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Research Article

Stability and Finite-Time Synchronization Analysis for Recurrent Neural Networks with Improved Integral-Type Time-Varying Delays

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This paper studies the stability criterion of integral time-varying recurrent neural networks (RNNs) with zero lower bound and finite-time synchronization based on improved sliding mode control (SMC). Firstly, a sufficient criterion for universal asymptotic stability of RNNs with integral time-varying delays is obtained by estimating a tight upper bound of augmented Lyapunov-Krasovskii functional (LKF) derivative with inequality scaling technique and mutually convex combined inequality. Secondly, in order to eliminate the time that error system state trajectory slides along sliding mode flow pattern until convergence at the origin, based on drive response and SMC theory, a suitable sliding mode controller is designed by considering that sliding mode flow pattern is equal to synchronization error. Finally, maximum allowable upper bound of delay under different delay derivatives are obtained by considering trajectory change of input function under different initial value. Synchronization trajectory of drive and response systems with mismatched parameters and activation functions under the influence of controller are studied, and synchronization time which is required for error system to reach stability is obtained. Simulation results show that the introduction of integral delay can be more comprehensive from both difference and area, so that drive system state is eventually steady at equilibrium point and synchronized with response system. Stability criterion of this paper not only has less conservative and computation complexity but also has shorter synchronization control time.

1. Introduction

For the past few years, RNNs have been intensively investigated in various research fields such as machine learning, medical diagnosis, natural language processing, and model prediction [1–8]. Due to a certain transmission time of signal or information, time delays inevitably exist in RNNs, which may lead to oscillation, and even degradation for systems performances. Hence, delayed RNNs have become a significant research topic on their stability and synchronization analysis [9, 10]. The integrating approach of terminal sliding mode control and RNNs is proposed [11], based on Lyapunov stability theorem, the convergence and stability of deep learning RNNs are proved. The stability and synchronization of Riemann-Liouville fractional-order inertial neural network with time delay are studied [12] and transform the original inertial system into a conventional system through appropriate variable substitution.

The stable neural network is a necessary prerequisite for practical engineering model; LKF is an effective method for stability analysis of delayed system, whose purpose is to obtain maximum upper bound of time delay and to guarantee the globally asymptotically stability; in other words, to find a positively definite LKF whose difference along the system orbit is less than zero. The main difficulty is how to estimate the upper bound of integral terms of LKF derivative, and three primary methods are proposed to solve this problem: the model transformation method, the free weighted matrix method, and integral inequality method [13].

SMC has been proposed for synchronization of delayed RNNs, which is very robust due to sliding mode invariance when the system state reaches the sliding mode surface [14, 15]. Nevertheless, delayed RNNs may be typically expected to attain synchronization in short time in practical applications; it is essential to introduce finite-time synchronization with the understanding that the orbits of two neural networks from different initial states under the control strategy eventually converge over time; finite-time synchronization has fast dynamic behavior and high accuracy synchronization compared with asymptotic and exponential synchronization [16].

Researchers also investigated the consistent asymptotic stability and integrability of nonregressive systems and boundedness of nonlinear delay-dependent perturbed systems based on new LKF method [17]. By considering appropriate LKF and feedback controllers, the finite/fixed-time synchronization of Clifford-valued RNNs with time-varying delays is derived by decomposing the proposed Clifford-valued drive-response models into real-valued drive-response models [18]. A finite-time synchronization for class of drive-response bidirectional associative memory (BAM) neural networks with time-varying delays is developed by employing maximum-value approach, and two new inequalities are also proposed [19]. Furthermore, a novel nonsingular integral terminal sliding control strategy for finite-time synchronization of category of hyperchaotic systems is proposed [20]; in addition, the planned controller scheme confirms that master-slave hyperchaotic systems arrive in existence of parameter uncertainty as quickly as possible by using Lyapunov stability theorem. The combination–combination synchronization of systems under multiple stochastic disturbances is studied by utilizing finite-time Lyapunov theory, and nonsingular terminal SMC technique [21], new fractional-order sliding surface, adaptive combination controller, and some parameter updating laws are deduced at the same time.

In terms of practical applications, an appropriate LKF with double and triple integral terms with details on both lower and upper bounds of delay is completely designed [22]; then, a new class of delay-dependent adequate condition is proposed, so that the error system is $(Q, S, R)-\gamma-$strict dissipative. The problem of sampled-data synchronization for delayed multiagent networks with fixed topology is investigated and applied to coupling circuit successfully [23]. The finite-time event-triggered control synchronization for delayed stochastic neural networks is studied with passivity and passification approach [24]; then, a nonfragile ETC is proposed to diminish the communication load during networked transmission.

Based on above findings in the existing literature, the problem of global asymptotic stability and finite-time simultaneous SMC of neural networks with integral nonzero lower bound time-varying delays is debated in depth in this paper. In the process of research, inspired by the integral-differential equation, the original distributed time-varying delay of integral term is replaced by a time-varying delay. The main contributions of paper are summarized as follows:

(i) By utilizing the inequality scaling technique and mutually convex combined inequality, a tight upper bound of augmented LKF derivative is estimated; as a result, a sufficient criterion for universal asymptotic stability of RNNs with the combination of distributed time-varying delay and time-varying delay is derived

(ii) In order to eliminate the time that error system state trajectory slides along sliding mode flow pattern until convergence at the origin, a sliding mode manifold is defined by synchronization error; as a result, a semiglobal practical finite-time synchronization of time-varying delayed RNNs is obtained

The rest of the remaining part is arranged as follows. The stability analysis of delayed RNNs is described, and some lemmas are given in Section 2. In Section 3, a new criterion which can be used to guarantee the stability of improved integral-type time-varying delayed RNNs is obtained. In Section 4, based on stability of drive system, the finite-time synchronization of drive-response system is derived. Simulation results are provided in Section 5, and the conclusion is drawn in Section 6.

For convenience of understanding, the mathematical notations are summarized in Table 1.

