Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 6559779, 13 pages
https://doi.org/10.1155/2022/6559779

Research Article

Noninteger Derivative Order Analysis on Plane Wave Reflection from Electro-Magneto-Thermo-Microstretch Medium with a Gravity Field within the Three-Phase Lag Model

S. H. Elhag,[1] Fatimah S. Bayones,[1] A. A. Kilany,[2] S. M. Abo-Dahab,[3] Emad A.-B. Abdel-Salam,[4] M. Elsagheer,[2] and A. M. Abd-Alla[2]

[1]*Department of Mathematics and Statistics, College of Science, Taif University, PO. Box 11099, Taif 21944, Saudi Arabia*
[2]*Department of Mathematics Faculty of Science, Sohag University, Egypt*
[3]*Department of Computer Science, Faculty of Computers and Information, Luxor University, Egypt*
[4]*Department of Mathematics, Faculty of Science, New Valley University, Egypt*

Correspondence should be addressed to A. A. Kilany; arabyatef@yahoo.com

Received 27 April 2021; Revised 28 June 2021; Accepted 18 November 2021; Published 6 January 2022

Academic Editor: Younes Menni

The present research paper illustrates how noninteger derivative order analysis affects the reflection of partial thermal expansion waves under the generalized theory of plane harmonic wave reflection from a semivacuum elastic solid material with both gravity and magnetic field in the three-phase lag model (3PHL). The main goal for this study is investigating the fractional order impact and the applications related to the orders, especially in biology, medicine, and bioinformatics, besides the integer order considering an external effect, such as electromagnetic, gravity, and phase lags in a microstretch medium. The problem fractional form was formulated, and the boundary conditions were applied. The results were displayed graphically, considering the 3PHL model with magnetic field, gravity, and relaxation time. These findings were an explicit comparison of the effect of the plane wave reflection amplitude with integer derivative order analysis and noninteger derivative order analysis. The fractional order was compared to the correspondence integer order that indicated to the difference between them and agreement with the applications in biology, medicine, and other related topics. This phenomenon has more applications in relation to the biology and biomathematics problems.

1. Introduction

Recently, due attention has been made to study the interaction between electromagnetic field, thermal field, gravity, and the influence of microstretch due to its utilitarian features within different domains, such as physics, geophysics, geology, astronomy, and engineering. Choudhuri [1] proposed the problem of thermoelasticity theory with 3PHL. Kumar et al. [2] explored the plane strain problem within microstretch elastic solid. Kumar and Partap [3] examined the reflection of plane waves in a heat flux-dependent microstretch thermoelastic solid half space. Lord and Shulman [4] established generalized theories in which they replaced the Fourier law of thermal conductivity by Maxwell-Cattaneo law introducing a thermal relaxation time parameter within Fourier's law. Green and Lindsay [5] illustrated binary relaxation parameters at the constitutive relationships regarding the entropy and stress tensor. After some years, Green and Naghdi [6–8] introduced new three models known as GN-I, GN-II, and GN-III. They argued that the linearized form of model-I agrees with the classical thermoelastic one. In model-II, the internal production rate of is typically zero, suggesting no dissipation of thermal energy. The model discloses undamped thermoelastic waves in a thermoelastic material. It is primarily indicated as the theory of thermoelasticity in the absence of the dissipation of energy. The former models are combined in model-III as special cases. The model discloses energy dissipation generally. Abbas and Kumar [9] discussed deformation because of the heat source in micropolar generalized thermoelastic half-space using the finite element technique. The authors of [10] explored the stabile solutions concerning the 3PHL thermal conductivity. Utilizing the theory, Kar and Kanoria [11] and Banik and Kanoria [12] resolved various issues.

The initial stresses in solids considerably affect the mechanical response of the material from a primarily stressed configuration. They are widely applied in soft biological tissues' performance, engineering structures, and geophysics. Elsagheer and Abo-Dahab [13] discussed the reflection of thermoelastic waves from insulated boundary fiber-reinforced half-space affected by the magnetic field and rotation. Abo-Dahab et al. [14] employed the Lord-Shulman and dual-phase lag models to explore how rotation and gravity affect an electro-magneto-thermoelastic medium in the presence of diffusion and voids. Using three thermoelastic theories, the authors of [15] discussed SV-waves occurrence at the interface between solid-liquid media under the initial stress and electromagnetic field. Abo-Dahab et al. [16] studied the impact of rotation and gravity on the reflection of P-waves from thermo-magneto-microstretch medium within the 3PHL model with initial stress.

Several definitions were introduced concerning fractional derivatives. For instance, Riemann–Liouville's definition is characterized by the fractional derivative of constant is not zero and applied for the non-differentiable functions. Caputo's definition can be applied to differentiable functions, and zero is the value of the fractional derivative of constant [17].

The fractional differential equations' theory and applications were studied by Kilbas et al. [18]. Hilfer [17] discussed the uses of fractional calculus in physics. Katugampola [19] used a novel method to a generalized fractional integral. The authors of [20] discussed analytically solving the space-time fractional nonlinear Schrödinger equation. Abdel-Salam and Hassan [21] studied solving the class of linear and nonlinear fractional differential equations. Abdel-Salam and Yousif [22] discussed solving the nonlinear space-time fractional differential equations by the fractional Riccati expansion approach. The approximate solution to the fractional Lane-Emden kind equations was studied by Nouh and Abdel-Salam [23]. Examining the fractional derivative for natural convection in a slanted cavity having porous media was discussed by Ahmed et al. [24]. The authors of [25] discussed the impacts of the rotation, voids, magnetic field, and initial stress on plane waves in generalized thermoelasticity. Marin et al. [26] explained extending the domain of the influence theory concerning the generalized thermoelasticity of anisotropic material in the presence of voids. Saeed et al. [27] discussed the GL model on the thermoelastic interaction in a poroelastic material by the finite element approach. Abo-Dahab et al. [28] discussed the impact of the electromagnetic field in the fiber-reinforced micropolar thermoelastic medium in the context of four models.

Many authors in [29–36] studied various applications of fractional calculus in mathematical modeling with a comparison to constant cases with physics properties. The analysis of the fractional derivative order and temperature-dependent characteristics on P- and SV-waves reflection influenced by initial stress and 3PHL model were considered by Abo-Dahab et al. [37]. Alotaibi et al. [38] studied the fractional calculus of thermoelastic P-wave reflection influenced by the electromagnetic fields and gravity. Hobiny and Abbas [39] analytically solved the fractional order photo-thermoelasticity within a nonhomogenous semiconductor medium. Povstenko and Kyrylych [40] illustrated the fractional thermoelasticity issue concerning a plane with a line crack influenced by heat flux loading.

Many researchers in [41–47] investigated the effect of several variables in thermoelasticity with different models and archived all conditions to these models.

In this paper, the reflection of the plane waves from a semivacuum elastic solid material with the electromagnetic field and gravitational under the influence of relaxation times was studied. The necessary comparisons were made to simplify and explain the phenomenon at the fractional and non-fractional differentiations. The reflection appeared in the effect of the amplitude of the plane waves where the presence of the fractional differential was evident in each of the different results of the phenomenon. The fractional order was compared to the correspondence integer order that indicated the application and agreement with the applications in biology, bioinformatics, medicine, and related topics.

1.1. Formulation of the Problem. We utilized the generalized fractional thermo-microstretch's equations in a rectangular coordinate scheme (x, y, z) with z-axis directed into the media. We take the constant magnetic field intensity $H = (0, H_0, 0)$ to represent the y-axis direction. We consider with linearized equations of electrodynamics in the presence of displacement current due to motion as (Ezzat [48])

$$\begin{aligned}
J &= \operatorname{curl}_\alpha h - \varepsilon_0 D_t^\alpha E, \nabla_\alpha \\
&= D_x^\alpha \underline{i} + D_y^\alpha \underline{j} + D_z^\alpha \underline{k}, \operatorname{curl}_\alpha h \\
&= \nabla_\alpha \times h = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ D_x^\alpha & D_y^\alpha & D_z^\alpha \\ 0 & h_2 & 0 \end{vmatrix}, \\
\operatorname{curl}_\alpha E &= -\mu_0 D_t^\alpha h, \operatorname{curl}_\alpha E \\
&= \nabla_\alpha \times E = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ D_x^\alpha & D_y^\alpha & D_z^\alpha \\ E_1 & 0 & E_3 \end{vmatrix}, \\
E &= -\mu_0 \left(D_t^\alpha u \times H \right), D_t^\alpha u \times H = | \\
&\underline{i}\underline{j}\underline{k} D_t^\alpha u_1 0 D_t^\alpha u_3 0 H_0 0 |, \nabla_\alpha \cdot h = 0, D_t^\alpha h_2 = 0.
\end{aligned} \tag{1}$$

The fractional motion equation with gravitational and Lorentz's body forces as (Paria [49], Lord and Shulman [4])

$$D_\ell^\alpha \sigma_{\ell i} + F_i + G_i = \rho D_t^{\alpha\alpha} u_i, \tag{2}$$

where

$$F_i = \mu_0 (J \times H)_i, G_i = \rho g (D_x^\alpha u_3, 0, -D_z^\alpha u_1). \tag{3}$$

Using the previous equations, the following equations are obtained:

$$E=\mu_0 H_0(D_t^{\alpha}u_3,0,-D_t^{\alpha}u_1), \tag{4}$$

$$h=(0,h_2,0)=(0,h,0)=(0,-H_0e,0)e=D_x^{\alpha}u_1+D_z^{\alpha}u_3, \tag{5}$$

$$\begin{aligned}J&=(-D_z^{\alpha}h-\varepsilon_0\mu_0H_0D_t^{\alpha\alpha}u_3,0,D_x^{\alpha}h+\varepsilon_0\mu_0H_0D_t^{\alpha\alpha}u_1)\\&=H_0(-D_z^{\alpha}D_x^{\alpha}u_1-D_z^{\alpha\alpha}u_3-\varepsilon_0\mu_0D_t^{\alpha\alpha}u_3,0,D_x^{\alpha\alpha}u_1\\&\quad+D_x^{\alpha}D_z^{\alpha}u_3+\varepsilon_0\mu_0D_t^{\alpha\alpha}u_1).\end{aligned} \tag{6}$$

From equations (3) and (5), we obtain

$$\begin{aligned}F&=(F_1^{\alpha},F_2^{\alpha},F_3^{\alpha})\\&=\left(\mu_0H_0^2D_x^{\alpha}e-\varepsilon_0\mu_0^2H_0^2D_t^{\alpha\alpha}u_1,0,\mu_0H_0^2D_z^{\alpha}e-\varepsilon_0\mu_0^2H_0^2D_t^{\alpha\alpha}u_3\right)\\&=\mu_0H_0^2(D_x^{\alpha\alpha}u_1+D_x^{\alpha}D_z^{\alpha}u_3-\varepsilon_0\mu_0D_t^{\alpha\alpha}u_1,0,D_z^{\alpha}D_x^{\alpha}u_1\\&\quad+D_z^{\alpha\alpha}u_3-\varepsilon_0\mu_0D_t^{\alpha\alpha}u_3).\end{aligned} \tag{7}$$

So, the displacement vector $u=(u_1,u_2,u_3)$ has the components $u_1=u_1(x,z,t), u_2=0, u_3=u_3(x,z,t)$.

