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Research Article

A Study on Regular Domination in Vague Graphs with Application

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Vague graphs (VGs), which are a family of fuzzy graphs (FGs), are a well-organized and useful tool for capturing and resolving a range of real-world scenarios involving ambiguous data. In graph theory, a dominating set (DS) for a graph $G^* = (X, E)$ is a subset $\mathfrak{S}$ of the vertices X such that every vertex not in $\mathfrak{S}$ is adjacent to at least one member of $\mathfrak{S}$. The concept of DS in FGs has received the attention of many researchers due to its many applications in various fields such as computer science and electronic networks. In this paper, we introduce the notion of $((\epsilon_1, \epsilon_2), 2)$-Regular vague dominating set and provide some examples to explain various concepts introduced. Also, some results were discussed. Additionally, the $((\epsilon_1, \epsilon_2), 2)$-Regular strong (weak) and independent strong (weak) domination sets for vague domination set (VDS) were presented with some theorems to support the context.

1. Introduction

Zadeh [1] introduced the subject of a fuzzy set (FS) in 1995. Rosenfeld [2] proposed the subject of FGs. The definitions of FGs from the Zadeh fuzzy relations in 1973 were presented by Kaufmann [3]. Akram et al. [4–6] introduced several concepts in FGs. Irregular VGs, domination in Pythagorean FGs, and 2-domination in VGs were studied by Banitalebi et al. [7–9]. Gau and Buehrer [10] introduced the notion of a vague set (VS) in 1993. The concept of VGs was defined by Ramakrishna [11]. Akram et al. [12] introduced vague hypergraphs. Rashmanlou et al. [13–15] investigated different subjects of VGs. Moreover, Akram et al. [16–18] developed several results on VGs. Kosari et al. [19] defined VG structure and studied its properties. The concepts of degree, order, and size were developed by Gani and Begum [20]. Borzooei and Rashmanlou [21] proposed the degree of vertices in VGs. Manjusha and Sunitha [22] studied the paired domination. Haynes et al. [23] expressed the fundamentals of domination in graphs. Nagoor Gani and Prasanna Devi [24] suggested the reduction in the domination number of an FG and the notion of 2-domination in FGs [25] as the extension of 2-domination in crisp graphs. The domination number and the independence number were introduced by Cockayne and Hedetniem [26]. In another study, A. Somasundaram and S. Somasundaram [27] proposed the notion of domination in FGs. Kosari et al. [28] studied new concepts in intuitionistic FG with an application in water supplier systems.

Parvathi and Thamizhendhi [29] introduced the domination in intuitionistic FGs. Domination in product FGs and intuitionistic FGs was studied by Mahioub [30, 31]. Karunambigai et al. [32] introduced the domination in bipolar FGs. Rao et al. [33–35] expressed certain properties of domination in vague incidence graphs. Shi and Kosari [36, 37] studied the domination of product VGs with an application in transportation. The concept of DS in FGs, both theoretically and practically, is very valuable. A DS in FGs is used for solving problems of different branches in applied sciences such as location problems. In this way, the study of new concepts such as DS is essential in FG. Domination in VGs has applications in several fields. Domination emerges in the facility location problems, where the number of facilities is fixed and one endeavors to minimize the distance that a person needs to travel to get to the closest facility. Qiang et al. [38] defined the novel concepts of domination in VGs. The notions of total domination, strong domination, and connected domination in FGs using strong arcs were studied by Manjusha and Sunitha [39–41]. Cockayne et al. [42] and Haynes et al. [43] investigated the independent and irredundance domination numbers in graphs. Natarajan and Ayyaswamy [44] introduced the notion of 2-strong (weak) domination in FGs. New results of irregular intuitionistic fuzzy graphs were presented by Talebi et al. [45, 46]. Talebi and Rashmanlou, in [47], presented the concepts of DSs in VFGs. Narayanan and Murugesan [48] expressed the regular domination in intuitionistic fuzzy graph. A few researchers studied other domination variations which are based on the above definitions such as independent domination [49], complementary nil domination [50], and efficient domination [51]. In this paper, we introduced a new notion of $((\epsilon_1, \epsilon_2), 2)$-Regular DS in VG. Finally, an application is given.

2. Preliminaries

In this section, we present some preliminary results which will be used throughout the paper.

Definition 1. A graph G^* is a pair (X, E), where X is called the vertex set and $E \subseteq X \times X$ is called the edge set.

Definition 2. A pair $\mathfrak{C} = (\psi, \zeta)$ is an FG on a graph $G^* = (X, E)$, where ψ is an FS on X and ζ is an FS on E, such that

$$\zeta(sv) \leq \min\{\psi(s), \psi(v)\}, \tag{1}$$

for all $sv \in E$.

Definition 3 (see [10]). A vague set (VS) $\mathfrak{M}$ is a pair $(t_{\mathfrak{M}}, f_{\mathfrak{M}})$ on set X, where $t_{\mathfrak{M}}$ and $f_{\mathfrak{M}}$ are real-valued functions which can be defined on $X \longrightarrow [0, 1]$ so that $t_{\mathfrak{M}}(s) + f_{\mathfrak{M}}(s) \leq 1, \forall s \in X$.

Definition 4 (see [11]). A pair $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ is called a VG on graph $G^* = (X, E)$, where $\mathfrak{M} = (t_{\mathfrak{M}}, f_{\mathfrak{M}})$ is a VS on X and $\mathfrak{Z} = (t_{\mathfrak{Z}}, f_{\mathfrak{Z}})$ is a VS on E such that

$$\begin{aligned} t_{\mathfrak{Z}}(sv) &\leq \min\{t_{\mathfrak{M}}(s), t_{\mathfrak{M}}(v)\}, \\ f_{\mathfrak{Z}}(sv) &\geq \max\{f_{\mathfrak{M}}(s), f_{\mathfrak{M}}(v)\}, \end{aligned} \tag{2}$$

for all $s, v \in X$. Note that $\mathfrak{Z}$ is called vague relation on $\mathfrak{M}$. A VG G is named strong if

$$\begin{aligned} t_{\mathfrak{Z}}(sv) &= \min\{t_{\mathfrak{M}}(s), t_{\mathfrak{M}}(v)\}, \\ f_{\mathfrak{Z}}(sv) &= \max\{f_{\mathfrak{M}}(s), f_{\mathfrak{M}}(v)\}, \end{aligned} \tag{3}$$

for all $sv \in E$.

