u = \sum_{i=0}^\infty u_i = f+N\left(\sum_{i=1}^\infty u_i\right). \quad (39)$$

As a result, the $m-$term approximate solution of Equation (31) is defined as follows:

$$u = u_0+u_1+u_2+\cdots+u_{m-1},\ m>1. \quad (40)$$

4.1. Convergence of the NIM. The conditions for the series (32) convergence are presented in this subsection. And [41] is a good place to start for more information.

Theorem 13. *If N is a continuously differentiable functional in a neighborhood of u_0 and $\|N^{(n)}(u_0)\| \le L$, for each n and for some real $L>0$ and $\|u_i\| \le M < 1/e, i=1,2,3,\cdots$, then the series $\sum_{n=0}^\infty G_n$ is absolutely convergent and moreover, $\|G_n\| \le LM^n e^{n-1}(e-1), n=1,2,\cdots$.*

Theorem 14. *If N is a continuously differentiable functional in a neighborhood of u_0 and $\|N^{(n)}(u_0)\| \leq M \leq 1/e$ for all n, then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent.*

5. Description of the Model

To solve the problem (2)-(5) by using the proposed method first, the source term $f((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ must be decomposed into two functions namely $f_1((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ and $f_2((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ as explained in the introduction section. The part $f_1((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ with the terms in Equation (2) always leads to the simple algebraic expression while applying the fractional conformable triple Laplace transform. The part $f_2((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$ is combined with the nonlinear term of Equation (2) to avoid noise terms in the iteration process.

The following are the steps to determine the analytical solution of Equations (2)-(6) using the proposed techniques:

Step 1. On both sides of Equation (2), we use the properties of the conformable triple Laplace transform and Theorem (12) to obtain

$$\begin{aligned} &s^2 U(k,p,s) - sU(k,p,0) - \frac{\partial}{\partial t}U(k,p,0) \\ &\quad + a(sU(k,p,s) - U(k,p,0)) \\ &\quad = bL_x^\beta L_y^\gamma L_t^\alpha \left(N\left(u\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right)\right)\right) \\ &\quad = c\left(k^2 U(k,p,s) - kU(0,p,s) - \frac{\partial}{\partial x}U(0,p,s)\right. \\ &\qquad \left. + p^2 U(k,p,s) - pU(k,0,s) - \frac{\partial}{\partial y}U(k,0,s)\right) \\ &\qquad + \bar{f}_1(k,p,s) + L_x^\beta L_y^\gamma L_t^\alpha \left(f_2\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right)\right). \end{aligned} \tag{41}$$

Here, $\bar{f}_1(k,p,s)$ is the conformable triple Laplace transform of $f_1((x^\beta/\beta), (y^\gamma/\gamma), (t^\alpha/\alpha))$..

Step 2. We get the following result by applying the conformable double Laplace transform to the initial circumstance (2):

$$U(k,p,0) = \overline{\varphi_1}(k,p), \quad \frac{\partial}{\partial t}U(k,p,0) = \overline{\varphi_2}(k,p). \tag{42}$$

Step 3. The conformable double Laplace transform is applied to the boundary Conditions (4) and (5) to obtain

$$U(0,p,s) = \overline{g_1}(p,s), \quad \frac{\partial}{\partial x}U(0,p,s) = \overline{g_2}(p,s), \tag{43}$$

$$U(k,0,s) = \overline{g_3}(k,s), \quad \frac{\partial}{\partial y}U(k,0,s) = \overline{g_4}(k,s). \tag{44}$$

Step 4. Using Equations (42), (43), and (44) into the Equation (41), and simplifying, we obtain

$$U(k,p,s) = \frac{1}{s^2 + as - ck^2 - cp^2} \left\{ \begin{array}{c} (s+a)\overline{\varphi_1}(k,p) + \overline{\varphi_2}(k,p) \\ -c(k\overline{g_1}(p,s) + \overline{g_2}(p,s) + p\overline{g_3}U(k,s) + \overline{g_4}U(k,s)) \\ +\bar{f}_1(k,p,s) + L_x^\beta L_y^\gamma L_t^\alpha \left(f_2\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) - bN\left(u\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right)\right)\right) \end{array} \right\}. \tag{45}$$