The basic governing equations become

$$\begin{aligned}&(\lambda+\mu)(D_x^{\alpha\alpha}u_1+D_x^{\alpha}D_z^{\alpha}u_3)+(\mu+k)(D_x^{\alpha\alpha}u_1+D_z^{\alpha\alpha}u_1)\\&\quad-kD_z^{\alpha}\varphi_2+\lambda_0D_x^{\alpha}\varphi^*-\widehat{\gamma}\,D_x^{\alpha}T+F_1+\rho gD_x^{\alpha}u_3=\rho D_t^{\alpha\alpha}u_1,\end{aligned} \tag{8}$$

$$\begin{aligned}&(\lambda+\mu)(D_x^{\alpha\alpha}u_3+D_x^{\alpha}D_z^{\alpha}u_1)+(\mu+k)(D_x^{\alpha\alpha}u_3+D_z^{\alpha\alpha}u_3)\\&\quad+kD_x^{\alpha}\varphi_2+\lambda_0D_z^{\alpha}\varphi^*-\widehat{\gamma}\,D_z^{\alpha}T+F_3-\rho gD_z^{\alpha}u_1=\rho D_t^{\alpha\alpha}u_3,\end{aligned} \tag{9}$$

$$\begin{aligned}&(a+b+c)\nabla_\alpha(\nabla_\alpha\cdot\varphi)-c\nabla_\alpha\times(\nabla_\alpha\times\varphi)+k(\nabla_\alpha\times u)\\&\quad-2k\varphi=\rho jD_t^{\alpha\alpha}\varphi,\end{aligned} \tag{10}$$

$$\alpha_0\nabla_\alpha^2\varphi^*-\frac{1}{3}b_1\varphi^*-\frac{1}{3}b_0(\nabla_\alpha\cdot u)+\frac{1}{3}\widehat{\gamma}_1T=\frac{3}{2}\rho\,jD_t^{\alpha\alpha}\varphi^*, \tag{11}$$

$$\begin{aligned}&(K^*+\tau_v^*D_t^{\alpha}+K\tau_TD_t^{\alpha\alpha})\nabla_\alpha^2T=\left(1+\tau_qD_t^{\alpha}+\frac{\tau_q^2}{2}D_t^{\alpha\alpha}\right)\\&\quad\cdot[\rho C_eD_t^{\alpha\alpha}T+\widehat{\gamma}T_0D_t^{\alpha\alpha}e+\widehat{\gamma}_1T_0D_t^{\alpha\alpha}\varphi^*].\end{aligned} \tag{12}$$

Such that $\tau_v^*=K+K^*\tau_v, \nabla_\alpha^2=D_x^{\alpha\alpha}+D_z^{\alpha\alpha}$.

$$\begin{aligned}\sigma_{i\ell}&=(b_0\varphi^*+\lambda D_r^{\alpha}u_r)\delta_{i\ell}+(\mu+k)D_i^{\alpha}u_\ell+\mu D_\ell^{\alpha}u_i\\&\quad-k\varepsilon_{i\ell r}\varphi_r-\widehat{\gamma}T\delta_{i\ell},\\m_{i\ell}&=aD_r^{\alpha}\varphi_r\delta_{i\ell}+bD_\ell^{\alpha}\varphi_i+cD_i^{\alpha}\varphi_\ell,\\\lambda_i&=a_0D_i^{\alpha}\varphi^*.\end{aligned} \tag{13}$$

The constitutive relations take the following form:

$$\begin{aligned}\sigma_{xx}&=(\lambda+2\mu+k)D_x^{\alpha}u_1+\lambda D_z^{\alpha}u_3+b_0\varphi^*-\widehat{\gamma}T,\\\sigma_{zz}&=(\lambda+2\mu+k)D_z^{\alpha}u_3+\lambda D_x^{\alpha}u_1+b_0\varphi^*-\widehat{\gamma}T,\\\sigma_{xz}&=\mu D_z^{\alpha}u_1+(\mu+k)D_x^{\alpha}u_3+k\varphi_2,\sigma_{zx}\\&=\mu D_x^{\alpha}u_3+(\mu+k)D_z^{\alpha}u_1-k\varphi_2,\\m_{xy}&=cD_x^{\alpha}\varphi_2,m_{zy}=cD_z^{\alpha}\varphi_2,\\\lambda_x&=a_0D_x^{\alpha}\varphi^*,\lambda_z=a_0D_z^{\alpha}\varphi^*.\end{aligned} \tag{14}$$

Moreover, we introduce these dimensionless quantities to make the solution easier

$$\begin{aligned}x_i'&=\frac{\omega^*}{c_0}x_i,u_i'=\frac{\rho c_0\omega^*}{\widehat{\gamma}\,T_0}u_i,\Theta'\\&=\frac{\widehat{\gamma}}{\rho c_0^2}(T-T_0),\left\{t',\tau_T{}',\tau_v{}',\tau_q{}'\right\}\\&=\omega^*\{t,\tau_T,\tau_v,\tau_q\},\sigma_{ij}{}'\\&=\frac{\sigma_{ij}}{\widehat{\gamma}T_0},m_{ij}'=\frac{\omega^*}{c_0\widehat{\gamma}T_0}m_{ij},\lambda_i'=\frac{\omega^*}{c_0\widehat{\gamma}T_0}\lambda_i,\varphi'^*\\&=\frac{\rho c_0^2}{\widehat{\gamma}\,T_0}\varphi^*,\varphi_2'=\frac{\rho c_0^2}{\widehat{\gamma}\,T_0}\varphi_2,g'=\frac{g}{c_0\omega^*},\omega^*\\&=\frac{\rho C_ec_0^2}{k},h'=\frac{h}{H_0},\end{aligned} \tag{15}$$

$$c_0^2=\frac{\lambda+2\mu+k}{\rho},i,j=1,2,3. \tag{16}$$

Utilizing Equation (16), the governing Equations (8)–(12) recast in this form (after suppressing the primes)

$$\begin{aligned}&\left(\frac{\mu+k}{\rho\,c_0^2}\right)\nabla_\alpha^2u_1+\left(\frac{\mu+\lambda}{\rho\,c_0^2}+R_H\right)D_x^{\alpha}e-\frac{k}{\rho\,c_0^2}D_z^{\alpha}\varphi_2+\frac{\lambda_0}{\rho\,c_0^2}D_x^{\alpha}\varphi^*\\&\quad-\frac{\rho\,c_0^2}{\widehat{\gamma}T_0}D_x^{\alpha}\Theta+gD_x^{\alpha}u_3=\beta^2D_t^{\alpha\alpha}u_1,\end{aligned} \tag{17}$$

$$\begin{aligned}&\left(\frac{\mu+k}{\rho\,c_0^2}\right)\nabla_\alpha^2u_3+\left(\frac{\mu+\lambda}{\rho\,c_0^2}+R_H\right)D_z^{\alpha}e+\frac{k}{\rho\,c_0^2}D_x^{\alpha}\varphi_2+\frac{\lambda_0}{\rho\,c_0^2}D_z^{\alpha}\varphi^*\\&\quad-\frac{\rho\,c_0^2}{\widehat{\gamma}T_0}D_z^{\alpha}\Theta-gD_x^{\alpha}u_1=\beta^2D_t^{\alpha\alpha}u_3,\end{aligned} \tag{18}$$

$$\nabla_\alpha^2\varphi_2-\frac{2kc_0^2}{c_2\omega^{*2}}\varphi_2+\frac{kc_0^2}{c_2\omega^{*2}}(D_z^{\alpha}u_1-D_x^{\alpha}u_3)=\frac{\rho jc_0^2}{c_2}D_t^{\alpha\alpha}\varphi_2, \tag{19}$$

$$\left(\frac{C_1^2}{c_0^2}\nabla_\alpha^2-\frac{C_2^2}{\omega^{*2}}-D_t^{\alpha\alpha}\right)\varphi^*-\frac{C_3^2}{\omega^{*2}}e+C_4\Theta=0, \tag{20}$$

$$(C_k + C_v D_t^{\alpha} + C_T D_t^{\alpha\alpha})\nabla_\alpha^2 \Theta = \left(1 + \tau_q D_t^{\alpha} + \frac{\tau_q^2}{2} D_t^{\alpha\alpha}\right) \cdot [D_t^{\alpha\alpha}\Theta + \varepsilon_1 D_t^{\alpha\alpha} e + \varepsilon_2 D_t^{\alpha\alpha}\varphi^*], \tag{21}$$

