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Research Article

On the Characterization of Antineutrosophic Subgroup

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This article gives some essential scopes to study the characterizations of the antineutrosophic subgroup and antineutrosophic normal subgroup. Again, several theories and properties have been mentioned which are essential for analyzing their mathematical framework. Moreover, their homomorphic properties have been discussed.

1. Introduction

Fuzzy set (FS) [1] theory was introduced to handle uncertain situations more precisely than crisp sets. But there may exist some complex uncertain situations for which even FS theory is insufficient. As a result, intuitionistic fuzzy set (IFS) [2] and neutrosophic set (NS) [3] theories evolved, where the latter is more capable of dealing with uncertainties. Apart from these, there exist several byproducts of these set theories, like interval-valued versions [4–6]; type-I, type-II, and type-III versions; and soft [7–9] and hard versions. Presently, these theories have been adopted by several researchers in different applied fields. Also, in several pure mathematical fields, these notions are being utilized. In abstract algebra, Rosenfeld [10] was the pioneer to do so. He defined and studied the characteristics of a fuzzy subgroup (FSG). Thereafter, Das [11] presented the concept of the level subgroup of a FSG and showed several beautiful relationships between them. Afterward, Anthony and Sherwood [12, 13] redefined FSG by applying general T-norms and defined function generated FSG and subgroup generated FSG. In 1984, Mukherjee and Bhattacharya [14] introduced normal versions of FSG and cosets. Furthermore, Biswas [15] established the concept of intuitionistic fuzzy subgroup (IFSG). Similarly, Çetkin and Aygün [16] developed the neutrosophic subgroup (NSG) and studied its homomorphic properties. They have also established some connections between an NSG and its level subgroup.

The concept of the antifuzzy subgroup (AFSG) [17] is a kind of dual to FSG. It was defined and characterized by Biswas in 1990. He has mentioned some relationships between FSG and AFSG and studied several other properties. Similarly, there is notion of the intuitionistic antifuzzy subgroup (IAFSG) [18], which was developed by Li et al. in 2009. They have also studied its homomorphic properties and established some connections with its intuitionistic fuzzy counterpart. Table 1 contains some contributions of various researchers involving different antialgebraic notions under uncertainty.

Hence, it is obvious that antiversions of FSG, IFSG, etc. have been adopted by different researchers for the anticipation of unique and impactful results. In neutrosophic group theory, so far, authors have discussed NSGs and some of their algebraic structures. But still, the antineutrosophic subgroup (ANSG) is undefined. Also, the relationship between NSG and ANSG are still unexplored. Hence, this can be a fruitful area which can generate some scope of future research. Based on the aforementioned gaps, the objectives of this paper are as follows:

(i) to introduce ANSG and investigate its algebraic features

(ii) to define the antineutrosophic normal subgroup (ANNSG) and explore its algebraic characteristics

Table 1: Desk research of different antialgebraic notions.

Author & references	Year	Contributions in various fields
Kim et al. [19]	2005	Introduced the concept of antifuzzy ideals of near-rings and investigated some of its properties.
Feng & Yao [20]	2012	Introduced (λ, μ)-antifuzzy subgroups and studied its properties.
Kausar [21]	2019	Introduced intuitionistic fuzzy normal subrings and intuitionistic anti fuzzy normal subrings over a nonassociative ring and studied their properties.
Ejegwa et al. [22]	2021	Studied antifuzzy multigroup and its characteristics.
Hoskova-Mayerova & Al Tahan [23]	2021	Introduced different operations on fuzzy multi-ideals of near-rings and defined antifuzzy multisubnear-rings of near-rings and study their properties.
Ahmad et al. [24]	2021	Defined kernel subgroup of a FSG and AFSG and presented several results involving them.
Kalaiarasi et al. [25]	2022	Studied the properties of γ-antifuzzy normal subgroup and γ-fuzzy normal subgroup and presented their application in gene mutation.
Hemabala & Kumar [26]	2022	Introduced and analyzed anti neutrosophic multifuzzy ideals of γ near-ring and studied their product.

(iii) to figure out the relationships between NSG and ANSG

(iv) to study several homomorphic attributes of ANSG and ANNSG

This article has been structured in the following manner. In Section 2, desk research of FSG, IFSG, and NSG and their normal versions are given. Also, antiversions of FSG and IFSG are discussed. In Section 3, the notions of ANSG and ANNSG are introduced along with some other essential definitions and theories are given. Finally, in Section 4, conclusion is given by mentioning some scopes of further research.

2. Preliminaries

Here, some elementary set theories under uncertainties are discussed which are required for our current study.

Definition 1 (see [1]). A FS λ of a crisp set V is defined as $\lambda : V \longrightarrow [0, 1]$.

Definition 2 (see [2]). An IFS γ of a crisp set V is defined as $\gamma = \{(r, t_\gamma(r), f_\gamma(r)) : r \in V\}$, where t_γ and f_γ are, respectively, known as the membership and nonmembership degrees.

Definition 3 (see [3]). A NS η of a crisp set V is defined as $\eta = \{(r, t_\eta(r), i_\eta(r), f_\eta(r)) : r \in V\}$, where $t_\eta, i_\eta,$ and f_η are, respectively, known as the truth, indeterminacy, and falsity degrees.

Definition 4 (see [1]). Let ψ be a FS of V. Then, the set $\psi_t = \{r \in V : \psi(r) \geq t\} \forall t \in [0, 1]$ is denoted as a level subset of ψ.

Definition 5 (see [17]). Let φ be a FS of V. Then, the set $\bar{\varphi}_t = \{r \in V : \varphi(r) \leq t\} \forall t \in [0, 1]$ is denoted as a lower level subset of φ.

Next, the notions of FSG, IFSG, NSG, and a few of their essential properties are addressed.

2.1. Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Subgroup

Definition 6 (see [10]). For a classical group V, a FS ψ is denoted as a FSG iff $\forall m, r \in V$, the subsequent conditions are fulfilled:

(i) $\psi(m \cdot r) \geq \min \{\psi(m), \psi(r)\}$

(ii) $\psi(r^{-1}) \geq \psi(r)$

Theorem 7 (see [10]). *ψ is a FSG of V iff $\forall m, r \in V$ $\psi(mr^{-1}) \leq \min \{\psi(m), \psi(r)\}$.*

Proposition 8 (see [10]). *Homomorphic image and preimage of a FSG is a FSG.*

Theorem 9 (see [11]). *Let V be a classical group and $\psi \in FSG(V)$, then $\forall t \in [0, 1]$ with $\psi(e) \geq t$, ψ_t are classical subgroups of V.*

Theorem 10 (see [11]). *Let V be a classical group and $\forall t \in [0, 1]$ with $\psi(e) \geq t$, ψ_t are classical subgroups of V, then $\psi \in FSG(V)$.*

Definition 11 (see [11]). Let ψ be a FSG of a classical group V. Then, $\forall t \in [0, 1]$ and $\psi(e) \geq t$ the subgroups ψ_t are termed as level subgroups of ψ.