2. Problem Description and Preparation

Consider the following RNNs with integral time-varying delays:

Drive system:

$$\begin{cases} \dot{x}(t) = -C_1 x(t) + A_1 \mathscr{k}(x(t)) + B_1 \mathscr{k}(x(t-\jmath(t))) + D_1 \int_{t-\jmath(t)}^{t} \mathscr{k}(x(v))dv + J_1, \\ x(t) = \varphi(t), t \in [-\jmath, 0]. \end{cases} \tag{1}$$

Response system:

$$\begin{cases} \dot{y}(t) = -C_2 y(t) + A_2 g(y(t)) + B_2 g(y(t-\jmath(t))) + D_2 \int_{t-\jmath(t)}^{t} g(y(v))dv + J_2 + u(t), \\ y(t) = \psi(t), t \in [-\jmath, 0], \end{cases} \tag{2}$$

where $x(\cdot)=[x_1(\cdot),x_2(\cdot),\cdots,x_n(\cdot)]^T\in\mathbb{R}^n$ and $y(\cdot)=[y_1(\cdot),y_2(\cdot),\cdots,y_n(\cdot)]^T\in\mathbb{R}^n$ represent the state vectors of neural network. $\hbar(x(\cdot))=[\hbar_1(x_1(\cdot)),\hbar_2(x_2(\cdot)),\cdots,\hbar_n(x_n(\cdot))]^T\in\mathbb{R}^n$and $g(y(\cdot))=[g_1(y_1(\cdot)),g_2(y_2(\cdot)),\cdots,g_n(y_n(\cdot))]^T\in\mathbb{R}^n$ denote the vectors of neuron activation functions. $C_1=\mathrm{diag}\{c_{11},c_{12},\cdots,c_{1n}\}>0$ and $C_2=\mathrm{diag}\{c_{21},c_{22},\cdots,c_{2n}\}>0$ denote the self-feedback matrices. $A_1\in\mathbb{R}^{n\times n}$ and $A_2\in\mathbb{R}^{n\times n}$ are connection weight matrices. $B_1\in\mathbb{R}^{n\times n}$, $B_2\in\mathbb{R}^{n\times n}$, $D_1\in\mathbb{R}^{n\times n}$, and $D_2\in\mathbb{R}^{n\times n}$ are delay connection weight matrices. $J_1=[J_{11},J_{12},\cdots,J_{1n}]^T\in\mathbb{R}^n$ and $J_2=[J_{21},J_{22},\cdots,J_n]^T\in\mathbb{R}^n$ are constant external input vectors. $u(t)$ is control input to be designed. $\varphi(t)$ and $\psi(t)$ are initial conditions of drive (1) and response system (2). $\jmath(t)$ denotes the internal time-varying delays and satisfies below constraints:

$$0\le\jmath(t)\le\jmath,\ \dot{\jmath}(t)\le\varrho, \tag{3}$$

where $\jmath>0$ and ϱ are constants. Initial vector $\varphi(t)$ is bounded and continuously differentiable on the interval $[-\jmath,0]$. Furthermore, assume that each neuron activation function $\hbar_i(\cdot)$ in Equation (1) is bounded and satisfies the following inequality:

$$\mu_i^-\le\frac{\hbar_i(a)-\hbar_i(b)}{a-b}\le\mu_i^+,\quad\forall a,b\in R,a\ne b,i=1,2,\cdots,n, \tag{4}$$

where $\mu_i^-,\mu_i^+,i=1,2,\dots,n$ are an arbitrary real constants. For facilitating the calculation, set $Y_1=\mathrm{diag}\{\mu_1^-,\mu_2^-,\cdots,\mu_n^-\}$, $Y_2=\mathrm{diag}\{\mu_1^+,\mu_2^+,\cdots,\mu_n^+\}$, by noting (4), the following definition can be obtained:

$$\begin{aligned}&\mu_i=\max\{|\mu_i^-|,|\mu_i^+|\},\quad i=1,2,\cdots,n,\\&L=\mathrm{diag}\{\mu_1,\mu_2,\cdots,\mu_n\}.\end{aligned} \tag{5}$$

According to Brouwer's fixed-point theorem, assume that $x^*=[x_1^*,x_2^*,\cdots x_n^*]^T$ is equilibrium point of drive system (1), which exists and is unique. Shift the equilibrium point of (1) to the origin and defining new state variable $\varpi(t)=x(t)-x^*$; transform (1) into the following form:

$$\begin{aligned}\dot{\varpi}(t)=&-C_1\varpi(t)+A_1f(\varpi(t))\\&+B_1f(\varpi(t-\jmath(t))+D_1\int_{t-\jmath(t)}^t f(\varpi(v))dv,\end{aligned} \tag{6}$$

where $\varpi(\cdot)=[\varpi_1(\cdot),\varpi_2(\cdot),\cdots,\varpi_n(\cdot)]^T\in\mathbb{R}^n$, $f(\varpi(\cdot))=[f_1(\varpi_1(\cdot)),f_2(\varpi_2(\cdot)),\cdots,f_n(\varpi_n(\cdot))]^T\in\mathbb{R}^n$, $f_i(\varpi_i(\cdot))=g_i(\varpi_i(\cdot)+y_i^*)-g_i(y_i^*)$.

According to (4), it is obvious that $f_i(\cdot)$, $i=1,2,\dots,n$ satisfies the following conditions:

$$\mu_i^-\le\frac{f_i(a)-f_i(b)}{a-b}\le\mu_i^+,\quad\forall a,b\in R,a\ne b,i=1,2,\cdots,n. \tag{7}$$

Table 1: Mathematical notations.

Notations	Definition
$\mathbb{R}^n$	n-dimensional euclidean space
$\mathbb{R}^{m\times n}$	$m\times n$-dimensional real matrix space
U^T	Transpose of the vector or matrix U
$U>0\,(U<0)$	Matrix U is positive definite (negative definite)
$U\ge0$	Matrix U is semipositive definite
$S^\perp\in\mathbb{R}^{n\times(n-r)}$	Right orthogonal complement of matrix $S\in\mathbb{R}^{m\times n}$, and $\mathrm{Rank}(S)=r<n$
*	Symmetric block of matrix
I	Unit matrix of appropriate dimensions
$\Leftrightarrow$	Mutual equivalence
$He\{X\}$	$X+X^T$
$\|\bullet\|$	Euclidean norm
$\|\bullet\|_1$	1-norm

If $b=0$ in (7), it follows that:

$$\mu_i^-\le\frac{f_i(a)}{a}\le\mu_i^+,\quad\forall a\ne0,i=1,2,\cdots,n. \tag{8}$$

According to hypothetical conditions (7) and (8), it can be derived that:

$$[f_i(a)-\mu_i^-a][\mu_i^+a-f_i(a)]\le0,i=1,2,\cdots,n \tag{9}$$

$$\begin{aligned}&[f_i(a)-f_i(b)-\mu_i^-(a-b)][\mu_i^+(a-b)-f_i(a)+f_i(b)]\\&\le0,i=1,2,\cdots,n\end{aligned} \tag{10}$$