Step 5. Taking Equation (43) and applying the inverse conformable triple Laplace transform, we get

$$u(x,y,t) = L_{xyt}^{-1}\left[\frac{1}{s^2 + as - ck^2 - cp^2} \left\{ \begin{array}{c} (s+a)\overline{\varphi_1}(k,p) + \overline{\varphi_2}(k,p) \\ -c(k\overline{g_1}(p,s) + \overline{g_2}(p,s) + p\overline{g_3}U(k,s) + \overline{g_4}U(k,s)) \\ +\bar{f}_1(k,p,s) + L_x^\beta L_y^\gamma L_t^\alpha \left(f_2\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) - bN\left(u\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right)\right)\right) \end{array} \right\}\right]. \tag{46}$$

Step 6. Applying the iterative process to Equation (46), assume that the given problem (2) has the series solution of the form:

$$u\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) = \sum_{i=0}^{\infty} u_i\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right). \tag{47}$$

Step 7. Plugging Equation (47) into Equation (46), we obtain

$$\sum_{i=0}^{\infty} u_i\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = L_{xyt}{}^{-1}\left[\frac{1}{s^2+as-ck^2-cp^2}\left\{\begin{array}{c}(s+a)\overline{\varphi_1}(k,p)+\overline{\varphi_2}(k,p)+\bar{f}_1(k,p,s)\\ -c(k\overline{g_1}(p,s)+\overline{g_2}(p,s)+p\overline{g_3}U(k,s)+\overline{g_4}U(k,s))\\ +L_x^\beta L_y^\gamma L_t^\alpha\left(f_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)-bN\left(u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right)\end{array}\right\}\right]. \quad (48)$$

Step 8. Decomposing the nonlinear term $N(u((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha)))$ in Equation (48) as follows:

$$N\left(\sum_{i=0}^{\infty} u_i\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right) = N\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right) + \sum_{i=1}^{\infty} u_i\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\left\{N\left(\sum_{r=0}^{i} u_r\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right) - N\left(\sum_{r=0}^{i-1} u_r\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right\}. \quad (49)$$

Step 9. Using the Equation (49) in Equation (48) produces

$$\sum_{i=0}^{\infty} u_i\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = L_{xyt}{}^{-1}\left\{\frac{1}{s^2+as-ck^2-cp^2}\left\{\begin{array}{c}(s+a)\overline{\varphi_1}(k,p)+\overline{\varphi_2}(k,p)+\bar{f}_1(k,p,s)\\ -c(k\overline{g_1}(p,s)+\overline{g_2}(p,s)+p\overline{g_3}U(k,s)+\overline{g_4}U(k,s))\\ +L_x^\beta L_y^\gamma L_t^\alpha\left(f_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)-b\left(\begin{array}{c}N(u_0)\\ +\sum_{i=1}^{\infty} u_i\left\{N\left(\sum_{r=0}^{i} u_r\right)-N\left(\sum_{r=0}^{i-1} u_r\right)\right\}\end{array}\right)\right)\end{array}\right\}\right\}. \quad (50)$$

Step 10. The following recurrence relations are defined from Equation (50) according to iteration ((35)-(37)):

$$u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = L_{xyt}{}^{-1}\left[\frac{1}{s^2+as-ck^2-cp^2}\left\{\begin{array}{c}(s+a)\overline{\varphi_1}(k,p)+\overline{\varphi_2}(k,p)+\bar{f}_1(k,p,s)\\ -c(k\overline{g_1}(p,s)+\overline{g_2}(p,s)+p\overline{g_3}U(k,s)+\overline{g_4}U(k,s))\end{array}\right\}\right], \quad (51)$$

$$u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = L_{xyt}{}^{-1}\left[\frac{1}{s^2+as-ck^2-cp^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(f_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)-bN\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right)\right\}\right], \quad (52)$$

$$u_{m+1}\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right) = L_{xyt}{}^{-1}\left[\frac{1}{s^2+as-ck^2-cp^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(\begin{array}{c}f_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\\ -b\left(\sum_{i=1}^{\infty} u_i\left\{N\left(\sum_{r=0}^{i} u_r\right)-N\left(\sum_{r=0}^{i-1} u_r\right)\right\}\right)\end{array}\right)\right\}\right], n \geq 1. \quad (53)$$

Step 11. The series solution to the given problem (2)–(5) can be found by using (40) as follows:

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+\cdots+u_m\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+.\cdots \tag{54}$$

6. Application

Two test instances will be examined in this part to assess the performance of the suggested method.

