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Research Article

Phase Portraits and Traveling Wave Solutions of Fokas System in Monomode Optical Fibers

Chao Tang and Zhao Li

College of Computer Science, Chengdu University, Chengdu 610106, China

Correspondence should be addressed to Zhao Li; lizhao10.26@163.com

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The main purpose of this paper is to investigate the bifurcation and traveling wave solution of the Fokas system in monomode optical fibers by using the method of planar dynamical system. Firstly, the Fokas systems are reduced to two-dimensional planar dynamic system by using the traveling wave transformation. Secondly, by selecting fixed parameters, the phase portraits are drawn by using the Maple software. Finally, the Jacobi elliptic function solutions, the hyperbolic function solutions, and trigonometric function solutions of the Fokas system are obtained.

1. Introduction

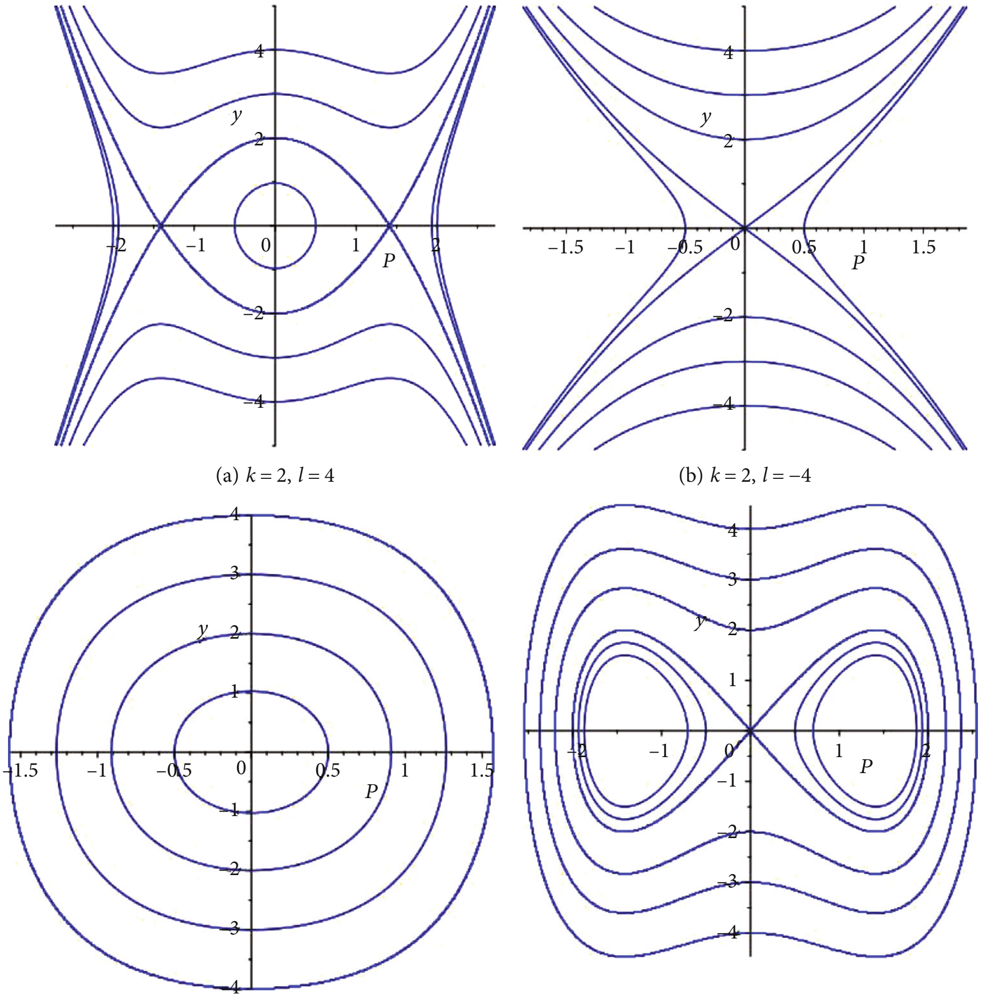
In recent years, the complex nonlinear partial differential equations [1–4] have been widely used in nonlinear optics, fluid mechanics, quantum mechanics, biology, communication, control, and other fields [5–9], which mainly include the well-known Schrödinger equation [10], the Radhakrishnan-Kundu-Lakshmanan equation [11], the Kundu-Mukherjee-Naskar equation [12], the Lakshmanan-Porsezian-Daniel equation [13], the Triki-Biswas equation [14], the Fokas-Lenells equation [15], the Gerdjikov-Ivanov equation [16], the Ginzburg-Landau equation [17], and the Biswas-Arshed equation [18]. The study of dynamic behavior and exact traveling wave solutions of complex nonlinear partial differential equations has always been a very important hot topic.

