Hindawi
Advances in Mathematical Physics
Volume 2023, Article ID 4009056, 8 pages
https://doi.org/10.1155/2023/4009056

Research Article

Analysis of Fuzzy Differential Equation with Fractional Derivative in Caputo Sense

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Received 11 June 2022; Revised 8 July 2022; Accepted 20 March 2023; Published 20 April 2023

Academic Editor: S. E. Najafi

In this article, the dynamics of the fuzzy fractional order enzyme Michaelis Menten model are investigated. To study problems with uncertainty, fuzzy fractional technique is applied. Using fuzzy theory, the sequential iterations of the model are calculated by applying fractional calculus theory and the homotopy perturbation method. A comparison is given for fractional and fuzzy results, and the numerical findings validate the fuzzy fractional case. Using MATLAB software, the results are simulated for various fractional orders, corresponding to the provided data. The simulations demonstrate the model's appropriateness.

1. Introduction

When doing an unusual experiment, such as putting a fraction into the sequence of differentiation, it is critical to remain intrigued about the results, as this is how many unique scientific studies are conducted. When venturing into unknown territory, however, one should be prepared to fore-go much of what is currently known and is taken normal and obvious. The fractional integral and derivative is not the only different-integral operators available; there is still a vast universe of generalizing differentiation and integration with which we are both comfortable and secure, such as chain and product laws. Another important attribute of fractional derivatives is nonlocality [1–3]. If the result of calculating the value of an integer-order derivative at a point is dependent on that point, we call this property as locality. With the fractional derivative, things are a little different. When studying physical systems, the case where $\alpha = 0$ is common because the dependent variable is always time. The fractional derivative is determined by the state of the system, which includes all moments after the experiment begins at $t = 0$. This nonlocality is one of the primary drivers of interest in fractional calculus in applications. Memory effects refer to a group of remarkable physical phenomena in which the state is influenced not only by time and place, but also by prior states. For example, consider a section of an electric circuit whose resistance is based on the total amount of charge that has gone through it over a set period of time. Memory effects can be difficult to represent and analyze using conventional differential equations, but nonlocality provides a built-in capacity for fractional derivatives to integrate memory effects. As a result, fractional calculus could be a valuable tool for analyzing this type of system.

In mathematical modeling, memory is used to explain the present by emphasizing what happened in the past. Previous experiences, for example, may indicate that, depending on the type of disease, social distance or additional hygiene practices are protective behaviors in the case of an infectious disease being transmitted to people. Because the vaccination has a long-lasting effect, it may also have a long-term memory effect. Fractional calculus is an excellent tool for understanding real-life phenomena involving the memory effect. For example, we assume $g(t, x_0)$ as the solution of an autonomous ODE of first order, provided x_0 at $t = 0$; hence, the property $g(t + s, x_0) = g(t, g(s, x_0))$ is assured, which implies that the results are unchanged by taking $g(s, x_0)$ as the initial

condition as $g(t, x_0)$ belongs to results. As a result, for any domain point, the solution is uniquely specified given an initial value. This statement is not true for fractional differential equations in general. Adjusting the order of a classical model's derivative such that it becomes noninteger is one approach for a mathematical model to integrate the memory effect.

Conventional mathematical optimization approaches for reactive biological systems include equations containing empirical or semiempirical expressions rather than the traditional mass-action law. By applying the memory effect, the kinetics of such reactive systems can be accurately represented using fractional calculus, providing forms similar to those given by the law of mass action. As a result, a great mechanism for explaining the dynamical behavior of many chemical and biological systems has been developed. Several research papers have been published on the use of FDEs in biological and chemical reactions. As a result, fractional derivative focused models have a greater potential for description accuracy. Theoretical advancements are also being made in order to expand the application of this technology in research and engineering. Hans-Jürgen [4] shows the validity and possibilities of fractional calculus as a tool for modeling dynamic systems in the field of process systems engineering. They developed a fractional calculus-based model for the fermentation problem and used experimental data to demonstrate the model's validity in biological reactions.

For examining the approximate solution of the Michaelis Menten enzymatic reaction equation, Manal and Saad [5] suggested an extension of the spectral homotopy analysis method. They compared the accuracy and efficiency of Runge-Kutta methods. He and Li [6] used the Laplace transformation and Adomian decomposition approach to analyze the semianalytical results of fractional time enzyme kinetics.

Further developments in the related areas can be seen in [7–13].