Definition 12 (see [15]). For a classical group V, an IFS $\gamma = \{(r, t_\gamma(r), f_\gamma(r)) : r \in V\}$ is denoted an IFSG iff $\forall m, r \in V$,

(i) $t_\gamma(m \cdot r) \geq \min \{t_\gamma(m), t_\gamma(r)\}$

(ii) $t_\gamma(r^{-1}) \geq t_\gamma(r)$

(iii) $f_\gamma(m \cdot r) \leq \max \{f_\gamma(m), f_\gamma(r)\}$

(iv) $f_\gamma(r^{-1}) \leq f_\gamma(r)$

Proposition 13 (see [15]). *For a classical group V, an IFS $\gamma = \{(m, t_\gamma(m), f_\gamma(m)): m \in V\}$ is an IFSG iff $\forall m, r \in V$*

(i) $t_\gamma(mr^{-1}) \geq \min\{t_\gamma(m), t_\gamma(r)\}$

(ii) $f_\gamma(mr^{-1}) \leq \max\{f_\gamma(m), f_\gamma(r)\}$

Theorem 14 (see [27]). *Let V and R be two classical groups and $l: V \longrightarrow R$ be a homomorphism. Also, let $\gamma \in IFSG(V)$ and $\gamma' \in IFSG(R)$. Then,*

(i) *If γ has the supremum property, then $l(\gamma) \in IFSG(R)$*

(ii) $l^{-1}(\gamma') \in IFSG(V)$

Definition 15 (see [27]). Let γ be an IFS of V and let $s_1, s_2 \in [0, 1]$ with $s_1 + s_2 \leq 1$. Then, the set $\gamma_{(s_1,s_2)} = \{m \in V : t_\gamma(m) \geq s_1 \& f_\gamma(m) \leq s_2\}$ is known as (s_1, s_2)-level set of γ.

Theorem 16 (see [27]). *Let V be a classical group and $\gamma \in IFSG(V)$. Then, $\forall s_1, s_2 \in [0, 1]$ with $t_\gamma(e) \geq s_1$ and $f_\gamma(e) \leq s_2$, $\gamma_{(s_1,s_2)}$ are classical subgroups of V.*

Theorem 17 (see [27]). *Let V be a classical group and $\forall s_1, s_2 \in [0, 1]$ with $t_\gamma(e) \geq s_1$ and $f_\gamma(e) \leq s_2$, $\gamma_{(s_1,s_2)}$ are classical subgroups of V. Then, $\gamma \in IFSG(V)$.*

Definition 18 (see [16]). For a classical group V, a NS δ is defined as an NSG of V iff the subsequent terms are fulfilled:

(i) $\delta(m \cdot r) \geq \min\{\delta(m), \delta(r)\}$, i.e., $t_\delta(m \cdot r) \geq \min\{t_\delta(m), t_\delta(r)\}$, $i_\delta(m \cdot r) \geq \min\{i_\delta(m), i_\delta(r)\}$ and $f_\delta(m \cdot r) \leq \max\{f_\delta(m), f_\delta(r)\}$

(ii) $\delta(m^{-1}) \geq \delta(m)$, i.e., $t_\delta(m^{-1}) \geq t_\delta(r)$, $i_\delta(m^{-1}) \geq i_\delta(r)$, and $f_\delta(m^{-1}) \leq f_\delta(r)$

A set of all the NSGs will be signified as $\mathrm{NSG}(V)$. Here, note that t_δ and i_δ are following Definition 6, i.e., they are FSGs of V whereas, f_δ is following Definition 24, i.e., it is an AFS of V.

Theorem 19 (see [16]). *For a classical group $V \delta \in NSG(V)$ iff $\forall m, r \in V$*

$$\delta(m \cdot r^{-1}) \geq \min\{\delta(m), \delta(r)\}, \quad (1)$$

i.e., $t_\delta(m \cdot r^{-1}) \geq \min\{t_\delta(m), t_\delta(r)\}$, $i_\delta(m \cdot r^{-1}) \geq \min\{i_\delta(m), i_\delta(r)\}$, and $f_\delta(m \cdot r^{-1}) \leq \max\{f_\delta(m), f_\delta(r)\}$.

Theorem 20 (see [16]). *$\delta \in NSG(V)$ iff the p-level sets $(t_\delta)_p$, $(i_\delta)_p$, and p-lower level set $(\bar{f}_\delta)_p$ are classical subgroups of $V \forall p \in [0, 1]$.*

Theorem 21 (see [16]). *Homomorphic image and preimage of any NSG is a NSG.*

Definition 22 (see [16]). For a classical group V, a neutrosophic δ is called an NNSG of V iff $\forall m, r \in V$

$$\delta(m \cdot r \cdot m^{-1}) \leq \delta(r), \quad (2)$$

i.e., $t_\delta(m \cdot r \cdot m^{-1}) \leq t_\delta(r)$, $i_\delta(m \cdot r \cdot m^{-1}) \leq i_\delta(r)$, and $f_\delta(m \cdot r \cdot m^{-1}) \geq f_\delta(r)$.

The set of all NNSG of V will be signified as $\mathrm{NNSG}(V)$. Also, notice that $\eta \in \mathrm{NNSG}(V)$ implies that t_δ and i_δ are fuzzy normal subgroups (FNSG) of V and f_δ is the antifuzzy normal subgroup (AFNSG) of V.

Theorem 23 (see [16]). *Homomorphic image and preimage of any NNSG is a NNSG.*

In the next segment, the notions of AFSG and IAFSG are discussed.

