

## Research Article

# Command Filter AILC for Finite Time Accurate Tracking of Aircraft Track Angle System Based on Fuzzy Logic

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In this paper, the longitudinal model of an uncertain aircraft is taken as the research object, and the aircraft path inclination is controlled by controlling the input rudder deflection angle. An adaptive iterative learning control (AILC) scheme is proposed to solve the accurate tracking control problem of the flight path inclination on a finite time interval. The aircraft track angle system is abstractly modeled to obtain a triangular model in the form of strict feedback. For the abstracted strict feedback model, the fuzzy logic is used to approximate the uncertain part of the model. A command filter and an error compensation mechanism are introduced to prevent the computational bloat problem caused by excessive system order, and a convergent series sequence is used to deal with the truncation error caused by the approximation of the fuzzy logic. Based on the Lyapunov stability theorem, all signals of the closed-loop system are bounded on the finite time interval  $[0, T]$ , and the output of the system can track the desired trajectory accurately. Finally, the feasibility and effectiveness of the method are verified by MATLAB simulation results.

## 1. Introduction

Nowadays, science and technology are constantly developing toward the direction of intelligence, the application scope of aircraft is becoming wider and wider, and flight missions are becoming more and more complex. There will also be a certain threshold for the performance indicators of aircraft control systems. The characteristics of aircraft include complex flight environment, strong nonlinearity of the model, and so on. Nonlinear control of aircraft [1–3] has always been a hot and challenging problem in the field of control. The tracking control of aircraft track angle [4–5] is a particularly important part of the aircraft control system, which has been widely used in aircraft system design.

When control system is an uncertain system, it is obviously difficult to meet the control requirements of the system if the conventional controller is applied to the system. However, adaptive control can automatically adjust parameters to compensate for uncertainties in models [6], parameters [7], and input signals [8–10]. Compared with other control methods, it has significant advantages in controlling systems with uncertainties. This is a control method with online parameter identification. With the continuous development of intelligent

control methods, more and more scholars combine an adaptive control method with other intelligent control methods to conduct the control research, and obtain many good results. The paper by Bao et al. [11] proposed an adaptive fuzzy control and optimization framework for nonlinear systems affected by uncertainties and disturbances. Zhuang et al. [12] applies robust adaptive control and backstepping method to attitude control of the aircraft system. An adaptive law combined with sliding mode control (SMC) is designed to compensate the unknown and uncertain parts of the aircraft system. In a study by Zhu et al. [13], an adaptive iterative learning disturbance observer based on adaptive notch filter is proposed to reduce the complex disturbance of unknown fundamental frequency caused by the spacecraft formation maneuver, so as to achieve high performance of spacecraft formation flight. The paper by Guo and Su [14] proposed a nonlinear optimal learning control method based on an adaptive dynamic programming for hypersonic vehicle control and designed speed and height adaptive control systems. The paper by Wang et al. [15] designed an adaptive dynamic inversion landing control law to solve the problems of parameter uncertainty and ship wake interference in the process of carrier-based UAV landing. From the

above discussion, it can be seen that the uncertainty problem in the aircraft track angle system is a widespread problem that cannot be ignored and is a very important research topic.

Generally speaking, the iterative learning control (ILC) method is that the current experience is the result of the previous learning, and the expected effect is achieved with the increase of learning times. ILC is widely used in tracking control of control systems with repetitive motions. Because ILC does not require a lot of prior knowledge, it can deal with uncertainty problem, but there are still some limitations in dealing with uncertainty problem. Therefore, adaptive control and ILC are combined to form an adaptive iterative learning control (AILC) method [16–18]. This method has the advantages of both adaptive control and ILC. It can not only solve the uncertainty problem of the system, but also solve the tracking problem of the system with repeated tracking. Therefore, it is a very good choice to apply this method to the tracking problem of aircraft track angle system on the finite time interval.

Fuzzy concepts have been widely recognized and applied by the scholars. Recently, Li et al. [19] proposed a nonlinear dynamic index-fuzzy dispersed entropy, which combines fuzzy with other methods and achieves good experimental results. For unknown nonlinear functions in the processing system, both fuzzy logic system [20] and neural network function [21–22] can be used for approximation. Because its fuzzy rules can ensure that it is not disturbed by noise, in fuzzy systems, the design of fuzzy sets, membership functions, and fuzzy rules is based on the empirical knowledge. This design method has great subjectivity. The learning mechanism is introduced into the fuzzy system [23], so that the fuzzy system can modify and improve the membership function and fuzzy rules through continuous learning. The fuzzy logic and adaptive iterative learning method are applied together. Fuzzy logic system approximation can fully utilize the information ability of language and is relatively easy to construct. Neural network approximators or more advanced machine learning algorithms may have better approximation accuracy than fuzzy approximations, but when fitting nonlinear functions, they rely on a large dataset (the richer the set, the higher the degree of fitting), and their construction is more complex, making practical applications more difficult. The fuzzy logic is used to approximate the unknown function of the system, and the fuzzy adaptive iterative learning controller is designed, which can better improve the control performance of the system.

With the rapid development of information technology, the development of aircraft technology is also faster and faster. For the research of aircraft track angle system, it is generally adopted to abstract the aircraft track angle model into the aircraft longitudinal model for research [24–26]. It can be further simplified into a strict feedback system model. The backstepping method is a powerful tool for dealing with high-order strict feedback systems. In the paper by Zhang et al. [27], the authors proposed a new AILC method for the finite time tracking control problem of uncertain aircraft track angle system based on the neural network. However, the quasi controller and the desired output are derived

continuously in the design process. With the increase of the order of the system, the realization of the controller will become extremely complex, which will lead to the explosion problem of complexity. Based on this paper by Zhang et al. [27], considering that the command filter [28, 29] can estimate the derivative of the virtual controller to deal with the corresponding explosion problem of complexity. We introduce it to solve the explosion problem of complexity in the process of designing the controller of aircraft track angle system.

To sum up, the longitudinal model of uncertain aircraft is taken as the research object in this paper. Aiming at the simplified strict feedback model, fuzzy logic is used to approximate the uncertain part of the simplified model. An adaptive iterative learning controller and the parameter adaptive laws are designed based on Lyapunov function. Through adjusting the parameters of the controller, the trajectory inclination of the aircraft can track the desired trajectory on a finite time interval. The simulation results show that the design method is correct and effective. The main contributions can be summarized as follows:

- (1) In this paper, ILC is introduced for the first time to solve the tracking control problem of aircraft track angle. Due to the repeated operation of the aircraft track angle system, it is feasible and practical to introduce the ILC method into the tracking control of this system.
- (2) The adaptive learning control for uncertain aircraft track angle system is proposed to tackle the output tracking problem on a finite time interval completely. There are few relevant references in this field of research.
- (3) Compared with the paper by Zhang et al. [27], the command filter is introduced to estimate the derivative of the virtual controller and solve the explosion problem of complexity in the process of designing the controller of aircraft track angle system. The controller becomes simpler.

The main content and structural arrangement of this article are as follows:

- (1) Introduction: the purpose and significance of the proposed method were briefly explained.
- (2) Model building: analyze the given controlled system model and approach the uncertain parts of the model using a fuzzy logic system.
- (3) Controller design and analysis, as well as simulation verification: the adaptive iterative learning controller is designed based on Lyapunov function, and the stability of the system is analyzed, so that the aircraft track inclination can track the expected trajectory in a limited time. Finally, the correctness and effectiveness of the proposed method are verified through simulation.
- (4) Discussion: summarize the work content and conclusions of this paper.