$$\sigma_{xx} = a_1\varphi^* + D_x^{\alpha} u_1 + a_2 D_z^{\alpha} u_3 - a_3\Theta - 1, \tag{22}$$

$$\sigma_{zz} = a_1\varphi^* + D_z^{\alpha} u_3 + a_2 D_x^{\alpha} u_1 - a_3\Theta - 1, \tag{23}$$

$$\sigma_{xz} = a_4 D_z^{\alpha} u_1 + a_5 D_x^{\alpha} u_3 + a_6\,\varphi_2, \tag{24}$$

$$\sigma_{zx} = a_4 D_z^{\alpha} u_3 + a_5 D_x^{\alpha} u_1 - a_6\varphi_2, \tag{25}$$

$$m_{xy} = a_7 D_x^{\alpha}\varphi_2, \tag{26}$$

$$m_{zy} = a_7 D_z^{\alpha}\varphi_2, \tag{27}$$

$$\lambda_x = a_8 D_x^{\alpha}\varphi^*, \tag{28}$$

$$\lambda_z = a_8 D_z^{\alpha}\varphi^*, \tag{29}$$

where

$$\begin{aligned} R_H &= \frac{\mu_0 H_0^2}{\rho c_0^2}, b^2 = \left(1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho}\right), (C_1^2, C_2^2, C_3^2) \\ &= \frac{2}{9\rho j}(3a_0, b_1, b_0), C_4 = \frac{2\rho\, c_0^4 \widehat{\gamma}_1}{9\, j\gamma\wedge^2 T_0 \omega^{*2}} \\ c_2 &= (2a + 2b + c), C = \frac{2\, c_0^2 \widehat{\gamma}_1}{9\, j\widehat{\gamma}\, \omega^{*2}}, (C_k, C_v, C_T) \\ &= \frac{1}{\rho c_0^2 C_e}(K^*, \tau_v^*, K\tau_T\omega^*), \varepsilon_1 = \frac{\gamma\wedge^3 T_0}{\rho^3 c_0^4 C_e}, \\ \varepsilon_2 &= \frac{\widehat{\gamma}_1 \gamma\wedge^2 T_0^2}{\rho^3 c_0^4 C_e}, (a_1, a_2) = \frac{1}{\rho c_0^2}(b_0, \lambda), a_3 \\ &= \frac{\rho c_0^2}{\widehat{\gamma}\, T}, (a_4, a_5, a_6) = \frac{1}{\rho c_0^2}(\mu, \mu + k, k), \\ (a_7, a_8) &= \frac{\omega^{*2}}{\rho c_0^4}(c, a_0). \end{aligned} \tag{30}$$

The components of the displacement $u_1(x, z, t)$ and $u_3(x, z, t)$ take the following form concerning the potential functions $\Phi(x, z, t)$ and $\Psi(x, z, t)$

$$u_1 = D_x^{\alpha}\Phi + D_z^{\alpha}\Psi, u_3 = D_z^{\alpha}\Phi - D_x^{\alpha}\Psi, \vec{\Psi} = (0, -\Psi, 0). \tag{31}$$

Using Equation (31) in Equations (17)–(21), we get

$$\left(\nabla_\alpha^2 - \zeta_0 D_t^{\alpha\alpha}\right)\Phi - \zeta_1 D_x^{\alpha}\Psi - \zeta_2\Theta + \zeta_3\varphi^* = 0, \tag{32}$$

$$\zeta_4 D_x^{\alpha}\Phi + \left(\nabla_\alpha^2 - \zeta_5 D_t^{\alpha\alpha}\right)\Psi - \zeta_6\varphi_2 = 0, \tag{33}$$

$$\frac{kc_0^2}{c\omega^{*2}}\nabla_\alpha^2\Psi + \left(\nabla_\alpha^2 - \frac{2kc_0^2}{c\omega^{*2}} - \frac{j\rho c_0^2}{c} D_t^{\alpha\alpha}\right)\varphi_2 = 0, \tag{34}$$

$$\left(\frac{C_1^2}{c_0^2}\nabla_\alpha^2 - \frac{C_2^2}{\omega^{*2}} - D_t^{\alpha\alpha}\right)\varphi^* - \frac{C_3^2}{\omega^{*2}}\nabla_\alpha^2\Phi + a_0\Theta = 0, \tag{35}$$

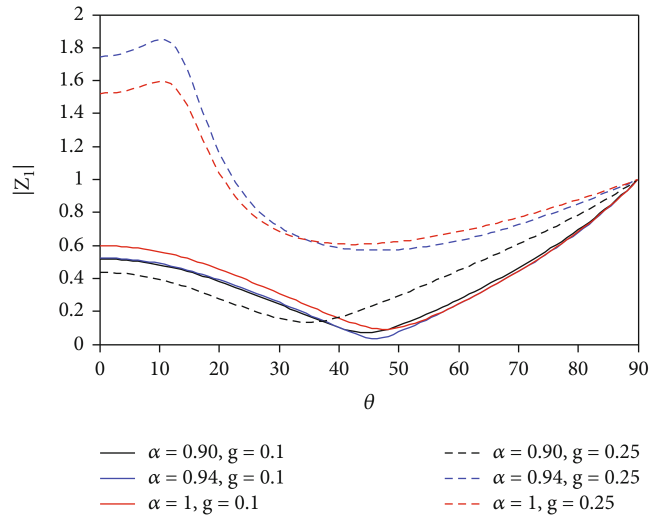

FIGURE 1: The various values of the amplitudes $|z_1|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of gravity g.

$$C_k\nabla_\alpha^2\Theta + C_v\nabla_\alpha^2(D_t^{\alpha}\Theta) + C_T\nabla_\alpha^2(D_t^{\alpha\alpha}\Theta) = \left(1 + \tau_q D_t^{\alpha} + \frac{\tau_q^2}{2} D_t^{\alpha\alpha}\right) \cdot \left[D_t^{\alpha\alpha}\Theta + \varepsilon_1\nabla_\alpha^2(D_t^{\alpha\alpha}\Phi) + \varepsilon_2 D_t^{\alpha\alpha}\varphi^*\right], \tag{36}$$

where

$$(\zeta_0, \zeta_1, \zeta_2, \zeta_3) = \frac{1}{1 + R_H}\left(b^2, g, \frac{\rho c_0^2}{\widehat{\gamma} T_0}, \frac{b_0}{\rho c_0^2}\right), (\zeta_4, \zeta_5, \zeta_6) = \frac{\rho c_0^2}{\mu + k}\left(g, b^2, \frac{k}{\rho c_0^2}\right). \tag{37}$$

1.2. Solution of the Problem. In this section, solving Equations (32)–(36) is assumed as

$$(\Phi, \Psi, \Theta, \varphi^*, \varphi_2) = \left(\bar{\Phi}, \bar{\Psi}, \bar{\Theta}, \overline{\varphi^*}, \overline{\varphi_2}\right) \exp \cdot [i\xi(x^{\alpha}\sin\theta + z^{\alpha}\cos\theta) - i\omega t^{\alpha}]. \tag{38}$$

Substituting from Equation (38) into Equtions (32)–(36), the result is

$$\alpha^2\left(-\xi^2 + \zeta_0\omega^2\right)\bar{\Phi} - i\alpha\zeta_1\xi\sin\theta\bar{\Psi} - \zeta_2\bar{\Theta} + \zeta_3\overline{\varphi^*} = 0, \tag{39}$$

$$i\alpha\xi\zeta_4\sin\theta\bar{\Phi} + \alpha^2\left(-\xi^2 + \omega^2\zeta_5\right)\bar{\Psi} - \zeta_6\overline{\varphi_2} = 0, \tag{40}$$

$$-\frac{k\alpha^2\xi^2 c_0^2}{c\omega^{*2}}\bar{\Psi} + \left(-\alpha^2\xi^2 + \frac{j\rho c_0^2\omega^2\alpha^2}{c} - \frac{2kc_0^2}{c\omega^{*2}}\right)\overline{\varphi_2} = 0, \tag{41}$$

$$\frac{\alpha^2\xi^2C_3^2}{\omega^{*2}}\bar{\Phi}+a_0\bar{\Theta}+\left(-\frac{\alpha^2\xi^2C_1^2}{c_0^2}-\frac{C_2^2}{\omega^{*2}}+\alpha^2\omega^2\right)\overline{\varphi^*}=0, \tag{42}$$

$$\begin{aligned}&-\varepsilon_1\alpha^4\xi^2\omega^2\tau_q^*\bar{\Phi}+\left[\left(-C_k+iC_v\alpha\omega+C_T\alpha^2\omega^2\right)\alpha^2\xi^2+\alpha^2\omega^2\tau_q^*\right]\bar{\Theta}\\&+\varepsilon_2\alpha^2\omega^2\tau_q^*\bar{\varphi}^*=0.\end{aligned} \tag{43}$$

where $\tau_q^*=1-i\alpha\omega\tau_q-((\alpha^2\omega^2\tau_q^2)/2)$.

To help identify the nontrivial solution, we change from Equation (39) into Equations (33)–(38) and get

$$\begin{vmatrix}\alpha^2\left(\zeta_0\omega^2-\xi^2\right) & -i\alpha\zeta_1\xi\sin\theta & -\zeta_2 & \zeta_3 & 0\\ i\alpha\xi\zeta_4\sin\theta & \alpha^2\left(-\xi^2+\omega^2\zeta_5\right) & 0 & 0 & -\zeta_6\\ 0 & -\dfrac{k\alpha^2\xi^2c_0^2}{c\omega^{*2}} & 0 & 0 & \alpha^2\left(\dfrac{j\rho c_0^2\omega^2}{b}-\xi^2\right)-\dfrac{2kc_0^2}{b\omega^{*2}}\\ \dfrac{\alpha^2\xi^2C_3^2}{\omega^{*2}} & 0 & a_0 & \alpha^2\left(\omega^2-\dfrac{C_1^2\xi^2}{C_0^2}\right)-\dfrac{C_2^2}{\omega^{*2}} & 0\\ -\varepsilon_1\alpha^4\xi^2\omega^2\tau_q^* & 0 & \alpha^2\xi^2\left(i\alpha\omega C_v+\alpha^2\omega^2C_T-C_k\right)+\alpha^2\omega^2\tau_q^* & \varepsilon_2\alpha^2\omega^2\tau_q^* & 0\end{vmatrix}=0, \tag{44}$$

which tends to

$$L\alpha^{10}\xi^{10}+M\alpha^8\xi^8+N\alpha^6\xi^6+O\alpha^4\xi^4+P\alpha^2\xi^2+Q=0. \tag{45}$$

where L, M, N, O, P, and Q are in the Appendix.

