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Research Article

Mixed Lump-Stripe Soliton Solutions to a New Extended Jimbo-Miwa Equation

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In this paper, the localized properties of lump and interaction solutions to a new extended Jimbo-Miwa (EJM) equation are studied. Based on the Hirota bilinear method and the test function method, the exact solutions of the EJM equation are discussed; the lump soliton solution, lump-kink soliton solution, and periodic lump solution are obtained. Furthermore, the dynamic properties of the obtained solutions are also discussed by graphical simulation. As far as we know, the obtained results have not been reported.

1. Introduction

The research of exact solutions plays an important role in nonlinear evolution equations (NLEEs). There are many kinds of exact solutions, such as soliton, multisoliton, rational, periodic, breather line, breather kinky, and lump and rogue wave solutions. A lot of work has been done by scholars [1–5]. Particularly, studying the lump solutions of nonlinear evolution equations becomes a hot topic in the mathematical physics field [6, 7]. It is well known that lump solutions are also a kind of rational solutions and have some good characters. The lump-like solutions, such as lump-kink solutions, rogue wave solutions, and periodic lump solutions, possess many physical phenomena, which have been studied by many researchers [8–11]. Kaur and Wazwaz analyzed a new form of $(3+1)$ dimensional generalized KP-Boussinesq equation for exploring lump solutions applying Hirota's bilinear form [12]. Tang et al. studied the interaction of a lump with a stripe of $(2+1)$-dimensional Ito equation and showed that the lump is drowned or swallowed by a stripe soliton [13]. In [14], the higher-order rogue wave solutions of a new integrable $(2+1)$-dimensional Boussinesq equation were derived utilizing a generalized polynomial test function. Using an extended homoclinic approach, new exact solutions including kinky periodic solitary wave solutions and line breathers periodic of the $(3+1)$-dimensional generalized BKP equation were also obtained [15]. In mathematical physics, the interaction of rogue wave with other solitons or periodic waves is a remarkable task in nonlinear sciences. Recently, kinky-lump, kinky-rogue, periodic-lump wave, periodic-rogue wave, and kinky-periodic rogue wave for the NLEEs and their nonlinear dynamics become a subject of interest [16–18].

The well-known $(3+1)$-dimensional Jimbo-Miwa (JM) equation

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0 \tag{1}$$

can be applied to demonstrate some interesting phenomena in nonlinear physics whose exact solutions are discussed by many authors [14, 19–22].

In [23], Wazwaz proposed the following two extended $(3+1)$-dimensional Jimbo-Miwa equation (EJM):

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3\left(u_{xz} + u_{yz} + u_{zz}\right) = 0, \tag{2}$$

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2\left(u_{xt} + u_{yt} + u_{zt}\right) - 3u_{xz} = 0. \tag{3}$$

These two forms are the generalization of equation (1).

Lump solutions and the dynamics of these two extended Jimbo-Miwa equations via bilinear forms and the lump-kink solution are also obtained [24]. The dynamics of the obtained lump solutions, including the amplitude and the locations of the lump, are also analyzed. Recently, many researchers pay more attention to the Jimbo-Miwa equation. In [25], Kaur and Wazwaz construct bilinear forms of equations (2) and (3) using truncated Painlev expansions along with the Bell polynomial approach. In [26], kink soliton, breather, and lump and line rogue wave solutions of extended $(3+1)$-dimensional Jimbo-Miwa equation are obtained by the Hirota bilinear method, whose mixed cases are discussed. The authors of [26] also analyze their dynamic behavior and vividly demonstrate their evolution process. In [27], applying the Hirota bilinear method and KP hierarchy reduction, the rational solution of the $(3+1)$-dimensional generalized shallow water wave equation is presented in the Grammian form. The lump soliton solutions are derived from the corresponding rational solutions. In [28], an extended $(3+1)$-dimensional Jimbo-Miwa equation with time-dependent coefficients is investigated. One, two, and three soliton solutions are obtained via the Hirota method. The periodic wave solutions are constructed via the Riemann theta function. The authors of [28] show that the interaction between the solitons is elastic, and the time-dependent coefficients can affect the soliton velocities, while the soliton amplitudes remain unchanged. Yin et al. substituted test functions into the bilinear equations to obtain the lump solutions, lump-kink solutions, and interaction solutions in [29].

Yan et al. [30] study the dynamics of lump solutions, lump-kink solutions, and periodic lump solutions in a $(3+1)$-dimensional generalized Jimbo-Miwa equation given by

$$\alpha\left(u_{xxxy}+3u_yu_{xx}+3u_{xy}u_x\right)+2u_{yt}+\beta\left(u_{xz}+u_{yz}+u_{zz}\right)=0. \tag{4}$$

In [31], Cheng et al. discuss a new extended Jimbo-Miwa equation in the development process of nonlinear physical phenomena,

$$u_{xxxy}+\chi\left(u_xu_y\right)_x+\rho_1u_{xy}+\rho_2u_{xz}+\rho_3u_{yt}+\rho_4u_{yy}=0, \tag{5}$$

where χ is a nonzero constant and ρ_i, $1\le i\le 4$, are real constants, and $\rho_2\rho_3\ne 0$. When $\chi=3$, $\rho_2=-3$, $\rho_3=2$, ρ_1, and ρ_4 are zero, equation (5) is reduced to the Jimbo-Miwa equation (1). Taking $\chi=-3.\rho_2=-3$, $\rho_3=-1$, $\rho_1=\rho_4=0$, equation (5) becomes the $(3+1)$-dimensional generalized BKP equation introduced in [19].

$$u_{ty}-u_{xxxy}+3\left(u_xu_y\right)_x+3u_{xz}=0. \tag{6}$$

In [31], the two-wave and complexiton solutions of (5) are developed through symbolic computations with Maple. There is relatively little research on the exact solution of this equation. Our purpose is to seek new lump and lump-like solutions of (5).