2.2. Antifuzzy Subgroup and Intuitionistic Antifuzzy Subgroup

Definition 24 (see [17]). For a classical group V, a FS φ is denoted as an AFSG of V if $\forall m, r \in V$, the subsequent terms are fulfilled:

(i) $\varphi(m \cdot r) \leq \max\{\varphi(m), \varphi(r)\}$

(ii) $\varphi(r^{-1}) \leq \varphi(r)$

Theorem 25 (see [17]). *φ is an AFSG of V iff $\forall m, r \in V$ $\varphi(mr^{-1}) \leq \max\{\varphi(m), \varphi(r)\}$.*

Proposition 26 (see [17]). *φ is a FSG of the group V iff its complement φ^c is an AFSG of V.*

Definition 27 (see [17]). Let φ be an AFSG of a group V. Then, $\forall t \in [0, 1]$ and $\varphi(e) \leq t$, the subgroups $\bar{\varphi}_t$ are called lower-level subgroups of φ.

Proposition 28 (see [17]). *Let φ be an AFSG of V. Then, $\forall t \in [0, 1]$ such that $t \geq \mu(e), \overline{\varphi_t}$ are classical subgroups of V.*

Proposition 29 (see [17]). *Let φ be a FS of a classical group V such that $\bar{\varphi}_t$ is a classical subgroup of $V \forall t \in [0, 1]$ with $t \geq \mu(e)$. Then, μ is an AFSG of V.*

Definition 30 (see [28]). For a classical group V, an IFS $\gamma = \{(m, t_\gamma(m), f_\gamma(m)): m \in V\}$ is called an IAFSG of V iff $\forall m, r \in V$

(i) $t_\gamma(mr^{-1}) \leq \max\{t_\gamma(m), t_\gamma(r)\}$

(ii) $f_\gamma(mr^{-1}) \geq \min\{f_\gamma(m), f_\gamma(r)\}$

Proposition 31 (see [28]). *γ is a IFSG of the group V iff its complement γ^c is an IAFSG of V.*

Theorem 32 (see [28]). *$\gamma \in IFSG(V)$ iff $\forall s_1, s_2 \in [0,1]$ with $t_\gamma(e) \geq s_1$ and $f_\gamma(e) \leq s_2$, (s_1, s_2)-level set of γ, i.e., $\gamma_{(s_1,s_2)}$ are classical subgroups of V.*

Theorem 33 (see [18]). *Homomorphic image and preimage of any IAFSG is a IAFSG.*

In the following section, the notions of ANSG and ANNSG have been introduced and some of their fundamental properties are discussed.

3. Antineutrosophic Subgroup

Definition 34. For a classical group V, a neutrosophic set η is called an ANSG of V iff the following terms are fulfilled:

(i) $\eta(m \cdot r) \leq \max\{\eta(m), \eta(r)\}$, i.e., $t_\eta(m \cdot r) \leq \max\{t_\eta(m), t_\eta(r)\}$, $i_\eta(m \cdot r) \leq \max\{i_\eta(m), i_\eta(r)\}$, and $f_\eta(m \cdot r) \geq \min\{f_\eta(m), f_\eta(r)\}$

(ii) $\eta(r^{-1}) \leq \eta(r)$, i.e., $t_\eta(r^{-1}) \leq t_\eta(r)$, $i_\eta(r^{-1}) \leq i_\eta(r)$, and $f_\eta(r^{-1}) \geq f_\eta(r)$

The set of all ANSGs will be signified as ANSG(V)

Proposition 35. *$\eta \in ANSG(V)$ iff t_η and i_η are AFSGs of V and f_η is FSG of V.*

Proof. Let $\eta \in$ ANSG(V) then from Definition 34, it is evident that t_η and i_η are following Definition 24, i.e., they are AFSGs of V. Whereas f_η is following Definition 6, i.e., it is a FSG of V. Again, if t_η and i_η are AFSGs of V and f_η is a FSG of V then $\eta \in$ ANSG(V). □

Example 36. Let $V = \{1, i, -1, -i\}$ be a classical group of order 4 and η be a neutrosophic set of V, where the memberships of truth (t_η), indeterminacy (i_η), and falsity (f_η) of elements in η are given in Figure 1.
Notice that t_η and i_η are following Definition 24, i.e., are AFSGs of V. Again, f_η is following Definition 6, i.e., is a FSG of V. Hence, η is an ANSG of V.

Example 37. Let $V = \{a, e\}$ be a classical group of order 2 and η be a NS of V, where considering $\theta \in [\pi/4, \pi/2]$, let $\eta = \{(a, \sin\theta/2, \sin\theta/4, (\sin\theta + \cos\theta)/2), (e, \cos\theta/2, \cos\theta/4, (\sin\theta + \cos\theta)/2)\}$. In Figures 2 and 3, memberships of a and e have been described graphically.

Here, η is following Definition 34 and hence it is an ANSG.

Theorem 38. *Let $\eta \in ANSG(V)$ where V is a classical group. Then, $\forall r \in V$*

(i) $\eta(r^{-1}) = \eta(r)$

(ii) $\eta(e) \leq \eta(r)$, where e is the neutral element of V

Proof.

(i) Here, f_η is a FSG and both t_η and i_η are AFSGs of V, by Definition 6. So, $f_\eta(r) = f_\eta((r^{-1})^{-1}) \geq f_\eta(r^{-1})$ and hence $f_\eta(r^{-1}) = f_\eta(r)$. Again, from Definition 24, $t_\eta(r^{-1}) \leq t_\eta(r)$. So, $t_\eta(r) = t_\eta((r^{-1})^{-1}) \leq t_\eta(r^{-1})$ and hence $t_\eta(r^{-1}) = t_\eta(r)$. Similarly, using Definition 24, we can prove $i_\eta(r^{-1}) = i_\eta(r)$. So, $\eta(r^{-1}) = \eta(r)$

(ii) Using Definition 6, we have $f_\eta(e) = f_\eta(r \cdot r^{-1}) \geq \min\{f_\eta(r), f_\eta(r^{-1})\} = f_\eta(r)$. Again, using Definition 24,

$$t_\eta(e) = t_\eta(r \cdot r^{-1}) \leq \max\{t_\eta(r), t_\eta(r^{-1})\} = t_\eta(r).$$

Similarly, using Definition 24, we have

$$i_\eta(e) = i_\eta(r \cdot r^{-1}) \leq \max\{i_\eta(r), i_\eta(r^{-1})\} = i_\eta(r).$$

Hence, $\eta(e) \leq \eta(r)$

□

Theorem 39. *$\eta \in ANSG(V)$ iff $\forall m, r \in V$ $\eta(m \cdot r^{-1}) \leq \max\{\eta(m), \eta(r)\}$.*