From Equation (45), we can identify five diverse velocities accompanying five waves.

After that, $\Phi,\Psi,\Theta,\varphi^*$, and φ_2 are written as

$$\begin{aligned}&(\Phi,\Psi,\Theta,\varphi^*,\varphi_2)=(1,\eta_0,\kappa_0,\chi_0,\vartheta_0)A_0\exp\\&\quad\cdot\left[i\xi(x^\alpha\sin\theta+z^\alpha\cos\theta)-i\omega t^\alpha\right]\\&\sum_{j=1}^{5}\left(1,\eta_j,\kappa_j,\chi_j,\vartheta_j\right)A_j\exp\left[i\xi_j\left(x^\alpha\sin\theta_j-z^\alpha\cos\theta_j\right)-i\omega t^\alpha\right].\end{aligned} \tag{46}$$

Based on Equations (40) and (41), the result is

$$\begin{aligned}\eta_j&=\frac{-\iota\xi_j\zeta_4\sin\theta_j\left(\xi_j^2-\left((j\rho c_0^2\alpha^2\omega^2)/\hat{\gamma}\right)+\left(2kc_0^2/c\omega^{*2}\right)\right)}{\left(\xi_j^2-\alpha^2\omega^2\zeta_5\right)\left(\xi_j^2-\left((j\rho c_0^2\alpha^2\omega^2)/\hat{\gamma}\right)+\left(2kc_0^2/c\omega^{*2}\right)\right)-p^*\xi_j^2\zeta_6},\kappa_j\\&=\frac{ip^*\xi_j^3\zeta_4\sin\theta_j}{\left(\xi_j^2-\alpha^2\omega^2\zeta_5\right)\left(\xi_j^2-\left((j\rho c_0^2\alpha^2\omega^2)/\hat{\gamma}\right)+\left(2kc_0^2/c\omega^{*2}\right)\right)-p^*\xi_j^2\zeta_6},\end{aligned} \tag{47}$$

where $p^*=kc_0^2/\gamma\omega^{*2}$.

Also, from Equations (42) and (43), we get

$$\chi_j=\frac{-\varepsilon_2\alpha^2\omega^2\tau_q^*\xi_j^2\left(C_3^2/\omega^{*2}\right)-\varepsilon_1\alpha^2\omega^2\tau_q^*\xi_j^2\left[-\left(C_1^2/c_0^2\right)\xi_j^2-\left(C_2^2/\omega^{*2}\right)+\alpha^2\omega^2\right]}{a_0\varepsilon_2\alpha^2\omega^2\tau_q^*-\left[\left(-C_k-i\alpha\omega C_v+\alpha^2\omega^2C_T\right)\xi_j^2+\omega^2\tau_q^*\right]\left[-\left(C_1^2/c_0^2\right)\xi_j^2-\left(C_2^2/\omega^{*2}\right)+\alpha^2\omega^2\right]},$$

$$\vartheta_j=\frac{a_0\varepsilon_1\alpha^2\omega^2\tau_q^*\xi_j^2+\left(C_3^2/\omega^{*2}\right)\xi_j^2\left[\left(-C_k-i\alpha\omega C_v+\alpha^2\omega^2C_T\right)\xi_j^2+\alpha^2\omega^2\tau_q^*\right]}{a_0\varepsilon_2\alpha^2\omega^2\tau_q^*-\left[\left(-C_k-i\alpha\omega C_v+\alpha^2\omega^2C_T\right)\xi_j^2+\alpha^2\omega^2\tau_q^*\right]\left[-\left(C_1^2/c_0^2\right)\xi_j^2-\left(C_2^2/\omega^{*2}\right)+\alpha^2\omega^2\right]}. \tag{48}$$

1.3. The Boundary Conditions. The problem's boundary conditions are written as follows:

$$\sigma_{zz}+\tau_{zz}=0,\sigma_{xz}+\tau_{xz}=0,D_z^\alpha T=0,m_{yz}=0,\lambda_x=0atz=0, \tag{49}$$

$$\begin{aligned}&\sum_{j=1}^{5}\left[a_1\vartheta_j-\mu_eH_0^2\xi_j^2-a_3\xi_j^2\left(\sin^2\theta_j+\frac{\eta_j}{2}\sin 2\theta_j\right)-a_2\xi_j^2\left(\cos^2\theta_j-\frac{\eta_j}{2}\sin 2\theta_j+\frac{k_j}{\xi_j^2}\right)\right]A_j\\&=\left[a_1\vartheta_1-\mu_eH_0^2\xi_1^2-a_3\xi_1^2\left(\sin^2\theta_0+\frac{\eta_1}{2}\sin 2\theta_0\right)-a_2\xi_1^2\left(\cos^2\theta_0-\frac{\eta_1}{2}\sin 2\theta_0+\frac{k_1}{\xi_1^2}\right)\right]A_0,\end{aligned} \tag{50}$$

$$\begin{aligned}&\sum_{j=1}^{5}\left[(a_4+a_5)\frac{\xi_j^2}{2}\sin 2\theta_j+a_4\xi_j^2\eta_j\cos^2\theta_j-a_5\xi_j^2\eta_j\sin^2\theta_j+a_6\xi_j\right]A_j\\&=\left[-(a_4+a_5)\frac{\xi_1^2}{2}\sin 2\theta_0+a_4\xi_1^2\eta_1\cos^2\theta_0-a_5\xi_1^2\eta_1\sin^2\theta_0+a_6\xi_1\right]A_0,\end{aligned} \tag{51}$$

$$\sum_{j=1}^{5}k_j\xi_j\cos\theta_jA_j=k_1\xi_1\cos\theta_0A_0, \tag{52}$$

$$\sum_{j=1}^{5}\zeta_j\xi_j\cos\theta_jA_j=\zeta_1\xi_1\cos\theta_0A_0. \tag{53}$$

Finally,

$$\sum_{j=1}^{5}\vartheta_j\xi_j\cos\theta_jA_j=-\vartheta_1\xi_1\cos\theta_0A_0. \tag{54}$$

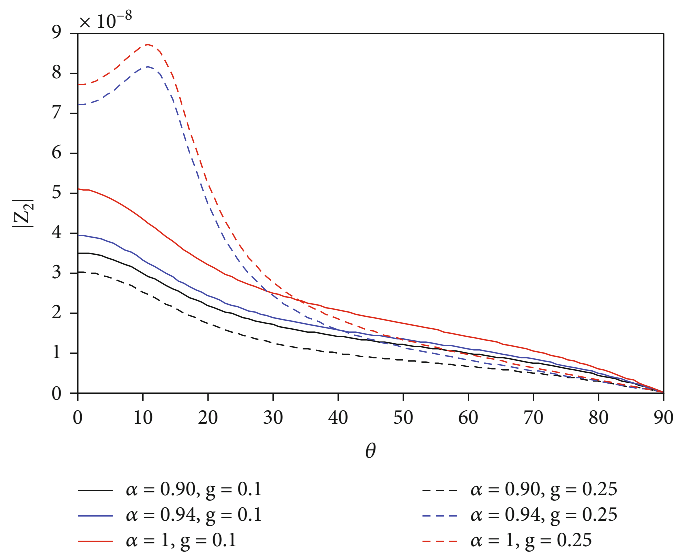

FIGURE 2: The various values of the amplitudes $|z_2|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of the values of gravity$g$.

Based on Equations (50)–(54), the result is

$$\begin{aligned}
&a_{ij}Z_j = B_i, i, j = (1, 2, \cdots, 5), Z_j = \frac{A_j}{A_0},\\
&a_{1j} = a_1\vartheta_j - \xi_j^2\left[a_3\left(\sin^2\theta_j + \frac{\eta_j}{2}\sin 2\theta_j\right) - a_2\left(\cos^2\theta_j - \frac{\eta_j}{2}\sin 2\theta_j + \frac{k_j}{\xi_j^2}\right) + \mu_e H_0^2\right],\\
&a_{2j} = (a_4 + a_5)\frac{\xi_j^2}{2}\sin 2\theta_j + a_4\xi_j^2\eta_j\cos^2\theta_j - a_5\xi_j^2\eta_j\sin^2\theta_j + a_6\xi_j,\\
&a_{3j} = k_j\xi_j\cos\theta_j, a_{4j} = \zeta_j\xi_j\cos\theta_j A_j, a_{5j} = \vartheta_j\xi_j\cos\theta_j,\\
&B_1 = -a_1\vartheta_1 + \xi_j^2\left[a_3\left(\sin^2\theta_0 + \frac{\eta_1}{2}\sin 2\theta_0\right) - a_2\left(\cos^2\theta_0 - \frac{\eta_1}{2}\sin 2\theta_0 + \frac{k_j}{\xi_j^2}\right) + \mu_e H_0^2\right],\\
&B_2 = -(a_4 + a_5)\frac{\xi_1^2}{2}\sin 2\theta_0 - a_4\xi_1^2\eta_1\cos^2\theta_1 + a_5\xi_1^2\eta_1\sin^2\theta_1 - a_6\xi_1,\\
&B_3 = k_1\xi_1\cos\theta_0, B_4 = \zeta_1\xi_1\cos\theta_0, B_5 = -\vartheta_1\xi_1\cos\theta_0.
\end{aligned} \tag{55}$$

1.4. Numerical Results and Discussion. In this section, some numerical findings are discussed based on the illustration of the results obtained in the former sections and a comparison of such results in different cases.