Definition 5. Assume $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ is a VG on G^*, the degree of a vertex s is denoted as $\mathfrak{d}(v) = (\mathfrak{d}_t(s), \mathfrak{d}_f(s))$, where

$$\begin{aligned} \mathfrak{d}_t(s) &= \sum_{s \neq v, v \in X} t_{\mathfrak{Z}}(sv), \\ \mathfrak{d}_f(s) &= \sum_{s \neq v, v \in X} f_{\mathfrak{Z}}(sv). \end{aligned} \tag{4}$$

The order of $\mathfrak{C}$ is defined as

$$O(\mathfrak{C}) = \left(\sum_{s \in X} t_{\mathfrak{M}}(s), \sum_{s \in X} f_{\mathfrak{M}}(s) \right). \tag{5}$$

Definition 6. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A vertex s is called to dominate a vertex v if $t_{\mathfrak{Z}}(sv) = \min\{t_{\mathfrak{M}}(s), t_{\mathfrak{M}}(v)\}$ and $f_{\mathfrak{Z}}(sv) = \max\{f_{\mathfrak{M}}(s), f_{\mathfrak{M}}(v)\}$.

Definition 7. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A subset $\mathfrak{S}$ of X is called to be VDS if there are some elements of $\mathfrak{S}$ that dominate every vertex $v \in X - \mathfrak{S}$.

Definition 8. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*.

(i) The vertex cardinality of $\mathfrak{C}$ is defined by

$$|X| = \sum_{s \in X} \left| \frac{1 + t_{\mathfrak{M}}(s) - f_{\mathfrak{M}}(s)}{2} \right|. \tag{6}$$

(ii) The edge cardinality of $\mathfrak{C}$ is defined by

$$|E| = \sum_{sv \in E} \left| \frac{1 + t_{\mathfrak{Z}}(sv) - f_{\mathfrak{Z}}(sv)}{2} \right|. \tag{7}$$

Definition 9. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*.

The vertex cardinality of $\mathfrak{S} \subseteq X$ of VG on G^* is defined by

$$|\mathfrak{S}| = \sum_{s \in \mathfrak{S}} \left| \frac{1 + t_{\mathfrak{M}}(s) - f_{\mathfrak{M}}(s)}{2} \right|. \tag{8}$$

Definition 10. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. The neighborhood of a vertex $s \in X$ is defined by

$$\mathfrak{N}(s) = \left\{ v \in X : t_{\mathfrak{Z}}(sv) = t_{\mathfrak{M}}(s) \wedge t_{\mathfrak{M}}(s) \text{ and } f_{\mathfrak{Z}}(sv) = f_{\mathfrak{M}}(s) \vee f_{\mathfrak{M}}(s) \right\}. \tag{9}$$

The neighborhood degree (ND) is denoted by $d_{\mathfrak{N}}(s)$ and defined by

$$d_{\mathfrak{N}}(s) = \sum_{v \in \mathfrak{N}(s)} \left| \frac{1 + t_{\mathfrak{Z}}(sv) - f_{\mathfrak{Z}}(sv)}{2} \right|. \tag{10}$$

The minimum ND is $\delta_{\mathfrak{N}}(\mathfrak{C}) = \wedge\{d_{\mathfrak{N}}(s) : s \in X\}$.

The maximum ND is $\Delta_{\mathfrak{N}}(\mathfrak{C}) = \vee\{d_{\mathfrak{N}}(s): s \in X\}$.

Definition 11 (see [48]). Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. Suppose s and v are any two vertices in $\mathfrak{C}$. Then, v is called to strongly dominate s (s weakly dominate v) if

(i) v dominate s

(ii) $d_{\mathfrak{N}}(v) \geq d_{\mathfrak{N}}(s)$

Definition 12 (see [48]). Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. $X_{\delta_{\mathfrak{N}}} = \{s \in X : d_{\mathfrak{N}}(v) = \delta_{\mathfrak{N}}(\mathfrak{C})\}$ and $X_{\Delta_{\mathfrak{N}}} = \{s \in X : d_{\mathfrak{N}}(v) = \Delta_{\mathfrak{N}}(\mathfrak{C})\}$.

In Table 1, we show the essential notations.

Table 1: Some essential notations.

Notation	Meaning
FS	Fuzzy set
FG	Fuzzy graph
VS	Vague set
VG	Vague graph
ND	Neighborhood degree
VDS	Vague domination set
VSDS	Vague strong dominating set
VWDS	Vague weak dominating set
TDS	Total dominating set
VDN	Vague domination number

3. $((\epsilon_1, \epsilon_2), 2)$-Regular Domination in Vague Graph

In this section, we define the notions of $((\epsilon_1, \epsilon_2), 2)$-Regular DS, independent DS, and strong (weak) DS of VG.

Definition 13. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A subset $\mathfrak{S}$ of X is called to be $((\epsilon_1, \epsilon_2), 2)$-Regular VDS if

(i) every $s \in X - \mathfrak{S}$ is dominated by two vertices in $\mathfrak{S}$

(ii) every vertex in $\mathfrak{S}$ has degree (ϵ_1, ϵ_2)

The minimum vague cardinality of $((\epsilon_1, \epsilon_2), 2)$-Regular VDS is named $((\epsilon_1, \epsilon_2), 2)$-Regular vague domination number (VDN) and denoted by $\eta_{rv}(\mathfrak{C})$.

Example 1. Consider a VG on G^*.