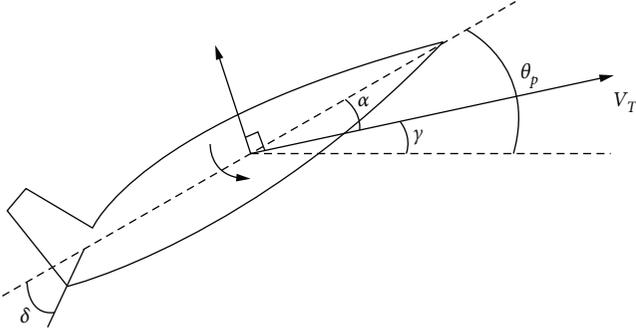


FIGURE 1: Longitudinal model of aircraft.

## 2. Model Building

The following simplified longitudinal model [21] of the aircraft is considered in Figure 1:

Consider the simplified model with external interference:

$$\begin{aligned}\dot{\gamma} &= \bar{L}_\alpha \alpha - \frac{g}{V_T} \cos \gamma + \bar{L}_0 + \Delta_1(t) \\ \dot{\alpha} &= q + \frac{g}{V_T} \cos \gamma - \bar{L}_\alpha \alpha - \bar{L}_0 + \Delta_2(t) \\ \dot{q} &= M_0 + M_\delta \delta + \Delta_3(t) \\ \dot{\theta}_p &= q,\end{aligned}\quad (1)$$

where  $\bar{L}_0 = L_0/mV_T$ ,  $\bar{L}_\alpha = L_\alpha/mV_T$ .  $\gamma$  is the aircraft track inclination angle;  $\alpha$  is the attack angle of the aircraft;  $\theta_p$  is the aircraft pitch angle;  $q$  is the change speed of pitch angle;  $V_T$  is the flight speed;  $m$  is the mass of the aircraft;  $g$  is the acceleration of gravity;  $M_\delta$  is pitch control torque;  $M_0$  is torque from other sources,  $M_0 = M_\alpha \alpha + M_q q$ ;  $L_\alpha$  is the slope of the lift curve;  $L_0$  is the influence factor of other lift;  $\delta$  is the control surface deflection angle. Here  $L_\alpha$ ,  $L_0$ ,  $M_\delta$ , and  $M_0$  are unknown constants,  $\Delta_i(t)$ ,  $i = 1, 2, 3$  are unknown external interference terms.

The model needs to meet the following assumptions:

**Assumption 1.** The speed  $V_T$  is regarded as a constant.

**Assumption 2.** All state variables are measurable.

**Assumption 3.** Unknown parameters are bounded, that is to say, for  $i = 1, 3$ , there exists known positive numbers  $a_{im}$ ,  $a_{iM}$  satisfying  $a_{im} \leq a_i \leq a_{iM}$ .

**Assumption 4.** The target tracking track is an ideal track, and its derivatives of all orders exist and are bounded.

Define states  $x_{1,k} = \gamma_k$ ,  $x_{2,k} = \alpha_k$ , and  $x_{3,k} = q_k$ . Control input is control surface deflection angle  $u_k = \delta_k$  and  $y_k = x_{1,k} = \gamma_k$  are the output of the aircraft track angle system, where  $k$  is the number of iterations. The following triangle model by Yue et al. [21] is obtained in strict feedback form by variable substitution:

$$\begin{aligned}\dot{x}_{1,k} &= a_1 x_{2,k} + W_{1,k}(x_{1,k}, t) \\ \dot{x}_{2,k} &= x_{3,k} + W_{2,k}(x_{1,k}, x_{2,k}, t) \\ \dot{x}_{3,k} &= a_3 u_k + W_{3,k}(x_{2,k}, x_{3,k}, t) \\ y_k &= x_{1,k},\end{aligned}\quad (2)$$

where  $W_{1,k} = f_{1,k}(x_{1,k}) + \Delta_{1,k}(t)$ ,  $W_{2,k} = f_{2,k}(x_{1,k}, x_{2,k}) + \Delta_{2,k}(t)$ , and  $W_{3,k} = f_{3,k}(x_{2,k}, x_{3,k}) + \Delta_{3,k}(t)$  are the uncertain parts of the system.  $|\Delta_{i,k}(t)| \leq \rho_i$ ,  $\rho_i$  are the unknown positive constants;  $a_1 = \bar{L}_\alpha > 0$ ,  $a_3 = M_\delta > 0$ .

$$\begin{aligned}f_{1,k}(x_{1,k}) &= -\frac{g}{V_T} \cos x_{1,k} + \bar{L}_0 \\ f_{2,k}(x_{1,k}, x_{2,k}) &= \frac{g}{V_T} \cos x_{1,k} - \bar{L}_0 - \bar{L}_\alpha x_{2,k} \\ f_{3,k}(x_{2,k}, x_{3,k}) &= M_\alpha x_{2,k} + M_q x_{3,k}.\end{aligned}\quad (3)$$

The control objective of this paper is to design an AILC law  $u_k$  for the simplified longitudinal model (1) of aircraft track angle with uncertain disturbances, making that the output  $y_k$  of the system can track the given smooth reference trajectory  $y_r(t)$  on finite time interval  $[0, T]$ .

## 3. AILC Design, Stability Analysis, and Simulation Research

In this part, we will give the specific controller design and stability analysis process and the relevant simulation results.

**3.1. Controller Design.** In the process of controller design, the following definition and lemma of convergence series sequence will be used.

**Definition 1.** [21]  $\{\Delta_k\}$  is a convergengce series sequence, which is defined as follows:

$$\Delta_k = \frac{a}{k^l},\quad (4)$$

where  $k = 1, 2, \dots$ ;  $a$  and  $l$  are constants,  $a > 0 \in \mathbb{R}$ ,  $l \geq 2 \in \mathbb{N}$ .

**Lemma 1.** [21] For the given sequence  $\{1/k^l\}$ ,  $k = 1, 2, \dots$ ,  $l \geq 2$  is a positive integer satisfying the following Equation:

$$\lim_{k \rightarrow \infty} \sum_{i=1}^k \frac{1}{i^l} \leq 2.\quad (5)$$

The specific design process of the controller is given below. Define the following three tracking error

$$z_{1,k} = y_k - y_r = x_{1,k} - y_r,\quad (6)$$

$$z_{2,k} = x_{2,k} - \beta_{2,k},\quad (7)$$

$$z_{3,k} = x_{3,k} - \beta_{3,k},\quad (8)$$

where  $\beta_{i,k}(t) \in R$  is the output of an instruction filter whose input is  $\alpha_{i-1,k}$ . Here  $\alpha_{i-1,k} \in R$  is a virtual control law, which is used to assist the design of actual control law  $u_k$ . Specifically, a command filter in the following form is adopted in the following controller design process:

$$\dot{\beta}_{i,k} = -\omega_i(\beta_{i,k} - \alpha_{i-1,k}), \quad (i = 2, 3), \quad (9)$$

where  $\omega_i > 0$  is a positive number. The initial condition of the filter is  $\beta_{i,k}(0) = \alpha_{i-1,k}(0)$ .