Therefore, we chose these materials:

$$i = \sqrt{-1}, a_0 = 0.779 \times 10^{-4}, b_0 = 0.5 \times 10^{11}, b_1 = 0.5 \times 10^{11}, j = 0.2 \times 10^{-15}, \rho = 8954,$$

$$C_e = 383.1, k = 386, T_0 = 293, \lambda = 7.76 \times 10^{10}, \mu = 3.86 \times 10^{10}, c = 0.779 \times 10^{-4},$$

$$K^* = 2.97 \times 10^{13}, \mu_0 = 0.1, \varepsilon_0 = 0.1, \omega = 0.034, \tau_T = 0.2, \tau_v = 0.1, \tau_q = 0.5. \tag{56}$$

Figure 1 highlights the various values of the amplitudes $|z_1|$ concerning the angle of incidence of the reflected waves θ for various values of the fractional parameter of two different values of gravity g. Clearly, the amplitudes of incidence waves $|z_1|$ decline when rising the gravity field. They decline then rise with the increase of θ to reach the unit at $\theta = 90^\circ$.

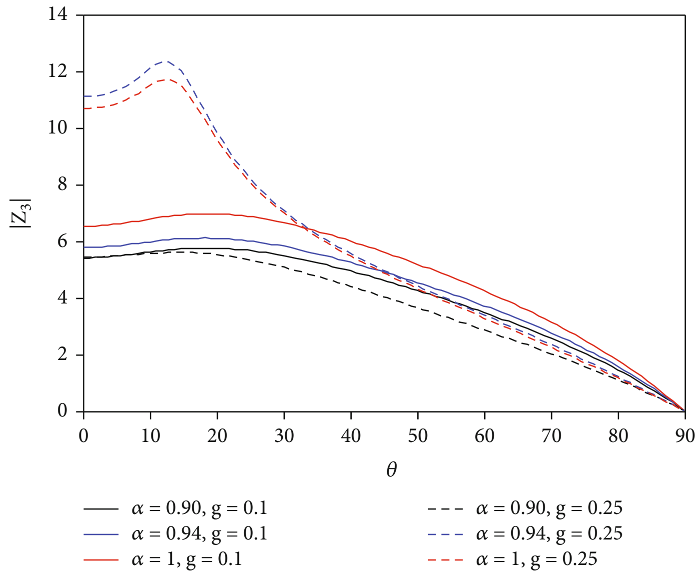

FIGURE 3: The various values of the amplitudes $|z_3|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of gravity g.

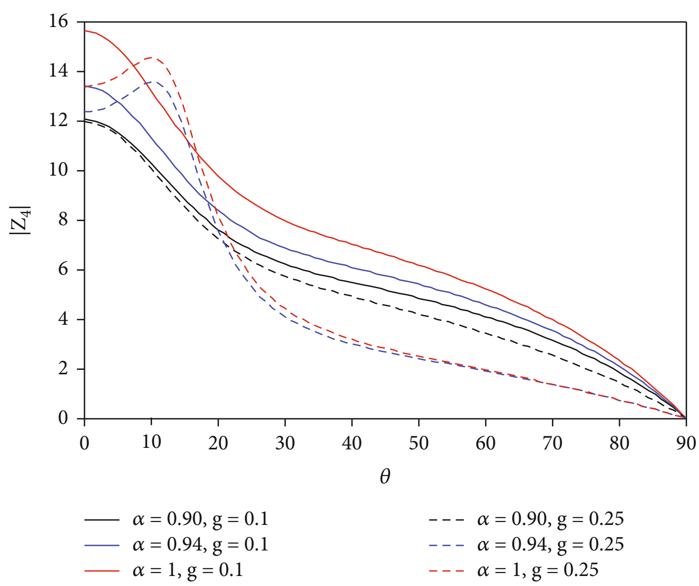

FIGURE 4: The various values of the amplitudes $|z_4|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of gravity g.

In the case of $g = 0.1$, the amplitude of the wave is less than in the case of $g = 0.25$, indicating that the greater the effect of gravity leads to an increase in the wave's amplitude.

Figure 2 displays the various values of amplitudes $|z_2|$ concerning the angle of incidence of the reflected waves θ for various values of the fractional parameter for the two diverse values of gravity g that shows the oscillatory performance in the entire range of angle θ. Clearly, the amplitudes of reflection waves $|z_2|$ rise when rising the gravity field, which rises then declines with increasing θ approaches zero at $\theta = 90^\circ$.

In the case of $|z_2|$, the wave amplitude started to increase significantly than $|z_1|$ at $g = 0.1$, and the effect of the fractional differential appears more clearly in the case of $\alpha = 0.90$ or $\alpha = 0.94$, both at $g = 0.1$ or $g = 0.25$.

Figure 3 highlights the various values of amplitudes $|z_3|$ concerning the angle of incidence of the reflected waves θ for various values of the fractional parameter for the two values of gravity g that shows the oscillatory performance in the entire range of angle θ. Clearly, the amplitudes of reflection waves $|z_3|$ rise when rising the gravity field and decline when rising θ until it equals zero at $\theta = 90^\circ$.

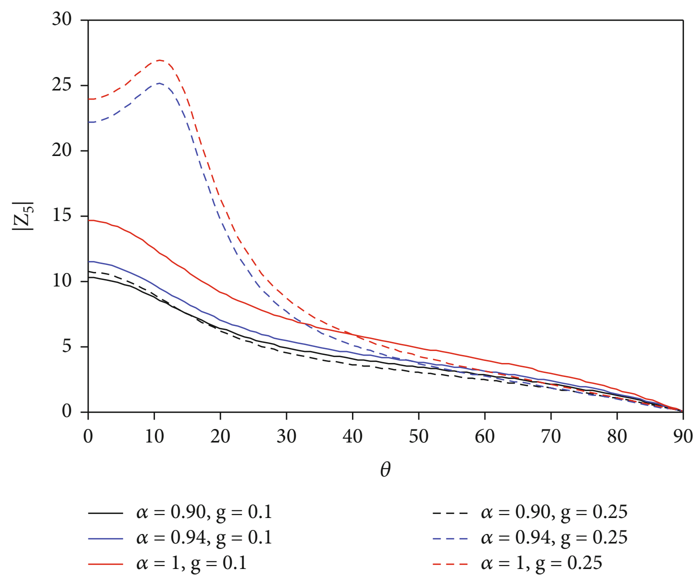

FIGURE 5: The various values of the amplitudes $|z_5|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of gravity g.

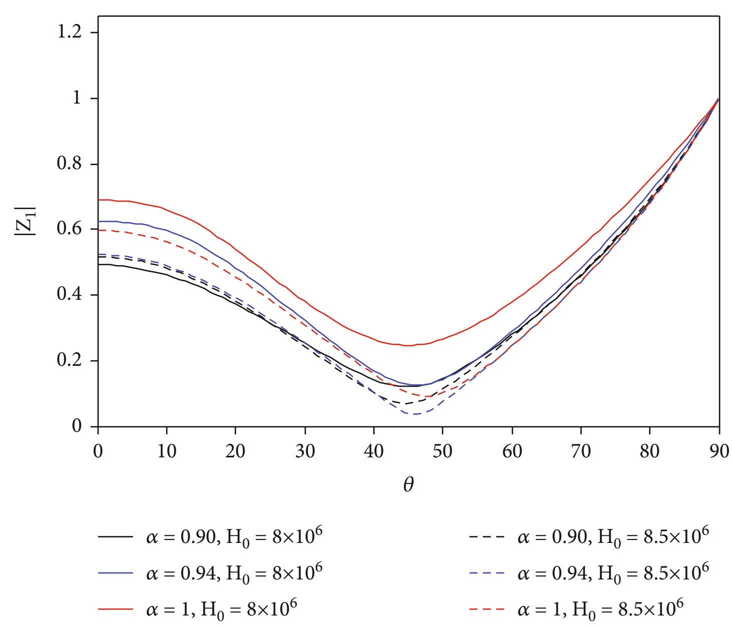

FIGURE 6: The various values of the amplitudes $|z_1|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of the magnetic field H_0.

In the case of $|z_3|$, the effect of the wave amplitude shows a slight change in $0 \le \theta \le 10$, and it is not affected by the gravity at the lowest value of the fractional differential parameter at $\alpha = 0.90$, while the rest of the values for the fractional differential parameter are clear.

Figures 4 and 5 display the various values of amplitudes $|z_4|, |z_5|$ concerning the angle of incidence of the reflected waves θ regarding various values of the fractional parameter for the two diverse values of gravity g that escalates in the whole entire of angle θ. Clearly, the amplitudes of the reflection waves $|z_4|, |z_5|$ increase with a higher gravity field. They rise then decline when rising θ until it equals zero at $\theta = 90°$. In the wave amplitude $|z_4|$, the effect of the fractional differential parameter at $\alpha = 0.90$ takes the same behavior whether gravitational $g = 0.1$ or $g = 0.25$. In the case of $|z_5|$, the effect of the wave amplitude shows a slight change in $10 \le \theta \le 20$,

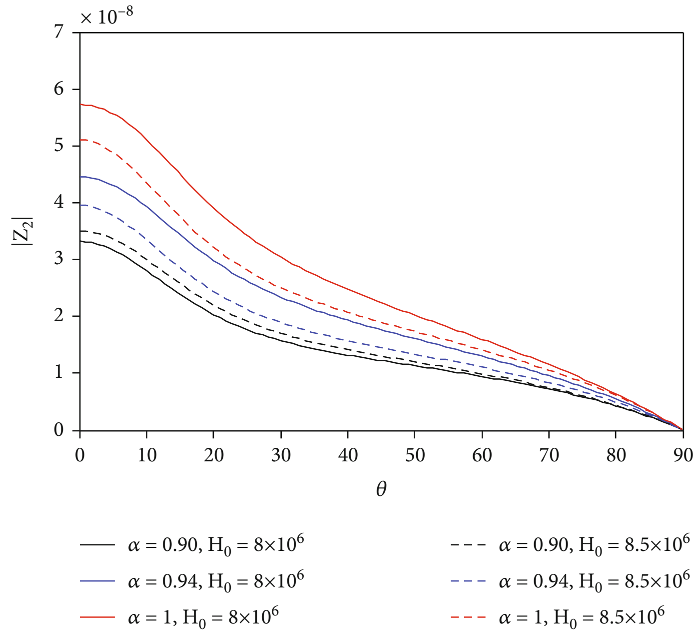

Figure 7: The various values of the amplitudes $|z_2|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of the magnetic field H_0.