The Fokas system first proposed by Fokas [19] is a very important class of complex nonlinear partial differential equations, which is a generalization of nonlinear Schrödinger equation. In this paper, we consider the following Fokas system [20, 21]:

$$\begin{cases} ip_t + a_1 p_{xx} + a_2 pq = 0, \\ a_3 q_y - a_4 \left(|p|^2\right)_x = 0, \end{cases} \tag{1}$$

where $p = p(t, x, y)$ and $q = q(t, x, y)$ are the complex valued functions, which represent the propagation of nonlinear pulse in monomode optical fibers. The parameters a_1, a_2, a_3, and a_4 are the nonzero constants. In recent years, many experts and scholars have studied the exact solution of Fokas system, many important methods have been proposed, such as the Jacobi elliptic function expansion method [22], the Hirota's bilinear method [23–26], the Painlevé analysis method [27], the bilinear Bäcklund transformation method [28], the exp $(-\psi(k))$-expansion method [29], the improved F-expansion method [30], the extended rational sine-cosine method [31], and the Exp-function method [32]. Although some important methods of constructing Fokas system have been established, the bifurcation of the dynamic system of Fokas system and the more general traveling wave solution have not been studied. In the paper, our main purpose is to study the bifurcation and traveling wave solutions of the Fokas system by using the theory of the planar dynamic system [33, 34].

The article is organized as follows: in Section 2, the phase portraits are drawn. Moreover, the traveling wave solution of (1) is constructed by using the method of planar dynamic system. In Section 3, a conclusion is presented.



(a) $k = 2$, $l = 4$ (b) $k = 2$, $l = -4$

(c) $k = -2$, $l = 4$ (d) $k = -2$, $l = -4$

Figure 1: The phase portraits of system (8).

2. Phase Portraits and Traveling Wave Solution of (1)

First, let us make a traveling wave transformation as follows:

$$p(t, x, y) = P(\xi)e^{i(k_1x+k_2y+k_3t+k_4)}, q(t, x, y) = Q(\xi), \xi = x + y - vt, \quad (2)$$

where k_1, k_2, and k_3 are real parameters and v stands for the wave transformation.

Applying the traveling wave transformation, we obtain

$$(-v + 2a_1k_1)iP' - k_3P + a_1P'' - a_1k_1^2P + a_2PQ = 0, \quad (3)$$

$$a_3Q' - 2a_4PP' = 0. \quad (4)$$

Integrating both sides of Equation (4) and making the integral constant be zero, we obtain

$$Q(\xi) = \frac{a_4}{a_3}P^2(\xi). \quad (5)$$

Then, substituting Equation (5) into Equation (3) and setting $v = 2a_1k_1$, we have

$$P'' + lP - kP^3 = 0, \quad (6)$$

where $l = -(k_3 + a_1k_1^2/a_1)$ and $k = -(a_2a_4/a_1a_3)$.

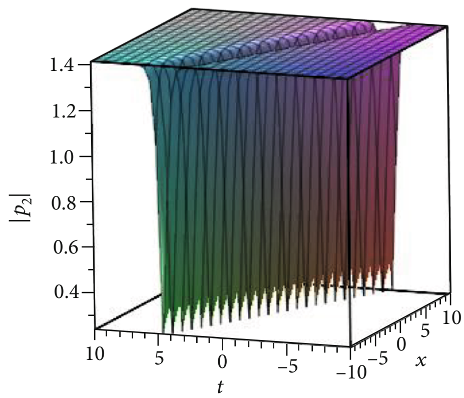
Let $dP(\xi)/d\xi = y$. System (6) can be transformed into a two-dimensional planar dynamic system:

$$\begin{cases} \dfrac{dP(\xi)}{d\xi} = y, \\ \dfrac{dy}{d\xi} = kP^3 - lP, \end{cases} \quad (7)$$