In fact, Equation (9) is a first-order low-pass filter whose unit DC gain and bandwidth can be adjusted by parameter  $\omega_i$ . Therefore, there must be a part of  $\alpha_{i-1,k}$  that cannot be passed by the filter, namely  $\beta_{i,k} - \alpha_{i-1,k}$ . As the order of the system increases, this part will become larger and larger. Finally, it may evolve into a key problem affecting the stability of the control system. Therefore, in order to overcome the impact of the command filter, an error compensation mechanism is introduced,

$$\dot{\xi}_{1,k} = a_1(-c_1\xi_{1,k} + \xi_{2,k} + (\beta_{2,k} - \alpha_{1,k})), \quad (10)$$

$$\dot{\xi}_{2,k} = -c_2\xi_{2,k} + \xi_{3,k} - \xi_{1,k} + (\beta_{3,k} - \alpha_{2,k}), \quad (11)$$

$$\dot{\xi}_{3,k} = a_3(-c_3\xi_{3,k} - \xi_{2,k}), \quad (12)$$

where  $i = 1, 2, 3$ ,  $c_i > 0$  are the controller parameters which are needed to be designed,  $\xi_{i,k} \in R$  is the deviation signal of  $\beta_{i,k} - \alpha_{i-1,k}$  processed by the error compensation mechanism. By removing the deviation generated by the command filter from  $z_{i,k}$ , that is to say  $\xi_{i,k}$ , the following compensation error signals can be obtained,

$$v_{i,k} = z_{i,k} - \xi_{i,k}, \quad i = 1, 2, 3. \quad (13)$$

Apply the above command filter and error compensation mechanism to the system (2) to assist in the design of controller, the specific steps are as follows.

Derive from Equations (6)–(8) and combine with Equation (2), then

$$\dot{z}_{1,k} = \dot{x}_{1,k} - \dot{y}_r = a_1\left(x_{2,k} + \frac{W_{1,k}}{a_1} - \frac{1}{a_1}\dot{y}_r\right), \quad (14)$$

$$\dot{z}_{2,k} = \dot{x}_{2,k} - \dot{\beta}_{2,k} = x_{3,k} + W_{2,k} - \dot{\beta}_{2,k}, \quad (15)$$

$$\dot{z}_{3,k} = \dot{x}_{3,k} - \dot{\beta}_{3,k} = a_3\left(u_k + \frac{W_{3,k}}{a_3} - \frac{1}{a_3}\dot{\beta}_{3,k}\right). \quad (16)$$

Combining Equations (10)–(12) and Equations (14)–(16), it is easy to obtain

$$\dot{v}_{1,k} = \dot{z}_{1,k} - \dot{\xi}_{1,k} = a_1\left(\alpha_{1,k} + v_{2,k} + \frac{W_{1,k}}{a_1} - \frac{1}{a_1}\dot{y}_r + c_1\xi_{1,k}\right), \quad (17)$$

$$\dot{v}_{2,k} = \dot{z}_{2,k} - \dot{\xi}_{2,k} = \alpha_{2,k} + v_{3,k} + W_{2,k} - \dot{\beta}_{2,k} + c_2\xi_{2,k} + \xi_{1,k}, \quad (18)$$

$$\dot{v}_{3,k} = \dot{z}_{3,k} - \dot{\xi}_{3,k} = a_3\left(u_k + \frac{W_{1,k}}{a_3} - \frac{1}{a_3}\dot{\beta}_{3,k} + c_3\xi_{3,k} + \xi_{2,k}\right). \quad (19)$$

Then, fuzzy logics are used to approximate  $W_{1,k}/a_1$ ,  $W_{2,k}$ ,  $W_{3,k}/a_3$ , we have

$$\begin{aligned} \frac{W_{1,k}}{a_1} &= \theta_1^{*T} \zeta_1(x_{1,k}, t) + \varepsilon_{1,k}(t) \\ W_{2,k} &= \theta_2^{*T} \zeta_2(x_{1,k}, x_{2,k}, t) + \varepsilon_{2,k}(t) \\ \frac{W_{3,k}}{a_3} &= \theta_3^{*T} \zeta_3(x_{2,k}, x_{3,k}, t) + \varepsilon_{3,k}(t), \end{aligned} \quad (20)$$

where  $\theta_1^*$ ,  $\theta_2^*$ , and  $\theta_3^*$  are the unknown weights,  $\varepsilon_{1,k}(t)$ ,  $\varepsilon_{2,k}(t)$ , and  $\varepsilon_{3,k}(t)$  are bounded approximation error, i.e.,  $|\varepsilon_{1,k}| \leq \varepsilon_{M1}$ ,  $|\varepsilon_{2,k}| \leq \varepsilon_{M2}$ , and  $|\varepsilon_{3,k}| \leq \varepsilon_{M3}$ .

Substitute Equation (20) into Equations (17)–(19), then

$$\dot{v}_{1,k} = \dot{z}_{1,k} - \dot{\xi}_{1,k} = a_1\left(\alpha_{1,k} + v_{2,k} + \theta_1^{*T} \zeta_1 + \varepsilon_{1,k} - \frac{1}{a_1}\dot{y}_r + c_1\xi_{1,k}\right), \quad (21)$$

$$\dot{v}_{2,k} = \dot{z}_{2,k} - \dot{\xi}_{2,k} = \alpha_{2,k} + v_{3,k} + \theta_2^{*T} \zeta_2 + \varepsilon_{2,k} - \dot{\beta}_{2,k} + c_2\xi_{2,k} + \xi_{1,k}, \quad (22)$$

$$\dot{v}_{3,k} = \dot{z}_{3,k} - \dot{\xi}_{3,k} = a_3\left(u_k + \theta_3^{*T} \zeta_3 + \varepsilon_{3,k} - \frac{1}{a_3}\dot{\beta}_{3,k} + c_3\xi_{3,k} + \xi_{2,k}\right). \quad (23)$$

Define

$$\theta_1^T = \theta_1^{*T}, \theta_2^T = \theta_2^{*T}, \theta_3^T = \theta_3^{*T}, \quad (24)$$

$$\frac{1}{a_1} = P_1, \frac{1}{a_3} = P_2. \quad (25)$$

Design the virtual control laws  $\alpha_{1,k}$ ,  $\alpha_{2,k}$  and actual control law  $u_k$  as follows:

$$\alpha_{1,k} = -\widehat{\theta}_{1,k}^T \zeta_1 - \frac{1}{\Delta_k} v_{1,k} \widehat{S}_{1,k} - c_1 z_{1,k} + \widehat{P}_{1,k} \dot{y}_r, \quad (26)$$

$$\alpha_{2,k} = -\widehat{\theta}_{2,k}^T \zeta_2 - \frac{1}{\Delta_k} v_{2,k} \widehat{S}_{2,k} - c_2 z_{2,k} + \dot{\beta}_{2,k} - z_{1,k}, \quad (27)$$

$$u_k = -\hat{\theta}_{3,k}^T \zeta_3 - \frac{1}{\Delta_k} v_{3,k} \hat{S}_{3,k} - c_3 z_{3,k} + \hat{P}_{2,k} \dot{\beta}_{3,k} - z_{2,k}, \quad (28)$$

where  $c_i, i = 1, 2, 3$  are positive adjustable gain,  $\hat{\theta}_{i,k}, \hat{S}_{i,k}, \hat{P}_{1,k}$ , and  $\hat{P}_{2,k}$  are estimation of  $\theta_i$  ( $i = 1, 2, 3$ ),  $S_i$  ( $i = 1, 2, 3$ ),  $P_1$  and  $P_2$ , respectively.

The design parameter update laws are as follows:

$$\dot{\hat{\theta}}_{1,k} = \Gamma_1 \zeta_1 v_{1,k}; \dot{\hat{S}}_{1,k} = \Gamma_2 \frac{1}{\Delta_k} v_{1,k}^2; \dot{\hat{P}}_{1,k} = -\Gamma_3 \dot{y}_r v_{1,k}, \quad (29)$$

$$\dot{\hat{\theta}}_{2,k} = \Gamma_4 \zeta_2 v_{2,k}; \dot{\hat{S}}_{2,k} = \Gamma_5 \frac{1}{\Delta_k} v_{2,k}^2, \quad (30)$$

$$\dot{\hat{\theta}}_{3,k} = \Gamma_6 \zeta_3 v_{3,k}; \dot{\hat{S}}_{3,k} = \Gamma_7 \frac{1}{\Delta_k} v_{3,k}^2; \dot{\hat{P}}_{2,k} = -\Gamma_8 \dot{\beta}_{3,k} v_{3,k}, \quad (31)$$

where  $\Gamma_i, i = 1, 2, 3, 4, 5, 6, 7, 8$  are positive definite diagonal gain matrix with an appropriate dimension and  $\Gamma_i = \Gamma_i^T$ .