2. Motivation

Many academics have given numerical, approximate approaches and applications to handle this problem in general, due to the difficulty that many researchers encounter in obtaining accurate solutions to fractional differential equations [14–17]. The authors of [5] looked into spectrum approaches in the context of fractal fractional differentiation. However, it only included research that used the Mittag-Leffler kernel. The significance of our research resides in the fact that we give a fuzzy solution to the uncertainty challenge. One of the major benefits of the Caputo fractional derivative is that it makes it possible to formulate the problem with conventional initial and boundary conditions. Its derivative for a constant is also zero. We take the Michaelis Menten differential equation system as

$$D_t R_1(t) = -\alpha R_1 R_2 + \beta R_3, \tag{1}$$

$$D_t R_2(t) = -\alpha R_1 R_2 + (\beta + \gamma) R_3, \tag{2}$$

$$D_t R_3(t) = \alpha R_1 R_2 (\beta + \gamma) R_3, \tag{3}$$

$$D_t R_4(t) = \gamma R_3. \tag{4}$$

The description of the model is given in Table 1.

$$R_1(\mathrm{t}) + R_2(t)(\mathrm{t}) \rightleftharpoons R_3(\mathrm{t}) \longrightarrow R_2(\mathrm{t}) + R_4(t). \tag{5}$$

According to this illustration, a complex R_3 is the result of a process involving a substrate R_1 and an enzyme R_2. Finally, the enzyme R_2 converts a complex R_3 into a product R_4.

3. Contribution

For more than a century, the Michaelis Menten equation has been used to predict the rate of product generation in enzymatic reactions. It specifically indicates that when substrate concentration increases, the rate of an enzymatic reaction increases, but greater unbinding of enzyme-substrate complexes decreases the reaction rate. This is the first investigation of the fractional Michaelis Menten enzymatic process using fuzzy approach. The fractional Michaelis Menten differential equation system can be written as

$$D_t^{\zeta} R_1(t) = -\alpha R_1 R_2 + \beta R_3, \tag{6}$$

$$D_t^{\zeta} R_2(t) = -\alpha R_1 R_2 + (\beta + \gamma) R_3, \tag{7}$$

$$D_t^{\zeta} R_3(t) = \alpha R_1 R_2 - (\beta + \gamma) R_3, \tag{8}$$

$$D_t^{\zeta} R_4(t) = \gamma R_3. \tag{9}$$

Introducing the fuzzy fractional parameters, the system can be written as

$$D_t^{\zeta}(F_{1k}) = -\tilde{\alpha} F_{1k} F_{2k} + \tilde{\beta} F_{3k}, \tag{10}$$

$$D_t^{\zeta}(F_{2k}) = -\tilde{\alpha} F_{1k} F_{2k} + \left(\tilde{\beta} + \tilde{\gamma}\right) F_{3k}, \tag{11}$$

$$D_t^{\zeta}(F_{3k}) = \tilde{\alpha} F_{1k} F_{2k} - \left(\tilde{\beta} + \tilde{\gamma}\right) F_{3k}, \tag{12}$$

$$D_t^{\zeta}(F_{4k}) = \tilde{\gamma} F_{3k}, \tag{13}$$

with fuzzy initial conditions given by

$$F_1^{\check{}}(0, p) = \left(\underline{F_1}(0, p), \overline{F_1}(0.p)\right), \tag{14}$$

$$F_2^{\check{}}(0, p) = \left(\underline{F_2}(0, p), \overline{F_2}(0.p)\right), \tag{15}$$

$$F_3^{\check{}}(0, p) = \left(\underline{F_3}(0, p), \overline{F_3}(0.p)\right), \tag{16}$$

$$F_4^{\check{}}(0, p) = \left(\underline{F_4}(0, p), \overline{F_4}(0.p)\right). \tag{17}$$

The following is an overview of the article's structure. The definitions of the fractional calculus and fuzzy operators are discussed in Section 2. In Section 3, we use the homotopy perturbation approach with fuzzy initial conditions to generate successive iterations of the fractional Michaelis Menten enzymatic reaction. The numerical results are presented in Section 4. Finally, in Sections 5 and 6, we explain and

Table 1: Parameters' values.

Parameters	Interpretation
$R_1(t)$	Concentration of substrate
$R_2(t)$	Concentration of enzyme
$R_3(t)$	Concentration of the resulting complex
$R_4(t)$	Concentration of resulting product
α, β, γ	Rate of reaction

examine the numerical results as well as make some final observations.

4. Preliminaries

In this section, we provide the definitions which will be used in the solution of the system [4, 18–20].