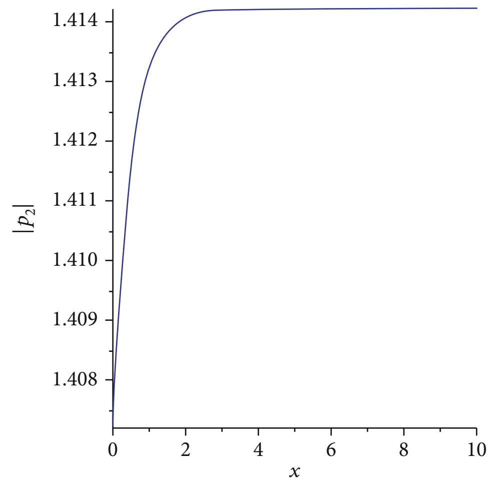
and the Hamiltonian system as follows:

$$H(P, y) = \frac{1}{2}y^2 - \frac{k}{4}P^4 + \frac{l}{2}P^2 = h, \quad h \in \mathbb{R}. \quad (8)$$

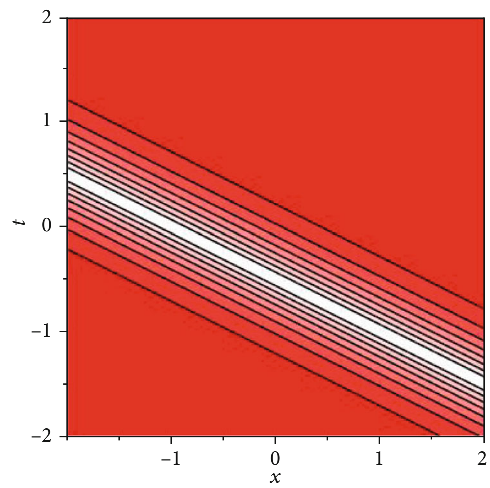
Assume that $M_i(P_i, 0)$ be the equilibrium points of system (7). Suppose further that $F(P) = kP^3 - lP$, $\lambda_{1,2} = \pm$



(a) Modulus of 3D graph



(b) Modulus of 2D graph



(c) Modulus of contour plot

FIGURE 2: Traveling wave solution of $p_2(t,x,y)$ with $a_1=-1, a_2=2, a_3=1, a_4=1/2, k_1=1, k_3=3, \xi_0=0, y=1$.

$\sqrt{F'(P)}$, $h_0=H(0,0)=0$, and $h_1=H(\pm\sqrt{l/k},0)=l^2/4k$. If k $l>0$, system (8) has three equilibrium points $M_1(0,0)$, $M_2(\sqrt{l/k},0)$, and $M_3(-\sqrt{l/k},0)$. Similarly, if $kl<0$, system (8) has one equilibrium point $M_4(0,0)$. If $F'(P_i)>0$, $F'(P_i)=0$, and $F'(P_i)<0$, we can obtain that the equilibrium point $M_i(P_i,0)$ is saddle point, degraded saddle point, and center point, respectively. The phase portraits of system (8) are drawn as shown in Figure 1.

Case 1 $k>0, l>0$.

(i) If $h\in(0,l^2/4k)$, Equation (8) can be rewritten as

$$y^2=\frac{k}{2}P^4-lP^2+2h=\frac{k}{2}\left(P^2-\mu_1^2\right)\left(P^2-\mu_2^2\right), \tag{9}$$