In Figure 1, we have $\mathfrak{S} = \{b, e, g, k\}$ and $X - \mathfrak{S} = \{a, c, d, f, h\}$. The vertices $\{e, b, k\}$ dominate $\{a, c, d\}$, and also, vertices $\{e, g\}$ dominate $\{f, h\}$. We have $\mathfrak{d}_{\mathfrak{S}} = (0.4{,}0.8)$. Therefore, $\mathfrak{S}$ is $((0.4{,}0.8), 2)$-Regular VDS. Thus, $\eta_{rv} = 1.9$.

Definition 14. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A set $\mathfrak{S} \subseteq X$ is called to be $((\epsilon_1, \epsilon_2), 2)$-Regular vague strong dominating set (VSDS) if every vertex s in $V - \mathfrak{S}$ is strongly dominated by two vertices of $\mathfrak{S}$ and each vertex in $\mathfrak{S}$ has degree (ϵ_1, ϵ_2).

The minimum vague cardinality of $((\epsilon_1, \epsilon_2), 2)$-Regular VSDS is named $((\epsilon_1, \epsilon_2), 2)$-Regular vague strong domination number and denoted by $\eta_{rvs}(\mathfrak{C})$.

Example 2. Consider a VG on G^*.

In Figure 2, we have $\mathfrak{S} = \{k, p\}$ which is a minimum size of VSDS, and each vertex in $\mathfrak{S}$ has $\mathfrak{d}_{\mathfrak{S}} = (0.8{,}0.6)$. Therefore, $\mathfrak{S}$ is $((0.8{,}0.6), 2)$-Regular VSDS. Thus, $\eta_{rvs} = 1.15$.

Definition 15. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A set $\mathfrak{S} \subseteq X$ is called to be $((\epsilon_1, \epsilon_2), 2)$-Regular vague weak dominating set (VWDS) if every vertex s in $V - \mathfrak{S}$ is weakly dominated by two vertices of $\mathfrak{S}$ and each vertex in $\mathfrak{S}$ has degree (ϵ_1, ϵ_2).

The minimum vague cardinality of $((\epsilon_1, \epsilon_2), 2)$-Regular VWDS is named $((\epsilon_1, \epsilon_2), 2)$-Regular vague weak domination number and is denoted by $\eta_{rvw}(\mathfrak{C})$.

Example 3. Consider a VG on G^*.

In Figure 3, $\mathfrak{S} = \{a, e, k, c, g\}$ have a minimum size of VWDS, and each vertex in $\mathfrak{S}$ has $\mathfrak{d}_{\mathfrak{S}} = (0.4{,}0.5)$. Therefore, $\mathfrak{S}$ is $((0.4{,}0.5), 2)$-Regular VSDS. Thus, $\eta_{rvw} = 2.55$.

Theorem 16. *For a VG $\mathfrak{M}$, we have*

(i) $\eta_{rv}(\mathfrak{C}) \leq \eta_{rvs}(\mathfrak{C})$

(ii) $\eta_{rv}(\mathfrak{C}) \leq \eta_{rvw}(\mathfrak{C})$

Proof. Since every $((\epsilon_1, \epsilon_2), 2)$-Regular VSDS is a $((\epsilon_1, \epsilon_2), 2)$-Regular VDS, we have $\eta_{rv}(\mathfrak{C}) \leq \eta_{rvs}(\mathfrak{C})$. Further, since every $((\epsilon_1, \epsilon_2), 2)$-Regular VWDS is a $((\epsilon_1, \epsilon_2), 2)$-Regular VDS, we have $\eta_{rv}(\mathfrak{C}) \leq \eta_{rvw}(\mathfrak{C})$. □

Definition 17. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A subset $\mathfrak{S}$ of X is called to be $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS if

(i) $\mathfrak{S}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular VDS

(ii) $t_{\mathfrak{Z}}(sv) < t_{\mathfrak{M}}(s) \wedge t_{\mathfrak{M}}(s)$ and $f_{\mathfrak{Z}}(sv) > f_{\mathfrak{M}}(s) \vee f_{\mathfrak{M}}(s)$, for all $s, v \in \mathfrak{S}$.

The minimum vague cardinality of $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS is denoted by $\iota_{rv}(\mathfrak{C})$.

Example 4. Consider a VG on G^*.

In Figure 4, we have $\mathfrak{S} = \{y, t\}$ that is $((0.5{,}0.9), 2)$-Regular independent VDS. Therefore, $\iota_{rv} = 1$.

Definition 18. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A $((\epsilon_1, \epsilon_2), 2)$-Regular VSDS is called to be $((\epsilon_1, \epsilon_2), 2)$-Regular independent VSDS if $\mathfrak{S}$ is independent.

The minimum vague cardinality of $((\epsilon_1, \epsilon_2), 2)$-Regular independent VSDS is named $((\epsilon_1, \epsilon_2), 2)$-Regular independent vague strong domination number and is denoted by $\iota_{rvs}(\mathfrak{C})$.

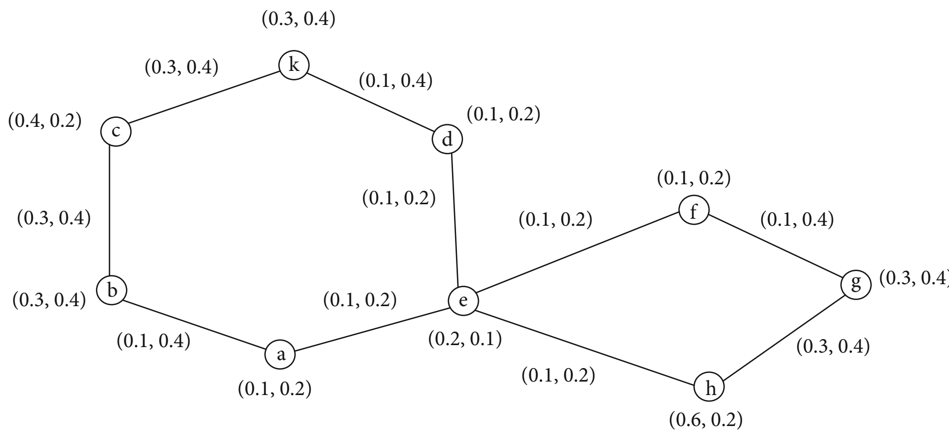

Figure 1: $((\epsilon_1, \epsilon_2), 2)$-Regular VDS of VG.