Definition 1. Let $\eta : \mathbb{R} \longrightarrow [0, 1]$ be a fuzzy set. η is said to be a fuzzy number if it satisfies the following properties:

(1) η is normal, i.e., $\eta(c_0) = 1$ for any $c_0 \in \mathbb{R}$

(2) η is semicontinuous on $\mathbb{R}$, i.e., for all $\varepsilon > 0$, there exists a $\delta > 0$ such that $|\eta(c) - \eta(c_0)| < \varepsilon$ for $|c - c_0| < \delta$

(3) η is convex

(4) $d1\{c \in \mathbb{R}\,; \eta(c) > 0\}$ is compact

Definition 2. If η is a fuzzy number, for $n \in (0, 1]$ and $c \in \mathbb{R}$, the n-th level set defined on η is given by

$$[\eta]n = \{c \in \mathbb{R} : \eta(a) \geq n\}. \tag{18}$$

Definition 3. Let $[\underline{\eta(\theta)}, \overline{\eta(\theta)}]$ for $0 \leq \theta \leq 1$ be the parametric form of a fuzzy number η, satisfying the following properties:

(1) $\underline{\eta(\theta)}$,is left continuous, bounded, and increasing over (0,1], and right continuous at 0

(2) $\overline{\eta(\theta)}$ is right continuous, bounded, and decreasing over [0,1], and right continuous at 0

(3) $\underline{\eta(\theta)} \leq \overline{\eta(\theta)}$

Also, if $\underline{\eta(\theta)} = \overline{\eta(\theta)}$; then, θ is called a crisp number.

Definition 4. Let ξ be the continuous fuzzy function on $[0, B] \subseteq R$, further if $\xi \in C^f[0, B] \cap L^f[0, B]$, where $C^f[0, B]$ is a fuzzy continuous space and $L^f[0, B]$ is a fuzzy Lebesgue integrable function such that $\xi = [\underline{\xi n(t)}, \overline{\xi n(t)}]$ for $0 \leq n \leq 1$ and $t \in (0, B)$; then, the fuzzy fractional derivative is defined as

$$[D^\kappa \xi(t_0)]n = \left[D^\kappa \underline{\xi_n}(t_0), D^\kappa \overline{\xi_n}(t_0)\right], \tag{19}$$

$$D^\kappa \underline{\xi_n}(t_0) = \left[\frac{1}{\Gamma(i-\kappa)}\right]\left[\int_0^t (t-\varsigma)^{i-\kappa-1}\left(\frac{d^i}{d\varsigma i}\right)\underline{\xi_n}(\varsigma)d\varsigma\right]_{t=t0}, \tag{20}$$

$$D^\kappa \overline{\xi_n}(t_0) = \left[\frac{1}{\Gamma(i-\kappa)}\right]\left[\int_0^t (t-\varsigma)^{i-\kappa-1}\left(\frac{d^i}{d\varsigma i}\right)\overline{\xi_n}(\varsigma)d\varsigma\right]_{t=t0}. \tag{21}$$

Definition 5. Let ξ be the continuous fuzzy function on $[0, B] \subseteq R$, further if $\xi \in C^f[0, B] \cap L^f[0, B]$, the Laplace transform of fuzzy fractional model derivative in Caputo sense is given as

$$L\left[D^\kappa \xi(t)_n\right] = s^k L[\xi(t)] - s^{k-1}[\xi(0)]. \tag{22}$$

Definition 6. We can construct a homotopy $v(r, P)$: $\Omega \times [0, 1] \longrightarrow R$

$$H(v, P) = (1 - P)[L(v) - L(v0)] + q[L(v) + N(v) - f(r)] = 0, \tag{23}$$

where L is the linear part, N is the nonlinear part, and $r \in \Omega$ and $P \in [0, 1]$ are the embedding parameter.

4.1. HPM for Fuzzy Fractional Model.

$$(1 - P)\left[D_t^\theta L\tilde{U} - D_t^\theta L\tilde{U}_0\right] + P\left[D_t^\theta L\tilde{U} + D_t^\theta N\tilde{U} + \widetilde{f}(r)\right] = 0, \tag{24}$$