Example 1. Consider the following two-dimensional nonlinear conformable telegraph equation in the region $\Omega=[0,2]^2$ as follows:

$$\frac{\partial^{2\alpha}u}{\partial t^{2\alpha}}+2\frac{\partial^\alpha u}{\partial t^\alpha}=\frac{\partial^{2\beta}u}{\partial x^{2\beta}}+\frac{\partial^{2\gamma}u}{\partial y^{2\gamma}}-u^2+e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}-2e^{\left(x^\beta/\beta\right)+\left(y^\gamma/\gamma\right)-2\left(t^\alpha/\alpha\right)}, \tag{55}$$

with preliminary conditions

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},0\right)=e^{x^\beta/\beta+y^\gamma/\gamma},\quad \frac{\partial^\alpha}{\partial t^\alpha}u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},0\right)=-2e^{x^\beta/\beta+y^\gamma/\gamma}, \tag{56}$$

and boundary conditions

$$u\left(0,\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=e^{\left(y^\gamma/\gamma\right)-2\left(t^\alpha/\alpha\right)},\quad \frac{\partial^\beta}{\partial x^\beta}u\left(0,\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=e^{\left(y^\gamma/\gamma\right)-2\left(t^\alpha/\alpha\right)}, \tag{57}$$

$$u\left(\frac{x^\beta}{\beta},0,\frac{t^\alpha}{\alpha}\right)=e^{\left(x^\beta/\beta\right)-2\left(t^\alpha/\alpha\right)},\quad \frac{\partial^\gamma}{\partial y^\gamma}u\left(\frac{x^\beta}{\beta},0,\frac{t^\alpha}{\alpha}\right)=e^{\left(x^\beta/\beta\right)-2\left(t^\alpha/\alpha\right)}. \tag{58}$$

Solution: To both sides of Equation (55), we use the properties of the conformable triple Laplace transform and Theorem (12) to get

$$\begin{aligned}
&s^2U(k,p,s)-sU(k,p,0)-\frac{\partial}{\partial t}U(k,p,0)\\
&\quad+2\left(sU(k,p,s)-U(k,p,0)\right)\\
&=k^2U(k,p,s)-kU(0,p,s)\\
&\quad-\frac{\partial}{\partial x}U(0,p,s)+p^2U(k,p,s)\\
&\quad-pU(k,0,s)-\frac{\partial}{\partial y}U(k,0,s)\\
&\quad-\frac{2}{(k-1)(p-1)(s+2)}\\
&\quad+L_x^\beta L_y^\gamma L_t^\alpha\left(e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}-u^2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right).
\end{aligned} \tag{59}$$

We get the following results by applying the conformable fractional double Laplace transform to the initial condition (56).

$$U(k,p,0)=\frac{1}{(k-1)(p-1)},\quad \frac{\partial}{\partial t}U(k,p,0)=-\frac{2}{(k-1)(p-1)}. \tag{60}$$

The boundary conditions (57) and (58) are transformed using the conformable double Laplace transform as follows:

$$U(0,p,s)=\frac{1}{(p-1)(s+2)},\quad \frac{\partial}{\partial x}U(0,p,s)=\frac{1}{(p-1)(s+2)}, \tag{61}$$

$$U(k,0,s)=\frac{1}{(k-1)(s+2)},\quad \frac{\partial}{\partial y}U(k,0,s)=\frac{1}{(k-1)(s+2)}. \tag{62}$$

Substituting Equations (60), (61), and (62) into Equation (59), and simplifying, we obtain

$$U(k,p,s)=\frac{1}{s^2+2s-k^2-p^2}\left\{\begin{array}{c}\dfrac{s+2}{(k-1)(p-1)}-\dfrac{k+1}{(p-1)(s+2)}-\dfrac{p+1}{(k-1)(s+2)}-\dfrac{2}{(k-1)(p-1)(s+2)}\\ +L_x^\beta L_y^\gamma L_t^\alpha\left(e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}-u^2\left(\dfrac{x^\beta}{\beta},\dfrac{y^\gamma}{\gamma},\dfrac{t^\alpha}{\alpha}\right)\right)\end{array}\right\}. \tag{63}$$

Again, performing some mathematical manipulation to Equation (63), it is reduced to

$$U(k,p,s)=\frac{2}{(k-1)(p-1)(s+2)}+\frac{1}{s^2+2s-k^2-p^2}\cdot\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}-u^2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right\}. \quad (64)$$

Taking the inverse conformable triple-Laplace transform to both sides of Equation (62), we get

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=e^{\left(x^\beta/\beta\right)+\left(y^\gamma/\gamma\right)-2\left(t^\alpha/\alpha\right)}+L_{xyt}{}^{-1}\left[\frac{1}{s^2+2s-k^2-p^2}\cdot\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}-u^2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right\}\right]. \quad (65)$$

Now, apply the new iterative method to Equation (65).