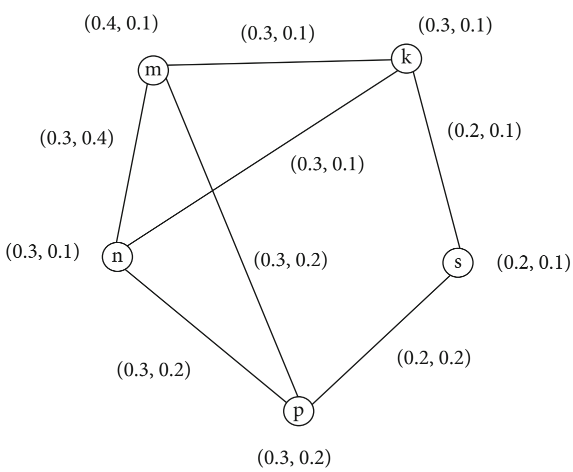

Figure 2: $((\epsilon_1, \epsilon_2), 2)$-Regular VSDS of VG.

Example 5. Consider a VG on G^*.

In Figure 5, we have $\mathfrak{S} = \{p, s, k, q\}$ of $((0.3,0.6), 2)$-Regular independent VDS which is a minimum size. Thus, $\iota_{rvs} = 1.8$.

Definition 19. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A $((\epsilon_1, \epsilon_2), 2)$-Regular VWDS is called to be $((\epsilon_1, \epsilon_2), 2)$-Regular independent VWDS if $\mathfrak{S}$ is independent.

The minimum vague cardinality of $((\epsilon_1, \epsilon_2), 2)$-Regular independent VWDS is named $((\epsilon_1, \epsilon_2), 2)$-Regular independent vague weak domination number and is denoted by $\iota_{rvw}(\mathfrak{C})$.

Example 6. Consider a VG on G^*. In Figure 6, we have six DS

$$\begin{aligned}
\mathfrak{S}_1 &= \{k, w, c, x\},\\
\mathfrak{S}_2 &= \{w, c, x, p\},\\
\mathfrak{S}_3 &= \{c, x, p, t\},\\
\mathfrak{S}_4 &= \{x, p, t, k\},\\
\mathfrak{S}_5 &= \{p, t, k, w\},\\
\mathfrak{S}_6 &= \{t, k, w, c\}.
\end{aligned} \tag{11}$$

We have $\mathfrak{S}_1 = 1.85$, $\mathfrak{S}_2 = 1.8$, $\mathfrak{S}_3 = 1.65$, $\mathfrak{S}_4 = 1.65$, $\mathfrak{S}_5 = 1.8$, and $\mathfrak{S}_6 = 1.85$.

Here, we see that $\mathfrak{S}_3 = \{c, x, p, t\}$ and $\mathfrak{S}_4 = \{x, p, t, k\}$ have minimum size of $((0.2,0.6), 2)$-Regular independent VDS. Thus, $\iota_{rvw} = 1.65$.

Theorem 20. *Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. If $\mathfrak{S}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular independent VWDS of $\mathfrak{C}$, then $\mathfrak{S} \cap X_{\delta_{\mathfrak{N}}} \neq \varnothing$.*

Proof. Assume $s \in X_{\delta_{\mathfrak{N}}}$. Since $\mathfrak{S}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular independent VWDS, either $s \in \mathfrak{S}$ or there exists a vertex $v \in \mathfrak{S}$ such that $t_{\mathfrak{Z}}(sv) = t_{\mathfrak{M}}(s) \wedge t_{\mathfrak{M}}(s)$ and $f_{\mathfrak{Z}}(sv) = f_{\mathfrak{M}}(s) \vee f_{\mathfrak{M}}(s)$, for which $d_{\mathfrak{N}}(v) \leq d_{\mathfrak{N}}(s)$. If $s \in \mathfrak{S}$, then clearly $\mathfrak{S} \cap X_{\delta_{\mathfrak{N}}} \neq \varnothing$. On the other hand, if$d_{\mathfrak{N}}(v) \leq d_{\mathfrak{N}}(s)$, then$d_{\mathfrak{N}}(s) = \delta_{\mathfrak{N}}(\mathfrak{C})$. Therefore, $\mathfrak{S} \cap X_{\delta_{\mathfrak{N}}} \neq \varnothing$. □

Theorem 21. *Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. If $\mathfrak{S}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular independent VSDS of $\mathfrak{C}$, then $\mathfrak{S} \cap X_{\Delta_{\mathfrak{N}}} \neq \varnothing$.*

Proof. Assume $s \in X_{\Delta_{\mathfrak{N}}}$. Since $\mathfrak{S}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular independent VSDS, either $s \in \mathfrak{S}$ or there exists a vertex $v \in \mathfrak{S}$ such that $t_{\mathfrak{Z}}(sv) = t_{\mathfrak{M}}(s) \wedge t_{\mathfrak{M}}(s)$ and $f_{\mathfrak{Z}}(sv) = f_{\mathfrak{M}}(s) \vee f_{\mathfrak{M}}(s)$, for which $d_{\mathfrak{N}}(v) \geq d_{\mathfrak{N}}(s)$. If $s \in \mathfrak{S}$, then clearly $\mathfrak{S} \cap X_{\Delta_{\mathfrak{N}}} \neq \varnothing$. On the other hand, if there exists a vertex $v \in \mathfrak{S}$, then $v \in X_{\Delta_{\mathfrak{N}}}(\mathfrak{M})$, because $d_{\mathfrak{N}}(s) = \Delta_{\mathfrak{N}}(\mathfrak{C})$. Therefore, $\mathfrak{S} \cap X_{\Delta_{\mathfrak{N}}} \neq \varnothing$. □