Here, we will apply the HPM to the considered model

$$(1 - P)\left[D_t^\theta R_1(t) - D_t^\theta R_{10}(t)\right] + P\left[D_t^\theta R_1(t) + \tilde{\alpha}R_1R_2 + \tilde{\beta}R_3\right] = 0, \tag{25}$$

$$(1 - P)\left[D_t^\theta R_2(t) - D_t^\theta R_{20}(t)\right] + P\left[D_t^\theta R_2(t) + \tilde{\alpha}R_1R_2 + \left(\tilde{\beta} + \tilde{\gamma}\right)R_3\right] = 0, \tag{26}$$

$$(1 - P)\left[D_t^\theta R_3(t) - D_t^\theta R_{30}(t)\right] + P\left[D_t^\theta R_3(t) - \tilde{\alpha}R_1R_2 - \left(\tilde{\beta} + \tilde{\gamma}\right)R_3\right] = 0, \tag{27}$$

$$(1 - P)\left[D_t^\theta R_4(t) - D_t^\theta R_{40}(t)\right] + P\left[D_t^\theta R_4(t) - \widetilde{\gamma}R_3\right] = 0. \tag{28}$$

If $P = 0$, we get

$$D_t^\theta R_1(t) - D_t^\theta R_{10}(t) = 0, \tag{29}$$

$$D_t^\theta R_2(t) - D_t^\theta R_{20}(t) = 0, \tag{30}$$

$$D_t^\theta R_3(t) - D_t^\theta R_{30}(t) = 0, \tag{31}$$

$$D_t^\theta R_4(t) - D_t^\theta R_{40}(t) = 0. \tag{32}$$

We define following sums,

$$\tilde{R}_1(t) = \sum_{n=0}^{\infty} P^n \tilde{R}_{1_n}, \tag{33}$$

$$\tilde{R}_2(t) = \sum_{n=0}^{\infty} P^n \tilde{R}_{2_n}(t), \tag{34}$$

$$\tilde{R}_3(t) = \sum_{n=0}^{\infty} P^n \tilde{R}_{3_n}(t), \tag{35}$$

$$\tilde{R}_4(t) = \sum_{n=0}^{\infty} P^n \tilde{R}_{4_n}(t). \tag{36}$$

Similarly,

$$\underline{R_{10}}(t) = \underline{R_1}(0,\vartheta), \bar{R_{10}}(t) = \bar{R_1}(0,\vartheta), \tag{37}$$

$$\underline{R_{20}}(t) = \underline{R_2}(0,\vartheta), \bar{R_{20}}(t) = \bar{R_2}(0,\vartheta), \tag{38}$$

$$\underline{R_{30}}(t) = \underline{R_3}(0,\vartheta), \bar{R_{30}}(t) = \bar{R_3}(0,\vartheta), \tag{39}$$

$$\underline{R_{40}}(t) = \underline{R_4}(0,\vartheta), \bar{R_{40}}(t) = \bar{R_4}(0,\vartheta). \tag{40}$$

Eventually, we get the following calculations

$$\underline{R}_{1_n}(t) = \underline{R_{10}}(t) + \underline{R_{11}}(t) + \underline{R_{12}}(t) + \cdots, \tag{41}$$

$$\bar{R}_{1n}(t) = \bar{R_{10}}(t) + \bar{R_{11}}(t) = \bar{R_{12}}(t) + \cdots, \tag{42}$$

$$\underline{R}_{2_n}(t) = \underline{R_{20}}(t) + \underline{R_{21}}(t) + \underline{R_{22}}(t) + \cdots, \tag{43}$$

$$\bar{R}_{2n}(t) = \bar{R_{20}}(t) + \bar{R_{21}}(t) = \bar{R_{22}}(t) + \cdots, \tag{44}$$

$$\underline{R}_{3_n}(t) = \underline{R_{30}}(t) + \underline{R_{31}}(t) + \underline{R_{32}}(t) + \cdots, \tag{45}$$

$$\bar{R}_{3n}(t) = \bar{R_{30}}(t) + \bar{R_{31}}(t) = \bar{R_{32}}(t) + \cdots, \tag{46}$$

$$\underline{R}_{4_n}(t) = \underline{R_{40}}(t) + \underline{R_{41}}(t) + \underline{R_{42}}(t) + \cdots, \tag{47}$$

$$\bar{R}_{4n}(t) = \bar{R_{40}}(t) + \bar{R_{41}}(t) = \bar{R_{42}}(t) + \cdots, \tag{48}$$

with following conditions,

$$\tilde{R}_1(0,\vartheta) = (2\vartheta - 1, 1 - 2\vartheta), \tag{49}$$

$$\tilde{R}_2(0,\vartheta) = (2\vartheta - 1, 1 - 2\vartheta), \tag{50}$$

$$\tilde{R}_3(0,\vartheta) = (2\vartheta - 1, 1 - 2\vartheta), \tag{51}$$

$$\tilde{R}_4(0,\vartheta) = (2\vartheta - 1, 1 - 2\vartheta). \tag{52}$$

Followed by iterations calculated as

$$\underline{R_{10}}(t,\vartheta) = (2\vartheta - 1), \bar{R_{10}}(t,\vartheta) = (1 - 2\vartheta), \tag{53}$$