Proof. Let $\eta \in$ ANSG(V). Then, by Definition 34, we have $\eta(m \cdot r^{-1}) \leq \max\{\eta(m), \eta(r^{-1})\}$. Again, by Definition 34, $\eta(r^{-1}) = \eta(r)$ and hence

$$\eta(m \cdot r^{-1}) \leq \max\{\eta(m), \eta(r^{-1})\} = \max\{\eta(m), \eta(r)\}. \quad (3)$$

Conversely, let $\eta(m \cdot r^{-1}) \leq \max\{\eta(m), \eta(r)\}$. So,

$$\begin{aligned} t_\eta(m \cdot r^{-1}) &\leq \max\{t_\eta(m), t_\eta(r)\}, \\ i_\eta(m \cdot r^{-1}) &\leq \max\{i_\eta(m), i_\eta(r)\}, \\ f_\eta(m \cdot r^{-1}) &\geq \min\{f_\eta(m), f_\eta(r)\}. \end{aligned} \quad (4)$$

Notice that,

$$\begin{aligned} t_\eta(r^{-1}) = t_\eta(e \cdot r^{-1}) &\leq \max\{t_\eta(e), t_\eta(r)\} = \max\{t_\eta(r \cdot r^{-1}), t_\eta(r)\} \\ &\leq \max\{t_\eta(r), t_\eta(r), t_\eta(r)\} = t_\eta(r). \end{aligned} \quad (5)$$

Similarly, $i_\eta(r^{-1}) \leq i_\eta(r)$ and $f_\eta(r^{-1}) \geq f_\eta(r)$.
Again,

$$\begin{aligned} t_\eta(m \cdot r) = t_\eta\left(m \cdot (r^{-1})^{-1}\right) &\leq \max\{i_\eta(m), i_\eta(r^{-1})\} \\ &\leq \max\{i_\eta(m), i_\eta(r)\}. \end{aligned} \quad (6)$$

Similarly, $i_\eta(m \cdot r) \leq \max\{i_\eta(m), i_\eta(r)\}$ and $f_\eta(m \cdot r) \geq$

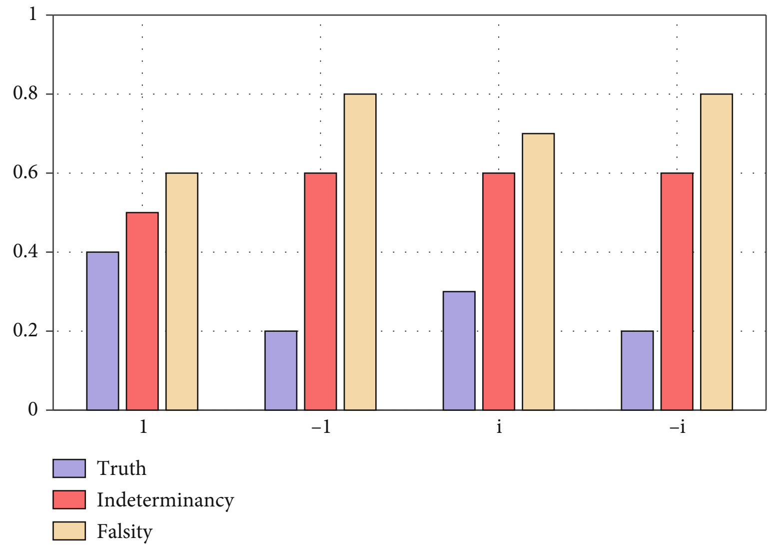

FIGURE 1: Memberships of elements in η.

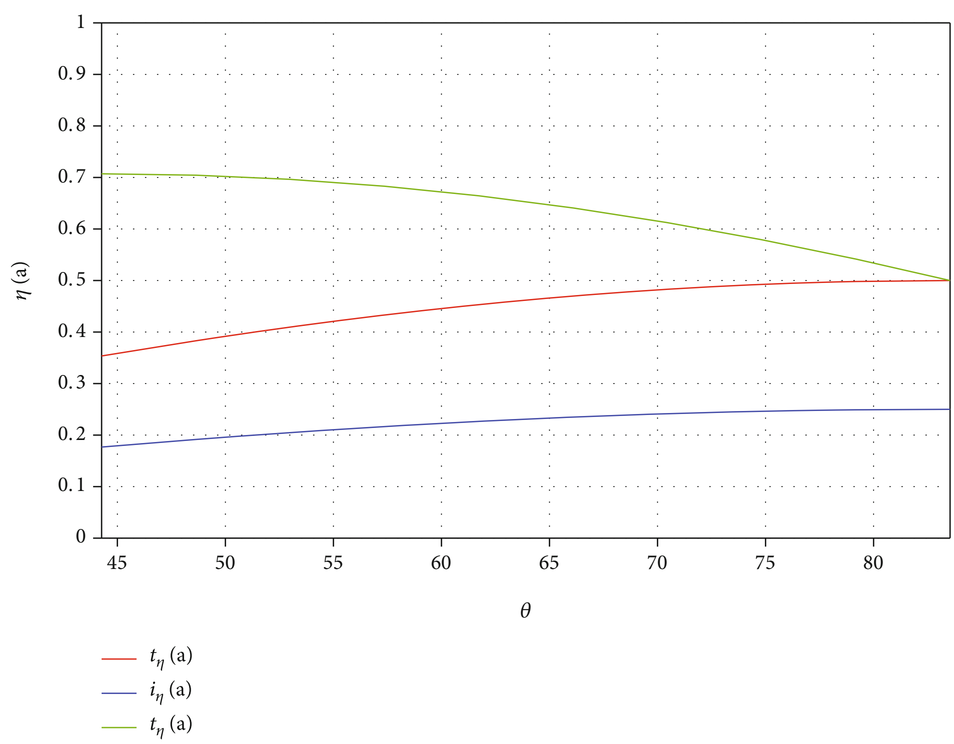

FIGURE 2: Memberships of elements in a.