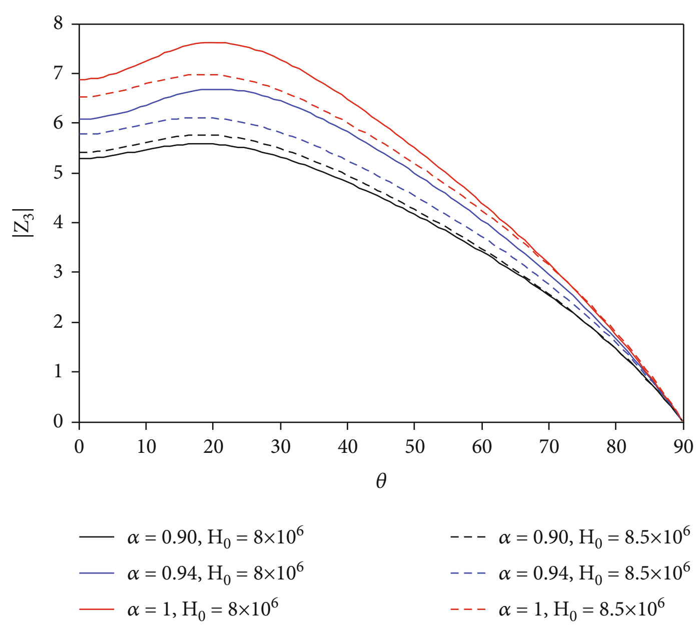

Figure 8: The various values of the amplitudes $|z_3|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of the magnetic field H_0.

and it is not affected by the gravity at the lowest value of the fractional differential parameter at $\alpha = 0.90$, while the rest of the values for the fractional differential parameter are clear.

Figure 6 illustrates the magnitude of amplitude ratios $|z_1|$ concerning the angle of incidence of the reflected waves θ regarding various values of the fractional parameter α of two diverse values of the magnetic field H_0. The amplitudes of incidence waves $|z_1|$ are lower when rising the magnetic field H_0 that decline then grows with rising θ to reach the unit at $\theta = 90^\circ$. It appears that the amplitude of the wave is lower in the case of the fractional differential parameter at $\alpha = 0.90$ along $0 \leq \theta \leq 90$, while it increases with an increase of $\alpha = 0.94$ and is more amplitude in the case of $\alpha = 1$.

Figures 7–10 display the various values of amplitudes $(|z_2|, |z_3|, |z_4|, |z_5|)$ concerning the angle of incidence of the reflected waves θ for various values of the fractional

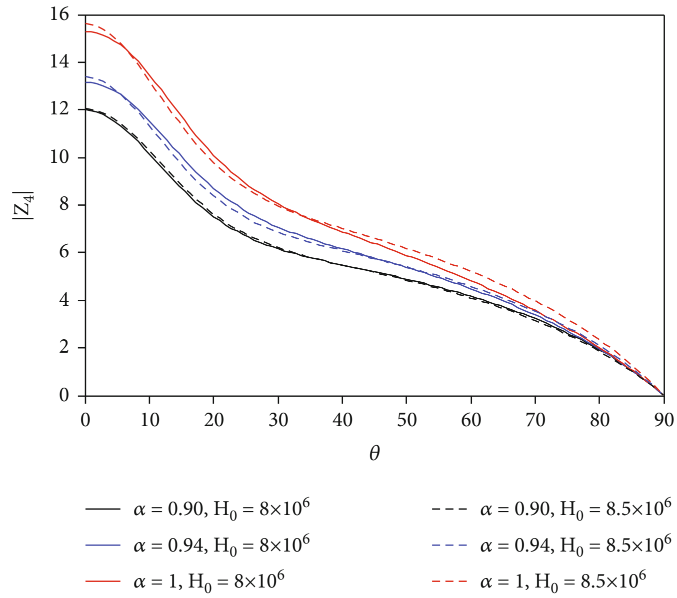

FIGURE 9: The various values of the amplitudes $|z_4|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of the magnetic field H_0.

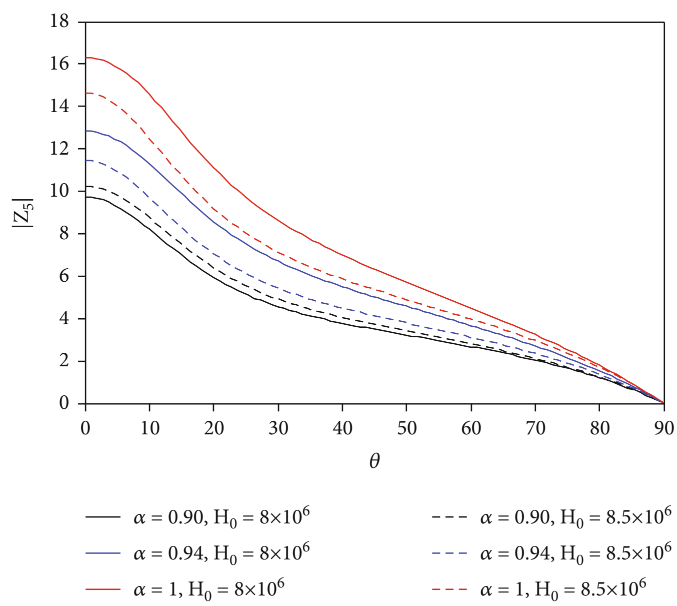

FIGURE 10: The various values of the amplitudes $|z_5|$ concerning the angle of incidence θ for various values of the fractional parameter of the diverse values of the magnetic field H_0.

parameter α of the two different values of the magnetic field H_0, which escalates in the entire range of angle θ. Obviously, the amplitude reflection waves $(|z_2|, |z_3|, |z_4|, |z_5|)$ decrease with an increase of the magnetic field H_0 and decrease with an increase of θ until equal zero at $\theta = 90^\circ$. In the case of amplitudes $(|z_2|, |z_3|, |z_5|)$, the behavior of the waves appears to be the same in all cases for different alpha values. At the same time, the effect of the fractal calculus parameter is less amplitude than the absence of fractional differential parameter. In the case of $|z_4|$, the impact of the magnetic field, whether $H_0 = 8 \times 10^6$ or $H_0 = 8.5 \times 10^6$ is not affected by the wave amplitude, as we see the effect of fractional differentiation in all the fractional differential parameter values.

In sum, the method of the fractional derivative technique applies to other relevant issues in geology, geophysics, physics, astronomy, and engineering, agreeing with the integer derivative if $\alpha = 1$.

If the fractional order considers only an integer without an electric field, the results obtained are deduced to the results given in Othman et al. [46].

2. Conclusion

The plane harmonic waves' reflection from a semi-infinite elastic solid of thermo-microstretch was studied in this article under the thermoelasticity theory with three-phase lag. Expressing the reflection coefficients that represent the relationships of the amplitudes of the reflected waves to the amplitude of the incidence waves obtained the reflection coefficient ratio variations with the angle of incidence with changing the dielectric constant, magnetic field, and gravity field. The results were compared in the cases of the existence and negligence of the parameter fractional.

Due to their practical issues, we recommend utilizing the research findings in other fields, such as geophysics, engineering, geology, volcanoes, earthquakes, and structures. Future works will consider taking into account the effect of radiation, rotation, and other external parameters related to the phenomenon topics applicable in the environment.

Appendix

$$L = -\frac{C_1^2}{c_0^2}A, M = \alpha^2\omega^2\left(A + \left[(\zeta_5 + \zeta_0)A - \tau_q^*(1 + \varepsilon_1\zeta_2)\right]\frac{C_1^2}{c_0^2}\right) + \frac{A}{\omega^{*2}}\left(\zeta_6\frac{kC_1^2}{c} - C_2^2 + \zeta_3C_3^2\right) + AB\frac{C_1^2}{c_0^2},$$

$$N = \alpha^2\omega^2\left(\frac{A}{\omega^{*2}}\left[C_2^2(\zeta_0 + \zeta_5) - \frac{\zeta_6 k}{c}\left(c_0^2 - \zeta_0C_1^2\right) - \zeta_3\zeta_5C_3^2\right] - AB\left(C_1^2\zeta_5 - 1 - \zeta_0\frac{C_1^2}{c_0^2}\right) + \tau_q^*B\frac{C_1^2}{c_0^2}(1 + \zeta_2\varepsilon_1)\right.$$

$$\left. + \alpha^4\omega^4\left(\tau_q^*[1 + \zeta_2\varepsilon_1] - A[\zeta_6 + \zeta_0] + \tau_q^*\frac{C_1^2}{c_0^2}\left[\zeta_5 + \zeta_0 + \zeta_2\zeta_5\varepsilon_1 - \zeta_0\zeta_5\frac{A}{\tau_q^*}\right]\right) + \frac{A}{\omega^{*2}}\left(BC_2^2 - \zeta_3bC_3^2\right) + \zeta_6A\frac{kc_0^2}{c\omega^{*4}}\left(C_2^2 - \zeta_3C_3^2\right) - \zeta_1\zeta_4A\sin^2\theta\frac{C_1^2}{c_0^2},\right.$$