Lemma 1. (Jensen's inequality [25]). *For any constant symmetric matrix $Q\in\mathbb{R}^{n\times n}>0$, $\beta\le\vartheta\le\alpha$ and the functions of variables $\mathfrak{x}$ and $\dot{\mathfrak{x}}$, then the following inequalities hold:*

$$-(\alpha-\beta)\int_\beta^\alpha\mathfrak{x}^T(\vartheta)Q\mathfrak{x}(\vartheta)d\vartheta\le-\left(\int_\beta^\alpha\mathfrak{x}(\vartheta)d\vartheta\right)^TQ\left(\int_\beta^\alpha\mathfrak{x}(\vartheta)d\vartheta\right), \tag{11}$$

$$\begin{aligned}&-\frac{(\alpha-\beta)^2}{2}\int_\beta^\alpha\int_\vartheta^\alpha\dot{\mathfrak{x}}^T(\varpi)Q\dot{\mathfrak{x}}(\varpi)d\varpi d\vartheta\\&\le-\left(\int_\beta^\alpha\int_\vartheta^\alpha\dot{\mathfrak{x}}(\varpi)d\varpi d\vartheta\right)^TQ\left(\int_\beta^\alpha\int_\vartheta^\alpha\dot{\mathfrak{x}}(\varpi)d\varpi d\vartheta\right).\end{aligned} \tag{12}$$

Lemma 2 (see [26]). *For a symmetric matrix $\Theta\in\mathbb{R}^{n\times n}$, matrix $\mathscr{D}\in\mathbb{R}^{m\times n}$, and vector $\xi\in\mathbb{R}^n$, then*

$$\xi^T\Theta\xi<0\Leftrightarrow\left(\mathscr{D}^\perp\right)^T\Theta\mathscr{D}^\perp. \tag{13}$$

Lemma 3. (Extended delays integral inequality). *For any symmetric matrix $L\in\mathbb{R}^{n\times n}>0$ and a matrix of any* *appropriate dimension X, there exist $\gamma_1 \le v \le \gamma_2$, $\gamma_1 \le \delta \le \gamma_2$, such that the following inequalities hold:*

$$-(\gamma_2-\gamma_1)\int_{t-\gamma_2}^{t-\gamma_1} \omega^T(v)L\omega(v)dv \le -\beta^T(t)\begin{bmatrix} L & X \\ * & L \end{bmatrix}\beta(t), \tag{14}$$

where $\begin{bmatrix} L & X \\ * & L \end{bmatrix} \ge 0$, $\beta^T(t) = [\int_{t-\gamma_2}^{t-\delta}\omega^T(v)dv \int_{t-\delta}^{t-\gamma_1}\omega^T(v)dv]$.

Proof. By using Jensen's inequality and extended mutually convex combination inequality [27], noting (11), one has

$$\begin{aligned}
&-(\gamma_2-\gamma_1)\int_{t-\gamma_2}^{t-\gamma_1} \omega^T(v)L\omega(v)dv \\
&\quad= -(\gamma_2-\gamma_1)\int_{t-\delta}^{t-\gamma_1} \omega^T(v)L\omega(v)dv \\
&\qquad - (\gamma_2-\gamma_1)\int_{t-\gamma_2}^{t-\delta} \omega^T(v)L\omega(v)dv \\
&\quad\le -\frac{1}{\mathfrak{S}}\left(\int_{t-\gamma_2}^{t-\delta} \omega^T(v)L\omega(v)dv\right)^T L\int_{t-\gamma_2}^{t-\delta} \omega^T(v)L\omega(v)dv \\
&\qquad - \frac{1}{1-\mathfrak{S}}\left(\int_{t-\delta}^{t-\gamma_1} \omega^T(v)L\omega(v)dv\right)^T L\int_{t-\delta}^{t-\gamma_1} \omega^T(v)L\omega(v)dv \\
&\quad= -\beta^T(t)\begin{bmatrix} \frac{L}{\mathfrak{S}} & 0 \\ 0 & \frac{L}{1-\mathfrak{S}} \end{bmatrix}\beta(t) \le -\beta^T(t)\begin{bmatrix} L & X \\ * & L \end{bmatrix}\beta(t),
\end{aligned} \tag{15}$$

where $\mathfrak{S} = (\delta-\gamma_1)/(\gamma_2-\gamma_1)$. When $\delta=\gamma_1$ or $\delta=\gamma_2$, we have $\int_{t-\gamma_2}^{t-\delta}\omega^T(v)dv = 0$ or $\int_{t-\delta}^{t-\gamma_1}\omega^T(v)dv = 0$, (15) still holds. □

Lemma 4 (see [28]). *Consider nonlinear systems $\dot{\eta} = F(\eta)$; suppose there exists a smooth positive definite function $\mho(\eta)$ satisfying the following inequality:*

$$\dot{\mho}(\eta) \le -\tau\mho^{\beta}(\eta) + \sigma, t \le 0, \tag{16}$$

where $\tau > 0$, $0 < \beta < 1$, $\sigma > 0$, and then, $\dot{\eta} = F(\eta)$ is semiglobal practical finite-time stable.

Remark 5. Due to the presence of external input J, equilibrium point in (1) is not located at the origin in most cases. Since changing the equilibrium point does not affect the stability, it is shifted to the origin for the convenience of calculation.

Remark 6. If the neuron activation function is derivable, μ_i^- and μ_i^+ can take the minimum and maximum values of their derivatives by employing the Lagrange's median theorem.

3. Time Delay-Dependent Stability Criterion

By constructing incremental LKF and applying Lemma 1, Lemma 2, and Lemma 3, the time delay global asymptotic stability criterion for time-varying delayed RNNs (6) can be obtained.

Theorem 7. *For a given scalar $\jmath$, any ϱ satisfies (3), if there exists symmetric matrices $E \in \mathbb{R}^{3n\times 3n} > 0$, $H \in \mathbb{R}^{2n\times 2n} > 0$, $K \in \mathbb{R}^{3n\times 3n} > 0$, $M \in \mathbb{R}^{3n\times 3n}$, $T_i \in \mathbb{R}^{n\times n}$ $(i=1,2,3)N_i \in \mathbb{R}^{3n\times 3n}$ $(i=1,2)$ and positive definite diagonal matrix $P_i \in \mathbb{R}^{n\times n}$ $(i=1,2)$, $\Lambda_i \in \mathbb{R}^{n\times n}$ $(i=1,2)$, and any matrix $Z \in \mathbb{R}^{3n\times 3n}$ $(i=1,2)$, the following inequalities hold:*

$$\Xi < 0, \tag{17}$$

$$\begin{bmatrix} M & X \\ * & M \end{bmatrix} \ge 0, \tag{18}$$

$$\mathfrak{R}_1 = \begin{bmatrix} T_2 & N_1 \\ * & T_3 \end{bmatrix} \ge 0, \tag{19}$$

$$\mathfrak{R}_2 = \begin{bmatrix} T_2 & N_2 \\ * & T_3 \end{bmatrix} \ge 0, \tag{20}$$

$$\begin{bmatrix} \mathfrak{R}_1 & Z \\ * & \mathfrak{R}_2 \end{bmatrix} \ge 0, \tag{21}$$