$\min\{f_\eta(m), f_\eta(r)\}$ can be proved. Hence, η satisfies Definition 34, i.e., $\eta \in \mathrm{ANSG}(V)$. □

Theorem 40. *$\eta \in ANSG(V)$ iff $\eta^c \in NSG(V)$.*

Proof. If we take the complement of η, i.e., η^c then corresponding degree of truth and degree of falsity will interchange their positions in η^c. Also, the degree of indeterminacy will have its complement, i.e., $i_\eta^c = 1 - i_\eta$. In other words, if

$$\eta = \left\{\left(r, t_\eta(r), i_\eta(r), f_\eta(r)\right) : r \in V\right\} \text{ then } \eta^c = \left\{\left(r, f_\eta(r), i_\eta^c(r), t_\eta(r)\right) : r \in V\right\}. \tag{7}$$

Let $\eta \in \mathrm{ANSG}(V)$ then by Proposition 35 t_η and i_η are AFSGs of V and f_η is FSG of V. So, in case of η^c, f_η and i_η^c

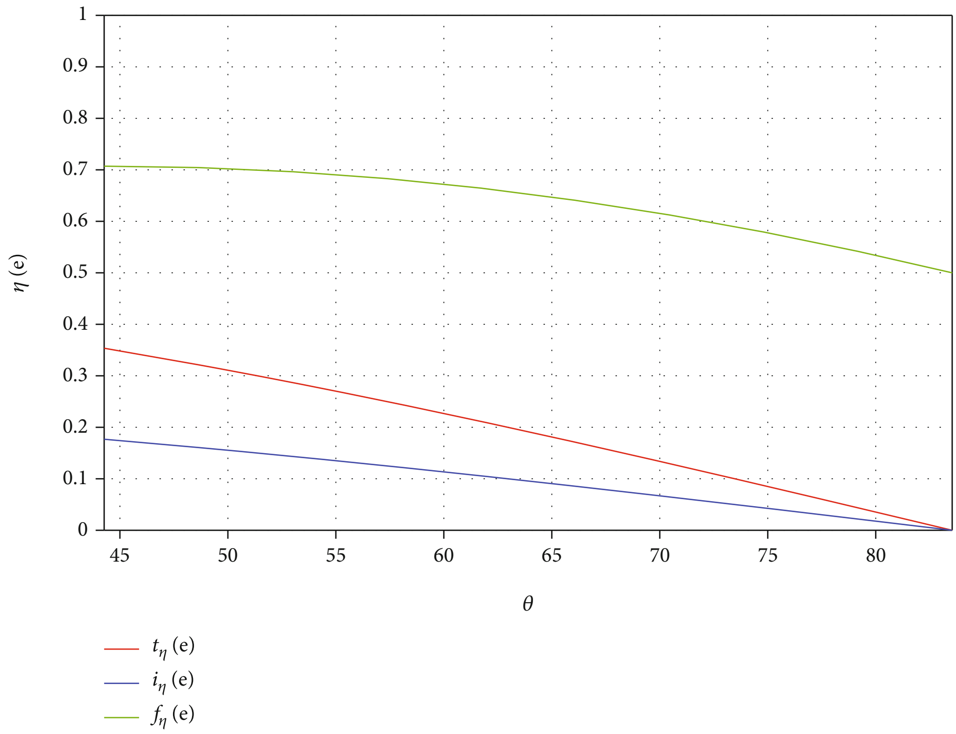

FIGURE 3: Memberships of elements in e.

will become FSGs and t_η will become AFS of V. Hence, they will follow Definition 18, i.e., $\eta^c \in \mathrm{NSG}(V)$. Similarly, the converse part can also be proved. □

Example 41. Let $(\mathbb{Z}_4, +)$ be the group of integers modulo 4 with usual addition and $\eta = \{(r, t_\eta(r), i_\eta(r), f_\eta(r)): r \in \mathbb{Z}_4\}$ is a NS of $\mathbb{Z}_4$, where t_η, i_η and f_η are mentioned in Table 2.

According to Definition 34, η is an ANSG of $\mathbb{Z}_4$.

Now $\eta^c = \{(r, t_{\eta^c}(r), i_{\eta^c}(r), f_{\eta^c}(r)): r \in \mathbb{Z}_4\}$, where t_{η^c}, i_{η^c}, and f_{η^c} are mentioned in Table 3.

Here, according to Definition 18, η^c is a NSG of $\mathbb{Z}_4$.

TABLE 2: Membership values of elements belonging to η.

η	t_η	i_η	f_η
$\bar{0}$	0.66	0.31	0.78
$\bar{1}$	0.85	0.35	0.59
$\bar{2}$	0.72	0.32	0.67
$\bar{3}$	0.85	0.35	0.59

TABLE 3: Membership values of elements belonging to η^c.

η^c	t_{η^c}	i_{η^c}	f_{η^c}
$\bar{0}$	0.78	0.69	0.66
$\bar{1}$	0.59	0.65	0.85
$\bar{2}$	0.67	0.68	0.72
$\bar{3}$	0.59	0.65	0.85

Theorem 42. *$\eta \in ANSG(V)$ iff the p-lower level sets $(\bar{t}_\eta)_p$, $(\bar{i}_\eta)_p$, and p-level set $(f_\eta)_p$ are classical subgroups of V $\forall p \in [0, 1]$.*

Proof. Let $\eta \in \mathrm{ANSG}(V)$, $p \in [0, 1]$ and $m, r \in (\bar{t}_\eta)_p$. Then, $t_\eta(m) \le p$ and $t_\eta(r) \le p$. Since $\eta \in \mathrm{ANSG}(V)$, we have $t_\eta(m \cdot r^{-1}) \le \max\{t_\eta(m), t_\eta(r)\} \le p$ and hence $m \cdot r^{-1} \in (\bar{t}_\eta)_p$. Similarly, it can be shown that $m \cdot r^{-1} \in (\bar{i}_\eta)_p$ and $m \cdot r^{-1} \in (f_\eta)_p$. So, $(\bar{t}_\eta)_p$, $(\bar{i}_\eta)_p$, and $(f_\eta)_p$ are classical subgroups of V.

Conversely, let $\forall p \in [0, 1] (\bar{t}_\eta)_p$ is a classical subgroup of V. Let $m, r \in V$ such that $t_\eta(m) = p_1$ and $t_\eta(r) = p_2$ for some $p_1, p_2 \in [0, 1]$. Then, $m \in (\bar{t}_\eta)_{p_1}$ and $r \in (\bar{t}_\eta)_{p_2}$.