$$Q = \zeta_0\zeta_5\alpha^6\omega^6\tau_q^*B\left(a_0\varepsilon_2 + \frac{C_2^2}{\omega^{*2}}\right) - \zeta_1\zeta_4\alpha^4\omega^4\tau_q^*B\sin^2\theta + \alpha^2\omega^2\zeta_1\zeta_4\tau_q^*B\sin^2\theta\left(a_0\varepsilon_2 + \frac{C_2^2}{\omega^{*2}}\right),$$

$$A = -C_k + i\alpha\omega C_v + \alpha^2\omega^2C_T, B = \frac{j\rho c_0^2\alpha^2\omega^2}{c} - \frac{2kc_0^2}{c\omega^{*2}},$$

$$O = \alpha^6\omega^6\left(\zeta_0\zeta_5\left[A - \tau_q^*\frac{C_1^2}{c_0^2}\right] - \tau_q^*[\zeta_5 + \zeta_0] - \zeta_2\zeta_5\varepsilon_1\tau_q^*\right) + \alpha^4\omega^4\left(\tau_q^*a_0[\zeta_5\varepsilon_2 - \zeta_0\varepsilon_2 - \zeta_3\zeta_5\varepsilon_1] + \tau_q^*\frac{C_2^2}{\omega^{*2}}[\zeta_5 + \zeta_0] + \zeta_0\frac{A}{\omega^{*2}}\left[\zeta_6\frac{kc_0^2}{c} - \zeta_5C_2^2\right]\right.$$

$$+AB\left[\zeta_5 + \zeta_0 + \zeta_0\zeta_5\frac{C_1^2}{c_0^2}\right] - \tau_q^*B\frac{C_1^2}{c_0^2}[\zeta_0 + \zeta_5 + \zeta_2\zeta_5\varepsilon_1] - \tau_q^*B[\zeta_2\varepsilon_1 + 1] - \zeta_6\tau_q^*\frac{kc_0^2}{c\omega^{*2}}[1 - \zeta_2\varepsilon_1] - \frac{\tau_q^*}{c\omega^{*2}}\left[\zeta_0\zeta_6kC_1^2 - \zeta_2\zeta_5\varepsilon_1cC_2^2\right]$$

$$\left. -\zeta_5\varepsilon_2\tau_q^*\frac{C_3^2}{\omega^{*2}}[\zeta_2\varepsilon_2 + \zeta_3]\right)$$

$$+\alpha^2\omega^2\left(\tau_q^*B\frac{C_2^2}{\omega^{*2}}[1 + \zeta_2\varepsilon_1] - \tau_q^*B\frac{C_3^2}{\omega^{*2}}[\zeta_3 + \zeta_2\varepsilon_2] + a_0\tau_q^*B[\varepsilon_2 - \zeta_3\varepsilon_1] - AB\frac{C_2^2}{\omega^{*2}}[\zeta_5 + \zeta_0]\right.$$

$$\left. -\zeta_6\frac{kc_0^2}{c\omega^{*4}}\left[\zeta_0AC_2^2 - a_0\varepsilon_2\tau_q^*\omega^{*2} + \zeta_2\varepsilon_1\tau_q^*C_2^2 - \zeta_2\varepsilon_2\tau_q^*C_3^2\right] - \zeta_1\zeta_4\sin^2\theta\left[\tau_q^*\frac{C_1^2}{c_0^2} - A\right] + \zeta_3\zeta_5Ab\frac{C_3^2}{\omega^{*2}} - \zeta_3\zeta_6a_0\varepsilon_1\tau_q^*\right)$$

$$+ \zeta_1\zeta_4A\sin^2\theta\left[B\frac{C_1^2}{c_0^2} - \frac{C_2^2}{\omega^{*2}}\right].$$

$$P = \zeta_0\zeta_5\alpha^8\omega^8\tau_q^* + \alpha^6\omega^6\left(\zeta_5\tau_q^*[\zeta_0a_0\varepsilon_2 + \zeta_2\varepsilon_1B] + \tau_q^*B[\zeta_5 + \zeta_0] + \zeta_0\zeta_5B\left[\tau_q^*\frac{C_1^2}{c_0^2} - A\right] - \zeta_0\frac{\tau_q^*}{\omega^{*2}}\left[\zeta_5C_2^2 - \zeta_6\frac{kc_0^2}{c}\right]\right)$$

$$+\alpha^4\omega^4\left(\zeta_5\tau_q^*B\frac{C_3^2}{\omega^{*2}}[\zeta_2\varepsilon_2 + \zeta_3] - \tau_q^*B\frac{C_2^2}{\omega^{*2}}[\zeta_5 + \zeta_0 + \zeta_2\zeta_5\varepsilon_1] - a_0\tau_q^*B[\zeta_5\varepsilon_2 + \zeta_0\varepsilon_2 - \zeta_3\zeta_5\varepsilon_1] - \zeta_0\zeta_6\tau_q^*\frac{kc_0^2}{c\omega^{*2}}\left[a_0\varepsilon_2 + \frac{C_2^2}{\omega^{*2}}\right] + \zeta_0\zeta_5AB\frac{C_2^2}{\omega^{*2}} + \zeta_1\zeta_4\tau_q^*\sin^2\theta\right)$$

$$+\alpha^2\omega^2\left(\zeta_1\zeta_4B\sin^2\theta\left[\tau_q^*\frac{C_1^2}{c_0^2} - A\right] - \zeta_1\zeta_4\tau_q^*\sin^2\theta\left[a_0\varepsilon_2 + \frac{C_2^2}{\omega^{*2}}\right] - \zeta_3\zeta_6\tau_q^*\frac{kc_0^2C_3^2}{c\omega^{*4}}\right) + \zeta_1\zeta_4AB\sin^2\theta\frac{C_2^2}{\omega^{*2}}, \tag{A.1}$$

Nomenclature

$\vec{B}$:	Magnetic induction vector
C_e:	Specific heat per unit mass
$\vec{E}$:	Electrical density vector
e_{ij}:	Strain tensor
$\vec{F_i}$:	Body force vector of Lorentz
g:	Gravitational constant
$\vec{H}$:	Magnetic vector
$\vec{h}$:	Perturbed magnetic vector
$\vec{H}_0$:	Elementary constant magnetic field vector
i:	$\sqrt{-1}$
j:	Microinertia moment
$\vec{J}$:	Electric current density vector
K, K^*:	Thermal conduction coefficients

m_{ij}: Coupled stress tensor
t: Time
T_0: The reference temperature
T: The temperature distribution
u: Displacement vector
a_0, b_0, b_1: Microstretch constants
$\alpha_{t_1}, \alpha_{t_2}$: Coefficient of linear thermal expansion
δ_{ij}: Kronecker delta
ε_0: Electric permeability
φ: Rotation vector
φ^*: Scalar microstretch
k, a, b, c: Micropolar constants
λ, μ: Lame' constants
μ_0: Magnetic permeability
$v = \omega/\xi$: Velocity of the coupled waves
ρ: Mass density
σ_{ij}: Stress tensor
τ_{ij}: Stress tensor of Maxwell
τ_q: The phase-lag of the temperature influx
τ_T: The phase-lag of the heat tendency
τ_v: Thermal displacement phase lag
ω: Complex frequency
ξ: Wave number
$\widehat{\gamma}$: $(3\lambda + 2\mu + k)\alpha_{t_1}$
$\widehat{\gamma}_1$: $(3\lambda + 2\mu + k)\alpha_{t_2}$.

Data Availability

The data supported from our published papers and all the results are new.

Conflicts of Interest

The authors declare no conflict of interests in publishing the present research paper.

Acknowledgments

The authors acknowledge the financial support of Taif University researchers, supporting project no. TURSP-2020/164, Taif University, Taif, Saudi Arabia.

References

[1] S. K. R. Choudhuri, "On a thermoelastic three-phase-lag model," *Journal of Thermal Stresses*, vol. 30, no. 3, article 231238, 2007.

[2] R. Kumar, R. Singh, and T. K. Chadha, "Plane strain problem in microstretch elastic solid," *Sadhana*, vol. 28, no. 6, pp. 975–990, 2003.

[3] R. Kumar and G. Partap, "Reflection of plane waves in a heat flux dependent microstretch Thermoelastic solid half space," *International Journal of Applied Mechanics and Engineering*, vol. 10, pp. 253–266, 2005.

[4] H. W. Lord and Y. Shulman, "A generalized dynamical theory of thermoelasticity," *Journal of the Mechanics and Physics of Solids*, vol. 15, no. 5, pp. 299–309, 1967.

[5] A. E. Green and A. Lindsay, "Thermoelasticity," *Journal of Elasticity*, vol. 2, no. 1, pp. 1–7, 1972.

[6] A. E. Green and P. M. Naghdi, "A re-examination of the basic postulates of thermomechanics," *Procedding of the Royal Society Londan A*, vol. 432, no. 1885, pp. 171–194, 1991.

[7] A. E. Green and P. M. Naghdi, "On undamped heat waves in an elastic solid," *Journal of Thermal Stresses*, vol. 15, no. 2, pp. 253–264, 1992.

[8] A. E. Green and P. M. Naghdi, "Thermoelasticity without energy dissipation," *Journal of Elasticity*, vol. 31, no. 3, pp. 189–208, 1993.

[9] I. A. Abbas and R. Kumar, "Deformation due to thermal source in micropolar generalized thermoelastic half-space by finite element method," *Journal of Computational and Theoretical Nanoscience*, vol. 11, no. 1, pp. 185–190, 2014.

[10] R. Quintanilla and R. Racke, "A note on stability in three-phase-lag heat conduction," *International Journal of Heat and Mass Transfer*, vol. 51, no. 1-2, pp. 24–29, 2008.