where

$$\Xi = \sum_{i=1}^{8}\Xi_i,$$

$$\Xi_1 = He\{q_1 E q_2^T\},$$

$$\Xi_2 = q_3 H q_3^T - q_4 H q_4^T,$$

$$\Xi_3 = He\{(e_4 - Y_1 e_1)\Lambda_1\mathfrak{A} + (Y_2 e_1 - e_4)\Lambda_2\mathfrak{A}\},$$

$$\Xi_4 = q_5 K q_5^T - (1-\varrho) q_6 K q_6^T,$$

$$\Xi_5 = \jmath^2 q_7 M q_7^T - q_8\begin{bmatrix} M & X \\ * & M \end{bmatrix} q_8^T,$$

$$\Xi_6 = \frac{\jmath^4}{4}\mathfrak{A}^T T_1 \mathfrak{A} - (\jmath e_1 - e_7 - e_8) T_1 (\jmath e_1 - e_7 - e_8)^T,$$

$$\begin{aligned}
\Xi_7 = {} & \jmath^2 e_1 T_2 e_1{}^T + \jmath^2\mathfrak{A}^T T_2\mathfrak{A} + \jmath\left[e_1 N_1 e_1{}^T\right. \\
& \left. + e_2(-N_1+N_2)e_2{}^T - e_3 N_2 e_3{}^T\right] \\
& - q_9\begin{bmatrix} \mathfrak{R}_1 & Z \\ * & \mathfrak{R}_2 \end{bmatrix} q_9^T,
\end{aligned}$$

$$\begin{aligned}
&\Xi_8 = He\Big\{-[e_4 - e_5 - Y_1(e_1 - e_2)]P_1[e_4 - e_5 - Y_2(e_1 - e_2)]^T \\
&\qquad - [e_5 - e_6 - Y_1(e_2 - e_3)]P_2[e_5 - e_6 - Y_2(e_2 - e_3)]^T\Big\}, \\
&\mathfrak{A} = [-C\,0\,0\,A\,B\,0\,0\,0\,D\,0], \\
&e_i^T = \left[O_{n\times(i-1)n}\, I_{n\times n}\, O_{n\times(10-i)n}\right], i = 1, 2, \cdots, 10, \\
&q_1 = [e_1\, e_7 + e_8\, e_9 + e_{10}], \\
&q_2 = \left[\mathfrak{A}^T\, e_1 - e_3\;\, e_4 - e_6\right], \\
&q_3 = [e_1\, e_4], \\
&q_4 = [e_3\;\, e_6], \\
&q_5 = [e_1\, e_4 - Y_1 e_1\, e_4 - Y_2 e_1], \\
&q_6 = [e_2\, e_5 - Y_1 e_2\, e_5 - Y_2 e_2], \\
&q_7 = \left[e_1\, e_4\, \mathfrak{A}^T\right], \\
&q_8 = [e_7\, e_9\, e_1 - e_2\, e_8\, e_{10}\, e_2 - e_3], \\
&q_9 = [e_7\, e_1 - e_2\, e_8\, e_2 - e_3],
\end{aligned} \tag{22}$$

Proof. Select the following augmented LKF candidate:

$$V(t) \triangleq \sum_{i=1}^{6} V_i(t), \tag{23}$$

where

$$\begin{aligned}
&V_1(t) = \Omega_1{}^T(t)E\Omega_1(t), \\
&V_2(t) = \int_{t-\jmath}^{t} \Omega_2{}^T(v)H\Omega_2(v)dv, \\
&V_3(t) = 2\sum_{i=1}^{n}\int_0^{w_i(t)} \{\eta_{1i}[\mu_i^+ v - f_i(v)] + \eta_{2i}[f_i(v) - \mu_i^- v]dv\}, \\
&V_4(t) = \int_{t-\jmath(t)}^{t} \Omega_3{}^T(v)K\Omega_3(v)dv, \\
&V_5(t) = \jmath\int_{t-\jmath}^{t}\int_v^t \Omega_4{}^T(u)M\Omega_4(u)dudv, \\
&V_6(t) = \frac{\jmath^2}{2}\int_{t-\jmath}^{t}\int_v^t\int_u^t \dot{w}^T(z)T_1\dot{w}(z)dzdudv, \\
&V_7(t) = \jmath\int_{t-\jmath}^{t}\int_v^t w^T(u)T_2 w(u)dudv \\
&\qquad + \jmath\int_{t-\jmath}^{t}\int_v^t \dot{w}^T(u)T_3\dot{w}(u)dudv,
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
&\Omega_1{}^T(t) = \left[w^T(t)\int_{t-\jmath}^{t} w^T(v)dv \int_{t-\jmath}^{t} f^T(w(v))dv\right], \\
&\Omega_2{}^T(t) = \left[w^T(t)f^T(w(t))\right], \\
&\Omega_3{}^T(t) = \left[w^T(t)f^T(w(t)) - Y_1 w^T(t)f^T(w(t)) - Y_2 w^T(t)\right], \\
&\Omega_4{}^T(t) = \left[w^T(t)f^T(w(t))\,\dot{w}^T(t)\right],
\end{aligned} \tag{25}$$

□

According to Jensen's inequality and noting (3), one can deduce that for $i = 1, 2, \ldots, 10$, the time-derivative of $V_i(t)$ is:

$$\dot{V}_1(t) = \dot{\Omega}_1^T(t)E\Omega_1(t) + \Omega_1^T(t)E\dot{\Omega}_1(t) = \mathfrak{B}^T(t)\Xi_1\mathfrak{B}(t), \tag{26}$$

$$\begin{aligned}
\dot{V}_2(t) &\le \Omega_2^T(t)H\Omega_2(t) - \Omega_2^T(t-\jmath)H\Omega_2(t-\jmath) \\
&= \mathfrak{B}^T(t)\Xi_2\mathfrak{B}(t),
\end{aligned} \tag{27}$$

$$\begin{aligned}
\dot{V}_3(t) &= He\Big\{[f(w(t)) - Y_1 w(t)]^T\Lambda_1\dot{w}(t) \\
&\qquad + [Y_2 w(t) - f(w(t))]^T\Lambda_2\dot{w}(t)\Big\} \\
&= \mathfrak{B}^T(t)\Xi_3\mathfrak{B}(t),
\end{aligned} \tag{28}$$

$$\begin{aligned}
\dot{V}_4(t) &\le \Omega_3{}^T(t)K\Omega_3(t) - (1-\varrho)\Omega_3{}^T(t-\jmath(t))K\Omega_3(t-\jmath(t)) \\
&= \mathfrak{B}^T(t)\Xi_4\mathfrak{B}(t).
\end{aligned} \tag{29}$$