The initial states satisfy the following assumptions.

**Assumption 5.** For any  $k$ , when  $t=0, x_{1,k}(0) = y_r(0); \hat{\theta}_{i,k}(0) = \hat{\theta}_{i,k-1}(T); \hat{S}_{i,k}(0) = \hat{S}_{i,k-1}(T); \hat{P}_{1,k}(0) = \hat{P}_{1,k-1}(T); \hat{P}_{2,k}(0) = \hat{P}_{2,k-1}(T)$ .

**3.2. Stability Analysis.** According to the control law and parameter adaptive laws designed in Section 3.1, the conclusion of this paper is given and analyzed in the form of the following theorem.

**Theorem 1.** Under assumptions 1–5, design the virtual control laws Equations (26) and (27), the actual control law Equation (28), and the parameter adaptive laws Equations (29)–(31) for the aircraft track angle system Equation (1), and obtain that all signals  $\hat{\theta}_{i,k}(t), \hat{S}_{i,k}(t), \hat{P}_{1,k}(t), \hat{P}_{2,k}(t), u_k(t), v_{1,k}(t), (i = 1, 2, 3)$  of the closed-loop system are bounded on  $[0, T]$ , and the error compensation signal  $v_{1,k}(t)$  converges to zero asymptotically, that is to say,  $\lim_{k \rightarrow \infty} v_{1,k}(t) = 0$ , so the tracking error  $z_{1,k}(t)$  will converge and remain in a bounded compact set.

*Proof.* According to the assumptions, definition, and lemma, it is easy to prove that the conclusion of the theorem is true. The specific process is as follows:

Choose the following Lyapunov functions for each subsystem, respectively:

$$V_{1,k} = \frac{1}{2} v_{1,k}^2 + \frac{a_1}{2} \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \tilde{\theta}_{1,k} + \frac{a_1}{2} \Gamma_2^{-1} \tilde{S}_{1,k}^2 + \frac{a_1}{2} \Gamma_3^{-1} \tilde{P}_{1,k}^2, \quad (32)$$

$$V_{2,k} = \frac{1}{2} v_{2,k}^2 + \frac{1}{2} \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \tilde{\theta}_{2,k} + \frac{1}{2} \Gamma_5^{-1} \tilde{S}_{2,k}^2 + \frac{1}{a_1} V_{1,k}, \quad (33)$$

$$V_{3,k} = \frac{1}{2} v_{3,k}^2 + \frac{a_3}{2} \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \tilde{\theta}_{3,k} + \frac{a_3}{2} \Gamma_7^{-1} \tilde{S}_{3,k}^2 + \frac{a_3}{2} \Gamma_8^{-1} \tilde{P}_{2,k}^2 + a_3 V_{2,k}, \quad (34)$$

where  $\tilde{\theta}_{i,k} = \hat{\theta}_{i,k} - \theta_i, \tilde{S}_{i,k} = \hat{S}_{i,k} - S_i, \tilde{P}_{1,k} = \hat{P}_{1,k} - P_1, \tilde{P}_{2,k} = \hat{P}_{2,k} - P_2, S_i = \sigma_{Mi}^2, i = 1, 2, 3$ .  $\square$

Take the derivation of Equation (32) and get:

$$\begin{aligned} \dot{V}_{1,k} &= v_{1,k} \dot{v}_{1,k} + a_1 \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k} + a_1 \Gamma_2^{-1} \tilde{S}_{1,k} \dot{\hat{S}}_{1,k} + a_1 \Gamma_3^{-1} \tilde{P}_{1,k} \dot{\hat{P}}_{1,k} \\ &= a_1 v_{1,k} (v_{2,k} + \alpha_{1,k} + \theta_1^T \zeta_1 + \varepsilon_{1,k} - P_1 \dot{y}_r + c_1 \xi_{1,k}) + a_1 \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k} + a_1 \Gamma_2^{-1} \tilde{S}_{1,k} \dot{\hat{S}}_{1,k} + a_1 \Gamma_3^{-1} \tilde{P}_{1,k} \dot{\hat{P}}_{1,k} \\ &= a_1 v_{1,k} (v_{2,k} + \alpha_{1,k} + \theta_1^T \zeta_1 - P_1 \dot{y}_r + c_1 \xi_{1,k}) + a_1 v_{1,k} \varepsilon_{1,k} + a_1 \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k} + a_1 \Gamma_2^{-1} \tilde{S}_{1,k} \dot{\hat{S}}_{1,k} + a_1 \Gamma_3^{-1} \tilde{P}_{1,k} \dot{\hat{P}}_{1,k} \\ &\leq a_1 v_{1,k} (v_{2,k} + \alpha_{1,k} + \theta_1^T \zeta_1 - P_1 \dot{y}_r + c_1 \xi_{1,k}) + a_1 \|v_{1,k}\| \|\varepsilon_{1,k}\| + a_1 \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k} + a_1 \Gamma_2^{-1} \tilde{S}_{1,k} \dot{\hat{S}}_{1,k} + a_1 \Gamma_3^{-1} \tilde{P}_{1,k} \dot{\hat{P}}_{1,k} \\ &\leq a_1 v_{1,k} (v_{2,k} + \alpha_{1,k} + \theta_1^T \zeta_1 - P_1 \dot{y}_r + c_1 \xi_{1,k}) + a_1 \|v_{1,k}\| \varepsilon_{M1} + a_1 \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k} + a_1 \Gamma_2^{-1} \tilde{S}_{1,k} \dot{\hat{S}}_{1,k} + a_1 \Gamma_3^{-1} \tilde{P}_{1,k} \dot{\hat{P}}_{1,k} \\ &\leq a_1 v_{1,k} (v_{2,k} + \alpha_{1,k} + \theta_1^T \zeta_1 - P_1 \dot{y}_r + c_1 \xi_{1,k}) + \frac{a_1}{\Delta_k} v_{1,k}^2 \varepsilon_{M1}^2 + \frac{a_1}{4} \Delta_k + a_1 \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k} + a_1 \Gamma_2^{-1} \tilde{S}_{1,k} \dot{\hat{S}}_{1,k} + a_1 \Gamma_3^{-1} \tilde{P}_{1,k} \dot{\hat{P}}_{1,k} \\ &= a_1 v_{1,k} (v_{2,k} + \alpha_{1,k} + \theta_1^T \zeta_1 - P_1 \dot{y}_r + c_1 \xi_{1,k}) + \frac{a_1}{\Delta_k} v_{1,k}^2 S_1 + \frac{a_1}{4} \Delta_k + a_1 \tilde{\theta}_{1,k}^T \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k} + a_1 \Gamma_2^{-1} \tilde{S}_{1,k} \dot{\hat{S}}_{1,k} + a_1 \Gamma_3^{-1} \tilde{P}_{1,k} \dot{\hat{P}}_{1,k} \end{aligned} \quad (35)$$