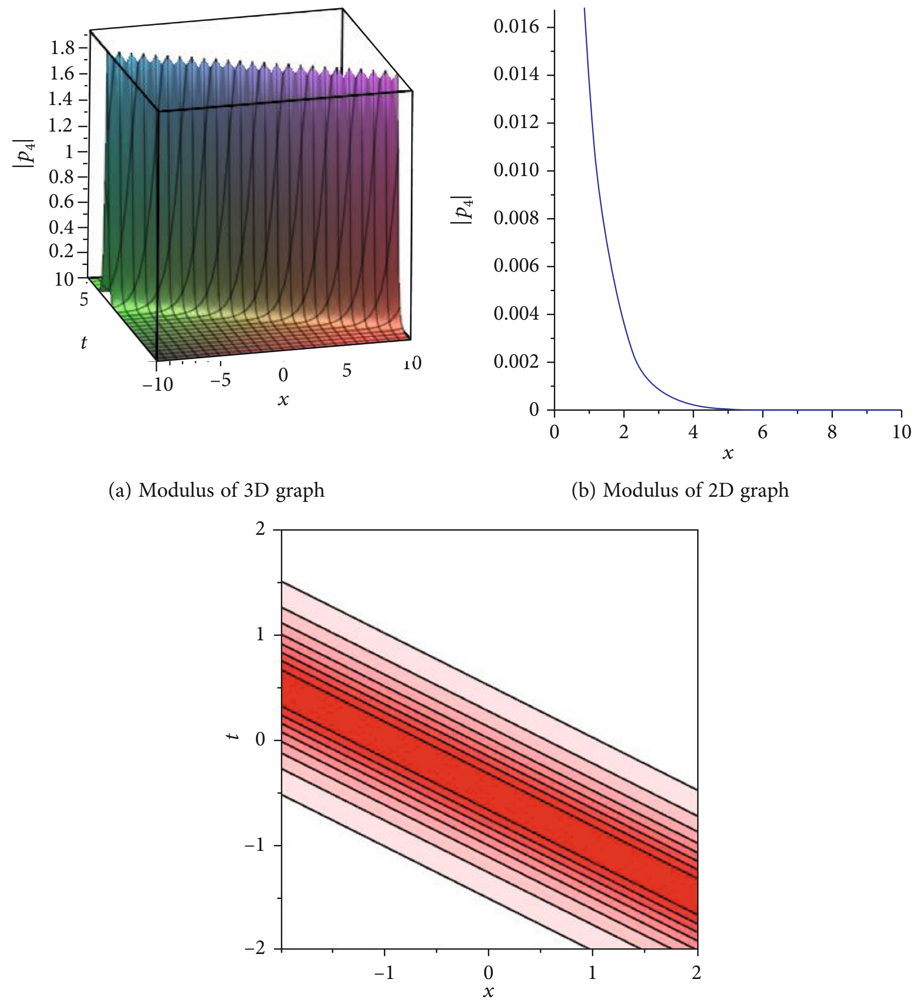
where $\mu_1=\sqrt{(l+\sqrt{l^2-4kh})/k}$ and $\mu_2=\sqrt{(l-\sqrt{l^2-4kh})/k}$.

Substituting (9) into $dP(\xi)/d\xi=y$, we can obtain

$$\int_P^{\mu_2}\frac{dP}{\sqrt{\left(P^2-\mu_1^2\right)\left(P^2-\mu_2^2\right)}}=\pm\sqrt{\frac{k}{2}}(\xi-\xi_0). \tag{10}$$

Integrating Equation (10) and combining Equation (2), we can obtain the Jacobi elliptic function solutions of Equation (1).

$$\begin{aligned} p_1(t,x,y)&=\pm\mu_1\,\mathbf{sn}\left(\mu_2\sqrt{\frac{k}{2}}(x+y-2a_1k_1t-\xi_0),\frac{\mu_1}{\mu_2}\right)\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)},\\ q_1(t,x,y)&=\frac{a_4\mu_1^2}{a_3}\left[\mathbf{sn}\left(\mu_2\sqrt{\frac{k}{2}}(x+y-2a_1k_1t-\xi_0),\frac{\mu_1}{\mu_2}\right)\right]^2\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)}. \end{aligned} \tag{11}$$



(c) Modulus of contour plot

Figure 3: Traveling wave solution of $p_4(t,x,y)$ with $a_1=1, a_2=2, a_3=1, a_4=1/2, k_1=-1, k_3=1, \xi_0=0, y=1$.

(ii) If $h=l^2/4k$, we obtain $\mu_1=\mu_2=\pm\sqrt{l/k}$. Then, we obtain the hyperbolic function solution of Equation (1).

$$p_2(t,x,y)=\pm\sqrt{\frac{l}{k}}\,\mathbf{tanh}\left(\sqrt{\frac{l}{2}}(x+y-2a_1k_1t-\xi_0)\right)\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)},$$
$$q_2(t,x,y)=\frac{a_4l}{a_3k}\left[\mathbf{tanh}\left(\sqrt{\frac{l}{2}}(x+y-2a_1k_1t-\xi_0)\right)\right]^2\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)}. \tag{12}$$

When $a_1=-1, a_2=2, a_3=1, a_4=1/2, k_1=1, k_3=3, \xi_0=0, y=1$, the modulus of solution $p_2(t,x,y)$ is plotted as shown in Figure 2.

Case 2 $k<0, l<0$.

(i) If $h\in(-(l^2/4k),0)$, Equation (8) can be rewritten as

$$y^2=\frac{k}{2}P^4-lP^2+2h=-\frac{k}{2}\left(P^2-\varrho_1^2\right)\left(\varrho_2^2-P^2\right), \tag{13}$$

where $\varrho_1=\sqrt{(l+\sqrt{l^2-4kh})/k}$ and $\varrho_2=\sqrt{(l-\sqrt{l^2-4kh})/k}$. Substituting (9) into $dP(\xi)/d\xi=y$, we can obtain

$$\int_P^{\varrho_2}\frac{dP}{\sqrt{\left(P^2-\varrho_1^2\right)\left(\varrho_2^2-P^2\right)}}=\pm\sqrt{-\frac{k}{2}}(\xi-\xi_0), \tag{14}$$

$$\int_{-\varrho_2}^{P}\frac{dP}{\sqrt{\left(P^2-\varrho_1^2\right)\left(\varrho_2^2-P^2\right)}}=\pm\sqrt{-\frac{k}{2}}(\xi-\xi_0). \tag{15}$$