For $\begin{bmatrix} M & X \\ * & M \end{bmatrix} \ge 0$, by using Lemma 3, it yields

$$\begin{aligned}
\dot{V}_5(t) &\le \jmath^2\Omega_4^T(t)M\Omega_4(t) - \jmath\int_{t-\jmath}^{t}\Omega_4^T(v)dvM\int_{t-\jmath}^{t}\Omega_4(v)dv \\
&\le \jmath^2\Omega_4^T(t)M\Omega_4(t) \\
&\quad - \begin{bmatrix} \int_{t-\jmath(t)}^{t}\Omega_4^T(v)dv \\ \int_{t-\jmath}^{t-\jmath(t)}\Omega_4^T(v)dv \end{bmatrix} \begin{bmatrix} M & X \\ * & M \end{bmatrix} \begin{bmatrix} \int_{t-\jmath(t)}^{t}\Omega_4(v)dv \\ \int_{t-\jmath}^{t-\jmath(t)}\Omega_4(v)dv \end{bmatrix} \\
&= \mathfrak{B}^T(t)\Xi_5\mathfrak{B}(t).
\end{aligned} \tag{30}$$

By using Lemma 1, it can be calculated that

$$\begin{aligned}\dot{V}_6(t) &= \frac{\jmath^4}{4}\dot{w}^T(t)T_1\dot{w}(t) - \frac{\jmath^2}{2}\int_{t-\jmath}^{t}\int_{v}^{t}\dot{w}^T(u)T_1\dot{w}(u)dudv \\ &\le \frac{\jmath^4}{4}\dot{w}^T(t)T_1\dot{w}(t) - \left(\int_{t-\jmath}^{t}[w(t)-w(v)]dv\right)^T \\ &\cdot T_1\int_{t-\jmath}^{t}[w(t)-w(v)]dv = \frac{\jmath^4}{4}\dot{w}^T(t)T_1\dot{w}(t) \\ &- \left(sw^T(t) - \int_{t-\jmath(t)}^{t}w^T(v)dv - \int_{t-\jmath}^{t-\jmath(t)}w^T(v)dv\right) \\ &\cdot T_1\left(sw(t) - \int_{t-\jmath(t)}^{t}w(v)dv - \int_{t-\jmath}^{t-\jmath(t)}w(v)dv\right) \\ &= \mathfrak{B}^T(t)\Xi_6\mathfrak{B}(t).\end{aligned} \tag{31}$$

Inspired by method of [29], the following two equalities hold:

$$\begin{aligned}0 &= \jmath\left[w^T(t)N_1w(t) - w^T(t-\jmath(t))N_1w(t-\jmath(t)) - 2\int_{t-\jmath(t)}^{t}w^T(v)N_1\dot{w}(v)dv\right], \\ 0 &= \jmath\left[w^T(t-\jmath(t))N_2w(t-\jmath(t)) - w^T(t-\jmath)N_2w(t-\jmath) - 2\int_{t-\jmath}^{t-\jmath(t)}w^T(v)N_2\dot{w}(v)dv\right].\end{aligned} \tag{32}$$

According to conditions (18)–(21), by using Lemma 1, Lemma 3, and combining with two zero equations, $\dot{V}_7(t)$ upper limit can be calculated:

$$\begin{aligned}\dot{V}_7(t) &\le \jmath^2\left[w^T(t)T_2w(t) + \dot{w}^T(t)T_3\dot{w}(t)\right] \\ &+ \jmath\left[w^T(t)N_1w(t) + w^T(t-\jmath(t))\right. \\ &\left.\cdot(-N_1+N_2)w(t-\jmath(t)) - w^T(t-\jmath)N_2w(t-\jmath)\right] \\ &- \begin{bmatrix}\int_{t-\jmath(t)}^{t}w^T(v)dv \\ \int_{t-\jmath(t)}^{t}\dot{w}^T(v)dv \\ \int_{t-\jmath}^{t-\jmath(t)}w^T(v)dv \\ \int_{t-\jmath}^{t-\jmath(t)}\dot{w}^T(v)dv\end{bmatrix}\begin{bmatrix}\mathfrak{R}_1 & Z \\ * & \mathfrak{R}_2\end{bmatrix}\begin{bmatrix}\int_{t-\jmath(t)}^{t}w(v)dv \\ \int_{t-\jmath(t)}^{t}\dot{w}(v)dv \\ \int_{t-\jmath}^{t-\jmath(t)}w(v)dv \\ \int_{t-\jmath}^{t-\jmath(t)}\dot{w}(v)dv\end{bmatrix} \\ &= \mathfrak{B}^T(t)\Xi_7\mathfrak{B}(t),\end{aligned} \tag{33}$$

where $\mathfrak{R}_1 = \begin{bmatrix}T_2 & N_1 \\ * & T_3\end{bmatrix} \ge 0$, $\mathfrak{R}_2 = \begin{bmatrix}T_2 & N_2 \\ * & T_3\end{bmatrix} \ge 0$, $\begin{bmatrix}\mathfrak{R}_1 & Z \\ * & \mathfrak{R}_2\end{bmatrix} \ge 0$, the positive definite diagonal matrix P_i ($i = 1, 2$) exists, and the following inequalities can be obtained by combining (8) and (9):

$$\begin{aligned}0 &\le 2\left[f(w(t)) - f(w(t-\jmath(t)))\right. \\ &\left.- Y_1(w(t) - w(t-\jmath(t)))\right]^T P_1 \\ &\cdot\left[Y_2(w(t) - w(t-\jmath(t))) - f(w(t)) + f(w(t-\jmath(t)))\right] \\ &+ 2\left[f(w(t-\jmath(t))) - f(w(t-\jmath_2))\right. \\ &\left.- Y_1(w(t-\jmath(t)) - w(t-\jmath_2))\right]^T P_2 \\ &\cdot\left[Y_2((w(t-\jmath(t)) - w(t-\jmath_2))\right. \\ &\left.- f(w(t-\jmath(t))) + f(w(t-\jmath_2))\right] = \mathfrak{B}^T(t)\Xi_8\mathfrak{B}(t).\end{aligned} \tag{34}$$

By combining (26)–(34), one can derive that:

$$\mathfrak{B}^T(t)\Xi\mathfrak{B}(t) < 0, \tag{35}$$

where

$$\begin{aligned}\mathfrak{B}^T(t) &= \left[w^T(t)\, w^T(t-\jmath(t))\, w^T(t-\jmath)\right. \\ &\cdot f^T(w(t))\, f^T(w(t-\jmath(t)))\, f^T(w(t-\jmath)) \\ &\cdot \int_{t-\jmath(t)}^{t}w^T(v)dv \int_{t-\jmath}^{t-\jmath(t)}w^T(v)dv \\ &\left.\cdot \int_{t-\jmath(t)}^{t}f^T(w(v))dv \int_{t-\jmath}^{t-\jmath(t)}f^T(w(v))dv\right], \\ \Lambda_1 &= \text{diag}\{\eta_{11}, \eta_{12}, \cdots, \eta_{1n}\}, \\ \Lambda_2 &= \text{diag}\{\eta_{21}, \eta_{22}, \cdots, \eta_{2n}\}.\end{aligned} \tag{36}$$

According to Lemma 2, noticing $\mathfrak{A}\mathfrak{B}(t) = 0$, $(\mathfrak{A}^\perp)^T\Xi\mathfrak{A}^\perp < 0$ is equivalent to (35). Therefore, if inequalities (17)–(21) hold, the time-varying delayed RNNs (6) that satisfies Equation (3) is globally asymptotically stable.