$$\underline{R_{20}}(t,\vartheta) = (2\vartheta - 1), \bar{R_{20}}(t,\vartheta) = (1 - 2\vartheta), \tag{54}$$

$$\underline{R_{30}}(t,\vartheta) = (2\vartheta - 1), \bar{R_{30}}(t,\vartheta) = (1 - 2\vartheta), \tag{55}$$

$$\underline{R_{40}}(t,\vartheta) = (2\vartheta - 1), \bar{R_{40}}(t,\vartheta) = (1 - 2\vartheta). \tag{56}$$

Second term of solution is calculated as

$$\underline{R_{11}}(t,\vartheta) = -\tilde{\alpha}\left\{(2\vartheta - 1)^2 + (2\vartheta - 1)\right\}\frac{t^\theta}{\Gamma(\theta + 1)}, \tag{57}$$

$$\bar{R}_{11}(t,\vartheta) = -\tilde{\alpha}\left\{(1 - 2\vartheta)^2 + (1 - 2\vartheta)\right\}\frac{t^\theta}{\Gamma(\theta + 1)}, \tag{58}$$

$$\underline{R_{21}}(t,\vartheta) = \tilde{\alpha}\left\{(2\vartheta - 1)^2 - \tilde{\beta}(2\vartheta - 1)\right\}\frac{t^\theta}{\Gamma(\theta + 1)}, \tag{59}$$

$$\bar{R}_{21}(t,\vartheta) = \tilde{\alpha}\left\{(1 - 2\vartheta)^2 - \tilde{\beta}(1 - 2\vartheta)\right\}\frac{t^\theta}{\Gamma(\theta + 1)}, \tag{60}$$

$$\underline{R_{31}}(t,\vartheta) = \tilde{\alpha}\left\{(2\vartheta - 1)^2 - \left(\tilde{\beta} + \tilde{\gamma}\right)(2\vartheta - 1)\right\}\frac{t^\theta}{\Gamma(\theta + 1)}, \tag{61}$$

$$\bar{R}_{31}(t,\vartheta) = \tilde{\alpha}\left\{(1 - 2\vartheta)^2 - \left(\tilde{\beta} + \tilde{\gamma}\right)(1 - 2\vartheta)\right\}\frac{t^\theta}{\Gamma(\theta + 1)}, \tag{62}$$

$$\underline{R_{41}}(t,\vartheta) = \tilde{\gamma}(2\vartheta - 1), \tag{63}$$

$$\bar{R}_{41}(t,\vartheta) = \tilde{\gamma}(1 - 2\vartheta). \tag{64}$$

Applying the same process, we can find the higher terms as follows

$$\begin{aligned}\underline{R_{12}}(t,\vartheta) = \Bigg(&\left[\left\{-\tilde{\alpha}(2\vartheta - 1)^2 + (2\vartheta - 1)\right\}2\vartheta - 1\right] \\ &- \left[\left\{\tilde{\alpha}(2\vartheta - 1)^2 + \tilde{\beta}(2\vartheta - 1)\right\}(2\vartheta - 1)\right] \\ &+ \left[\left\{-\tilde{\alpha}(2\vartheta - 1)^2 + \left(\tilde{\beta} + \tilde{\gamma}\right)(2\vartheta - 1)\right\}(2\vartheta - 1)\right]\Bigg)\frac{t^{2\theta}}{\Gamma(2\theta + 1)},\end{aligned} \tag{65}$$

$$\begin{aligned}\bar{R}_{21}(t,\vartheta) = \Bigg(&\left[\left\{-\tilde{\alpha}(1 - 2\vartheta)^2 + (1 - 2\vartheta)\right\}(1 - 2\vartheta)\right] \\ &- \left[\left\{\tilde{\alpha}(1 - 2\vartheta)^2 + \tilde{\beta}(c)\right\}(1 - 2\vartheta)\right] \\ &+ \left[\left\{-\tilde{\alpha}(1 - 2\vartheta)^2 + \left(\tilde{\beta} + \tilde{\gamma}\right)(1 - 2\vartheta)\right\}(1 - 2\vartheta)\right]\Bigg)\frac{t^{2\theta}}{\Gamma(2\theta + 1)},\end{aligned} \tag{66}$$