2. Bilinear Representation

By transformation

$$u=\frac{6}{\chi}(\ln f)_x, \tag{7}$$

equation (5) owns a Hirota bilinear formulation

$$\left(D_x^3+\rho_1D_xD_y+\rho_2D_xD_z+\rho_3D_yD_t+\rho_4D_y^2\right)f\cdot f=0, \tag{8}$$

where $f=f(x,y,z,t)$ is a real function and D_x, D_y, D_z and D_t are Hirota's differential operators [32]. The above bilinear equation is equivalent to the following form:

$$\begin{aligned}&\left(\rho_1f_{xy}+\rho_2f_{xz}+\rho_3f_{yt}+\rho_4f_{yy}+f_{xxxy}\right)f-\rho_1f_yf_x-\rho_2f_xf_z\\&-\rho_3f_yf_t-\rho_4f_y^2-3f_{xxy}f_x+3f_{xx}f_{xy}-f_{xxx}f_y=0.\end{aligned} \tag{9}$$

3. Lump Soliton Solution

In order to seek the lump soliton solutions of equation (5), we let f be

$$f=g^2+h^2+a_5, \tag{10}$$

where g and h are, respectively, expressed by

$$g=a_0+a_1x+a_2y+a_3z+a_4t, \tag{11}$$

$$h=b_0+b_1x+b_2y+b_3z+b_4t, \tag{12}$$

where a_i, $(i=0\cdots 5)$, b_j, $(j=0\cdots 4)$ are real constants to be determined. Substituting (10)–(12) into equation (9), by means of the symbolic computation methods [24, 26–29], one can obtain several groups of constraint conditions with respect to a_i, b_j, ρ_j, which are listed as follows:

Case 1.

$$\begin{aligned}&a_2=0,\ a_3=-\frac{3b_1b_2\left(a_1^2+b_1^2\right)}{a_1a_5\rho_2},\\&a_4=-\frac{a_1^2a_5(b_2\rho_1+b_3\rho_2)-3b_1^2b_2\left(a_1^2+b_1^2\right)}{a_1a_5b_2\rho_3},\\&b_4=-\frac{3a_1^2b_1b_2+a_5b_1b_2\rho_1+a_5b_1b_3\rho_2+a_5b_2^2\rho_4+3b_1^3b_2}{a_5b_2\rho_3},\end{aligned} \tag{13}$$

where $a_1a_5b_2\rho_2\rho_3\ne 0$; the other parameters are arbitrary constants.

Case 2.

$$a_2 = 0, a_5 = 0, b_1 = 0, a_4 = -\frac{a_1(b_2\rho_1 + b_3\rho_2)}{b_2\rho_3}, b_4 = \frac{a_1a_3\rho_2 - b_2^2\rho_4}{b_2\rho_3}, \tag{14}$$

where $b_2\rho_3 \neq 0$; the other parameters are arbitrary constants.

Case 3.

$$a_1 = -\frac{b_1b_2}{a_2}, a_4 = -\frac{a_2^2\rho_4 - b_1b_2\rho_1 - b_1b_3\rho_2}{a_2\rho_3},$$

$$a_5 = 0, b_4 = -\frac{a_2b_1\rho_1 + a_2b_2\rho_4 + a_3b_1\rho_2}{a_2\rho_3}, \tag{15}$$

where $a_2\rho_3 \neq 0$; the other parameters are arbitrary constants.

Case 4.

$$\begin{aligned} &a_2 = 0, a_5 = 0, b_1 = 0,\\ &a_4 = -\frac{a_1(b_2\rho_1 + b_3\rho_2)}{b_2\rho_3},\\ &b_4 = \frac{a_1a_3\rho_2 - b_2^2\rho_4}{b_2\rho_3}, \end{aligned} \tag{16}$$

where $b_2\rho_3 \neq 0$; the other parameters are arbitrary constants.

Case 5.

$$\begin{aligned} &a_1 = -\frac{b_1b_2}{a_2}, a_3 = -\frac{a_2(b_1\rho_1 + b_2\rho_4 + b_4\rho_3)}{b_1\rho_2},\\ &a_4 = -\frac{a_2^2\rho_4 + b_2^2\rho_4 + b_2b_4\rho_3}{a_2\rho_3}, \end{aligned} \tag{17}$$

$$b_0 = \frac{a_0b_2}{a_2}, b_3 = -\frac{(b_1\rho_1 + b_2\rho_4 + b_4\rho_3)b_2}{b_1\rho_2}, \tag{18}$$

where $a_2b_1\rho_2\rho_3 \neq 0$; the other parameters are arbitrary constants.

Substituting equations (14)–(17) along with equations (11) and (12) into equation (10), one obtains the corresponding lump soliton solution as follows:

Case 6.

$$u = \frac{12(a_1g + b_1h)}{\chi(g^2 + h^2 + a_5)}, \tag{19}$$

where

$$\begin{aligned} g &= -\frac{\left(a_1^2a_5b_2\rho_1 + a_1^2a_5b_3\rho_2 - 3a_1^2b_1^2b_2 - 3b_1^4b_2\right)t}{a_1a_5b_2\rho_3}\\ &\quad + a_1x - \frac{3b_1b_2\left(a_1^2 + b_1^2\right)z}{a_1a_5\rho_2} + a_0,\\ h &= -\frac{\left(3a_1^2b_1b_2 + a_5b_1b_2\rho_1 + a_5b_1b_3\rho_2 + a_5b_2^2\rho_4 + 3b_1^3b_2\right)t}{a_5b_2\rho_3}\\ &\quad + b_1x + b_2y + b_3z + b_0. \end{aligned} \tag{20}$$

Case 7.

$$u = \frac{12a_1g)}{\chi\left(g^2 + h^2\right)}, \tag{21}$$

where

$$\begin{aligned} g &= -\frac{a_1(b_2\rho_1 + b_3\rho_2)t}{b_2\rho_3} + a_1x + a_3z + a_0,\\ h &= \frac{\left(a_1a_3\rho_2 - b_2^2\rho_4\right)t}{b_2\rho_3} + b_2y + b_3z + b_0. \end{aligned} \tag{22}$$

Case 8.