Remark 8. It showed no information of the delayed neuron state derivatives for seeking the augmented LKF derivative in (30)–(33); at the same time, in order to reduce the computation complexity and conservative of the stability criterion, the constraints of neuron activation function and the information of time-varying delays lower bound are fully considered in (34).

Remark 9. The upper bound of delay can be calculated by using LMI toolbox in MATLAB; the smaller step, the higher precision, but with constraint of the running time.

4. Design of Sliding Mode Controller

Based on stability of drive system (1), the finite-time synchronization of drive-response system is further analyzed. Assume that the external input vectors in drive system (1) is equivalent to response system (2), which represents $J_1 = J_2$, the synchronization error system can be obtained by defining error signal $k(t) = x(t) - y(t)$:

$$\begin{aligned}\dot{k}(t) = &-C_1 k(t) + A_1 \varsigma(k(t)) + B_1 \varsigma(k(t - \jmath(t)))\\ &- u(t) + (C_2 - C_1) y(t) - A_2 g(y(t))\\ &- B_2 g(y(t - \jmath(t))) + A_1 f(y(t))\\ &+ B_1 f(y(t - \jmath(t))),\end{aligned} \tag{37}$$

where $\varsigma(k(\bullet)) = f(x(\bullet)) - f(y(\bullet))$.

Remark 10. Assuming $\|k(\bullet)\| \le \varepsilon$ [30], it is easy to derive that $\|\varsigma(k(\bullet))\| \le \|L\| \bullet \|k(\bullet)\|$ and $\|f(a)\| \le \|L\| \bullet |a|$ from (7) and (8).

Sliding mode manifold is defined as

$$\hbar(t) = k(t). \tag{38}$$

Sliding mode controller is designed as

$$u(t) = \rho(t)\ \mathrm{sgn}\ (\hbar(t)) + \xi \hbar^{\alpha}(t) - \jmath, \tag{39}$$

where $\rho(t) = [\|C_1\| + (\|A_1\| + \|B_1\|)\|L\|]\varepsilon + (\|C_2 - C_1\| + \|A_1\| \bullet \|L\|)\|y(t)\| + \|B_1\| \bullet \|L\| \bullet \|y(t - \jmath(t))\| + \|A_2\| \bullet \|g(y(t))\| + \|B_2\| \bullet \|g(y(t - \jmath(t)))\|$, $\hbar^{\alpha}(t) = [\hbar_1^{\alpha}(t), \hbar_2^{\alpha}(t), \cdots, \hbar_n^{\alpha}(t)]^T$, $\xi > 0$, $\jmath > 0$, $\alpha \in (0, 1)$.

Theorem 11. *The drive system (1) and response system (2) are semiglobal practical synchronized in a finite time for given sliding mode manifold defined in (38) and controller defined in (39).*

Proof. Choose the following Lyapunov function candidate:

$$Y = \frac{1}{2} \hbar^T(t) \hbar(t). \tag{40}$$

Combining Equation (37) with Equation (39), the time derivative of Y is

$$\begin{aligned}\dot{Y} = \hbar^T(t)\dot{\hbar}(t) \le \|\hbar(t)\| \big[&\|C_1\| \bullet \|\mathrm{k}(t)\|\\ &+ \|A_1\| \bullet \|\varsigma(\mathrm{k}(t))\| + \|B_1\| \bullet \|\varsigma(\mathrm{k}(t - \jmath(t)))\|\\ &+ \|C_2 - C_1\| \bullet \|y(t)\| + \|A_2\| \bullet \|g(y(t))\|\\ &+ \|B_2\| \bullet \|g(y(t - \jmath(t)))\| + \|A_1\| \bullet \|f(y(t))\|\\ &+ \|B_1\| \bullet \|f(y(t - \jmath(t)))\|\big] - \rho(t)\|\hbar(t)\|_1\\ &- \xi \sum_{i=1}^{n} \hbar_i^{\alpha+1}(t) + \jmath.\end{aligned} \tag{41}$$

Table 2: Maximum allowable upper bound $\jmath$.

ϱ	0.1	0.5	0.9
[33]	3.9151	2.8049	2.5583
Theorem 7	3.4272	2.9714	2.6477
Step	0.07616	0.07619	0.07565

According to Remark 10, by employing the inequality $\|\hbar(t)\| \le \|\hbar(t)\|_1$, one has

$$\begin{aligned}\dot{Y} \le \big[&(\|C_1\| + (\|A_1\| + \|B_1\|)\|L\|)\varepsilon\\ &+ (\|C_2 - C_1\| + \|A_1\| \bullet \|L\|)\|y(t)\|\\ &+ \|B_1\| \bullet \|L\| \bullet \|y(t - \jmath(t))\| + \|A_2\| \bullet \|g(y(t))\|\\ &+ \|B_2\| \bullet \|g(y(t - \jmath(t)))\| - \rho(t)\big] \|\hbar(t)\|_1\\ &- \xi \sum_{i=1}^{n} \hbar_i^{\alpha+1}(t) + \jmath \le -\xi 2^{(\alpha+1)/2} Y^{(\alpha+1)/2} + \jmath.\end{aligned} \tag{42}$$

□

Lemma 4 implies that for $\forall t \le 0$, the inequality $\dot{Y} \le -\xi 2^{(\alpha+1)/2} Y^{(\alpha+1)/2} + \jmath$ holds, then $k(t) = \hbar(t)$ is semiglobal practical finite-time stable. After the state trajectory of the error system (37) under the condition of the sliding mode controller (39) reaches the sliding mode manifold $\hbar(t) = 0$ in a finite time, the system state satisfies $\hbar(t) = k(t) = 0$, which means that the synchronization error $k(t)$ will converge to zero in a finite time; in other words, drive system (1) and response system (2) are globally synchronized in a finite time. This completes the proof of Theorem 11.

Remark 12. In order to reduce the chattering on the switching control action, the sign function sgn $(\hbar(t))$ in (39) is replaced by $\hbar(t)/(\|\hbar(t)\| + 0.01)$ by using the method of [31].

Remark 13. The method proposed in this paper does not require solving for the feedback control gain, and the switching gain term $\rho(t)$ does not need to apply the neuron activation function term $\hbar(\cdot)$ in the driving system Equation (1).

Remark 14. In this paper, Sigmoid is chosen as the activation function of drive system, due to Sigmoid function is flatter than tanh and its derivative decreases at a slower rate, which effectively alleviate the gradient disappearance problem and make the neural network learn more efficiently.