Using (49) into (65) and using (51), (52), and (53), the components of the solution are obtained as follows:

$$u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=e^{\left(x^\beta/\beta\right)+\left(y^\gamma/\gamma\right)-2\left(t^\alpha/\alpha\right)},$$

$$u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=L_{xyt}{}^{-1}\left[\frac{1}{s^2+2s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}-u_0{}^2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right\}\right]=L_{xyt}{}^{-1}\left[\frac{1}{s^2+2s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}-e^{2\left(x^\beta/\beta\right)+2\left(y^\gamma/\gamma\right)-4\left(t^\alpha/\alpha\right)}\right)\right\}\right]=0, \quad (66)$$

$u_{m+1}((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))=L_{xyt}{}^{-1}[(1/s^2+2s-k^2-p^2)\{L_x^\beta L_y^\gamma L_t^\alpha(\sum_{r=0}^{i}u_r{}^2((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))-\sum_{r=0}^{i-1}u_r{}^2((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha)))\}]$, $n\geq 1$.As a result,

$$u_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=L_{xyt}{}^{-1}\left[\frac{1}{s^2+2s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^2-\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^2\right)\right\}\right]=0,$$

$$u_3\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=L_{xyt}{}^{-1}\left[\frac{1}{s^2+2s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^2-\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^2\right)\right\}\right]=0. \quad (67)$$

Similarly, we get $u_4((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))=u_5((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))=0$ and so on.

Therefore, using Equation (54), the solution to Example 1 is given by

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=e^{\left(x^\beta/\beta\right)+\left(y^\gamma/\gamma\right)-2\left(t^\alpha/\alpha\right)}. \quad (68)$$

Remark 15. For $\alpha=\beta=1$, in two dimensions, the conformable fractional telegraph in Equation (55) reduces to the classical (or non-fractional) space-time telegraph equation, and its exact solution is $u(x,t)=e^{x+y-2t}$, which is the same as the result obtained in [28, 30, 49].

For $\beta=\gamma=1\&\alpha=0.5$, the exact solution is $u(x,y,t)=e^{x+y}(e^{2t}+e^{2t}(\mathrm{erf}\ c(\sqrt{2t}-1)))$.

For $\beta=\alpha=1\&\gamma=0.5$, the exact solution is $u(x,y,t)=e^{x+2t}(e^{y}+e^{y}(-\mathrm{erf}\ c(\sqrt{y}+1)))$.

For $\alpha=\gamma=1\&\beta=0.5$, the exact solution is $u(x,y,t)=e^{y+2t}(e^{x}+e^{x}(-\mathrm{erf}\ c(\sqrt{x}+1)))$.

Whenever $\mathrm{erf}\ (t)=2/\sqrt{\pi}\int_0^t(e^{-z^2})dz$ and $\mathrm{erf}\ c(t)=1-\mathrm{erf}\ (t)$.