$$u = \frac{12b_1(a_2h - b_2g)}{\chi a_2\left(g^2 + h^2\right)}, \tag{23}$$

where

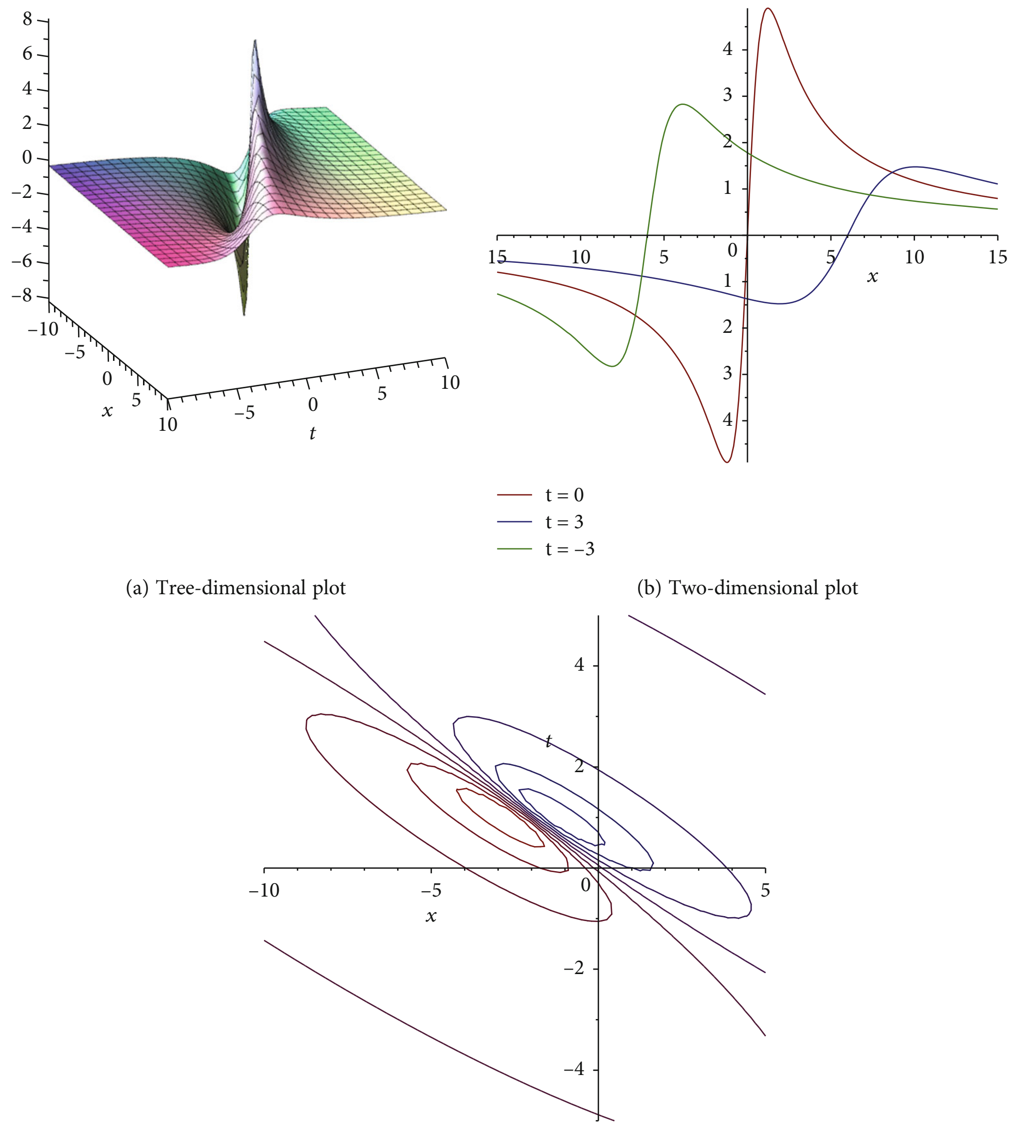
$$\begin{aligned} g &= -\frac{\left(a_2^2\rho_4 - b_1b_2\rho_1 - b_1b_3\rho_2\right)t}{a_2\rho_3} - \frac{b_1b_2x}{a_2} + a_2y + a_3z + a_0,\\ h &= -\frac{(a_2b_1\rho_1 + a_2b_2\rho_4 + a_3b_1\rho_2)t}{a_2\rho_3} + b_1x + b_2y + b_3z + b_0. \end{aligned} \tag{24}$$

Case 9.

$$u = \frac{12(a_1g + b_1h)}{\chi\left(g^2 + h^2\right)}, \tag{25}$$

where

$$\begin{aligned} g &= -\frac{\left(a_2a_1\rho_1 + a_2^2\rho_4 - b_1b_3\rho_2\right)t}{a_2\rho_3} + a_1x + a_2y - \frac{b_2b_3z}{a_2} + a_0,\\ h &= -\frac{(a_1b_3\rho_2 + a_2b_1\rho_1 + a_2b_2\rho_4)t}{a_2\rho_3} + b_1x + b_2y + b_3z + \frac{a_0b_2}{a_2}. \end{aligned} \tag{26}$$



(a) Tree-dimensional plot

(b) Two-dimensional plot

(c) Contour plot

FIGURE 1: Figures on (x, t) of solution (27) with $a_0 = 1$, $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $b_0 = 1$, $b_1 = 1$, $b_2 = 1$, $b_3 = 1$, $b_4 = 1$, $\chi = 1$, $\rho_1 = 1$, $\rho_2 = 1$, $\rho_3 = 1$, $\rho_4 = 1$, $y = 0$, $z = 0$.

Case 10.

$$u = \frac{12(b_1 h - b_2 g)}{\chi(g^2 + h^2 + a_5)}, \quad (27)$$

where

$$g = -\frac{(a_2^2\rho_4 + b_2^2\rho_4 + b_2 b_4 \rho_3)t}{a_2\rho_3} - \frac{b_1 b_2 x}{a_2} + a_2 y - \frac{a_2(b_1\rho_1 + b_2\rho_4 + b_4\rho_3)z}{b_1\rho_2} + a_0,$$