Take the derivation of Equation (33) and get:

$$\begin{aligned}
\dot{V}_{2,k} &= v_{2,k}\dot{v}_{2,k} + \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \hat{\theta}_{2,k} + \Gamma_5^{-1} \tilde{S}_{2,k} \hat{S}_{2,k} + \frac{1}{a_1} \dot{V}_{1,k} \\
&= v_{2,k}(v_{3,k} + \alpha_{2,k} + \theta_2^T \zeta_2 + \varepsilon_{2,k} - \dot{\beta}_{2,k} + c_2 \xi_{2,k} + \xi_{1,k}) + \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \hat{\theta}_{2,k} + \Gamma_5^{-1} \tilde{S}_{2,k} \hat{S}_{2,k} + \frac{1}{a_1} \dot{V}_{1,k} \\
&= v_{2,k}(v_{3,k} + \alpha_{2,k} + \theta_2^T \zeta_2 - \dot{\beta}_{2,k} + c_2 \xi_{2,k} + \xi_{1,k}) + v_{2,k} \varepsilon_{2,k} + \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \hat{\theta}_{2,k} + \Gamma_5^{-1} \tilde{S}_{2,k} \hat{S}_{2,k} + \frac{1}{a_1} \dot{V}_{1,k} \\
&\leq v_{2,k}(v_{3,k} + \alpha_{2,k} + \theta_2^T \zeta_2 - \dot{\beta}_{2,k} + c_2 \xi_{2,k} + \xi_{1,k}) + \|v_{2,k}\| \|\varepsilon_{2,k}\| + \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \hat{\theta}_{2,k} + \Gamma_5^{-1} \tilde{S}_{2,k} \hat{S}_{2,k} + \frac{1}{a_1} \dot{V}_{1,k} \quad (36) \\
&\leq v_{2,k}(v_{3,k} + \alpha_{2,k} + \theta_2^T \zeta_2 - \dot{\beta}_{2,k} + c_2 \xi_{2,k} + \xi_{1,k}) + \|v_{2,k}\| \varepsilon_{M2} + \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \hat{\theta}_{2,k} + \Gamma_5^{-1} \tilde{S}_{2,k} \hat{S}_{2,k} + \frac{1}{a_1} \dot{V}_{1,k} \\
&\leq v_{2,k}(v_{3,k} + \alpha_{2,k} + \theta_2^T \zeta_2 - \dot{\beta}_{2,k} + c_2 \xi_{2,k} + \xi_{1,k}) + \frac{1}{\Delta_k} v_{2,k}^2 \varepsilon_{M2}^2 + \frac{1}{4} \Delta_k + \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \hat{\theta}_{2,k} + \Gamma_5^{-1} \tilde{S}_{2,k} \hat{S}_{2,k} + \frac{1}{a_1} \dot{V}_{1,k} \\
&= v_{2,k}(v_{3,k} + \alpha_{2,k} + \theta_2^T \zeta_2 - \dot{\beta}_{2,k} + c_2 \xi_{2,k} + \xi_{1,k}) + \frac{1}{\Delta_k} v_{2,k}^2 S_2 + \frac{1}{4} \Delta_k + \tilde{\theta}_{2,k}^T \Gamma_4^{-1} \hat{\theta}_{2,k} + \Gamma_5^{-1} \tilde{S}_{2,k} \hat{S}_{2,k} + \frac{1}{a_1} \dot{V}_{1,k}.
\end{aligned}$$

Take the derivation of Equation (34) and get:

$$\begin{aligned}
\dot{V}_{3,k} &= v_{3,k}\dot{v}_{3,k} + a_3 \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \hat{\theta}_{3,k} + a_3 \Gamma_7^{-1} \tilde{S}_{3,k} \hat{S}_{3,k} + a_3 \Gamma_8^{-1} \tilde{P}_{2,k} \hat{P}_{2,k} + a_3 \dot{V}_{2,k} \\
&= a_3 v_{3,k}(u_k + \theta_3^T \zeta_3 + \varepsilon_{3,k} - P_2 \dot{\beta}_{3,k} + c_3 \xi_{3,k} + \xi_{2,k}) + a_3 \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \hat{\theta}_{3,k} + a_3 \Gamma_7^{-1} \tilde{S}_{3,k} \hat{S}_{3,k} + a_3 \Gamma_8^{-1} \tilde{P}_{2,k} \hat{P}_{2,k} + a_3 \dot{V}_{2,k} \\
&= a_3 v_{3,k}(u_k + \theta_3^T \zeta_3 - P_2 \dot{\beta}_{3,k} + c_3 \xi_{3,k} + \xi_{2,k}) + a_3 v_{3,k} \varepsilon_{3,k} + a_3 \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \hat{\theta}_{3,k} + a_3 \Gamma_7^{-1} \tilde{S}_{3,k} \hat{S}_{3,k} + a_3 \Gamma_8^{-1} \tilde{P}_{2,k} \hat{P}_{2,k} + a_3 \dot{V}_{2,k} \\
&\leq a_3 v_{3,k}(u_k + \theta_3^T \zeta_3 - P_2 \dot{\beta}_{3,k} + c_3 \xi_{3,k} + \xi_{2,k}) + a_3 \|v_{3,k}\| \|\varepsilon_{3,k}\| + a_3 \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \hat{\theta}_{3,k} + a_3 \Gamma_7^{-1} \tilde{S}_{3,k} \hat{S}_{3,k} + a_3 \Gamma_8^{-1} \tilde{P}_{2,k} \hat{P}_{2,k} + a_3 \dot{V}_{2,k} \\
&\leq a_3 v_{3,k}(u_k + \theta_3^T \zeta_3 - P_2 \dot{\beta}_{3,k} + c_3 \xi_{3,k} + \xi_{2,k}) + a_3 \|v_{3,k}\| \varepsilon_{M3} + a_3 \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \hat{\theta}_{3,k} + a_3 \Gamma_7^{-1} \tilde{S}_{3,k} \hat{S}_{3,k} + a_3 \Gamma_8^{-1} \tilde{P}_{2,k} \hat{P}_{2,k} + a_3 \dot{V}_{2,k} \\
&\leq a_3 v_{3,k}(u_k + \theta_3^T \zeta_3 - P_2 \dot{\beta}_{3,k} + c_3 \xi_{3,k} + \xi_{2,k}) + \frac{a_3}{\Delta_k} v_{3,k}^2 \varepsilon_{M3}^2 + \frac{a_3}{4} \Delta_k + a_3 \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \hat{\theta}_{3,k} + a_3 \Gamma_7^{-1} \tilde{S}_{3,k} \hat{S}_{3,k} + a_3 \Gamma_8^{-1} \tilde{P}_{2,k} \hat{P}_{2,k} + a_3 \dot{V}_{2,k} \\
&= a_3 v_{3,k}(u_k + \theta_3^T \zeta_3 - P_2 \dot{\beta}_{3,k} + c_3 \xi_{3,k} + \xi_{2,k}) + \frac{a_3}{\Delta_k} v_{3,k}^2 S_3 + \frac{a_3}{4} \Delta_k + a_3 \tilde{\theta}_{3,k}^T \Gamma_6^{-1} \hat{\theta}_{3,k} + a_3 \Gamma_7^{-1} \tilde{S}_{3,k} \hat{S}_{3,k} + a_3 \Gamma_8^{-1} \tilde{P}_{2,k} \hat{P}_{2,k} + a_3 \dot{V}_{2,k}. \quad (37)
\end{aligned}$$

Substitute Equations (26) and (29) into Equation (35), we have

$$\dot{V}_{1,k} \leq -a_1 c_1 v_{1,k}^2 + a_1 v_{1,k} v_{2,k} + \frac{a_1}{4} \Delta_k. \quad (38)$$

Substitute Equations (27) and (30) into Equation (36), we have

$$\dot{V}_{2,k} \leq -c_1 v_{1,k}^2 - c_2 v_{2,k}^2 + v_{2,k} v_{3,k} + \frac{2}{4} \Delta_k. \quad (39)$$

Substitute Equations (28) and (31) into Equation (37), we have

$$\dot{V}_{3,k} \leq -a_3 c_1 v_{1,k}^2 - a_3 c_2 v_{2,k}^2 - a_3 c_3 v_{3,k}^2 + \frac{3a_3}{4} \Delta_k. \quad (40)$$

Here, for any  $r > 0$ ,  $mn \leq \frac{1}{r} m^2 + \frac{1}{4} n^2 r$  ( $r = \Delta_k$ ).