[11] A. Kar and M. Kanoria, "Generalized thermoelastic functionally graded orthotropic hollow sphere under thermal shock with three-phase-lag effect," *European Journal of Mechanics A–Solids*, vol. 28, no. 4, pp. 757–767, 2009.

[12] S. Banik and M. Kanoria, "Generalized thermoelastic interaction in a functionally graded isotropic unbounded medium due to varying heat source with three-phase-lag effect," *Mathematics and Mechanics of Solids*, vol. 18, no. 3, pp. 231–245, 2013.

[13] M. Elsagheer and S. M. Abo-Dahab, "Reflection of thermoelastic waves from insulated boundary fibre-reinforced half-space under influence of rotation and magnetic field," *Applied Mathematics & Information Sciences*, vol. 10, no. 3, pp. 1129–1140, 2016.

[14] S. M. Abo-Dahab, A. M. Abd-Alla, and A. A. Kilany, "Effects of rotation and gravity on an electro-magneto-thermoelastic medium with diffusion and voids by using the Lord-Shulman and dual-phase-lag models," *Applied Mathematics and Mechanics (English Edition)*, vol. 40, no. 8, pp. 1135–1154, 2019.

[15] A. M. Abd-Alla, S. M. Abo-Dahab, and A. A. Kilany, "SV-waves incidence at interface between solid-liquid media under electromagnetic field and initial stress in the context of three thermoelastic theories," *Journal of Thermal Stresses*, vol. 39, no. 8, pp. 960–976, 2016.

[16] S. M. Abo-Dahab, A. M. Abd-Alla, A. A. Kilany, and M. Elsagheer, "Effect of rotation and gravity on the reflection of P-waves from thermo-magneto-microstretch medium in the context of three phase lag model with initial stress," *Journal of Microsystem Technologies*, vol. 24, no. 8, pp. 3357–3369, 2018.

[17] R. Hilfer, *Applications of Fractional Calculus in Physics*, World Science, Singapore, 2000.

[18] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Elsevier B.V, Amsterdam, UK, 2006.

[19] U. N. Katugampola, "New approach to a generalized fractional integral," *Applied Mathematics and Computation*, vol. 218, no. 3, pp. 860–865, 2011.

[20] E.-A.-B. Abdel-Salam, E. A. Yousif, and M. A. El-Aasser, "Analytical solution of the space-time fractional nonlinear Schrödinger equation," *Reports on Mathematical Physics*, vol. 19, p. 77, 2016.

[21] E.-A.-B. Abdel-Salam and G. F. Hassan, "Solutions to class of linear and nonlinear fractional differential equations,"

Communications in Theoretical Physics, vol. 65, no. 2, pp. 127–135, 2016.

[22] E. A.-B. Abdel-Salam and E. A. Yousif, "Solution of nonlinear space-time fractional differential equations using the fractional Riccati expansion method," *Mathematical Problems in Engineering*, vol. 2013, Article ID 846283, 6 pages, 2013.

[23] M. I. Nouh and E.-A.-B. Abdel-Salam, "Approximate solution to the fractional Lane-Emden type equations," *Iranian Journal of Science and Technology Transaction a Science*, vol. 42, no. 4, pp. 2199–2206, 2018.

[24] S. E. Ahmed, M. A. Mansour, E.-A.-B. Abdel-Salam, and E. F. Mohamed, "Studying the fractional derivative for natural convection in slanted cavity containing porous media," *SN Applied Sciences*, vol. 1, no. 9, p. 1117, 2019.

[25] A. M. El-Naggar, Z. Kishka, A. M. Abd-Alla, I. A. Abbas, S. M. Abo-Dahab, and M. Elsagheer, "On the initial stress, magnetic field, voids and rotation effects on plane waves in generalized thermoelasticity," *Journal of Computational and Theoretical Nanoscience*, vol. 10, no. 6, pp. 1408–1417, 2013.

[26] M. Marin, M. I. A. Othman, and I. A. Abbas, "An extension of the domain of influence theorem for generalized thermoelasticity of anisotropic material with voids," *Journal of Computational and Theoretical Nanoscience*, vol. 12, no. 8, pp. 1594–1598, 2015.

[27] T. Saeed, I. Abbas, and M. Marin, "A GL model on thermoelastic interaction in a poroelastic material using finite element method," *Symmetry*, vol. 12, no. 3, p. 488, 2020.

[28] S. M. Abo-Dahab, A. M. Abd-Alla, and A. A. Kilany, "Electromagnetic field in fiber-reinforced micropolar thermoelastic medium using four models," *Journal of Ocean Engineering and Science*, vol. 5, no. 3, pp. 230–248, 2020.

[29] J. H. He, S. K. Elagan, and Z. B. Li, "Geometrical explanation of the fractional complex transform and derivative chain rule for fractional calculus," *Physics Letters A*, vol. 376, no. 4, pp. 257–259, 2012.

[30] K. M. Kolwankar and A. D. Gangal, "Fractional differentiability of nowhere differentiable functions and dimensions," *Chaos*, vol. 6, no. 4, pp. 505–513, 1996.

[31] K. M. Kolwankar and A. D. Gangal, "Local fractional Fokker-Planck equation," *Physical Review Letters*, vol. 80, no. 2, pp. 214–217, 1998.

[32] Y. Chen, Y. Yan, and K. Zhang, "On the local fractional derivative," *Journal of Mathematical Analysis and Applications*, vol. 362, no. 1, pp. 17–33, 2010.

[33] R. Khalil, M. al Horani, A. Yousef, and M. Sababheh, "A new definition of fractional derivative," *Journal of Computational and Applied Mathematics*, vol. 264, pp. 65–70, 2014.

[34] T. Abdeljawad, "On conformable fractional calculus," *Journal of Computational and Applied Mathematics*, vol. 279, pp. 57–66, 2015.

[35] F. Jarad, T. Abdeljawad, and Alzabut, "Generalized fractional derivatives generated by a class of local proportional derivatives," *The European Physical Journal Special Topics*, vol. 226, no. 16-18, pp. 3457–3471, 2017.

[36] T. Abdeljawad, F. Jarad, and Alzabut, "Fractional proportional differences with memory," *The European Physical Journal Special Topics*, vol. 226, no. 16-18, pp. 3333–3354, 2017.

[37] S. M. Abo-Dahab, A. A. Kilany, E. A.-B. Abdel-Salam, and A. Hatem, "Fractional derivative order analysis and temperature-dependent properties on p- and SV-waves reflection under initial stress and three-phase-lag model," *Results in Physics*, vol. 18, article 103270, 2020.

[38] H. Alotaibi, S. M. Abo-Dahab, H. R. Abdlrahim, and A. A. Kilany, "Fractional calculus of Thermoelastic p-waves reflection under influence of gravity and electromagnetic fields," *Fractals*, vol. 28, no. 8, article 2040037, 2020.

[39] A. Hobiny and I. Abbas, "Analytical solution of fractional order photo-thermoelasticity in a non-homogenous semiconductor medium," *Multidiscipline Modeling in Materials and Structures*, vol. 14, no. 5, pp. 1017–1030, 2018.

[40] Y. Povstenko and T. Kyrylych, "Fractional thermoelasticity problem for a plane with a line crack under heat flux loading," *Journal of Thermal Stresses*, vol. 41, no. 10-12, pp. 1313–1328, 2018.

[41] S. M. Abo-Dahab, S. Z. Rida, R. A. Mohamed, and A. A. Kilany, "Rotation, initial stress, gravity and electromagnetic field effect on P wave reflection from stress-free surface elastic half-space with voids under three thermoelastic models," *Mechanics and Mechanical Engineering*, vol. 22, no. 1, pp. 313–328, 2018.

[42] A. M. Abd-Alla, S. M. Abo-Dahab, and A. A. Kilany, "Effect of several fields on a generalized thermoelastic medium with voids in the context of Lord-Shulman or dual-phase-lag models," *Mechanics Based Design of Structures and Machines*, pp. 1–24, 2020.

[43] F. S. Bayones, S. M. Abo-Dahab, A. M. Abd-Alla, and A. A. Kilany, "Electromagnetic field and three-phase lag in a compressed rotating isotropic homogeneous micropolar thermo-viscoelastic half-space," *Mathematical Methods in the Applied Sciences*, vol. 44, no. 13, pp. 9944–9965, 2021.

[44] F. S. Bayones, S. M. Abo-Dahab, A. M. Abd-Alla, S. H. Elhag, A. A. Kilany, and M. Elsagheer, "Initial stress and gravity on P-wave reflection from electromagneto-Thermo- microstretch medium in the context of three-phase lag model," *Complexity*, vol. 2021, Article ID 5560900, 15 pages, 2021.

[45] F. S. Bayones, S. Mondal, S. M. Abo-Dahab, and A. A. Kilany, "Effect of moving heat source on a magneto-thermoelastic rod in the context ofEringen'snonlocal theory under three-phase lag with a memory dependent derivative," *Mechanics Based Design of Structures and Machines*, pp. 1–17, 2021.

[46] M. I. A. Othman, S. M. Abo-Dahab, and K. Lotfy, "Gravitational effect and initial stress on generalized magneto-thermo-microstretch elastic solid for different theories," *Applied Mathematics and Computation*, vol. 230, pp. 597–615, 2014.

[47] S. M. Abo-Dahab, A. A. Kilany, M. N. M. Allam, R. A. Mohamed, and S. Z. Rida, "Influence of several fields on Rayleigh waves propagation in a fiber-reinforced orthotropic half-space material under four thermoelastic models," *Waves in Random and Complex Media*, pp. 1–24, 2020.

[48] M. A. Ezzat, "State space approach to generalized magneto-thermoelasticity with two relaxation times in a medium of perfect conductivity," *International Journal of Engineering Science*, vol. 35, pp. 741–752, 1997.

[49] G. Paria, "Magneto-elasticity and magneto-thermoelasticity," *Advances in Applied Mechanics*, vol. 10, p. 73, 1967.