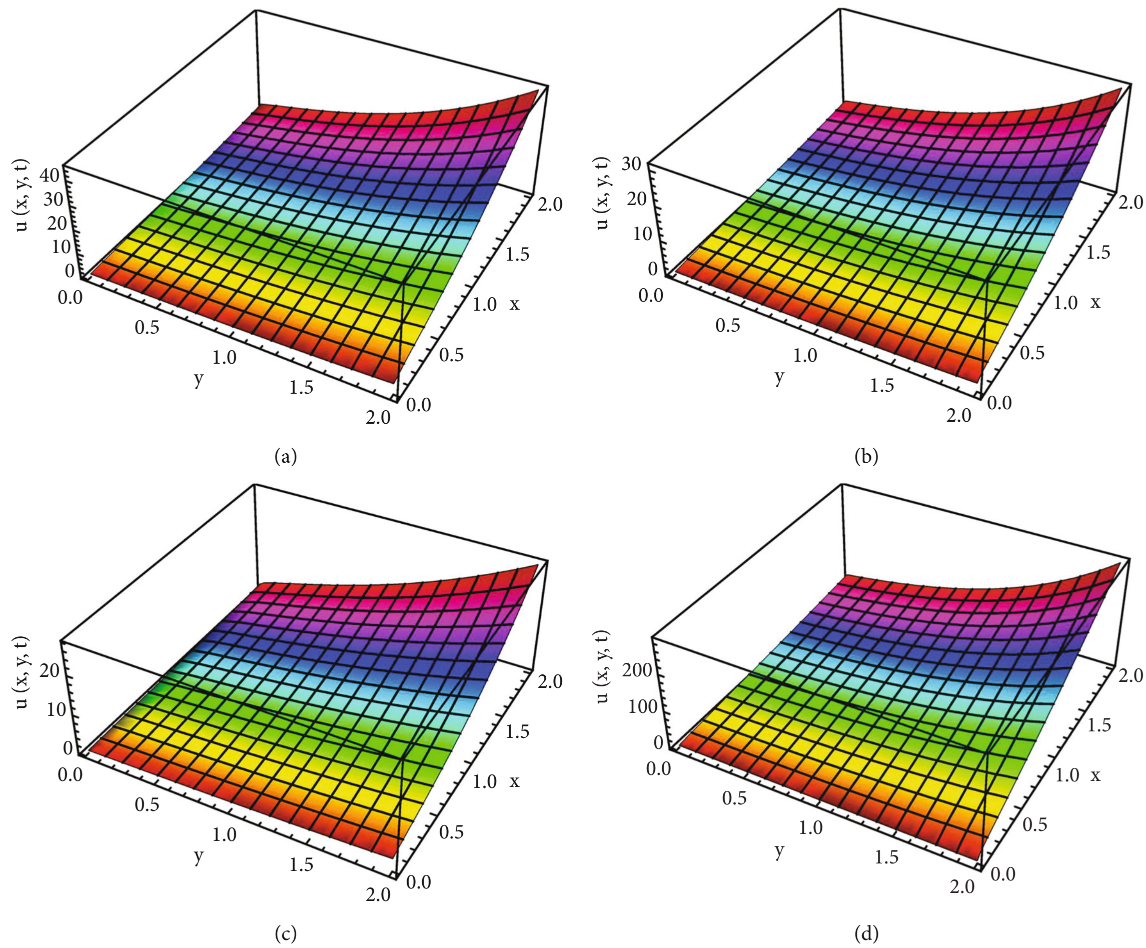

FIGURE 1: 3D solution plots of Example 1 at (a) $\alpha = 0.5, \beta = 0.8, \gamma = 0.6, t = 0.1$, (b) $\alpha = 0.6, \beta = 0.9, \gamma = 0.5, t = 0.4$, (c) $\alpha = 0.8, \beta = 0.5, \gamma = 0.7, t = 0.8$, and (d) $\alpha = 0.9, \beta = 1, \gamma = 0.2, t = 1$.

The 3D graphical simulation of exact solution corresponding to Example 1 for different fractional-order values α, β, and γ are depicted in Figure 1.

Example 2. Consider the time fractional-order nonlinear telegraph equation with the external source term as follows:

$$\begin{aligned} \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}} + 5\frac{\partial^{\alpha} u}{\partial t^{\alpha}} + u^3 = {} & \frac{\partial^{2\beta} u}{\partial x^{2\beta}} + \frac{\partial^{2\gamma} u}{\partial y^{2\gamma}} \\ & - 2e^{-t^{\alpha}/\alpha} \sin\left(\frac{x^{\beta}}{\beta}\right) \cos\left(\frac{y^{\gamma}}{\gamma}\right) \\ & + e^{-3t^{\alpha}/\alpha} \sin^3\left(\frac{x^{\beta}}{\beta}\right) \cos^3\left(\frac{y^{\gamma}}{\gamma}\right), \end{aligned} \tag{69}$$

under the initial conditions

$$\begin{aligned} u\left(\frac{x^{\beta}}{\beta}, \frac{y^{\gamma}}{\gamma}, 0\right) = \sin\left(\frac{x^{\beta}}{\beta}\right) \cos\left(\frac{y^{\gamma}}{\gamma}\right), \frac{\partial^{\alpha}}{\partial t^{\alpha}} u\left(\frac{x^{\beta}}{\beta}, \frac{y^{\gamma}}{\gamma}, 0\right) \\ = - \sin\left(\frac{x^{\beta}}{\beta}\right) \cos\left(\frac{y^{\gamma}}{\gamma}\right), \end{aligned} \tag{70}$$

and boundary conditions

$$u\left(0, \frac{y^{\gamma}}{\gamma}, \frac{t^{\alpha}}{\alpha}\right) = 0, \frac{\partial^{\beta}}{\partial x^{\beta}} u\left(0, \frac{y^{\gamma}}{\gamma}, \frac{t^{\alpha}}{\alpha}\right) = e^{-t^{\alpha}/\alpha} \cos\left(\frac{y^{\gamma}}{\gamma}\right), \tag{71}$$

