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Research Article

Double-Periodic Soliton Solutions of the (2+1)-Dimensional Ito Equation

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In this work, a $(2+1)$-dimensional Ito equation is investigated, which represents the generalization of the bilinear KdV equation. Abundant double-periodic soliton solutions to the $(2+1)$-dimensional Ito equation are presented by the Hirota bilinear form and a mixture of exponentials and trigonometric functions. The dynamic properties are described through some 3D graphics and contour graphics.

1. Introduction

"Three-wave method" is an algebraic method developed by Wang et al. [1], when studying the soliton interaction, to find the three-solitary wave from the extended homoclinic test method. With this method, they successfully found the double-periodic soliton solution and breathing type solitary wave solution of Korteweg-de Vries (KdV) equation [1]. Subsequently, this method has been applied to many other nonlinear systems, such as the $(3+1)$-Dimensional Kadomtsev-Petviashvili-Boussinesq-Like equation [2], the $(3+1)$-dimensional Boiti–Leon–Manna–Pempinelli (BLMP) equation [3], the third-order $(2+1)$-dimensional equation [4], the forced KdV equation [5], the $(3+1)$-dimensional soliton equation [6], the fifth-order Sawada–Kotera equation [7] etc. Recently, this method has been extended to solve the nonlinear partial differential equations (NLPDES) with variable coefficients by Liu and Zhu [8] and Liu et al. [9], and good results have been obtained.

The $(2+1)$-dimensional Ito equation is a generalization of the bilinear KdV equation, which has attracted the attention of many scholars. Tang et al. [10] presented two classes of lump and interaction solutions, which included rational, periodic, and hyperbolic functions. Zhou and Lan [11] derived nonelastic interactional solutions composed of three different types of waves by two new test functions and the bilinear form. Du and Lou [12] investigated the interactions of lump and solitons. Feng et al. [13] obtained the exact analytical solutions and novel interaction solutions by Hirota bilinear method and symbolic computation. Tan and Zhaqilao [14] studied the effect of three-wave mixing by the long wave limit approach. Ma et al. [15] obtained the localized interaction solutions based on a Hirota bilinear transformation. Zhang et al. [16] presented three kinds of high-order localized waves. Sulaiman et al. [17] obtained some two-wave and breather wave solutions for the $(2+1)$-dimensional Ito equation. However, the double-periodic soliton solutions of this equation has not been studied in other literature, and this will be the main work of our paper. In this paper, we will discuss the following $(2+1)$-dimensional Ito equations [17]:

$$u_{tt} + u_{xxxt} + 6\, u_x\, u_t + 3u\, u_{xt} + 3v_t\, u_{xx} + \mu\, u_{yt} + \nu\, u_{xt} = 0, \tag{1}$$

where $u = u(x, y, t)$ and $v = v(x, y, t) = \int u\, dx$, μ and ν are arbitrary constants.

Performing the following logarithmic transformation:

$$u = 2\,[ln\,\Theta(x,y,t)]_{xx}, \quad v = 2\,[ln\,\Theta(x,y,t)]_{x}, \tag{2}$$

the bilinear form for Equation (1) can be written as follows:

$$\begin{aligned}&\left(D_t^2 + D_x^3 D_t + \mu D_y D_t + \nu D_x D_t\right)\Theta \cdot \Theta \\ &= \left(\Theta_{xxxt} + \Theta_{tt} + \mu\Theta_{ty} + \nu\Theta_{xt}\right)\,\Theta - 3\Theta_{xxt}\Theta_x + 3\Theta_{xt}\,\Theta_{xx} \\ &\quad - \Theta_t\,\Theta_{xxx} - \mu\Theta_t\Theta_y - \nu\Theta_t\Theta_x - \Theta_t^2 = 0.\end{aligned} \tag{3}$$

The structure of this paper is as follows: Section 2 obtains abundant double-periodic soliton solutions to the $(2+1)$-dimensional Ito equation based on a mixture of exponentials and trigonometric functions. Section 3 analyzes the dynamic properties for the derived results by some 3D graphics and contour graphics. Section 4 makes a conclusion.