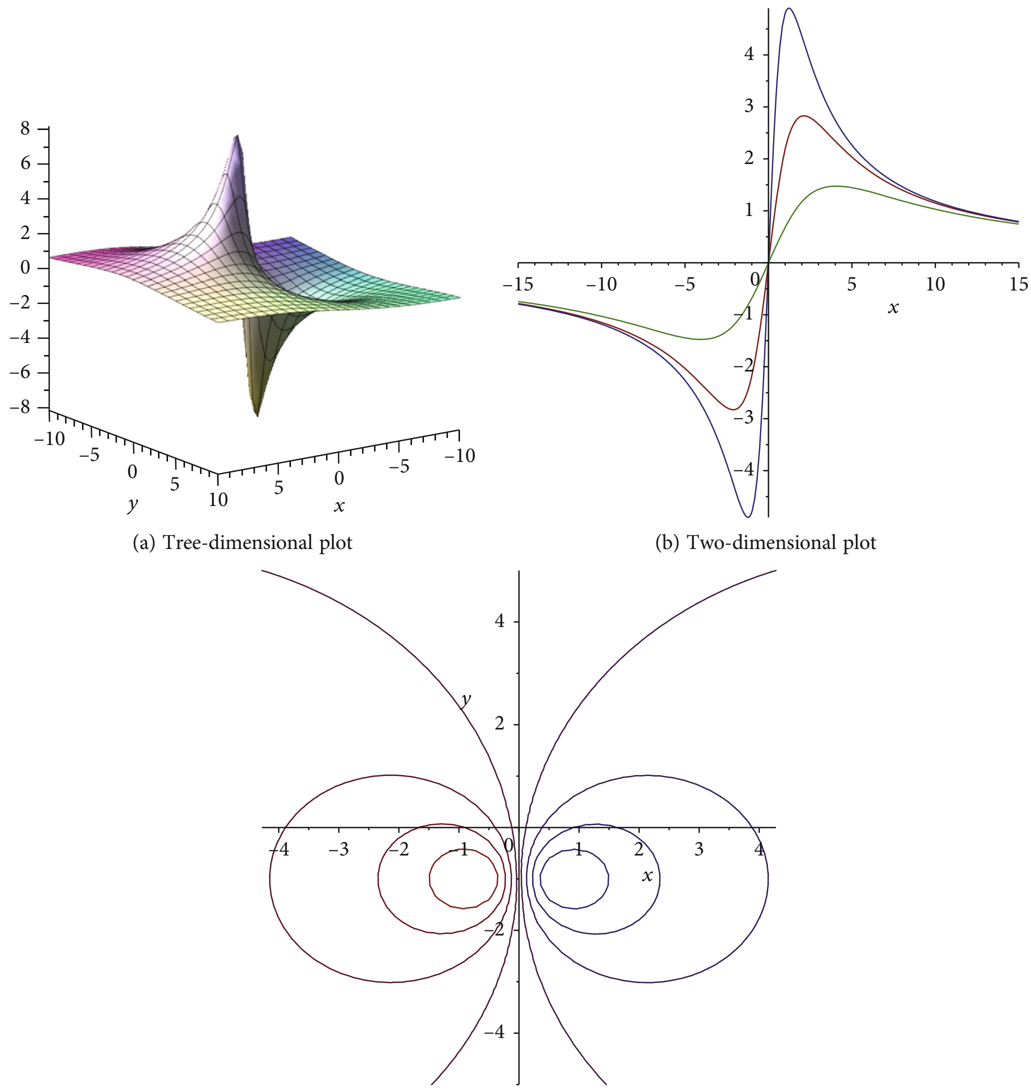
$$h = b_4 t + b_1 x + b_2 y - \frac{(b_1\rho_1 + b_2\rho_4 + b_4\rho_3)b_2 z}{b_1\rho_2} + \frac{a_0 b_2}{a_2}. \quad (28)$$

Figures 1 and 2 display the lump solution (27) of equation (5). By choosing suitable parameters, we can obtain two different shapes of lumps in x-t and x-y plane. Actually, from Figures 1 and 2, we can find that the lump solution (27) possesses localization character in both x, t- and x, y-plane, respectively. When $x^2 + y^2 \longrightarrow \infty$ is satisfied, the wave u will tend to zero along any direction.

4. Lump-Kink Soliton Solution

4.1. Type of Exp. In this section, we add an exponential function to the quadratic function solution (10), letting f be

$$f = g^2 + h^2 + e^{\xi} + a_5, \quad (29)$$



(a) Tree-dimensional plot

(b) Two-dimensional plot

(c) Contour plot

Figure 2: Figures on (x, y) of solution (27) with $a_0 = 1$, $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $b_0 = 1$, $b_1 = 1$, $b_2 = 1$, $b_3 = 1$, $b_4 = 1$, $\chi = 1$, $\rho_1 = 1$, $\rho_2 = 1$, $\rho_3 = 1$, $\rho_4 = 1$, $t = 0$, $z = 0$.

where g and h are, respectively, expressed by

$$g = a_0 + a_1 x + a_2 y + a_3 z + a_4 t, \tag{30}$$

$$h = b_0 + b_1 x + b_2 y + b_3 z + b_4 t, \tag{31}$$

$$\xi = c_0 + c_1 x + c_2 y + c_3 z + c_4 t, \tag{32}$$

where a_i, $(i = 0 \cdots 5)$, b_j, $c_j (j = 0 \cdots 4)$ are real constants to be determined and $a_5 > 0$. Substituting (47)–(32) into equation (9), by means of the symbolic computation methods, one can obtain several groups of constraint conditions with respect to a_i, b_j, c_j, ρ_j, which are listed as follows.

Case 11.

$$a_1 = 0, a_4 = -\frac{3b_2^2 c_1^3 + a_2^2 c_2 \rho_4}{a_2 c_2 \rho_3}, a_5 = \frac{b_2^2}{c_2^2}, b_1 = \frac{b_2 c_1}{c_2},$$
$$b_3 = -\frac{b_2 \left(3a_2^2 c_1^2 + 3b_2^2 c_1^2 - a_2 a_3 \rho_2\right)}{a_2^2 \rho_2}, \tag{33}$$

$$b_4 = -\frac{\left(-3b_2^2 c_1^3 + a_2^2 c_1 \rho_1 + a_2^2 c_2 \rho_4 + a_2 a_3 c_1 \rho_2\right) b_2}{a_2^2 c_2 \rho_3},$$
$$c_3 = \frac{\left(-3b_2^2 c_1^2 + a_2 a_3 \rho_2\right) c_2}{a_2^2 \rho_2}, \tag{34}$$

$$c_4 = -\frac{a_2^2 c_1^3 - 3b_2^2 c_1^3 + a_2^2 c_1 \rho_1 + a_2^2 c_2 \rho_4 + a_2 a_3 c_1 \rho_2}{a_2^2 \rho_3}. \tag{35}$$