$$\begin{aligned}\underline{R_{22}}(t,\vartheta) = \Bigg(&\Bigg[-\Bigg\{\tilde{\alpha}\Bigg\{\tilde{\alpha}(2\vartheta - 1)^3 + \tilde{\alpha}\left[(2\vartheta - 1)^3 - \tilde{\beta}(2\vartheta - 1)^2\right] \\ &- \tilde{\beta}(2\vartheta - 1)\Bigg\} + \tilde{\beta}\left[\tilde{\alpha}(2\vartheta - 1)^2 - \tilde{\beta} + \tilde{\gamma}(2\vartheta - 1)\right]\Bigg\}\Bigg]\Bigg)\frac{t^{2\theta}}{\Gamma(2\theta + 1)},\end{aligned} \tag{67}$$

$$\begin{aligned}\bar{R}_{22}(t,\vartheta) = \Bigg(&\Bigg[-\Bigg\{\tilde{\alpha}\Bigg\{\tilde{\alpha}(1 - 2\vartheta)^3 + \tilde{\alpha}\left[(1 - 2\vartheta)^3 - \tilde{\beta}(1 - 2\vartheta)^2\right] \\ &- \tilde{\beta}(1 - 2\vartheta)\Bigg\} + \tilde{\beta}\left[\tilde{\alpha}(1 - 2\vartheta)^2 - \tilde{\beta} + \tilde{\gamma}(1 - 2\vartheta)\right]\Bigg\}\Bigg]\Bigg)\frac{t^{2\theta}}{\Gamma(2\theta + 1)},\end{aligned} \tag{68}$$

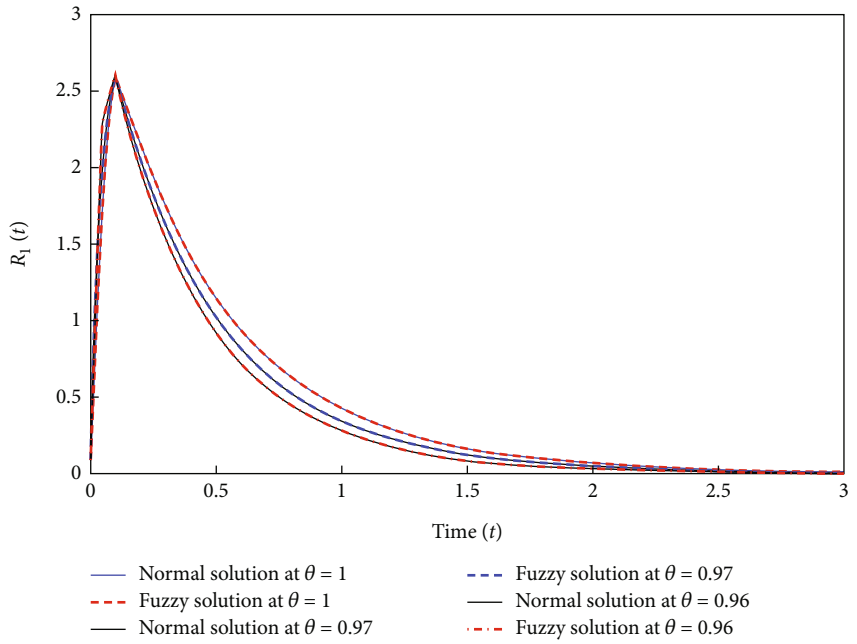

Figure 1: Fractional dynamics of reactants R_1 in the enzymatic reaction.