2. Double-Periodic Soliton Solutions

Recently, "Three-wave method" was revised for obtaining the double-periodic soliton solutions to NLPDES [18], such as the $(2+1)$- and $(3+1)$-dimensional BLMP equation [19, 20], the $(2+1)$-dimensional breaking soliton equation [21], the new $(2+1)$-dimensional KdV equation [22], the $(2+1)$-dimensional generalized Hirota–Satsuma–Ito equation [23] etc. Following the steps of this method, $\Theta(x,y,t)$ has a solution of the following form:

$$\begin{aligned}\Theta(x,y,t) &= k_1 e^{2(\delta_1 t+\alpha_1 x+\beta_1 y)} + e^{\delta_1 t+\alpha_1 x+\beta_1 y}[\gamma_2 \sin\left(\delta_2 t+\alpha_2 x+\beta_2 y\right) \\ &+ \gamma_1 \cos\left(\delta_2 t+\alpha_2 x+\beta_2 y\right)] + e^{\delta_3 t+\alpha_3 x+\beta_3 y}[\gamma_4 \sin\left(\delta_4 t+\alpha_4 x+\beta_4 y\right) \\ &+ \gamma_3 \cos\left(\delta_4 t+\alpha_4 x+\beta_4 y\right)] + k_2 e^{2(\delta_4 t+\alpha_4 x+\beta_4 y)},\end{aligned} \tag{4}$$

where α_i, β_i, and δ_i $(i = 1, 2, 3, 4)$ are constants to be determined later. The assumptions used in the "Three-wave method" are special cases of Equation (4). Substituting Equation (4) into Equation (3), a set of algebraic equations about α_i, β_i, and δ_i $(i = 1, 2, 3, 4)$ are obtained. With the aid of Mathematica software, we have the following results:

Case 1

$$\begin{aligned}&\delta_4 = \delta_2 = k_1 = 0, \beta_4 = -\frac{\alpha_4(3\alpha_3^2 - 12\alpha_4\alpha_3 + 11\alpha_4^2 + \nu)}{\mu}, \\ &\delta_3 = -\alpha_3\nu - \alpha_3^3 + 15\alpha_4^2\alpha_3 - 20\alpha_4^3 - \beta_3\mu, \\ &\delta_1 = -\alpha_1(\alpha_1^2 - 3\alpha_2^2 + \nu) - 14\alpha_4^3 - 12(\alpha_1 - 2\alpha_3)\alpha_4^2 \\ &\qquad + 6(\alpha_1^2 - \alpha_2^2 - \alpha_3^2)\alpha_4 - \beta_1\mu, \beta_2 = \frac{\alpha_2(\alpha_2^2 - 3(\alpha_1 - 2\alpha_4)^2 - \nu)}{\mu}, \\ &\beta_3 = [\alpha_1(-3\alpha_2^2 + 12\alpha_4^2 + \nu) - \alpha_3(\alpha_3^2 + \nu) + \alpha_1^3 \\ &\qquad - 6\alpha_4\alpha_1^2 - 6\alpha_4^3 - 9\alpha_3\alpha_4^2 + 6(\alpha_2^2 + \alpha_3^2)\alpha_4 + \beta_1\mu]/\mu.\end{aligned} \tag{5}$$

Case 2

$$\begin{aligned}&\delta_4 = \delta_2 = k_1 = \delta_3 = \delta_1 = 0, \beta_4 = -\frac{\alpha_4(3\alpha_3^2 - 12\alpha_4\alpha_3 + 11\alpha_4^2 + \nu)}{\mu}, \\ &\beta_2 = [\gamma_1[-\alpha_1(\alpha_1^2 - 3\alpha_2^2 + \nu) - 14\alpha_4^3 - 12(\alpha_1 - 2\alpha_3)\alpha_4^2 \\ &\qquad - 6(-\alpha_1^2 + \alpha_2^2 + \alpha_3^2)\alpha_4 - \beta_1\mu] + \alpha_2\gamma_2[\alpha_2^2 - 3(\alpha_1 - 2\alpha_4)^2 - \nu]]/(\gamma_2\mu), \\ &\beta_3 = -\frac{\alpha_3(\nu - 15\alpha_4^2) + \alpha_3^3 + 20\alpha_4^3}{\mu}.\end{aligned} \tag{6}$$