Remark 15. This paper investigates a class of continuous differentiable time delay for stability analysis of RNNs; its advantage is to ensure the stronger robustness of time-delay systems by taking into consideration that both the differential and the area tend to be zero, which makes the system stable, but for the case that the time delay is discontinuous, the improved delay-dependent integral inequality presented in this paper cannot be directly employed; then, it is necessary to construct proper LKFs and new integral inequalities.

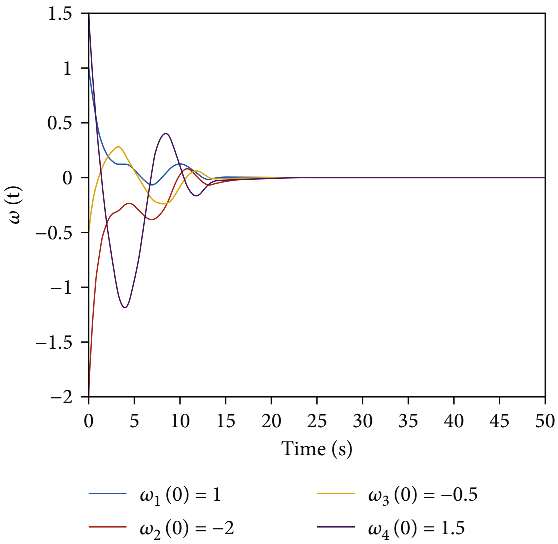

FIGURE 1: The state trajectories of $w_1(t)$, $w_2(t)$, $w_3(t)$, and $w_4(t)$.

5. Numerical Example

Consider the time-varying delayed RNNs (6) with the parameters given in [32]:

$$
\begin{aligned}
Y_1 &= \mathrm{diag}\,\{0, 0, 0, 0\},\\
Y_2 &= \mathrm{diag}\,\{0.1137, 0.1279, 0.7994, 0.2368\},\\
C_1 &= \mathrm{diag}\,\{1.2769, 0.6231, 0.9230, 0.4480\},\\
A_1 &= \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},\\
B_1 &= \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},\\
D_1 &= \begin{bmatrix} 0.6452 & 0.6974 & 1.4273 & 0.5138 \\ -0.3584 & 0.7583 & -0.5269 & 1.0145 \\ 0.1927 & -0.3286 & 0.1713 & -0.2485 \\ 0.6235 & 0.1754 & 0.4534 & -1.5015 \end{bmatrix}.
\end{aligned}
\tag{43}
$$

For different upper bounds on time-varying delay derivatives ϱ, the maximum upper bound that guarantees the stability of system $\jmath$ can be obtained according to Theorem 7 as shown in Table 2.

Literature [33] showed the experimental results without adding any integral term. For $\jmath = 3.4272$, $\varrho = 0.1$, and $\jmath(t) = 1.7136 + 1.7136\sin(t/3.4272)$, the satisfied activation function can be chosen as $f_1(w_1(t)) = 0.1137\mathrm{softsign}(w_1(t))$, $f_2(w_2(t)) = 0.1279\mathrm{softsign}(w_2(t))$, $f_3(w_3(t)) = 0.7994\,\mathrm{softsign}(w_3(t))$, $f_4(w_4(t)) = 0.2368\mathrm{softsign}(w_4(t))$, and $\mathrm{softsign}(t) = t/(1+|t|)$; the initial condition is: chosen as $[1\ {-2}\ {-0.5}\ 1.5]^T$; the simulation results are shown in Figure 1, and it is obvious that system (6) is globally asymptotically stable.

Considering that drive system of Theorem 7 is stable, it is investigated whether the error between drive and response system can converge to zero under the action of controller, therefore, to determine whether Theorem 11 is synchronous. Mismatched parameters in response system (2) are randomly taken as

$$
\begin{aligned}
J_1 &= J_2 = [0.1, 0.3, -0.1, -0.3]^T,\\
C_2 &= \mathrm{diag}\,\{1.18, 0.52, 0.74, 0.62\},\\
A_2 &= \begin{bmatrix} -0.38 & -0.88 & -0.56 & -0.57 \\ 0.53 & 0.55 & 0.22 & 1.62 \\ -0.05 & -0.04 & -0.89 & -0.97 \\ 1.81 & -1.00 & 0.87 & 0.17 \end{bmatrix},\\
B_2 &= \begin{bmatrix} -1.36 & 0.84 & 0.71 & -0.88 \\ -0.65 & 0.27 & 0.10 & 0.68 \\ -0.53 & -0.33 & 0.51 & -0.59 \\ 0.36 & 0.73 & 0.37 & -0.54 \end{bmatrix},\\
D_2 &= \begin{bmatrix} 0.42 & 0.17 & 0.95 & 0.88 \\ -1.50 & 0.35 & -0.76 & 1.21 \\ 0.82 & -0.04 & 0.65 & 0.38 \\ 0.53 & 0.21 & 1.53 & -0.35 \end{bmatrix}.
\end{aligned}
\tag{44}
$$

The neuron activation function in drive system (1) is *Sigmoid* function, while response system (2) is $y(t) = 1/(1+e^{-t})$. Inspired by literature [34], if time-varying delays satisfy $\jmath(t) = 2.3779 + 0.3536\sin(0.4620t) + 0.7725\cos(0.5\sqrt{3}t)$, $\jmath = 3.4995$, $\varrho = 0.8324$, drive system (1) is globally asymptotically stable at the equilibrium point. For $\jmath \in [-3.4995, 0]$, the initial conditions of (1) and (2) are taken as $\varphi(t) = [-1.3, 0.3, -0.8, 0.1]^T$ and $\psi(t) = [-0.1, 0.2, -0.5, -0.6]^T$. The parameters of controller in (39) are taken as $\varepsilon = 1.5$, $\xi = 0.1$, $\jmath = 2$, $\alpha = 5/6$, and $u(t) = (8.8614 + 1.9715\|y(t)\| + 2.8422\|y(t-\jmath(t))\| + 2.577\|g(y(t))\| + 2.1456\|g(y(t-\jmath(t)))\|)\,\mathrm{sgn}(\jmath(t)) + 0.1\jmath^{5/6}(t) - 2$.

Synchronization trajectory of drive and response systems under the influence of controller are simulated in Figures 2 and 3. Figure 2 shows that drive system (1) does not synchronize with response system (2), and it is clearly seen that drive and response systems achieve synchronization in a finite time under controller in Figure 3.

The corresponding synchronization error $k(t)$, which is equivalent to the sliding mode manifold $\hbar(t)$, is shown in Figure 4.

From Figures 2 to 4, it is clearly seen that drive system (1) and response system (2) can achieve finite-time global synchronization with a synchronization time about 0.07 s.