Theorem 22. *Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^* of order $\mathfrak{P}$. Then, $\iota_{rvs}(\mathfrak{C}) \leq \mathfrak{P} - \Delta_{\mathfrak{N}}(\mathfrak{C})$.*

Proof. Let $\mathfrak{S}$ be $((\epsilon_1, \epsilon_2), 2)$-Regular independent VSDS. Then, $\mathfrak{S} \cap X_{\Delta_{\mathfrak{N}}} \neq \varnothing$. Suppose $s \in \mathfrak{S} \cap X_{\Delta_{\mathfrak{N}}}$. Since $\mathfrak{S}$ is independent, $\mathfrak{S} \cap \mathfrak{N}(s) = \varnothing$. So, $\mathfrak{S} \subseteq X - \mathfrak{N}(s)$, then, $|\mathfrak{S}| \leq |X - \mathfrak{N}(s)|$. Thus, $\iota_{rvs}(\mathfrak{C}) \leq \mathfrak{P} - \Delta_{\mathfrak{N}}(\mathfrak{C})$. □

Theorem 23. *Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. Then, $\iota_{rvw}(\mathfrak{C}) \leq \mathfrak{P} - \delta_{\mathfrak{N}}(\mathfrak{C})$.*

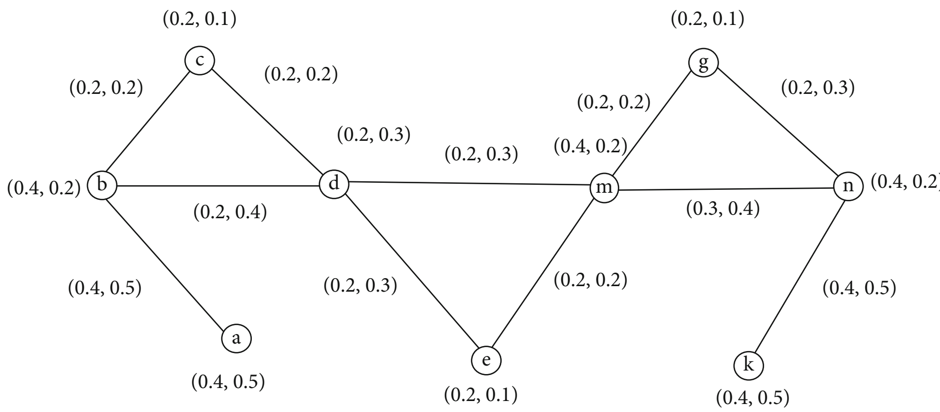

Figure 3: $((\epsilon_1, \epsilon_2), 2)$-Regular VWDS of VG.

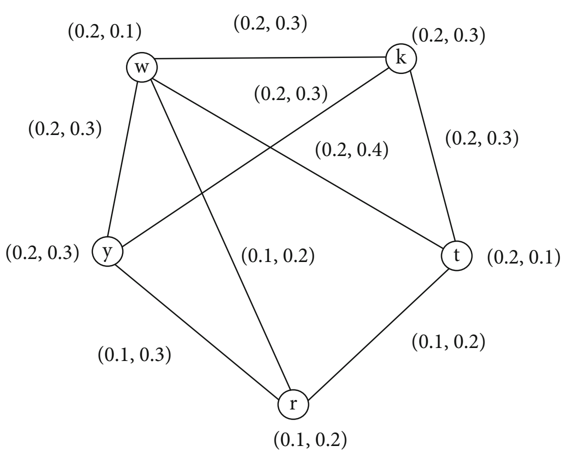

Figure 4: $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS of VG.

Proof. Suppose $\mathfrak{S}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular independent VWDS. Then, $\mathfrak{S} \cap X_{\delta_{\mathfrak{N}}} \neq \varnothing$. Suppose $s \in \mathfrak{S} \cap X_{\delta_{\mathfrak{N}}}$. Since $\mathfrak{S}$ is independent, $\mathfrak{S} \cap \mathfrak{N}(s) = \varnothing$. So, $\mathfrak{S} \subseteq X - \mathfrak{N}(s)$, then, $|\mathfrak{S}| \leq |X - \mathfrak{N}(s)|$. Thus, $\iota_{rvw}(\mathfrak{C}) \leq \mathfrak{P} - \delta_{\mathfrak{N}}(\mathfrak{C})$. □

Theorem 24. *Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a complete VG on G^* with $X = \{s_1, s_2, \cdots, s_n\}$ such that $t_{\mathfrak{M}}(s_1) = t_{\mathfrak{M}}(s_2) \leq \cdots \leq t_{\mathfrak{M}}(s_{n-1}) \leq t_{\mathfrak{M}}(s_n)$ and $f_{\mathfrak{Z}}(s_1) = f_{\mathfrak{Z}}(s_2) \geq \cdots \geq f_{\mathfrak{Z}}(s_{n-1}) \geq f_{\mathfrak{Z}}(s_n)$, and then,*

$$\eta_{rvw}(\mathfrak{C}) = 1 + t_{\mathfrak{M}}(s_1) - f_{\mathfrak{M}}(s_1). \tag{12}$$

Proof. Suppose $\mathfrak{C}$ is a complete VG. Then, all edges are strong. By hypothesis, $d_{\mathfrak{N}}(s_1) = d_{\mathfrak{N}}(s_2) \leq \cdots \leq d_{\mathfrak{N}}(s_{n-1}) = d_{\mathfrak{N}}(s_n)$. Then, we get that $\{s_1, s_2\}$ is a $((\epsilon_1, \epsilon_2), 2)$-Regular VWDS with the minimum vague cardinality. Therefore,

$$\eta_{rvw}(\mathfrak{C}) = |\{s_1, s_2\}| = 1 + t_{\mathfrak{M}}(s_1) - f_{\mathfrak{M}}(s_1). \tag{13}$$

□

Definition 25. A $((\epsilon_1, \epsilon_2), 2)$-Regular VDS of a graph $\mathfrak{C}$ is called to be minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VDS if it contains no $((\epsilon_1, \epsilon_2), 2)$-Regular VDS as a proper subset. The maximum size of minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VDS is denoted by $Y_{rv}(\mathfrak{C})$.