Case 3

$$\begin{aligned}&\delta_4 = \delta_2 = k_2 = \delta_1 = \delta_3 = 0, \beta_4 = -\frac{\alpha_4(3\alpha_3^2 - 12\alpha_4\alpha_3 + 11\alpha_4^2 + \nu)}{\mu}, \\ &\beta_2 = \frac{\alpha_2(-3\alpha_1^2 + \alpha_2^2 - \nu)}{\mu}, \beta_1 = -\frac{\alpha_1(\alpha_1^2 - 3\alpha_2^2 + \nu)}{\mu}.\end{aligned} \tag{7}$$

Case 4

$$\begin{aligned}
&\delta_4=\delta_2=k_2=0,\beta_2=\frac{\alpha_2(-3\alpha_1^2+\alpha_2^2-\nu)}{\mu},\\
&\delta_1=\delta_3=-\alpha_3(\alpha_3^2-3\alpha_4^2+\nu)+6\alpha_1^3-12\alpha_3\alpha_1^2+6(\alpha_2^2+\alpha_3^2-\alpha_4^2)\alpha_1-\beta_3\mu,\\
&\beta_1=[-\alpha_1(3\alpha_2^2+6\alpha_3^2-6\alpha_4^2+\nu)+\alpha_3(\nu-3\alpha_4^2)-7\alpha_1^3\\
&\qquad+12\alpha_3\alpha_1^2+\alpha_3^3+\beta_3\mu]/\mu,\beta_4=\frac{\alpha_4(-3(\alpha_3-2\alpha_1)^2-\nu)+\alpha_4^3}{\mu}.
\end{aligned}\tag{8}$$

Case 5

$$\begin{aligned}
&\delta_4=k_1=\alpha_2=0,\alpha_1=2\alpha_4,\beta_3=-\frac{\alpha_3(\nu-15\alpha_4^2)+\alpha_3^3+20\alpha_4^3+\delta_3}{\mu},\\
&\delta_2=-\beta_2\mu,\beta_4=-\frac{\alpha_4(3\alpha_3^2-12\alpha_4\alpha_3+11\alpha_4^2+\nu)}{\mu},\\
&\beta_1=-\frac{2\alpha_4(3\alpha_3^2-12\alpha_4\alpha_3+11\alpha_4^2+\nu)+\delta_1}{\mu}.
\end{aligned}\tag{9}$$

Case 6

$$\begin{aligned}
&\delta_4=k_2=\alpha_2=0,\beta_4=\frac{\alpha_4(-3(\alpha_1-\alpha_3)^2-\nu)+\alpha_4^3}{\mu},\delta_2=-\beta_2\mu,\delta_3=2\delta_1,\\
&\delta_1=\frac{1}{2}(-\alpha_3(3\alpha_1^2-3\alpha_3\alpha_1+\alpha_3^2+\nu)+3(\alpha_3-\alpha_1)\alpha_4^2-\beta_3\mu),\\
&\beta_1=\frac{(\alpha_3-2\alpha_1)(\alpha_1^2-\alpha_3\alpha_1+\alpha_3^2+\nu)+3(\alpha_1-\alpha_3)\alpha_4^2+\beta_3\mu}{2\mu}.
\end{aligned}\tag{10}$$

Case 7

$$\begin{aligned}
&\delta_4=k_2=\alpha_2=0,\beta_1=\frac{\beta_2\gamma_2}{\gamma_1}-\frac{\alpha_1(\alpha_1^2+\nu)}{\mu},\delta_2=-\beta_2\mu,\\
&\beta_3=\frac{\gamma_1(-\alpha_3(3\alpha_1^2-3\alpha_3\alpha_1+\alpha_3^2+\nu)-3(\alpha_1-\alpha_3)\alpha_4^2)+2\beta_2\gamma_2\mu}{\gamma_1\mu},\\
&\beta_4=\frac{\alpha_4(-3(\alpha_1-\alpha_3)^2-\nu)+\alpha_4^3}{\mu},\delta_3=\frac{-2\,\beta_2\gamma_2\mu}{\gamma_1}.
\end{aligned}\tag{11}$$