Case 12.

$$a_1=-\frac{3b_0^2a_3c_1^3\rho_2c_2}{9b_0^2c_1^4c_2^2+a_3^2\rho_2^2},\ a_2=0,$$
$$a_4=-\frac{a_3c_1\rho_2\left(-3b_0^2c_1^2c_2^2\rho_1-3b_0^2c_1^2c_2c_3\rho_2+a_3^2\rho_2^2\right)}{\left(9b_0^2c_1^4c_2^2+a_3^2\rho_2^2\right)c_2\rho_3}, \tag{36}$$

$$b_1=\frac{a_3^2b_0c_1\rho_2^2}{9b_0^2c_1^4c_2^2+a_3^2\rho_2^2},$$
$$b_4=-\frac{b_0\left(9b_0^2c_1^4c_2^4\rho_4+3a_3^2c_1^3c_2\rho_2^2+a_3^2c_1c_2\rho_1\rho_2^2+a_3^2c_1c_3\rho_2^3+a_3^2c_2^2\rho_2^2\rho_4\right)}{\left(9b_0^2c_1^4c_2^2+a_3^2\rho_2^2\right)c_2\rho_3}, \tag{37}$$

$$c_4=-\frac{c_1^3c_2+c_1c_2\rho_1+c_1c_3\rho_2+\rho_4c_2^2}{c_2\rho_3},\ a_5=\frac{a_3^2b_0^2\rho_2^2}{9b_0^2c_1^4c_2^2+a_3^2\rho_2^2}. \tag{38}$$

Case 13.

$$a_2=0, a_3=0, a_4=-\frac{a_1(c_2\rho_1+c_3\rho_2)}{c_2\rho_3}, b_2=0, b_3=0,$$
$$b_4=-\frac{b_1(c_2\rho_1+c_3\rho_2)}{c_2\rho_3}, c_1=0, c_4=-\frac{c_2\rho_4}{\rho_3}. \tag{39}$$

Substituting equations (33)–(39) along with equations (30), (31), and (32) into equation (47), one obtains the corresponding lump-kink soliton solution as follows:

$$u=\frac{12c_1/c_2h+6c_1e^{\xi}}{\chi\left(g^2+h^2+e^{\xi}+\left(b_2^2/c_2^2\right)\right)}, \tag{40}$$

where

$$g=-\frac{\left(3b_2^2c_1^3+a_2^2c_2\rho_4\right)t}{a_2c_2\rho_3}+a_2y+a_3z+a_0, \tag{41}$$

$$h=-\frac{\left(-3b_2^2c_1^3+a_2^2c_1\rho_1+a_2^2c_2\rho_4+a_2a_3c_1\rho_2\right)b_2t}{a_2^2c_2\rho_3}+\frac{b_2c_1x}{c_2}+b_2y-\frac{b_2\left(3a_2^2c_1^2+3b_2^2c_1^2-a_2a_3\rho_2\right)z}{a_2^2\rho_2}+b_0, \tag{42}$$

$$\xi=-\frac{t\left(a_2^2c_1^3-3b_2^2c_1^3+a_2^2c_1\rho_1+a_2^2c_2\rho_4+a_2a_3c_1\rho_2\right)}{a_2^2\rho_3}+xc_1+yc_2+\frac{z\left(-3b_2^2c_1^2+a_2a_3\rho_2\right)c_2}{a_2^2\rho_2}+c_0. \tag{43}$$

$$u=\frac{12\left(a_1g+b_1h+c_1e^{\xi}\right)}{\chi\left(g^2+h^2+e^{\xi}+a_5\right)}, \tag{44}$$

where a_1, b_1, c_1 satisfy equation (36) and

$$g=-\frac{a_3c_1\rho_2\left(-3b_0^2c_1^2c_2^2\rho_1-3b_0^2c_1^2c_2c_3\rho_2+a_3^2\rho_2^2\right)t}{\left(9b_0^2c_1^4c_2^2+a_3^2\rho_2^2\right)c_2\rho_3}-\frac{3b_0^2a_3c_1^3\rho_2c_2x}{9b_0^2c_1^4c_2^2+a_3^2\rho_2^2}+a_3z+a_0,$$
$$h=-\frac{b_0\left(9b_0^2c_1^4c_2^4\rho_4+3a_3^2c_1^3c_2\rho_2^2+a_3^2c_1c_2\rho_1\rho_2^2+a_3^2c_1c_3\rho_2^3+a_3^2c_2^2\rho_2^2\rho_4\right)t}{\left(9b_0^2c_1^4c_2^2+a_3^2\rho_2^2\right)c_2\rho_3}+\frac{a_3^2b_0c_1\rho_2^2x}{9b_0^2c_1^4c_2^2+a_3^2\rho_2^2}+b_0c_2y+b_0c_3z+b_0,$$
$$\xi=-\frac{\left(c_1^3c_2+c_1c_2\rho_1+c_1c_3\rho_2+\rho_4c_2^2\right)t}{c_2\rho_3}+xc_1+yc_2+c_3z+c_0,$$
$$u=\frac{12(a_1g+b_1h)}{\chi\left(g^2+h^2+e^{\xi}+a_5\right)}, \tag{45}$$

where

$$g=-\frac{a_1(c_2\rho_1+c_3\rho_2)t}{c_2\rho_3}+a_1x+a_0, h=-\frac{b_1(c_2\rho_1+c_3\rho_2)t}{c_2\rho_3}+b_1x+b_0,$$
$$\xi=-\frac{tc_2\rho_4}{\rho_3}+yc_2+c_3z+c_0. \tag{46}$$

Choosing a proper suitable value, Figure 3 presents the nonlinear dynamic propagation behaviors of the lump-kink wave solution (44). We can show that the interaction phenomena between a lump wave and a kink wave exist. Figures 3(a)–3(c) display the lump wave locating different places in the wave plane. When y is increased, then the lump soliton propagates from left to right in the (x, y)-plane. In fact, the lump will be swallowed by the kink wave at some time.