Let $p_1 \le p_2$. Then, $m, r \in (\bar{t}_\eta)_{p_2}$ and hence $mu^{-1} \in (\bar{t}_\eta)_{p_2}$. So, $t_\eta(mr^{-1}) \le p_2 \le \max\{t_\eta(m), t_\eta(r)\}$, i.e., t_η is an AFSG of V. Similarly, it can be proved that i_η is an AFSG and f_η is a FSG of V. So, $\eta \in \mathrm{ANSG}(V)$. □

Theorem 43. *Intersection of any two ANSG of any group is an ANSG.*

Proof. Let $\eta_1, \eta_2 \in \mathrm{ANSG}(V)$. To prove this, using Theorem 39, we can show that

$$\begin{aligned}(\eta_1 \cap \eta_2)(m \cdot r^{-1}) &\le \max\{(\eta_1 \cap \eta_2)(m), (\eta_1 \cap \eta_2)(r)\}, \text{ i.e.,}\\ t_{(\eta_1\cap\eta_2)}(m \cdot r^{-1}) &\le \max\left\{t_{(\eta_1\cap\eta_2)}(m), t_{(\eta_1\cap\eta_2)}(r)\right\},\\ i_{(\eta_1\cap\eta_2)}(m \cdot r^{-1}) &\le \max\left\{i_{(\eta_1\cap\eta_2)}(m), i_{(\eta_1\cap\eta_2)}(r)\right\},\\ f_{(\eta_1\cap\eta_2)}(m \cdot r^{-1}) &\ge \min\left\{f_{(\eta_1\cap\eta_2)}(m), f_{(\eta_1\cap\eta_2)}(r)\right\}.\end{aligned} \tag{8}$$

Here,

$$\begin{aligned}t_{(\eta_1\cap\eta_2)}(m \cdot r^{-1}) &= \max\left\{t_{\eta_1}(m \cdot r^{-1}), t_{\eta_2}(m \cdot r^{-1})\right\}\\ &\le \max\left\{\max\left\{t_{\eta_1}(m), t_{\eta_1}(r)\right\}, \max\left\{t_{\eta_2}(m), t_{\eta_2}(r)\right\}\right\}\\ &= \max\left\{\max\left\{t_{\eta_1}(m), t_{\eta_2}(m)\right\}, \max\left\{t_{\eta_1}(r), t_{\eta_2}(r)\right\}\right\}\\ &= \max\left\{t_{(\eta_1\cap\eta_2)}(m), t_{(\eta_1\cap\eta_2)}(r)\right\}.\end{aligned} \tag{9}$$

Similarly, we can show that

$$i_{(\eta_1\cap\eta_2)}(m \cdot r^{-1}) \le \max\left\{i_{(\eta_1\cap\eta_2)}(m), i_{(\eta_1\cap\eta_2)}(r)\right\}. \tag{10}$$

Again,

$$\begin{aligned}f_{(\eta_1\cap\eta_2)}(m \cdot r^{-1}) &= \min\left\{f_{\eta_1}(m \cdot r^{-1}), f_{\eta_2}(m \cdot r^{-1})\right\}\\ &\ge \min\left\{\min\left\{f_{\eta_1}(m), f_{\eta_1}(r)\right\}, \min\left\{f_{\eta_2}(m), f_{\eta_2}(r)\right\}\right\}\\ &= \min\left\{\min\left\{f_{\eta_1}(m), f_{\eta_2}(m)\right\}, \min\left\{f_{\eta_1}(r), f_{\eta_2}(r)\right\}\right\}\\ &= \min\left\{f_{(\eta_1\cap\eta_2)}(m), f_{(\eta_1\cap\eta_2)}(r)\right\}.\end{aligned} \tag{11}$$

Hence, $\eta_1 \cap \eta_2 \in \mathrm{ANSG}(V)$. □

Theorem 44. *Homomorphic image of any ANSG is an ANSG.*

Proof. Let U_1 and U_2be two classical groups and $s : U_1 \longrightarrow U_2$ be a homomorphism. Let $\eta \in \mathrm{ANSG}(U_1)$. Then, $\forall$ $m_1, m_2 \in U_1$, we have

$$\begin{aligned}t_\eta(m_1 \cdot m_2{}^{-1}) &\le \max\left\{t_\eta(m_1), t_\eta(m_2)\right\},\\ i_\eta(m_1 \cdot m_2{}^{-1}) &\le \max\left\{i_\eta(m_1), i_\eta(m_2)\right\},\\ f_\eta(m_1 \cdot m_2{}^{-1}) &\ge \min\left\{f_\eta(m_1), f_\eta(m_2)\right\}.\end{aligned} \tag{12}$$

Here, we have to show that $s(\eta)$ is an ANSG of U_2.

Let $\exists n_1, n_2 \in U_2$ such that $n_1 = s(m_1)$ and $n_2 = s(m_2)$. Now, as s is a group homomorphism, we have

$$\begin{aligned}s(t_\eta)(n_1 \cdot n_2^{-1}) = \min_{m\in s^{-1}(n_1\cdot n_2^{-1})} t_\eta(m) &\le t_\eta(m_1 \cdot m_2^{-1})\\ &\le \max\left\{t_\eta(m_1), t_\eta(m_2)\right\}.\end{aligned} \tag{13}$$

Again, $s(t_\eta)(n_1) = \min_{m\in s^{-1}(n_1)} t_\eta(m) \le t_\eta(m_1)$. Where-from $\max s(t_\eta)(n_1) = t_\eta(m_1)$ and hence,

$$\begin{aligned}s(t_\eta)(n_1 \cdot n_2^{-1}) &\le \max\left\{t_\eta(m_1), t_\eta(m_2)\right\}\\ &= \max\left\{\max s(t_\eta)(n_1), \max s(t_\eta)(n_2)\right\}\\ &= \max\left\{s(t_\eta)(n_1), s(t_\eta)(n_2)\right\}.\end{aligned} \tag{14}$$

Similarly, it can be shown that $s(i_\eta)(n_1 \cdot n_2^{-1}) \le \max\{s(i_\eta)(n_1), s(i_\eta)(n_2)\}$.