Case 8

$$\begin{aligned}
&\delta_4=k_2=\alpha_2=\alpha_1=0,\beta_1=\frac{\alpha_3(\alpha_3^2-3\alpha_4^2+\nu)}{\mu}+\frac{\beta_2\gamma_1}{\gamma_2}+\beta_3,\\
&\delta_3=-\alpha_3(\alpha_3^2-3\alpha_4^2+\nu)-\beta_3\mu,\beta_4=\frac{\alpha_4(-3\alpha_3^2+\alpha_4^2-\nu)}{\mu},\\
&\gamma_4=\frac{2\alpha_3\alpha_4\gamma_3}{\alpha_3^2-\alpha_4^2},\delta_1=-\alpha_3(\alpha_3^2-3\alpha_4^2+\nu)-\frac{\beta_2\gamma_1\mu}{\gamma_2}-\beta_3\mu.
\end{aligned}\tag{12}$$

Case 9

$$\begin{aligned}
&k_1=\delta_2=\alpha_4=0,\delta_4=-\beta_4\mu,\beta_3=-\frac{\alpha_3\nu+\alpha_3^3+\delta_3}{\mu},\\
&\delta_1=-2\beta_4\mu,\beta_2=\frac{-\alpha_2\nu+\alpha_2^3-3\alpha_1^2\alpha_2-3\alpha_3^2\alpha_2+6\alpha_1\alpha_3\alpha_2}{\mu},\\
&\beta_4=\frac{\alpha_1(-3\alpha_2^2+3\alpha_3^2+\nu)+\alpha_1^3-3\alpha_3\alpha_1^2+3\alpha_2^2\alpha_3+\beta_1\mu}{2\mu}.
\end{aligned}\tag{13}$$

Case 10

$$k_1=\delta_2=\alpha_4=0, \delta_4=-\beta_4\mu, \beta_3=-\frac{\alpha_3\nu+\alpha_3^3+\delta_3}{\mu},$$
$$\delta_1=\frac{2\gamma_3\delta_3}{2\gamma_3-\gamma_4}, \beta_2=\frac{\alpha_2(\alpha_2^2-3(\alpha_1-\alpha_3)^2-\nu)}{\mu}, \beta_4=\frac{\gamma_3\delta_3}{\gamma_4\mu-2\gamma_3\mu},$$
$$\beta_1=-\frac{\alpha_1(-3\alpha_2^2+3\alpha_3^2+\nu)+\alpha_1^3-3\alpha_3\alpha_1^2+3\alpha_2^2\alpha_3+\frac{2\gamma_3\delta_3}{2\gamma_3-\gamma_4}}{\mu}. \tag{14}$$

Case 11

$$k_2=\delta_2=\alpha_4=0, \delta_4=-\beta_4\mu, \beta_2=\frac{\alpha_2(-3\alpha_1^2+\alpha_2^2-\nu)}{\mu}, \alpha_3=2\alpha_1,$$
$$\beta_1=\frac{-\alpha_1\nu-\alpha_1^3+3\alpha_2^2\alpha_1-\delta_1}{\mu}, \beta_3=-\frac{2\alpha_1(\alpha_1^2-3\alpha_2^2+\nu)+\delta_3}{\mu}. \tag{15}$$

Case 12

$$k_1=\alpha_4=\alpha_2=0, \alpha_3=\alpha_1, \delta_2=-\beta_2\mu, \delta_4=-\beta_4\mu,$$
$$\beta_3=-\frac{\alpha_1\nu+\alpha_1^3+\delta_3}{\mu}, \delta_1=-\alpha_1\nu-\alpha_1^3-\beta_1\mu. \tag{16}$$

Case 13

$$k_2=\alpha_4=\alpha_2=0, \alpha_3=2\alpha_1, \delta_2=-\beta_2\mu, \delta_4=-\beta_4\mu,$$
$$\beta_3=-\frac{2\alpha_1(\alpha_1^2+\nu)+\delta_3}{\mu}, \beta_1=-\frac{\alpha_1\nu+\alpha_1^3+\delta_1}{\mu}. \tag{17}$$

Case 14

$$\alpha_2=\alpha_4=0, \delta_1=-\beta_4\mu, \alpha_3=\alpha_1, \delta_2=-\beta_2\mu, \delta_4=-\beta_4\mu,$$
$$\beta_3=-\frac{\alpha_1\nu+\alpha_1^3+\delta_3}{\mu}, \beta_1=\beta_4-\frac{\alpha_1(\alpha_1^2+\nu)}{\mu}. \tag{18}$$

Substituting these results into Equations (2) and (4), we can obtain 14 different double-periodic soliton solutions to Equation (1). These results have not been obtained in the other literature.