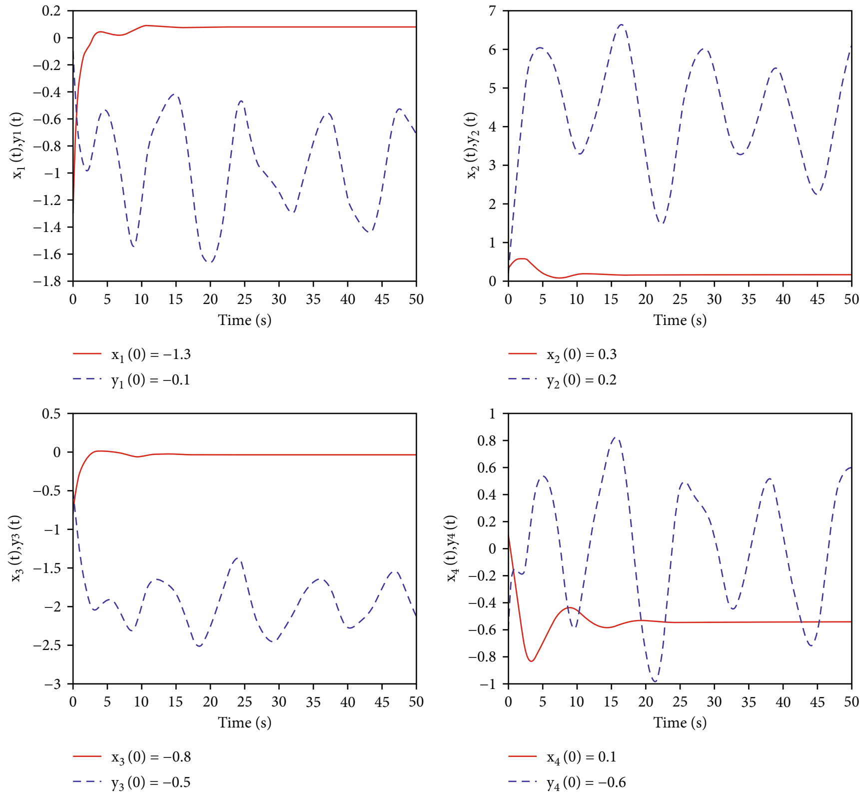

Figure 2: State trajectories of drive and response system without controller.

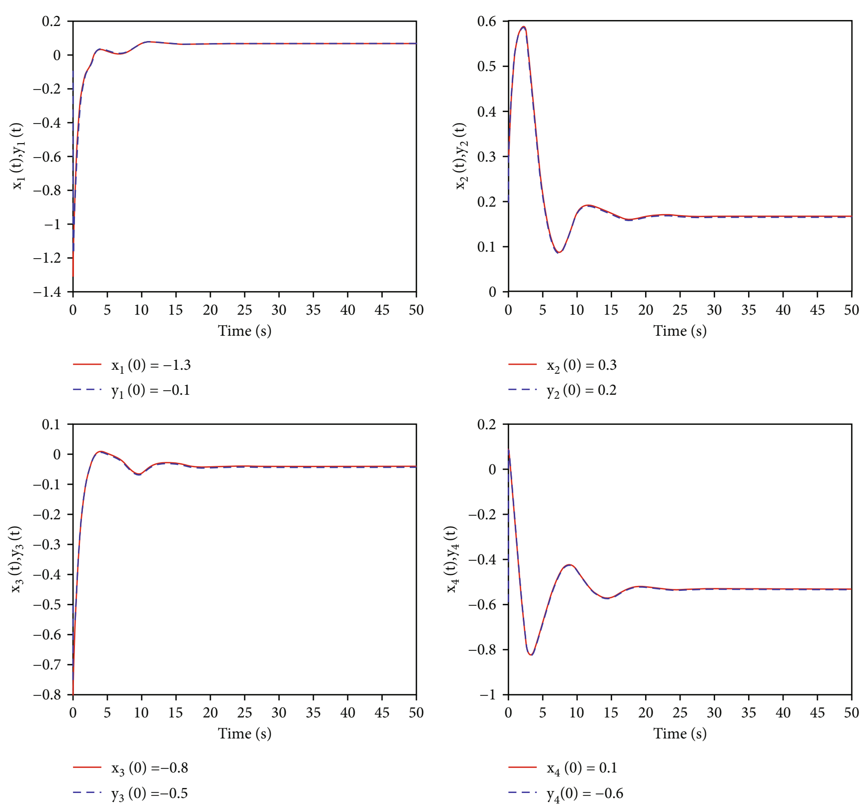

Figure 3: State trajectories of drive and response system with controller.

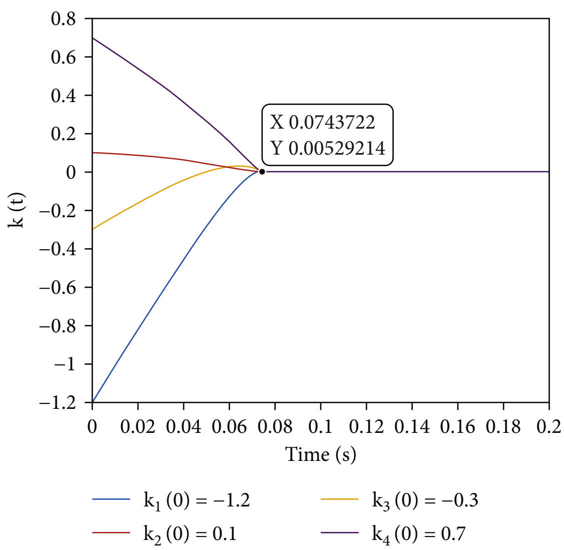

Figure 4: Synchronization error state trajectories with controller.

Simulation results verify the effectiveness and feasibility of synchronization control method proposed in this paper.

6. Conclusion

Stability and a finite-time synchronization problem for delayed RNNs with integral time-varying delays are studied in this paper. Firstly, an augmented LKF is developed without involving the information of delayed neural states, and a stability criterion of LMI is obtained. Secondly, a sliding mode flow pattern which is equivalent to synchronization error is presented; then, a suitable sliding mode controller is designed. Finally, the simulation results clearly verify the superiority of presented approach to stability analysis of delayed RNNs and effectiveness of developed SMC method to finite-time synchronization. In the future work, the proposed methods will be extended to coupling circuit, dissipation analysis, chaotic network, filter issues, and practical applying for signal processing.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Authors' Contributions

Meng Li and Gulijiamali Maimaitiaili are responsible for the conceptualization, methodology, and software; Meng Li for the validation; Gulijiamali Maimaitiaili for the formal analysis; Meng Li and Gulijiamali Maimaitiaili for the investigation; Meng Li for the writing—original draft preparation; and Gulijiamali Maimaitiaili for the writing—review and editing, project administration, and funding acquisition.

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