Theorem 26. *If $\mathfrak{C}$ is (ϵ_1, ϵ_2)-Regular VG and $\mathfrak{S}$ is minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VDS, then $X - \mathfrak{S}$ is (ϵ_1, ϵ_2)-Regular VDS.*

Proof. Suppose $\mathfrak{S}$ is a minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VDS. Suppose $X - \mathfrak{S}$ is not (ϵ_1, ϵ_2)-Regular VDS. Then, there is $v \in \mathfrak{S}$ that is not dominated by a vertex in $X - \mathfrak{S}$. Since $\mathfrak{C}$ is a (ϵ_1, ϵ_2)-Regular VG, v must be dominated by two vertices in $\mathfrak{S} - \{v\}$. Then, $\mathfrak{S} - \{v\}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular VDS which is a contradiction. Hence, every vertex in $\mathfrak{S}$ is dominated by two vertices in $X - \mathfrak{S}$. Therefore, $X - \mathfrak{S}$ is (ϵ_1, ϵ_2)-Regular VDS. □

Definition 27. Let $\mathfrak{C} = (\mathfrak{M}, \mathfrak{Z})$ be a VG on G^*. A set $\mathfrak{S} \subseteq X$ is named to be minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VSDS (VWDS) if $\mathfrak{S} - \{s\}$ is not a VSDS (VWDS).

Theorem 28. *A $((\epsilon_1, \epsilon_2), 2)$-Regular VDS $\mathfrak{C}$ of a VG $\mathfrak{C}$ is minimal if and only if for each $s \in \mathfrak{S}$, one of the following two conditions holds:*

(i) $|\mathfrak{N}(s) \cap \mathfrak{S}| \leq 1$

(ii) There exists a vertex $v \in X - \mathfrak{S}$ such that $\mathfrak{N}(v) \cap \mathfrak{S} = \{s, t\}$, for a $t \in \mathfrak{S}$

Proof. Suppose $\mathfrak{S}$ is a minimal $((\epsilon_1, \epsilon_2), 2)$-Regular dominating set. Then, for every vertex $s \in \mathfrak{S}, \mathfrak{S} - \{s\}$ is not a $((\epsilon_1, \epsilon_2), 2)$-Regular VDS. This means that some vertex $v \in X - (\mathfrak{S} - \{s\})$ is not dominated by two vertices in $\mathfrak{S} - \{s\}$. Then, either $s = v$ or $v \in X - \mathfrak{S}$. If $s = v$, then $|\mathfrak{N}(s) \cap \mathfrak{S}| \leq 1$. If $s \neq v$, then $v \in X - \mathfrak{S}$. Since v is not dominated by $\mathfrak{S} - \{s\}$, v is dominated by two vertices of s and t of $\mathfrak{S}$. Then, the vertex v is adjacent to s, t in $\mathfrak{S}$. Therefore, $\mathfrak{N}(v) \cap \mathfrak{S} = \{s, t\}$. Conversely, let $\mathfrak{S}$ be $((\epsilon_1, \epsilon_2), 2)$-Regular VDS, and for every $s \in \mathfrak{S}$, one of the two conditions holds. Suppose $\mathfrak{S}$ is not a minimal dominating set. Then, there exists $s \in \mathfrak{S}$ such that $\mathfrak{S} - \{s\}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular VDS. Hence, s is dominated by at least two vertices in $\mathfrak{S} - \{s\}$. Therefore, condition (i) does not

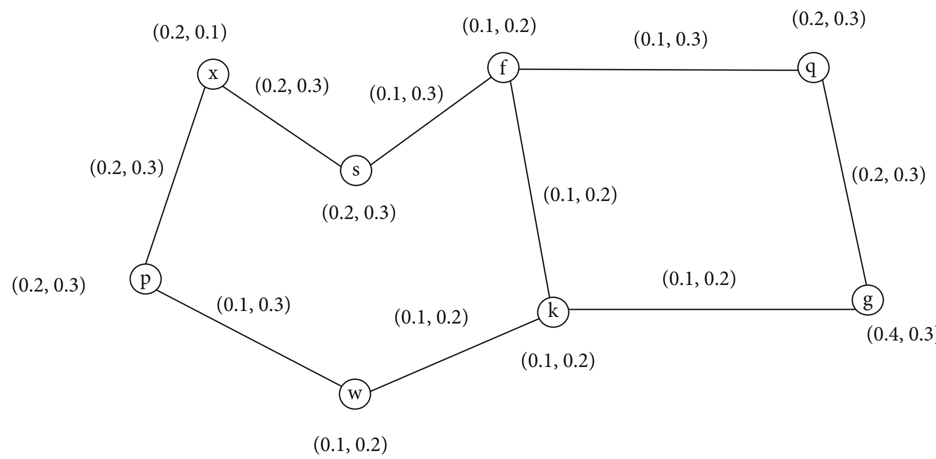

Figure 5: $((\epsilon_1, \epsilon_2), 2)$-Regular independent VSDS of VG.

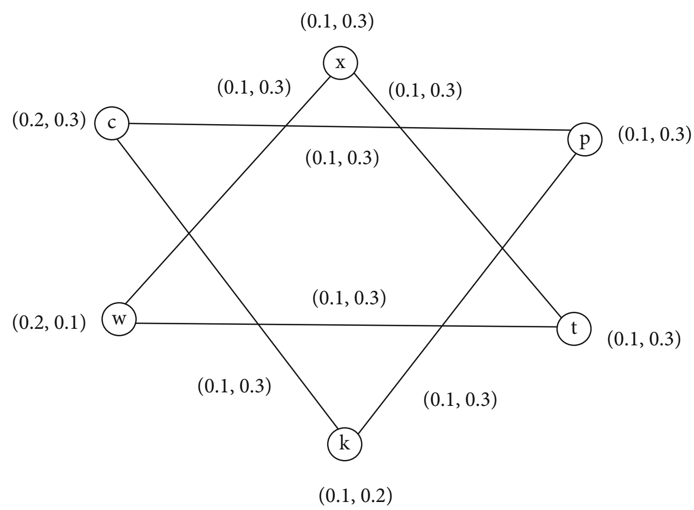

Figure 6: $((\epsilon_1, \epsilon_2), 2)$-Regular independent VWDS of VG.

hold. Also, if $\mathfrak{S} - \{s\}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular VDS, then every vertex v in $X - (\mathfrak{S} - \{s\})$ is dominated by at least two vertices in $\mathfrak{S} - \{s\}$. Therefore, condition (ii) does not hold. This leads to a contradiction. Thus, $\mathfrak{S}$ must be minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VDS. □

Similarly, we have the following theorem.