3. Discussion and Results

In this section, we discuss the dynamic properties by setting some special values for the free parameters in these solutions. For example, substituting:

$$\alpha_1=\beta_1=\mu=\nu=\alpha_2=1, \alpha_3=\gamma_2=\alpha_4=\gamma_4=-1,$$
$$k_2=\gamma_1=\gamma_3=2, \tag{19}$$

into Equation (5), we can obtain the following double-periodic solito solution:

$$u=[40e^{16t+3x+5y}[\cos(x-27y)-2\sin(x-27y)]+8e^{16t+x+27y}[\cos(x-3y)$$
$$-2\sin(x-3y)]-10(e^{2x+44y}+e^{6x})+4e^{4x+22y}[5\cos[2(x-15y)]$$
$$+6\sin(24y)-8\cos(24y)]]/[2e^{16t+5y}+e^{3x}[2\cos(x-27y)-\sin(x-27y)]$$
$$+e^{x+22y}[\sin(x-3y)+2\cos(x-3y)]]^2, \tag{20}$$

$$v=[-8e^{16t+5y}+2e^{3x}[\cos(x-27y)-3\sin(x-27y)]$$
$$-2e^{x+22y}[3\sin(x-3y)+\cos(x-3y)]]/[2e^{16t+5y}+e^{3x}]2\cos(x-27y)$$
$$-\sin(x-27y)]+e^{x+22y}[\sin(x-3y)+2\cos(x-3y)]]. \tag{21}$$

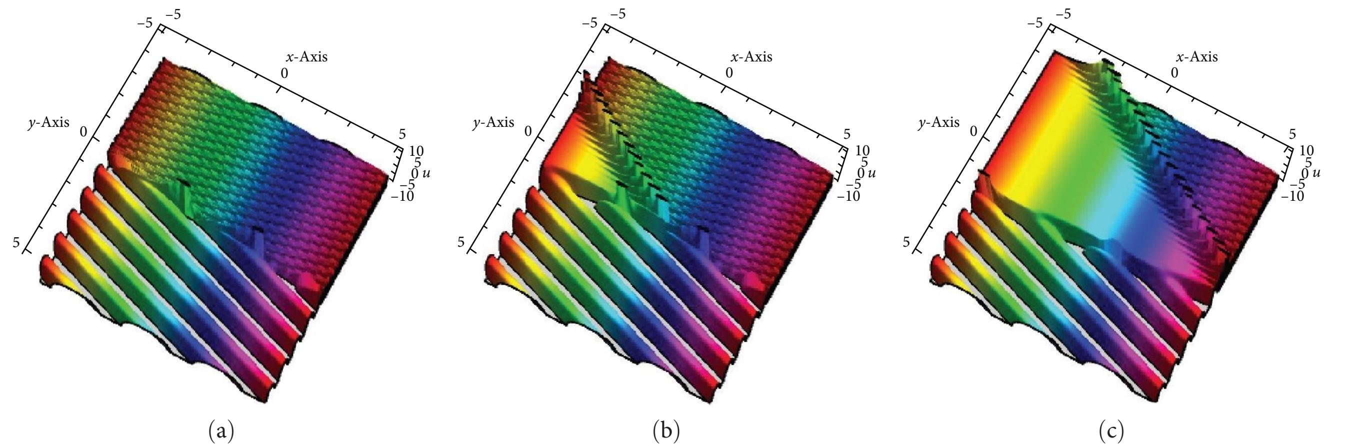


FIGURE 1: Solution (20) when (a) $t = -1$, (b) $t = 0$, and (c) $t = 1$.