4.2. Type of Cosh. In this section, we add an exponential function to the quadratic function solution (10), letting f be

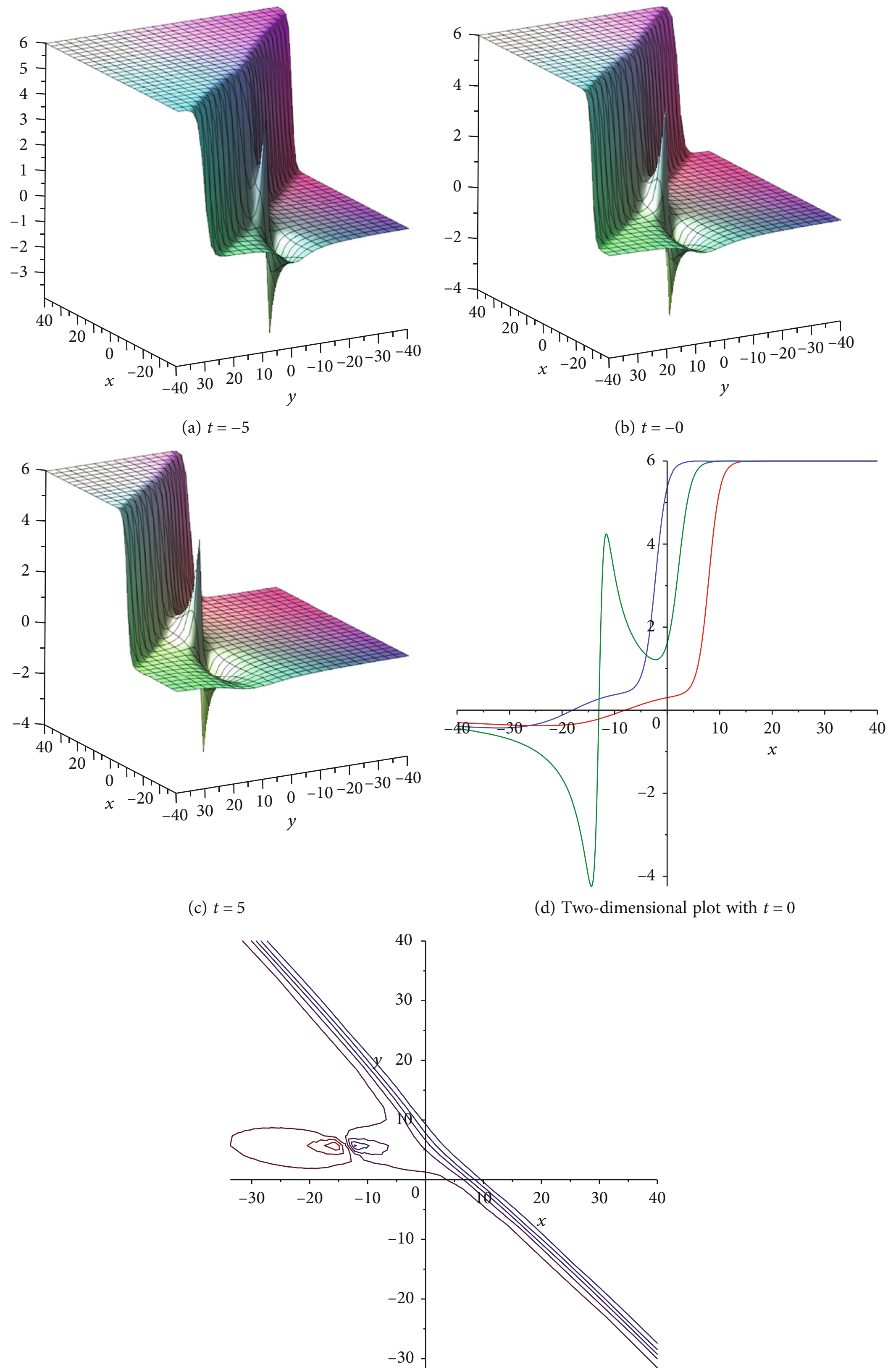
$$f=g^2+h^2+\cosh(\xi)+a_5, \tag{47}$$

where g, h, and ξ are defined by (30), (31), and (32), $a_i, (i=0\cdots5), b_j, c_j (j=0\cdots4)$ are real constants to be determined. By the same procedure, one can obtain several groups of constraint conditions with respect to a_i, b_j, c_j, ρ_j, which are listed as follows.

Case 14.

$$a_2=-\frac{b_1b_2}{a_1}, a_3=\frac{b_1b_2\left(a_1^2\rho_1+a_1a_4\rho_3-b_1b_2\rho_4\right)}{a_1^3\rho_2},$$
$$b_3=-\frac{b_2\left(a_1^2\rho_1+a_1a_4\rho_3-b_1b_2\rho_4\right)}{\rho_2a_1^2}, \tag{48}$$

$$b_4=\frac{-a_1^2b_2\rho_4+a_1a_4b_1\rho_3-b_1^2b_2\rho_4}{\rho_3a_1^2}, c_2=0, c_3=0,$$
$$c_4=\frac{c_1\left(-a_1^2c_1^2+a_1a_4\rho_3-b_1b_2\rho_4\right)}{a_1^2\rho_3}. \tag{49}$$



(a) $t = -5$

(b) $t = -0$

(c) $t = 5$

(d) Two-dimensional plot with $t = 0$

(e) Contour plot

Figure 3: Figures on (x, y) of solution (44) with $a_0 = 1$, $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $b_0 = 1$, $b_1 = 1$, $b_2 = 1$, $b_3 = 1$, $b_4 = 1$, $\chi = 1$, $\rho_1 = 1$, $\rho_2 = 1$, $\rho_3 = 1$, $\rho_4 = 1$, $z = -5$.