According to Assumption 1,  $V_{i,k}(0)^2 = 0 \leq V_{i,k}(T)^2$ ,  $i = 1, 2, 3$ , by Equation (37),

$$V_{3,k} \left( v_{i,k}(0), \hat{\theta}_{i,k}(T), \hat{S}_{i,k}(T), \hat{P}_{1,k}(T), \hat{P}_{2,k}(T) \right) \leq V_{3,k} \left( v_{i,k}(0), \hat{\theta}_{i,k}(0), \hat{S}_{i,k}(0), \hat{P}_{1,k}(0), \hat{P}_{2,k}(0) \right) + \int_0^T V_{3,k} dt. \quad (41)$$

Substitute Equation (40) into Equation (41), we have

$$V_{3,k}\left(v_{i,k}(0), \widehat{\theta}_{i,k}(T), \widehat{S}_{i,k}(T), \widehat{P}_{1,k}(T), \widehat{P}_{2,k}(T)\right) \leq V_1\left(v_{i,k}(0), \widehat{\theta}_{i,k}(0), \widehat{S}_{i,k}(0), \widehat{P}_{1,k}(0), \widehat{P}_{2,k}(0)\right) - \sum_{i=1}^3 \sum_{j=1}^k \int_0^T a_3 c_i v_{i,j}^2 dt + \frac{3a_3}{4} \sum_{j=1}^k \Delta_j T. \quad (42)$$

Take  $V_0 = V_1(v_{i,k}(0), \widehat{\theta}_{i,k}(0), \widehat{S}_{i,k}(0), \widehat{P}_{1,k}(0), \widehat{P}_{2,k}(0)) + 3a_3/4 \sum_{j=1}^k \Delta_j T$ . Substitute it into Equation (42), then we get

$$\sum_{i=1}^3 \sum_{j=1}^k \int_0^T a_3 c_i v_{i,j}^2 dt \leq V_0(k) - V_{3,k}\left(v_{i,k}(0), \widehat{\theta}_{i,k}(T), \widehat{S}_{i,k}(T), \widehat{P}_{1,k}(T), \widehat{P}_{2,k}(T)\right). \quad (43)$$

By Equation (5),  $\lim_{k \rightarrow \infty} V_0(t) \leq V_1 + 2a/4(3a_3)T$ ,  $V_0(t)$  is bounded. And  $V_k(v_{i,k}(0), \widehat{\theta}_{i,k}(T), \widehat{S}_{i,k}(T), \widehat{P}_{1,k}(T), \widehat{P}_{2,k}(T)) \geq 0$ , so

$$\lim_{k \rightarrow \infty} \sum_{i=1}^3 \int_0^T a_3 c_i v_{i,k}^2 dt = 0. \quad (44)$$

By Equation (34), for any  $k$ ,  $V_{3,k}(t) = V_{3,k}(0) + \int_0^t \dot{V}_{3,k}(\tau) d\tau$ , according to Equation (32), we get

$$V_{3,k}(t) = V_{3,k}(0) - \sum_{i=1}^3 \int_0^t a_3 c_i v_{i,k}^2 d\tau + t \frac{3a_3}{4} \Delta_k. \quad (45)$$

According to Definition 1,  $\Delta_k$  is bounded,  $t \in [0, T]$ , so  $t3a_3/4\Delta_k$  is bounded. And  $\widehat{\theta}_{i,k}(0) = \widehat{\theta}_{i,k-1}(T)$ ,  $\widehat{S}_{i,k}(0) = \widehat{S}_{i,k-1}(T)$ ,  $\widehat{P}_{1,k}(0) = \widehat{P}_{1,k-1}(T)$ ,  $\widehat{P}_{2,k}(0) = \widehat{P}_{2,k-1}(T)$ , ( $i = 1, 2, 3$ ).

By Equation (42), for any  $k$ ,  $V_{3,k}(0, \widehat{\theta}_{i,k}(T), \widehat{S}_{i,k}(T), \widehat{P}_{1,k}(T), \widehat{P}_{2,k}(T))$  is bounded, so  $V_{3,k}(0, \widehat{\theta}_{i,k}(0), \widehat{S}_{i,k}(0), \widehat{P}_{1,k}(0), \widehat{P}_{2,k}(0)) = V_{3,k-1}(0, \widehat{\theta}_{i,k-1}(T), \widehat{S}_{i,k-1}(T), \widehat{P}_{1,k-1}(T), \widehat{P}_{2,k-1}(T))$  is bounded. For any  $k$ ,  $V_{3,k}(t)$  is bounded. We get  $\widehat{\theta}_{i,k}(t), \widehat{S}_{i,k}(t), \widehat{P}_{1,k}(t), \widehat{P}_{2,k}(t)$  are bounded. So  $u_k$  is bounded,  $\dot{v}_{1,k}$  is bounded, and  $v_{1,k}$  is consistent and continuous, so  $\lim_{k \rightarrow \infty} v_{1,k}(t) = 0$ .

**3.3. Simulation Research.** According to the control model established in Section 2:

$$\begin{aligned} \dot{x}_{1,k} &= a_1 x_{2,k} + W_{1,k} \\ \dot{x}_{2,k} &= x_{3,k} + W_{2,k} \\ \dot{x}_{3,k} &= a_3 u_k + W_{3,k}, \end{aligned} \quad (46)$$

where  $W_{1,k} = f_{1,k}(x_{1,k}) + \Delta_{1,k}(t)$ ,  $W_{2,k} = f_{2,k}(x_{1,k}, x_{2,k}) + \Delta_{2,k}(t)$ ,  $W_{3,k} = f_{3,k}(x_{2,k}, x_{3,k}) + \Delta_{3,k}(t)$ ,  $i = 1, 2, 3$  are uncertain parts,  $|\Delta_{i,k}(t)| \leq \rho_i$ ,  $\rho_i$  are positive constants, and

$$\begin{aligned} f_{1,k}(x_{1,k}) &= -\frac{g}{V_T} \cos x_{1,k} + \bar{L}_0 \\ f_{2,k}(x_{1,k}, x_{2,k}) &= \frac{g}{V_T} \cos x_{1,k} - \bar{L}_0 - \bar{L}_\alpha x_{2,k} \\ f_{3,k}(x_{2,k}, x_{3,k}) &= M_\alpha x_{2,k} + M_q x_{3,k}. \end{aligned} \quad (47)$$

Here  $a_1 = \bar{L}_\alpha > 0$ ,  $a_3 = M_\delta > 0$ .

Take  $T = 2\pi$ ,  $\Delta_1 = 0.01 \sin 2t$ ,  $\Delta_2 = 0.1 \cos 2t$ ,  $\Delta_3 = 0.05 \sin t \cos 2t$ . The expected trajectory is  $y_r = \sin t$ .