Theorem 29. *Suppose $\mathfrak{S}$ is minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VWDS. Then, for every $s \in \mathfrak{S}$, one of the following two conditions holds:*

(i) There is no vertex other than $s \in \mathfrak{S}$

(ii) There exists vertex $v \in X - \mathfrak{S}$ such that s and other vertex t are the only vertices in $\mathfrak{S}$ that weakly dominate v

Definition 30. A $((\epsilon_1, \epsilon_2), 2)$-Regular VDS of a graph $\mathfrak{C}$ is called to be minimum $((\epsilon_1, \epsilon_2), 2)$-Regular VDS if it is DS of minimum size.

Definition 31. A $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS is called to be maximal $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS, if every proper subset of $\mathfrak{S}$ is not independent VDS.

Theorem 32. *A $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS is maximal if and only if it is independent and $((\epsilon_1, \epsilon_2), 2)$-Regular VDS.*

Proof. Suppose $\mathfrak{S}$ is maximal $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS. It is trivial that $\mathfrak{S}$ is $((\epsilon_1, \epsilon_2), 2)$-Regular VDS and $\mathfrak{S}$ is independent. Conversely, consider that $\mathfrak{S}$ is independent and $((\epsilon_1, \epsilon_2), 2)$-Regular VDS. Suppose $\mathfrak{S}$ is not maximal; then, there exists $v \in X - \mathfrak{S}$ such that $\mathfrak{S} \cup \{v\}$ is independent. Then, $v \in X - \mathfrak{S}$ is not adjacent to any vertex in $\mathfrak{S}$ which is a contradiction. Therefore, $\mathfrak{S}$ is maximal. □

Theorem 33. *In any VG $\mathfrak{C}$, $\eta_{rv} \le \iota_{rv} \le Y_{rv}$.*

Proof. Since each minimum $((\epsilon_1, \epsilon_2), 2)$-Regular independent VDS is $((\epsilon_1, \epsilon_2), 2)$-Regular VDS, we have $\eta_{rv} \le \iota_{rv}$. Since each minimum $((\epsilon_1, \epsilon_2), 2)$-Regular VDS is a minimal $((\epsilon_1, \epsilon_2), 2)$-Regular VDS, we have $\iota_{rv} \le Y_{rv}$. □

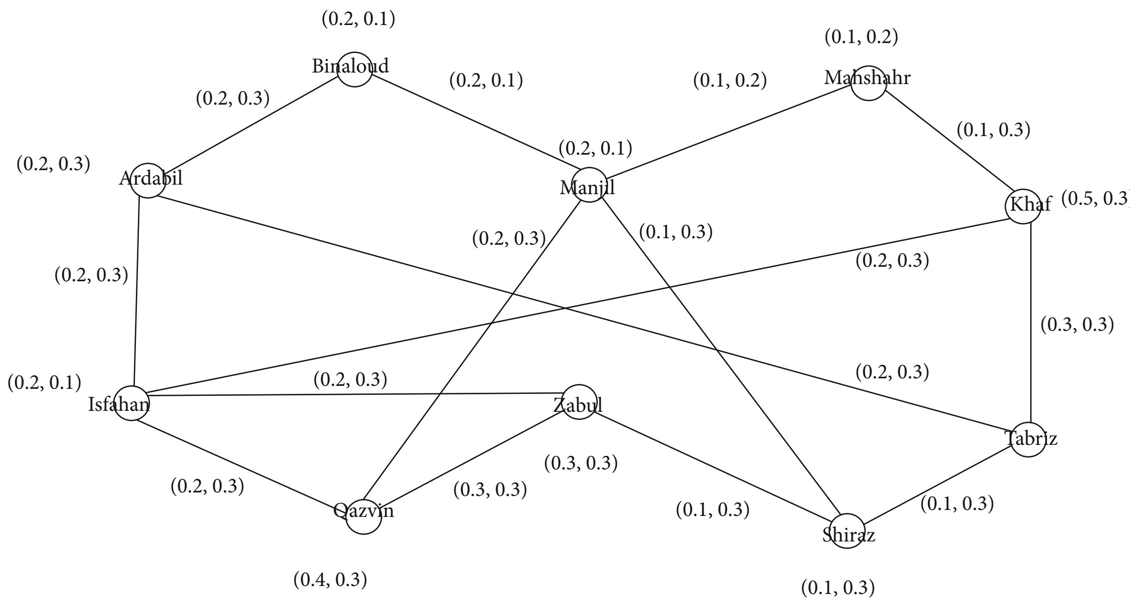

Figure 7: VG of wind turbines.

Table 2: The weights of vertices.

	Zabul	Mahshahr	Shiraz	Isfahan	Tabriz
t_w	0.3	0.1	0.1	0.2	0.3
f_w	0.3	0.2	0.3	0.1	0.3
	Manjil	Binaloud	Khaf	Qazvin	Ardebil
t_w	0.2	0.2	0.5	0.4	0.2
f_w	0.1	0.1	0.3	0.3	0.3

Table 3: The weights of edges.