Integrating Equation (14) and Equation (15), we can obtain the Jacobi elliptic function solutions of Equation (1).

$$p_3(t,x,y) = \pm \varrho_2 \mathbf{dn}\left(\varrho_2\sqrt{-\frac{k}{2}}(x+y-2a_1k_1t-\xi_0), \frac{\sqrt{\varrho_2^2-\varrho_1^2}}{\varrho_2}\right)\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)},$$
$$q_3(t,x,y) = \frac{a_4\varrho_2^2}{a_3}\left[\mathbf{dn}\left(\varrho_2\sqrt{-\frac{k}{2}}(x+y-2a_1k_1t-\xi_0), \frac{\sqrt{\varrho_2^2-\varrho_1^2}}{\varrho_2}\right)\right]^2\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)}. \quad (16)$$

(ii) If $h=0$, we obtain $\varrho_1=0$ and $\varrho_2=\sqrt{2l/k}$. Then, we obtain the solution of Equation (1).

$$p_4(t,x,y) = \pm\sqrt{\frac{2l}{k}}\,\mathbf{sech}\left(\sqrt{-l}(x+y-2a_1k_1t-\xi_0)\right)\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)},$$
$$q_4(t,x,y) = \frac{2la_4}{ka_3}\left[\mathbf{sech}\left(\sqrt{-l}(x+y-2a_1k_1t-\xi_0)\right)\right]^2\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)}. \quad (17)$$

When $a_1=1, a_2=2, a_3=1, a_4=1/2, k_1=-1, k_3=1, \xi_0=0, y=1$, the modulus of solution $p_4(t,x,y)$ is plotted as shown in Figure 3.

(iii) If $0<h<+\infty$, Equation (8) can be rewritten as

$$y^2 = \frac{k}{2}P^4 - lP^2 + 2h = -\frac{k}{2}\left(\varrho_3^2+P^2\right)\left(\varrho_4^2-P^2\right), \quad (18)$$

where $\varrho_3=\sqrt{-(l+\sqrt{l^2-4kh})/k}$ and $\varrho_4=\sqrt{(l-\sqrt{l^2-4kh})/k}$.

Substituting (18) into $dP(\xi)/d\xi=y$, we can obtain

$$\int_0^P \frac{dP}{\sqrt{\left(\varrho_3^2+P^2\right)\left(\varrho_4^2-P^2\right)}} = \pm\sqrt{-\frac{k}{2}}(\xi-\xi_0). \quad (19)$$

Integrating Equation (19), we can obtain the Jacobi elliptic function solutions of Equation (1).

$$p_5(t,x,y) = \pm\varrho_3\mathbf{cn}\left(\varrho_2\sqrt{-\frac{k(\varrho_3^2+\varrho_4^2)}{2}}(x+y-2a_1k_1t-\xi_0), \frac{\varrho_4}{\sqrt{\varrho_3^2+\varrho_4^2}}\right)\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)},$$
$$q_5(t,x,y) = \frac{a_4\varrho_3^2}{a_3}\left[\mathbf{cn}\left(\varrho_2\sqrt{-\frac{k(\varrho_3^2+\varrho_4^2)}{2}}(x+y-2a_1k_1t-\xi_0), \frac{\varrho_4}{\sqrt{\varrho_3^2+\varrho_4^2}}\right)\right]^2\mathbf{e}^{i(k_1x+k_2y+k_3t+k_4)}. \quad (20)$$

3. Conclusion

In this paper, we study the bifurcation and traveling wave solution of the Fokas system in monomode optical fibers by using the method of planar dynamical system. By selecting fixed parameters, the phase portraits, three-dimensional graphs, two-dimensional graphs, and contour plot are drawn by using the Maple software, which explains the propagation of optical solitons in nonlinear optical fibers from different angles. Compared with the published literature [20, 21], the solutions obtained in this paper are more abundant; we also get the Jacobi elliptic function solutions, the hyperbolic function solutions, and trigonometric function solutions of the Fokas system.

Data Availability

No underlying data was collected or produced in this study.

Conflicts of Interest

The author declares no conflict of interest.

Acknowledgments

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