$$u\left(\frac{x^{\beta}}{\beta}, 0, \frac{t^{\alpha}}{\alpha}\right) = e^{-t^{\alpha}/\alpha} \sin\left(\frac{x^{\beta}}{\beta}\right), \frac{\partial^{\gamma}}{\partial y^{\gamma}} u\left(\frac{x^{\beta}}{\beta}, 0, \frac{t^{\alpha}}{\alpha}\right) = 0. \tag{72}$$

Solution. On both sides of Equation (69), we use the properties of the conformable triple Laplace transform and Theorem (12) to obtain

$$\begin{aligned} s^2 U(k, p, s) - sU(k, p, 0) - \frac{\partial}{\partial t} U(k, p, 0) + 5(sU(k, p, s) - U(k, p, 0)) \\ = k^2 U(k, p, s) - kU(0, p, s) - \frac{\partial}{\partial x} U(0, p, s) + p^2 U(k, p, s) \\ - pU(k, 0, s) - \frac{\partial}{\partial y} U(k, 0, s) - \frac{2p}{(k^2+1)(p^2+1)(s+1)} \\ + L_x^{\beta} L_y^{\gamma} L_t^{\alpha}\left(e^{-3(t^{\alpha}/\alpha)} \sin^3\left(\frac{x^{\beta}}{\beta}\right) \cos^3\left(\frac{y^{\gamma}}{\gamma}\right) - u^3\left(\frac{x^{\beta}}{\beta}, \frac{y^{\gamma}}{\gamma}, \frac{t^{\alpha}}{\alpha}\right)\right). \end{aligned} \tag{73}$$