	(t_Z, f_Z)
Ardebil-Binaloud	(0.2, 0.3)
Manjil-Mahshahr	(0.1, 0.2)
Khaf-Tabriz	(0.3, 0.3)
Shiraz-Zabul	(0.1, 0.3)
Qazvin-Isfahan	(0.2, 0.3)
Khaf-Isfahan	(0.2, 0.3)
Zabul-Isfahan	(0.2, 0.3)
Manjil-Shiraz	(0.1, 0.3)
Binaloud-Manjil	(0.2, 0.1)
Mahshahr-Khaf	(0.1, 0.3)
Tabriz-Shiraz	(0.1, 0.3)
Zabul-Qazvin	(0.3, 0.3)
Isfahan-Ardebil	(0.2, 0.3)
Tabriz-Ardebil	(0.2, 0.3)
Manjil-Qazvin	(0.2, 0.3)

4. Application

4.1. Application of a $((\epsilon_1, \epsilon_2), 2)$-Regular Independent VDS. The extensive activities of many countries in the world to produce electricity from wind energy have become an example for other countries. The economic exploitation of wind energy in electricity production is one of the new production methods in the world's electricity industry. The trend of wind power plant expansion shows a significant increase to reduce the cost of produced electricity. A wind turbine is a turbine that is used to convert the kinetic energy of the wind into mechanical or electrical energy, which is called wind power. It is made in two types: a horizontal axis and a vertical axis.

Small wind turbines are used for applications such as charging batteries or auxiliary power in yachts, while larger wind turbines are used as a source of electrical energy by turning a generator and converting mechanical energy into electrical energy. In Iran, this capacity is also used to produce electricity. Wind power turbines have been installed and operated in the cities of Zabul, Mahshahr, Shiraz, Isfahan, Tabriz, Manjil, Binaloud, Khaf, Qazvin vineyards, and Ardebil.

The problem is how can we increase the amount of electricity produced with minimal wind turbines and have lower fuel costs. Which turbines are better to activate to reach the answer to the problem? To solve this problem, we first need to model the graph. The terms "amount of electricity produced" and "fuel cost reduction" are ambiguous in nature. Therefore, we need fuzzy graph modeling. Consider the vertices where the wind turbine is located and the edges denote the amount of energy production between them.

In Figure 7, the VG model shows the turbine installation locations and the amount of energy production between them. Consider $T = \{\text{Mahshahr, Zabul, Shiraz, Isfahan, Tabriz, Manjil, Binaloud, Khaf, Qazvin, Ardabil}\}$ as a set of

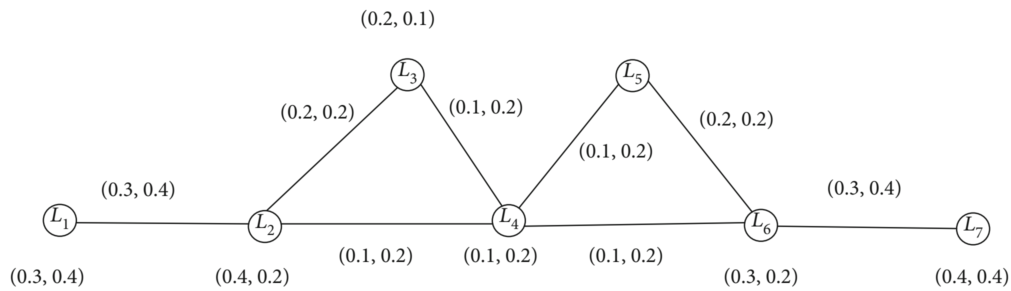

Figure 8: VG of proposed locations.

cities where the turbine is installed. The weights of the vertices and edges are given in Tables 2 and 3.

In this VG, a DS $\mathfrak{S}$ can be interpreted as a set of wind turbines that have more electricity production.

We have $\mathfrak{S} = \{\text{Manjil, Ardebil, Zabul, Khaf}\}$ that is a minimum size of $((0.6,0.9), 2)$-Regular independent VDS. Thus, $\iota_{rvs} = 2.15$.

In this example, by activating at least wind turbines installed in the cities of Manjil, Ardebil, Zabul, and Khaf, the amount of electricity production can be increased, and the cost of fuel can be reduced.

4.2. Application of a $((\epsilon_1, \epsilon_2), 2)$-Regular VWDS. In graph theory, the DS is an important issue in graphs. In this section, we explain the application of weak domination set in VG, and we present this concept in the form of an example. Suppose $\mathfrak{C}$ is a VG (see Figure 8). In this example, we considered seven proposed points of a region for the construction of a clinic. From these seven suggested points, we are going to choose the minimum place for this work that meets the following conditions: good geographical location, facilities of the area, easy access to other places, and the possibility of development and expansion of the desired place. Suppose that $X = \{L_1, L_2, L_3, L_4, L_5, L_6, L_7\}$ are vertices and $E = \{L_1L_2, L_2L_3, L_3L_4, L_2L_4, L_4L_5, L_5L_6, L_4L_6, L_6L_7\}$ are the edges of graph $\mathfrak{C}$.

In this VG, a DS $\mathfrak{S}$ can be interpreted as a set of locations that have the best conditions. We have $\mathfrak{S} = \{L_1, L_3, L_5, L_7\}$ that is a minimum size of $((0.3,0.4), 2)$-Regular VWDS. Thus, $\eta_{rvw} = 2.05$.

In this example, we can build clinics by choosing places that have the best conditions.

5. Conclusion

A VG is suitable for modeling problems with uncertainty which necessitates human knowledge and human evaluation. Moreover, dominations have a wide range of applications in VGs for the analysis of vague information and also serve as one of the most widely used topics in VGs in various sciences. In this research, we described a new concept of domination in VG called $((\epsilon_1, \epsilon_2), 2)$-Regular VDS. We also defined an independent strong (weak) DS in VG. Finally, an application of $((\epsilon_1, \epsilon_2), 2)$-Regular VDS was presented. In future work, we will define a VG structure and study new types of domination, such as $((\epsilon_1, \epsilon_2), 2)$-Regular DS and $((\epsilon_1, \epsilon_2), 2)$-Regular independent DS on VG structure.

Data Availability

No data is used in this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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