Also,

$$\begin{aligned}s(f_\eta)(n_1 \cdot n_2^{-1}) = \max_{m\in s^{-1}(n_1\cdot n_2^{-1})} f_\eta(m) &\ge f_\eta(m_1 \cdot m_2^{-1})\\ &\ge \min\left\{f_\eta(m_1), f_\eta(m_2)\right\}.\end{aligned} \tag{15}$$

Again $s(f_\eta)(n_1) = \max_{m\in s^{-1}(n_1)} f_\eta(m) \ge t_\eta(m_1)$. Where-from $\min s(f_\eta)(n_1) = f_\eta(m_1)$ and hence

$$\begin{aligned}s(f_\eta)(n_1 \cdot n_2^{-1}) &\ge \min\left\{f_\eta(m_1), f_\eta(m_2)\right\}\\ &= \min\left\{\min s(f_\eta)(n_1), \min s(f_\eta)(n_2)\right\}\\ &= \min\left\{s(f_\eta)(n_1), s(f_\eta)(n_2)\right\}.\end{aligned} \tag{16}$$

So, $s(\eta)$ is an ANSG of U_2. □

Theorem 45. *Homomorphic preimage of any ANSG is an ANSG.*

Proof. Let U_1 and U_2 be two classical groups and $s : U_1 \longrightarrow U_2$ be a homomorphism. Let $\delta \in \mathrm{ANSG}(U_2)$. Then, $\forall$ $n_1, n_2 \in U_2$, we have

$$\begin{aligned}t_\delta(n_1 \cdot n_2{}^{-1}) &\le \max\{t_\delta(n_1), t_\delta(n_2)\},\\ i_\delta(n_1 \cdot n_2{}^{-1}) &\le \max\{i_\delta(n_1), i_\delta(n_2)\},\\ f_\delta(n_1 \cdot n_2{}^{-1}) &\ge \min\{f_\delta(n_1), f_\delta(n_2)\}.\end{aligned} \tag{17}$$

Here, we have to show that $s^{-1}(\delta)$ is an ANSG of U_1.

Let $m_1, m_2 \in U_1$. Since s is a group homomorphism,

$$\begin{aligned} s^{-1}(t_\delta)\left(m_1 \cdot m_2^{-1}\right) &= t_\delta\left(s\left(m_1 \cdot m_2^{-1}\right)\right) = t_\delta\left(s(m_1) \cdot s\left(m_2^{-1}\right)\right) \\ &= t_\delta\left(s(m_1) \cdot s(m_2)^{-1}\right) \le \max\{t_\delta(s(m_1)), t_\delta(s(m_2))\} \\ &= \max\left\{s^{-1}(t_\delta(m_1)), s^{-1}(t_\delta(m_2))\right\}. \end{aligned} \tag{18}$$

Similarly, we can show that

$$\begin{aligned} s^{-1}(i_\delta)\left(m_1 \cdot m_2^{-1}\right) &\le \max\left\{s^{-1}(i_\delta(m_1)), s^{-1}(i_\delta(m_2))\right\}, \\ s^{-1}(f_\delta)\left(m_1 \cdot m_2^{-1}\right) &\ge \min\left\{s^{-1}(f_\delta(m_1)), s^{-1}(f_\delta(m_2))\right\}. \end{aligned} \tag{19}$$

Hence, $s^{-1}(\delta)$ is an ANSG of U_1. □

Theorem 46. *Let $\eta \in ANSG(V)$ and l be a homomorphism on V. Let $\eta^{-1} : V \longrightarrow [0,1] \times [0,1] \times [0,1]$ is defined as $\eta^{-1}(r) = \eta(r^{-1})$ for any $r \in V$ then $\eta^{-1} \in ANSG(V)$ and $(l(\eta))^{-1} = l(\eta^{-1})$.*

Proof. Here,

$$\begin{aligned} \eta^{-1}\left(m \cdot r^{-1}\right) &= \eta\left(m \cdot r^{-1}\right)^{-1} = \eta\left(\left(r^{-1}\right)^{-1} \cdot m^{-1}\right) \\ &= \eta\left(r \cdot m^{-1}\right) \le \max\left\{\eta(r), \eta\left(m^{-1}\right)\right\} \\ &= \max\left\{\eta\left(r^{-1}\right), \eta\left(m^{-1}\right)\right\} [\text{as } \eta \text{ is an ANSG}] \\ &= \max\left\{\eta^{-1}(m), \eta^{-1}(r)\right\}. \end{aligned} \tag{20}$$

Hence, by Theorem 39, $\eta^{-1} \in \text{ANSG}(V)$.
Again, notice that,

$$\begin{aligned} l(t_\eta)^{-1}(q) &= l(t_\eta)\left(q^{-1}\right) = l(t_\eta)(q) \left[\text{as } l(t_\eta) \text{ is an ANSG}\right] \\ &= \min_{m \in l^{-1}(q)} t_\eta(m) = \min_{m \in l^{-1}(q)} t_\eta\left(m^{-1}\right) = \min_{m \in l^{-1}(q)} t_{\eta^{-1}}(m) \\ &= l\left(t_{\eta^{-1}}\right)(q). \end{aligned} \tag{21}$$

Similarly, it can be shown that $l(i_\eta)^{-1} = l(i_{\eta^{-1}})$ and $l(f_\eta)^{-1} = l(f_{\eta^{-1}})$.
Hence, $(l(\eta))^{-1} = l(\eta^{-1})$ □

Theorem 47. *Let $\eta \in ANSG(V)$ and l be an isomorphism on V, then $l^{-1}(l(\eta)) = \eta$.*

Proof. Here

$$l^{-1}\left(l(t_\eta)\right)(p) = l(t_\eta)(l(p)) = \min_{m \in l^{-1}(l(p))} t_\eta(m) = t_\eta(p). \tag{22}$$

Similarly, it can be shown that $l^{-1}(l(i_\eta)) = i_\eta$ and $l^{-1}(l(f_\eta)) = f_\eta$.
Hence, $l^{-1}(l(\eta)) = \eta$. □

In the next segment, ANNSG has been introduced. Also, its homomorphic characteristics are mentioned.

3.1. Antineutrosophic Normal Subgroup

Definition 48. For a classical group V, a neutrosophic set η is called an ANNSG of V iff $\forall m, r \in V \eta(m \cdot r \cdot m^{-1}) \le \eta(r)$, i.e., $t_\eta(m \cdot r \cdot m^{-1}) \le t_\eta(r)$, $i_\eta(m \cdot r \cdot m^{-1}) \le i_\eta(r)$, and $f_\eta(m \cdot r \cdot m^{-1}) \ge f_\eta(r)$.

The set of all ANNSGs of V will be signified as ANNSG(V).

Example 49. Let $V = \{e, m, r, mr\}$ be the Klien's 4-group and $\eta = \{(r, t_\eta(r), i_\eta(r), f_\eta(r)) : r \in V\}$ is a NS of V, where t_η, i_η, and f_η are mentioned in Table 4.
Here, η follows Definition 48, i.e., it is an ANNSG.