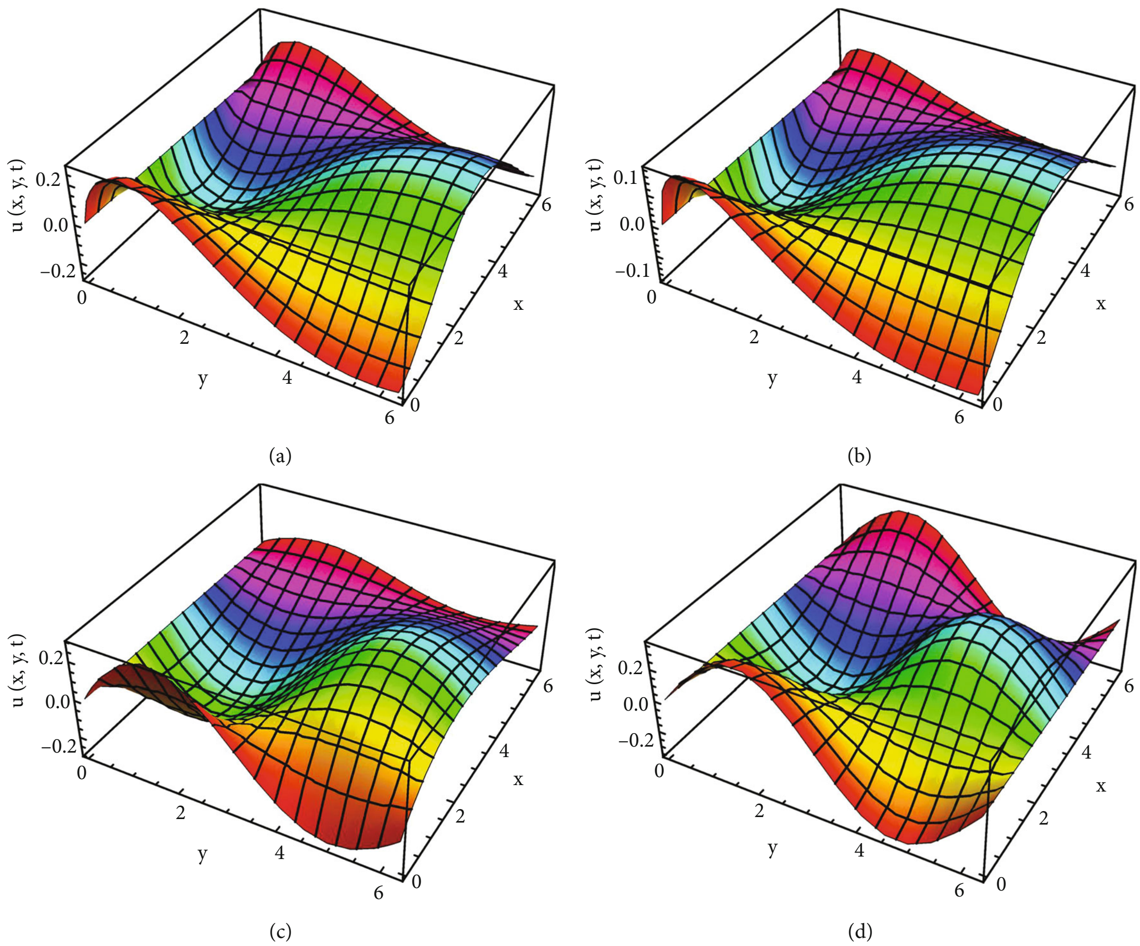

Figure 2: 3D solution plots of Example 1 at (a) $\alpha = 0.4, \beta = 0.5, \gamma = 0.7, t = 0.2$, (b) $\alpha = 0.5, \beta = 0.6, \gamma = 0.8, t = 0.4$, (c) $\alpha = 0.7, \beta = 0.9, \gamma = 0.5, t = 0.8$, and (d) $\alpha = 1, \beta = 1, \gamma = 0.9, t = 1$.

We get the following result by applying the conformable double Laplace transform to the initial condition (70):

$$U(k, p, 0) = \frac{p}{(k^2+1)(p^2+1)}, \frac{\partial}{\partial t} U(k, p, 0) = -\frac{p}{(k^2+1)(p^2+1)}. \tag{74}$$

The boundary conditions (70) and (71) are transformed using the conformable fractional double Laplace transform as follows:

$$U(0, p, s) = 0, \frac{\partial}{\partial x} U(0, p, s) = \frac{p}{(p^2+1)(s+1)}, \tag{75}$$

$$U(k, 0, s) = \frac{1}{(k^2+1)(s+1)}, \frac{\partial}{\partial y} U(k, 0, s) = 0. \tag{76}$$

Using Equations (74), (75), and (76) into the Equation (73), and simplifying, we get

$$U(k, p, s) = \frac{1}{s^2 + 5s - k^2 - p^2} \left\{ \begin{array}{c} \dfrac{p(s+5) - p}{(k^2+1)(p^2+1)} - \dfrac{p}{(p^2+1)(s+1)} - \dfrac{p}{(k^2+1)(s+1)} - \dfrac{2p}{(k^2+1)(p^2+1)(s+1)} \\ + L_x^\beta L_y^\gamma L_t^\alpha \left(e^{-3t^\alpha/\alpha} \sin^3 \left(\dfrac{x^\beta}{\beta} \right) \cos^3 \left(\dfrac{y^\gamma}{\gamma} \right) - u^3 \left(\dfrac{x^\beta}{\beta}, \dfrac{y^\gamma}{\gamma}, \dfrac{t^\alpha}{\alpha} \right) \right) \end{array} \right\}. \tag{77}$$

By simplifying Equation (77), we obtain

$$U(k,p,s)=\frac{p}{(k^2+1)(p^2+1)(s+1)}+\frac{1}{s^2+5s-k^2-p^2}\cdot\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(e^{-3t^\alpha/\alpha}\sin^3\left(\frac{x^\beta}{\beta}\right)\cos^3\left(\frac{y^\gamma}{\gamma}\right)-u^3\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right\}. \tag{78}$$

We get the following results by applying the inverse conformable triple-Laplace transform to both sides of Equation (78):

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=e^{-t^\alpha/\alpha}\sin\left(\frac{x^\beta}{\beta}\right)\cos\left(\frac{y^\gamma}{\gamma}\right)+L_{xyt}^{-1}\left[\frac{1}{s^2+5s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\cdot\left(e^{-3t^\alpha/\alpha}\sin^3\left(\frac{x^\beta}{\beta}\right)\cos^3\left(\frac{y^\gamma}{\gamma}\right)-u^3\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right\}\right]. \tag{79}$$