The physical parameters are selected as:  $\bar{L}_0 = -0.1$ ,  $\bar{L}_\alpha = 0.74$ ,  $M_\alpha = 0.1$ ,  $M_q = -0.02$ ,  $M_\delta = 1.36$ ,  $V_T = 200 \text{ m/s}$ ,  $g = 9.8 \text{ m/s}^2$ . The initial state of the model is selected as  $x(0) = [000]^T$ . When the function is approximated by fuzzy logic, the following five membership functions are taken:  $m_{NM}(x_i) = \exp[-((x_i + \pi/3)/(\pi/12))^2]$ ,  $m_{NS}(x_i) = \exp[-((x_i + \pi/6)/(\pi/12))^2]$ ,  $m_Z(x_i) = \exp[-(x_i/(\pi/12))^2]$ ,  $m_{PS}(x_i) = \exp[-((x_i - \pi/6)/(\pi/12))^2]$ ,  $m_{PM}(x_i) = \exp[-((x_i - \pi/3)/(\pi/12))^2]$ . The initial values of the optimal parameter  $\theta_i, i = 1, 2, 3$  are, respectively, selected as

$$\theta_{10} = [0.10.10.100]^T, \theta_{20} = \begin{bmatrix} 0.10.1 & \underbrace{0 \dots 0}_{23} \end{bmatrix}^T, \text{ and } \theta_{30} = \begin{bmatrix} 0.010.1 & \underbrace{0 \dots 0}_{23} \end{bmatrix}^T.$$

Choose all the initial values of  $S_{1,0}(0), S_{2,0}(0), S_{3,0}(0)$  as 0.1. All the initial values of  $P_{1,0}(0), P_{2,0}(0)$  are 0.01. Select the following control parameters:  $c_1 = 20, c_2 = 5, c_3 = 20, \omega_2 = 10, \omega_3 = 1, \Gamma_1 = [100010010.10]I_5, \Gamma_2 = 100, \Gamma_3 = 5.03,$

$$\Gamma_4 = \begin{bmatrix} 10001000.1 & \underbrace{0 \dots 0}_{22} \end{bmatrix} I_{25}, \Gamma_5 = 10, \Gamma_6 = \begin{bmatrix} 100500.1 & \underbrace{0 \dots 0}_{22} \end{bmatrix} I_{25},$$

$$\Gamma_7 = 0.0001, \Gamma_8 = 0.00000001.$$

The simulation is carried out through the actual control law Equation (28), parameter adaptive laws Equations (29)–(31) and the given initial states. The simulation results are as follows:

It can be seen from Figure 2 that the flight path inclination error of the designed controller tends to zero with the

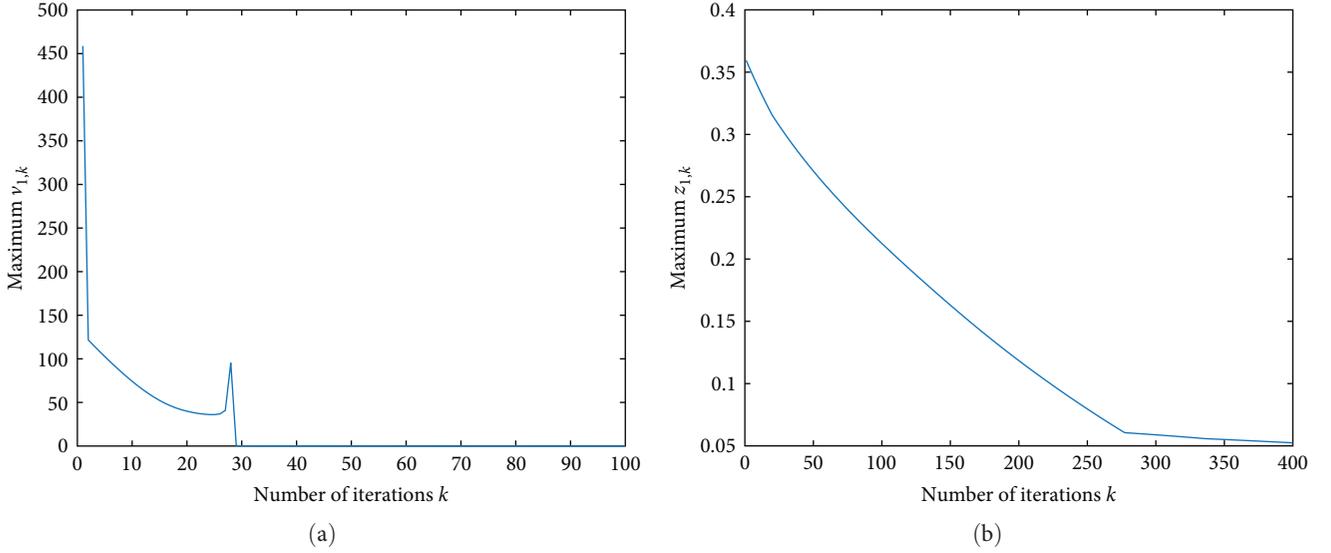


FIGURE 2: (a) Variation curve of maximum error  $v_{1,k}$  with the number of iterations and (b) curve of maximum error  $z_{1,k}$  with the number of iterations in the paper [27].

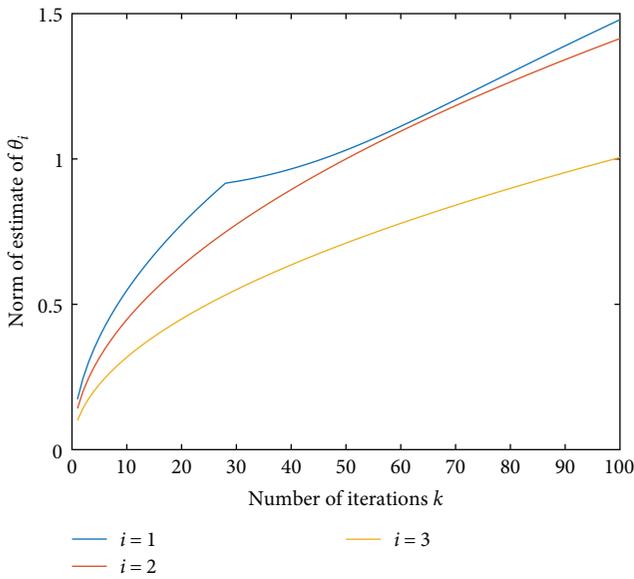


FIGURE 3: Norm variation curve of  $\|\hat{\theta}_{i,k}\|$  with the number of iterations.

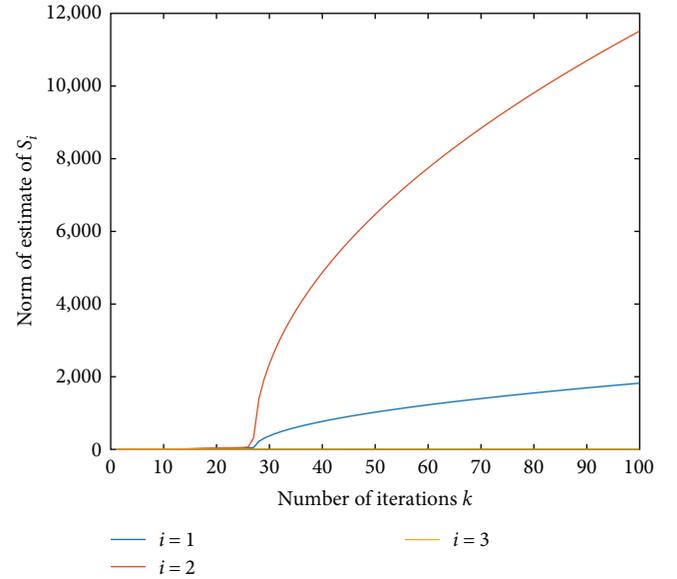


FIGURE 4: Norm variation curve of  $\|\hat{S}_{i,k}\|$  with the number of iterations.

increase of the number of iterations gradually whether using the method of this article or the method of article [27]. Due to the parameter definition of fuzzy logic systems has a clear physical meaning, make it an effective method for selecting initial parameters. The determination of initial parameters in this paper is more appropriate, so compared with article [27], the error rate of convergence can be faster. In addition, compared with the study of Zhang et al. [27], the controller in this article has a simpler structure due to the use of filters.