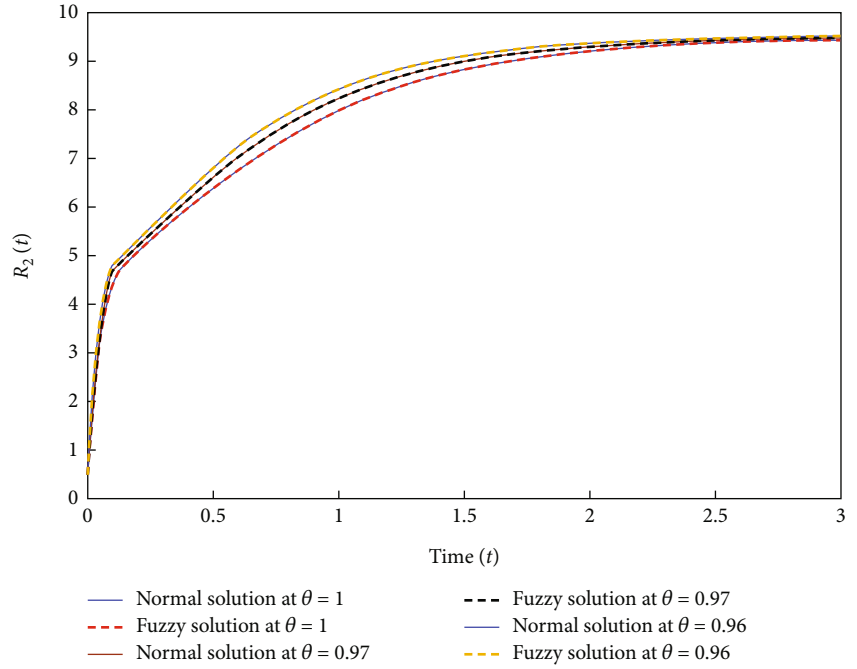

Figure 2: Fractional dynamics of reactants R_2 in the enzymatic reaction.

$$\underline{R_{32}}(t,\vartheta) = \left(\left[\left\{\tilde{\alpha}\left\{\tilde{\alpha}(2\vartheta-1)^3 + \tilde{\alpha}\left[(2\vartheta-1)^3 - \tilde{\beta}(2\vartheta-1)^2\right] - \tilde{\beta}(2\vartheta-1)\right\}\right.\right.\right.$$
$$\left.\left.\left. + \tilde{\beta}\left[\tilde{\alpha}(2\vartheta-1)^2 - \tilde{\beta} + \tilde{\gamma}(2\vartheta-1)\right]\right\}\right]\right)\frac{t^{2\theta}}{\Gamma(2\theta+1)}, \tag{69}$$

$$\bar{R_{32}}(t,\vartheta) = \left(\left[\left\{\tilde{\alpha}\left\{\tilde{\alpha}(1-2\vartheta)^3 + \tilde{\alpha}\left[(1-2\vartheta)^3 - \tilde{\beta}(1-2\vartheta)^2\right] - \tilde{\beta}(1-2\vartheta)\right\}\right.\right.\right.$$
$$\left.\left.\left. + \tilde{\beta}\left[\tilde{\alpha}(1-2\vartheta)^2 - \tilde{\beta} + \tilde{\gamma}(1-2\vartheta)\right]\right\}\right]\right)\frac{t^{2\theta}}{\Gamma(2\theta+1)}, \tag{70}$$

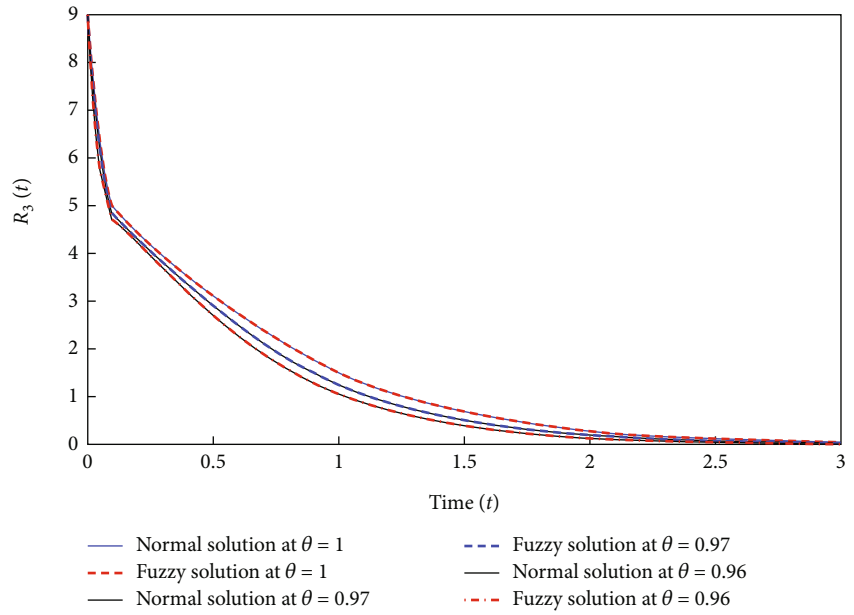

Figure 3: Fractional dynamics of reactants R_3 in the enzymatic reaction.