Case 15.

$$a_2=-\frac{b_1b_2}{a_1},\ a_3=-\frac{b_1b_2c_3}{a_1c_2},\ a_4=\frac{-a_1^2c_2\rho_1-a_1^2c_3\rho_2+b_1b_2c_2\rho_4}{c_2\rho_3a_1},\ b_3=\frac{b_2c_3}{c_2}, \tag{50}$$

$$b_4=-\frac{b_1c_2\rho_1+b_1c_3\rho_2+b_2c_2\rho_4}{c_2\rho_3},\ c_1=0,\ c_4=-\frac{c_2\rho_4}{\rho_3}. \tag{51}$$

Substituting equations (48) and (50) along with equations (30), (31), and (32) into equation (47), one obtains the respective lump-kink soliton solution as follows:

$$u=\frac{6(2a_1g+2b_1h-c_1\sinh(\xi))}{\chi\left(g^2+h^2+\cosh(\xi)+a_5\right)}, \tag{52}$$

where

$$g=a_4t+a_1x-\frac{b_1b_2y}{a_1}+\frac{b_1b_2\left(a_1^2\rho_1+a_1a_4\rho_3-b_1b_2\rho_4\right)z}{a_1^3\rho_2}+a_0,$$

$$h=\frac{\left(-a_1^2b_2\rho_4+a_1a_4b_1\rho_3-b_1^2b_2\rho_4\right)t}{\rho_3a_1^2}+b_1x+b_2y-\frac{b_2\left(a_1^2\rho_1+a_1a_4\rho_3-b_1b_2\rho_4\right)z}{\rho_2a_1^2}+b_0,$$

$$\xi=\frac{tc_1\left(-a_1^2c_1^2+a_1a_4\rho_3-b_1b_2\rho_4\right)}{a_1^2\rho_3}+c_1x+c_0,$$

$$u=\frac{12(a_1g+b_1h)}{\chi\left(g^2+h^2+\cosh(\xi)+a_5\right)}, \tag{53}$$

where

$$g=\frac{\left(-a_1^2c_2\rho_1-a_1^2c_3\rho_2+b_1b_2c_2\rho_4\right)t}{c_2\rho_3a_1}+a_1x-\frac{b_1b_2y}{a_1}-\frac{b_1b_2c_3z}{a_1c_2}+a_0,$$

$$h=\frac{\left(-a_1^2c_2\rho_1-a_1^2c_3\rho_2+b_1b_2c_2\rho_4\right)t}{c_2\rho_3a_1}+a_1x-\frac{b_1b_2y}{a_1}-\frac{b_1b_2c_3z}{a_1c_2}+a_0,$$

$$\xi=\frac{c_2\rho_4t}{\rho_3}+c_2y+c_3z+c_0. \tag{54}$$

Figure 4 illustrates the interaction phenomena between lump and kink. We obtain the wave consisting of two parts including the lump wave and the kink wave. With the increase of t, the lump first appears in the form of one kink, then it begins to move towards the other kink and finally, the lump disappears.

5. Periodic Lump Solution

In order to get the periodic lump wave solutions of equation (9), we take

$$f=\cosh(a_1x+a_2y+a_3z+a_4t)+\cos(b_1x+b_2y+b_3z+b_4t)+\cosh(c_1x+c_2y+c_3z+c_4t). \tag{55}$$

Substituting (55) into (9), we collect all the coefficients of hyperbolic functions and trigonometric functions. Computing these coefficients, one can obtain several groups of constraint conditions with respect to a_i, b_j, c_j, ρ_j, which are listed as follows.

Case 16.

$$a_1=0,\ a_2=0,\ a_3=0,\ a_4=0,\ b_2=0,\ b_3=0,\ b_4=\frac{b_1\left(b_1^2c_2-c_2\rho_1-c_3\rho_2\right)}{c_2\rho_3},\ c_1=0,\ c_4=-\frac{c_2\rho_4}{\rho_3}, \tag{56}$$

where $c_2\rho_3\neq 0$; the other parameters are arbitrary constants.

Case 17.

$$a_1=0,\ a_4=-\frac{a_2\rho_4}{\rho_3},\ b_1=0,\ b_3=\frac{a_3b_2}{a_2},\ b_4=-\frac{b_2\rho_4}{\rho_3},\ c_2=0,\ c_3=0,\ c_4=-\frac{c_1\left(a_2c_1^2+a_2\rho_1+a_3\rho_2\right)}{a_2\rho_3}, \tag{57}$$

where $a_2\rho_3\neq 0$; the other parameters are arbitrary constants.

Case 18.

$$a_1=0,\ a_2=a_2,\ a_3=a_3,\ a_4=-\frac{a_2\rho_4}{\rho_3},\ b_1=b_1,\ b_2=0,\ b_3=0,\ b_4=\frac{b_1\left(a_2b_1^2-a_2\rho_1-a_3\rho_2\right)}{a_2\rho_3}, \tag{58}$$

$$c_1=c_1,\ c_2=0,\ c_3=0,\ c_4=-\frac{c_1\left(a_2c_1^2+a_2\rho_1+a_3\rho_2\right)}{a_2\rho_3}, \tag{59}$$

where $a_2\rho_3\neq 0$; other parameters are arbitrary constants.

Substituting equations (56)–(58) along with equations (30), (31), and (32) into equation (55), one obtains the corresponding periodic lump solution as follows:

$$u=\frac{-6k_2b_1\sin(h)}{\chi(k_1+k_2\cos(h)+k_3\cosh(\xi))}, \tag{60}$$

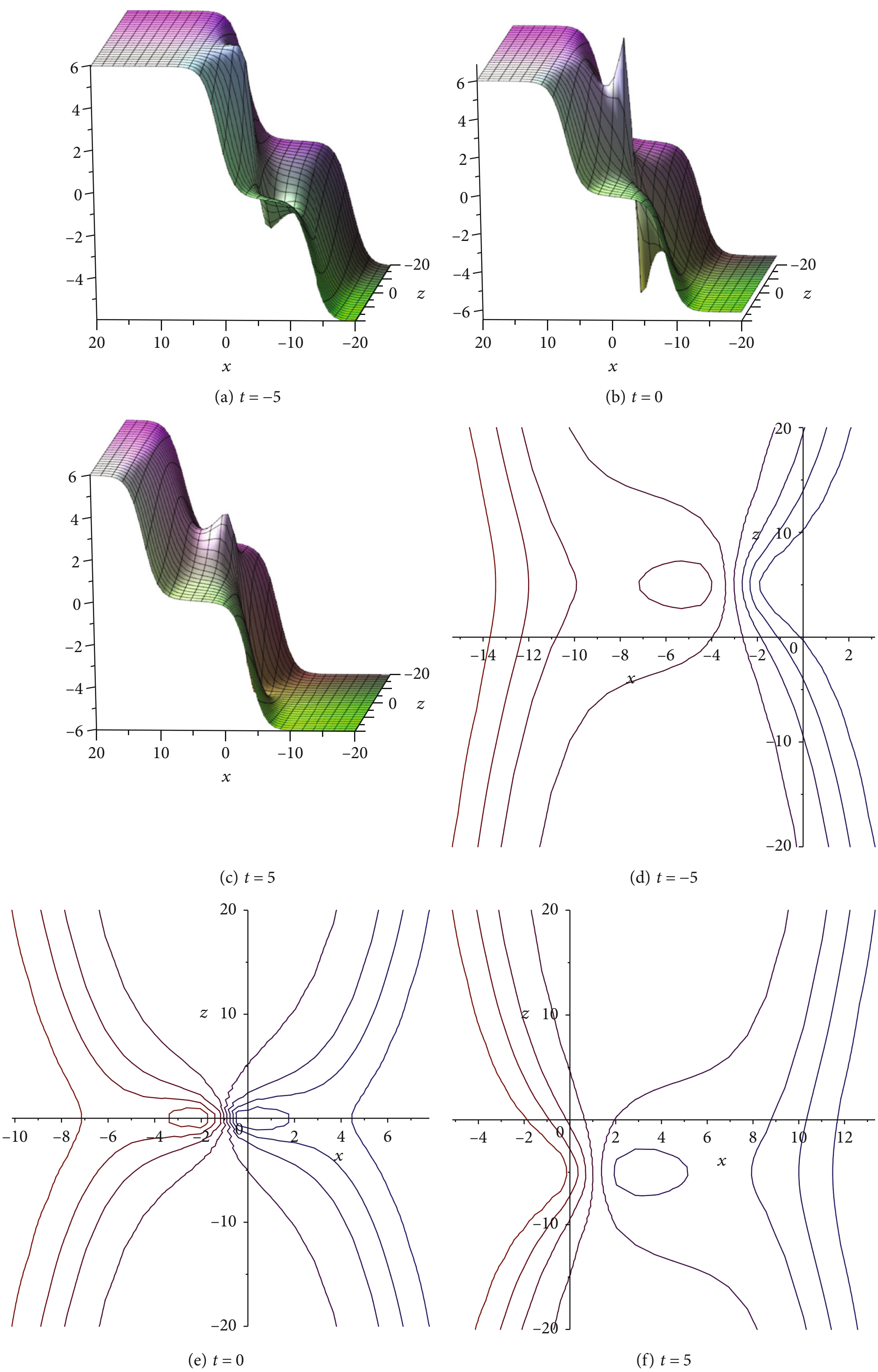


Figure 4: The figures on (x, z) of solution (52) with $a_0 = 1$, $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $b_0 = 1$, $b_1 = 1$, $b_2 = 1$, $b_3 = 1$, $b_4 = 1$, $\chi = 1$, $\rho_1 = 1$, $\rho_2 = 1$, $\rho_3 = 1$, $\rho_4 = 1$, $y = 0$.

where

$$h = \frac{b_1\left(b_1^2 c_2 - c_2\rho_1 - c_3\rho_2\right)t}{c_2\rho_3} + b_1 x, \xi = -\frac{c_2\rho_4 t}{\rho_3} + c_2 y + c_3 z,$$
$$u = \frac{-6k_3 c_1 \sinh(\xi)}{\chi(k_1 \cosh(g) + k_2 \cos(h) + k_3 \cosh(\xi))}, \tag{61}$$

where

$$g = -\frac{a_2\rho_4 t}{\rho_3} + a_2 y + a_3 z, h = -\frac{b_2\rho_4 t}{\rho_3} + b_2 y + \frac{a_3 b_2 z}{a_2},$$
$$\xi = -\frac{c_1\left(a_2 c_1^2 + a_2\rho_1 + a_3\rho_2\right)t}{a_2\rho_3} + c_1 x,$$
$$u = \frac{-6(k_2 b_1 \sin(h) + k_3 c_1 \sinh(\xi)}{\chi(k_1 \cosh(g) + k_2 \cos(h) + k_3 \cosh(\xi))}, \tag{62}$$

where

$$g = \frac{a_2\rho_4 t}{\rho_3} + a_2 y + a_3 z, h = \frac{b_1\left(a_2 b_1^2 - a_2\rho_1 - a_3\rho_2\right)t}{a_2\rho_3} + b_1 x,$$
$$\xi = -\frac{c_1\left(a_2 c_1^2 + a_2\rho_1 + a_3\rho_2\right)t}{a_2\rho_3} + c_1 x. \tag{63}$$

6. Conclusion

In this paper, a new extended Jimbo-Miwa (EJM) equation is studied, which is an extension of the Jimbo-Miwa equation and $(3+1)$-dimensional generalized BKP equation. It can be used to describe the propagation of three-dimensional nonlinear waves with weak dispersion. The exact solutions of the Jimbo-Miwa equation (1) and the extended Jimbo-Miwa equations (2) and (3) have been studied by many scholars; however, there are not many researches on the new Jimbo-Miwa equation (5). In our paper, the bilinear representation of (5) is shown. First, the exact solutions of the equation are studied, including lump soliton solution, lump-kink soliton solution, and periodic lump solution. The bright-dark lump wave solutions are directly obtained by taking the solution F in the bilinear equation as a quadratic function. Second, the lump-kink between one lump wave and one stripe wave are also presented by taking F as a combination of quadratic function and exponential function. Furthermore, the periodic lump solutions are also derived by taking F as a combination of the hyperbolic cosine function and cosine function. The properties of the solutions are also discussed by graphical simulation. The solutions obtained in our paper are different from that in [31]. As far as we know, our results have not been reported in other studies. It is hoped that our results can enrich the dynamic behaviors of the studied equation.

Data Availability

Data is available on request from the author.

Conflicts of Interest

The authors declare no conflicts of interest.

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