The simulation results from Figures 3–7 show the boundedness of the closed-loop system parameters. By comparing the tracking effect between Figures 8 and 9, with the increase of the number of iterations, the real trajectory of the aircraft

track inclination can track the desired trajectory very well. These results verify the effectiveness of the proposed controller.

#### 4. Discussion

In this paper, an AILC scheme based on fuzzy logic is proposed according to the feedback system in the strict feedback form abstracted from the longitudinal model of aircraft. Fuzzy approximation is used to approximate the unknown nonlinear part in the system design. The command filter is introduced to prevent the computational expansion caused by high-order index. The AILC law and parameter adaptive

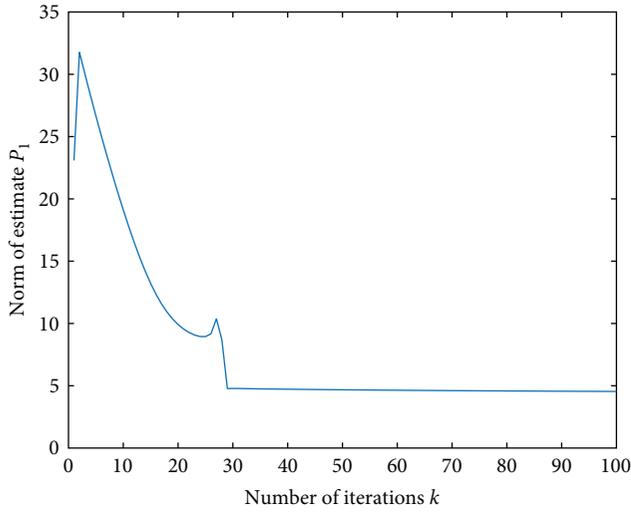


FIGURE 5: Norm variation curve of  $\|\widehat{P}_{1,k}\|$  with the number of iterations.

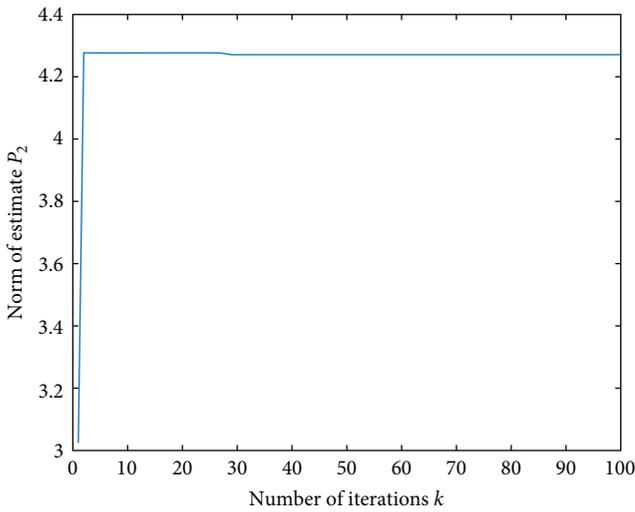


FIGURE 6: Norm variation curve of  $\|\widehat{P}_{2,k}\|$  with the number of iterations.

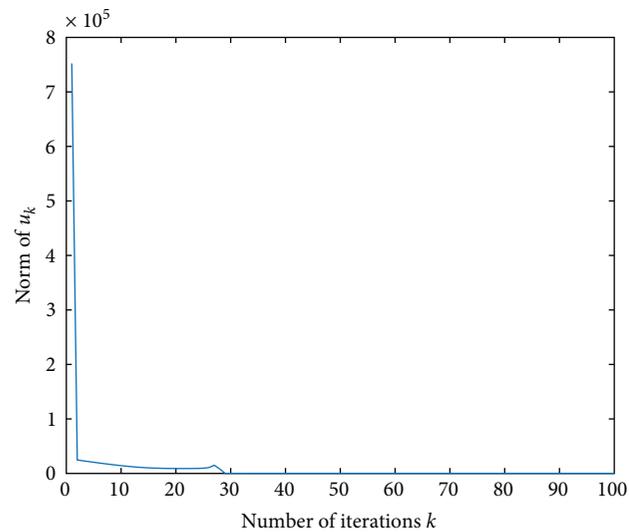
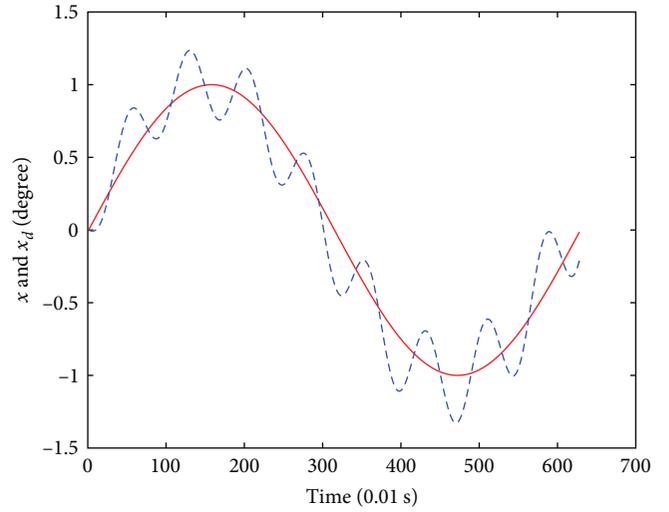
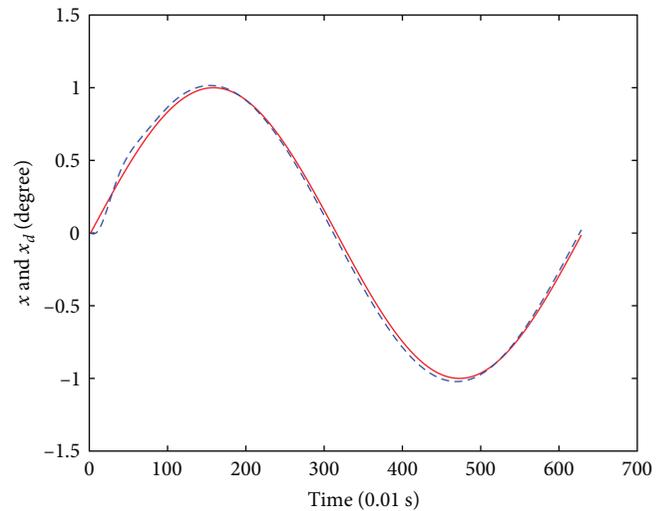


FIGURE 7: Norm curve of controller  $\|u_k\|$  with the number of iterations.



—  $y_r$   
 - - -  $y$

FIGURE 8: Trajectory trace curve of iteration 30.



—  $y_r$   
 - - -  $y$

FIGURE 9: Trajectory trace curve of iteration 100.

laws are designed, and the stability of the system is analyzed based on the Lyapunov function. Finally, the fuzzy AILC law is applied to the aircraft track angle system, and the effectiveness and feasibility of this theoretical method are verified by the simulation. From the simulation results, fuzzy logic is very effective in approaching the unknown part, and the tracking curve of the actual track is almost the same as that of the expected track. The flight path inclination is successfully controlled by the rudder surface deflection angle to achieve the goal of tracking the desired trajectory in a limited time.

### Data Availability

Previously reported modeling and simulation research model data were used to support this study and are available

at (DOI: <https://doi.org/10.3389/fphy.2022.1048942>). These prior studies (and datasets) are cited at relevant places within the text as reference [27].

## Disclosure

The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of Hindawi and/or the editor(s). Hindawi and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Chunli Zhang contributed in the conceptualization and methodology; Xu Tian contributed in the software and writing—original draft preparation; Xu Tian and Yangjie Gao contributed in the validation; Chunli Zhang and Fucai Qian contributed in writing—review and editing. All authors have read and agreed to the published version of the manuscript.

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