Now, apply the new iterative method to Equation (79). Substituting (49) into (79) and using (51), (52), and (53), we obtain the components of the solution as follows:

$$\begin{aligned}u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)&=e^{-t^\alpha/\alpha}\sin\left(\frac{x^\beta}{\beta}\right)\cos\left(\frac{y^\gamma}{\gamma}\right),\\ u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)&=L_{xyt}^{-1}\left[\frac{1}{s^2+5s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(e^{-3t^\alpha/\alpha}\sin^3\left(\frac{x^\beta}{\beta}\right)\cdot\cos^3\left(\frac{y^\gamma}{\gamma}\right)-{u_0}^3\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)\right\}\right]\\ &=L_{xyt}^{-1}\left[\frac{1}{s^2+5s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(e^{-3t^\alpha/\alpha}\sin^3\left(\frac{x^\beta}{\beta}\right)\cdot\cos^3\left(\frac{y^\gamma}{\gamma}\right)-e^{-3t^\alpha/\alpha}\sin^3\left(\frac{x^\beta}{\beta}\right)\cos^3\left(\frac{y^\gamma}{\gamma}\right)\right)\right\}\right]=0,\end{aligned} \tag{80}$$

$u_{m+1}((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))=L_{xyt}^{-1}[(1/s^2+5s-k^2-p^2)\{L_x^\beta L_y^\gamma L_t^\alpha(\sum_{r=0}^{i}{u_r}^3((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))-\sum_{r=0}^{i-1}{u_r}^3((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha)))\}]$, $n\geq 1$.As a result,

$$\begin{aligned}u_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)&=L_{xyt}^{-1}\left[\frac{1}{s^2+5s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(\begin{array}{c}\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^3\\-\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^3\end{array}\right)\right\}\right]=0,\\ u_3\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)&=L_{xyt}^{-1}\left[\frac{1}{s^2+5s-k^2-p^2}\left\{L_x^\beta L_y^\gamma L_t^\alpha\left(\begin{array}{c}\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_2\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^3\\-\left(u_0\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)+u_1\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)\right)^3\end{array}\right)\right\}\right]=0.\end{aligned} \tag{81}$$

Similarly, we get $u_4((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))=u_5((x^\beta/\beta),(y^\gamma/\gamma),(t^\alpha/\alpha))=0$ and so on.

Therefore, the solution to the Example 2 using Equation (54) is

$$u\left(\frac{x^\beta}{\beta},\frac{y^\gamma}{\gamma},\frac{t^\alpha}{\alpha}\right)=e^{-t^\alpha/\alpha}\sin\left(\frac{x^\beta}{\beta}\right)\cos\left(\frac{y^\gamma}{\gamma}\right). \tag{82}$$

Remark 16. For $\alpha=\beta=1$, in two dimensions, the conformable fractional telegraph equation (69) reduces to the classical (or non-fractional) space-time telegraph equation, and its exact solution is $u(x,t)=e^{-t}\sin(x)\cos(y)$, which is the same as the result obtained by [29, 30].

For $\beta=\gamma=1\&\alpha=0.5$, the exact solution is $u(x,y,t)=e^{x+y}(e^t+e^t(\text{erf}\,c(\sqrt{t}-1)))$.

The 3D graphical simulation of exact solution corresponding to Example 2 for different fractional-order values α, β, and γ are depicted in Figure 2.

7. Conclusion

The conformable triple Laplace transform has been examined in this study using all of our newly discovered results and theorems. The solution of the conformable fractional nonlinear Telegraph equation in two dimensions is found using the new conformable triple Laplace transform iterative approach. We give the basic definitions and properties of the conformable fractional derivative, conformable triple Laplace transform method, and the new iterative method. The proposed method CTLTIM has been put to the test in a numerical experiment, and the outcome is also supported by a 3D graphical representation for different values of fractional orders $\alpha, \beta,$ and γ as shown in Figures 1 and 2. Note that in Examples 1 and 2, if we use $\alpha = \beta = \gamma$, we obtain an exact solution which was considered in [28, 30, 49], and further, nontrivial problems that are solved using earlier methods become trivial in the sense that the decomposition,

$$u\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) = u_0\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) + u_1\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) + u_2\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) + \cdots + u_m\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) + \cdots, \tag{83}$$

consists of only one term, i.e.,

$$u\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right) = u_0\left(\frac{x^\beta}{\beta}, \frac{y^\gamma}{\gamma}, \frac{t^\alpha}{\alpha}\right). \tag{84}$$

Due to the critical need to explore new analytical solutions to understand the dynamics of solutions for such important equations in physics and engineering, our findings highlight the importance of exploring new generalized methods for solving fractional partial differential equations, mainly the nonlinear ones.

Data Availability

No data were used to support the study.

Conflicts of Interest

There are no conflicts of interest in the development of this research work, according to the author.

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