Proposition 50. *$\eta \in ANNSG(V)$ iff t_η and i_η are AFNSs of V and f_η is FNS of V.*

Proof. Using Definition 48, this can be observed. □

Theorem 51. *Intersection of any two ANNSG of any group is an ANNSG.*

Proof. Using Theorem 43, this can be proved. □

Theorem 52. *Let $\eta \in ANNSG(V)$. Then, the subsequent conditions are equivalent:*

(i) $\eta \in ANNS(U)$

(ii) $\eta(m \cdot r \cdot m^{-1}) = \eta(r)$, $\forall m, r \in V$

(iii) $\eta(m \cdot r) = \eta(V \cdot m)$, $\forall m, r \in V$

Proof. Let (i) be true. Then, by Definition 48, we have $\eta(m \cdot r \cdot m^{-1}) \le \eta(r)$, i.e., $t_\eta(m \cdot r \cdot m^{-1}) \le t_\eta(r)$, $i_\eta(m \cdot r \cdot m^{-1}) \le i_\eta(r)$, and $f_\eta(m \cdot r \cdot m^{-1}) \ge f_\eta(r)$.

To prove (ii), we need to show

$$\begin{aligned} t_\eta\left(m \cdot r \cdot m^{-1}\right) &\ge t_\eta(r), \\ i_\eta\left(m \cdot r \cdot m^{-1}\right) &\ge i_\eta(r), \\ f_\eta\left(m \cdot r \cdot m^{-1}\right) &\le f_\eta(r). \end{aligned} \tag{23}$$

In other words, we need to prove

$$\begin{aligned} t_\eta\left(m \cdot r \cdot m^{-1}\right) &= t_\eta(r), \\ i_\eta\left(m \cdot r \cdot m^{-1}\right) &= i_\eta(r), \\ f_\eta\left(m \cdot r \cdot m^{-1}\right) &= f_\eta(r). \end{aligned} \tag{24}$$

Table 4: Membership values of elements belonging to η.

η	t_η	i_η	f_η
e	0.1	0.5	0.9
a	0.3	0.6	0.7
b	0.4	0.5	0.6
ab	0.4	0.3	0.6

Notice that

$$t_\eta(m^{-1}\cdot r\cdot m)=t_\eta\left(m^{-1}\cdot r\cdot(m^{-1})^{-1}\right)\leq t_\eta(r). \quad (25)$$

Again,

$$t_\eta(r)=t_\eta(m^{-1}\cdot(m\cdot r\cdot m^{-1})\cdot m)\leq t_\eta(m\cdot r\cdot m^{-1}). \quad (26)$$

Hence, $t_\eta(m\cdot r\cdot m^{-1})=t_\eta(r)$.

Similarly, it can be shown that $i_\eta(m\cdot r\cdot m^{-1})=i_\eta(r)$ and $f_\eta(m\cdot r\cdot m^{-1})=f_\eta(r)$. Hence (i)⇒(ii).

Let condition (ii) be true. In (ii), substituting r in place of $r\cdot m^{-1}$ (iii) can easily be proved. So, (ii)⇒(iii).

Let condition (iii) be true. Applying $\eta(m\cdot r)=\eta(r\cdot m)$ in $t_\eta(m\cdot r\cdot m^{-1})$, we have

$$t_\eta(m\cdot r\cdot m^{-1})=t_\eta(r\cdot m^{-1}\cdot m)=t_\eta(r)\leq t_\eta(r). \quad (27)$$

So, (iii) ⇒ (i). □

Theorem 53. *$\eta\in ANNSG(V)$ iff the p-lower level sets $(\bar{t}_\eta)_p$, $(\bar{i}_\eta)_p$, and p-level set $(f_\eta)_p$ are classical normal subgroups of $V\forall p\in[0,1]$.*

Proof. Using Theorem 42, this can be proved. □

Theorem 54. *Let $\eta\in ANNSG(V)$. The set $U_\eta=\{m\in V:\eta(m)=\eta(e)\}$ is a classical normal subgroup of V, where e is the identity element of V.*

Proof. Since $\eta\in \mathrm{ANNSG}(V)$, we have $\eta\in \mathrm{ANSG}(V)$. Let $m,r\in U_\eta$ then by Theorem 39

$$\eta(m\cdot r^{-1})\leq \max\{\eta(m),\eta(r)\}=\max\{\eta(e),\eta(e)\}=\eta(e). \quad (28)$$

Again, by Theorem 38, we have $\eta(m\cdot r^{-1})\geq\eta(e)$ and hence $\eta(m\cdot r^{-1})=\eta(e)$, i.e., $m\cdot r^{-1}\in U_\eta$. Since $\eta\in \mathrm{ANNSG}(V)$, we have

$$\eta(m\cdot r\cdot m^{-1})=\eta(r\cdot m\cdot m^{-1})=\eta(r)=\eta(e), \quad (29)$$

i.e., $m\cdot r\cdot m^{-1}\in U_\eta$ or U_η is a normal subgroup of V. □

Theorem 55. *Let $\eta\in ANNSG(V)$ and l be a homomorphism on V. Then, the homomorphic pre-image of η, i.e., $l^{-1}(\eta)\in ANNSG(V)$.*

Proof. Using Theorem 44, we have $l^{-1}(\eta)\in \mathrm{ANSG}(V)$. Then, by Proposition 50, we can easily prove normality of $l^{-1}(\eta)$. Hence, $l^{-1}(\eta)\in \mathrm{ANNSG}(V)$. □

Theorem 56. *Let $\eta\in ANNSG(V)$ and l be a surjective homomorphism on V. Then the homomorphic image of η, i.e., $l(\eta)\in ANNSG(V)$.*

Proof. Using Theorem 44, we have $l(\eta)\in \mathrm{ANSG}(V)$. Again, by Proposition 50, the normality condition can easily be proved. So, $l(\eta)\in \mathrm{ANNSG}(V)$. □

4. Conclusion

The studies of ANSG and its normal version might open some new directions of research. Here, homomorphism has been introduced in ANSG and ANNSG to understand their algebraic characteristics. Moreover, connections with their nonantiversions are provided. For these, numerous examples, theories, and propositions are given. In the future, these studies can be further extended by introducing various notions like the antineutrosophic ideal, antineutrosophic ring, antineutrosophic field, and antineutrosophic topological space. Furthermore, their interval-valued versions can be introduced and studied.

Data Availability

This work is a contribution towards the theoretical development of fuzzy algebra and its generalizations. The data that support the findings of this study are not publicly available due to the fact that they were created specifically for this study. We have not used any additional data set for drafting this manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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