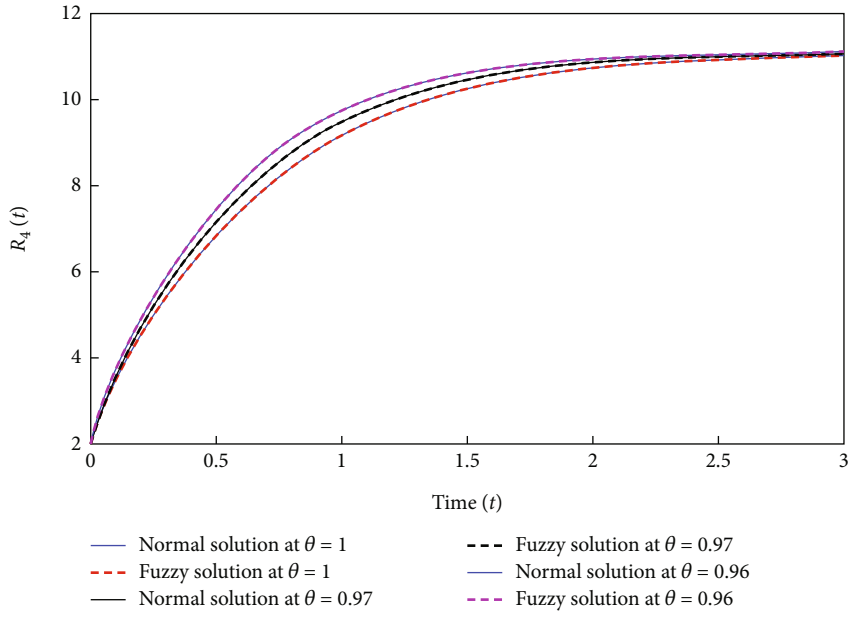

Figure 4: Fractional dynamics of reactants R_4 in the enzymatic reaction.

$$\underline{R_{42}}(t,\vartheta) = \tilde{\beta}\left[\tilde{\alpha}\left\{(2\vartheta-1)^2 - \tilde{\beta} + \tilde{\gamma}(2\vartheta-1)\right\}\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)}, \tag{71}$$

$$\overline{R_{42}}(t,\vartheta) = \tilde{\beta}\left[\tilde{\alpha}\left\{(1-2\vartheta)^2 - \tilde{\beta} + \tilde{\gamma}(1-2\vartheta)\right\}\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)}. \tag{72}$$

5. Numerical Results

Now we analyze the dynamics of a substrate's concentration, the enzyme's concentration, the concentration of the resulting complex, and the concentration of the resulting product in terms of fractional operators using the Homotopy perturbation method solution of fractional order. In Figures 1–4, we evaluated by comparing fuzzy and normal approximate solutions for the problem under discussion at various fractional orders against the observed uncertainty. The figures show that fuzzy logic, when combined with fractional calculus, provides global dynamics to nonlinear problems with uncertain data. Given that stochastic and random parameters are far more harder to resolve, and that uncertainty may lead to increases in estimation costs, modeling such physical problems using fuzzy notions is the right approach.

The values of R_1 first increases and then decreases drastically with a decrease in the fractional parameter θ.

The values of R_2 first increases at a slower rate and then increases exponentially with an increase in the fractional parameter θ.

The values of R_3 first decrease at a slower rate and then decrease exponentially with an increase in the fractional parameter θ.

The values of R_4 increase exponentially with a decrease in the fractional parameter θ.

6. Conclusion

Many academics have given numerical, approximate approaches and applications to handle Michaelis Menten enzymatic reaction model in general, due to the difficulty that appeared in obtaining accurate solutions to fractional differential equations. For examining the approximate solution of the Michaelis Menten enzymatic reaction equation, extension of the spectral homotopy analysis method, Runge-Kutta method, Laplace transformation, and Adomian decomposition approach has been used by researchers. We have developed a proper strategy for obtaining an approximate solution for the suggested model using the fuzzy theory and Homotopy perturbation method. To demonstrate the effectiveness of this strategy, we compared fuzzy and normal solutions up to three iterations. We discovered that fuzzy theory combined with fractional calculus technique yielded outstanding dynamics of Michaelis Menten enzymatic reaction model in instances where data uncertainty exists. By substituting classical differential derivatives with fractional derivatives based on fuzzy theory, we have suggested the new approach to Michaelis Menten enzymatic reaction model. The sequential iterations were built using fractional calculus theory and homotopy perturbation method in fuzzy sense. The numerical findings validated the fuzzy fractional case when compared to fractional order results.

7. Future Recommendations

As a result, developing various approaches known in the sense of fuzzy fractional differentials remains a future aim for us and many other scholars [21–25]. Finally, using the homotopy perturbation approach, the impacts of a wide range of fuzzy theory values and fractional order on the dynamics of fractional enzymatic reactions were examined. We propose that in future work, we concentrate on expanding this study with the help of other special functions and the use of two-scale fractal dimension. In addition, we can get additional results by using the modified homotopy perturbation method and He's fractal derivative.

Data Availability

All the data are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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