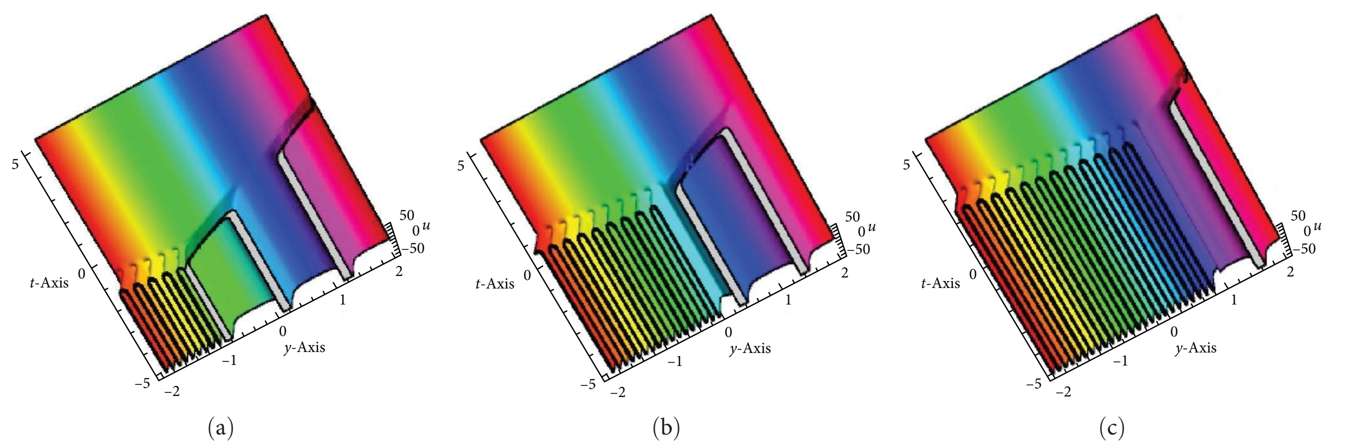


FIGURE 2: Solution (20) when (a) $x = -10$, (b) $x = 0$, and (c) $x = 10$.

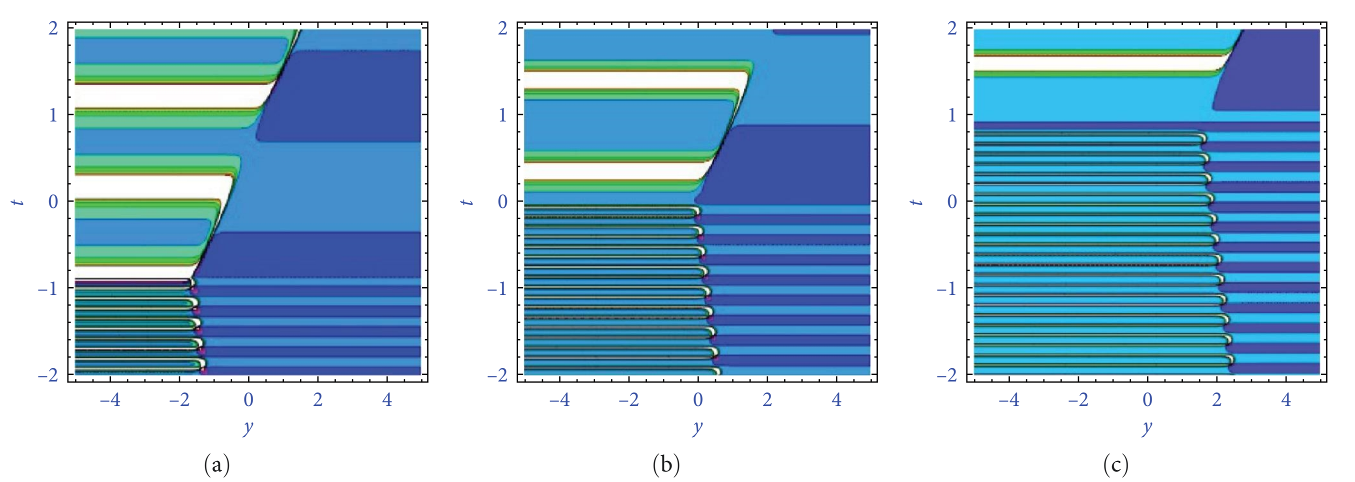
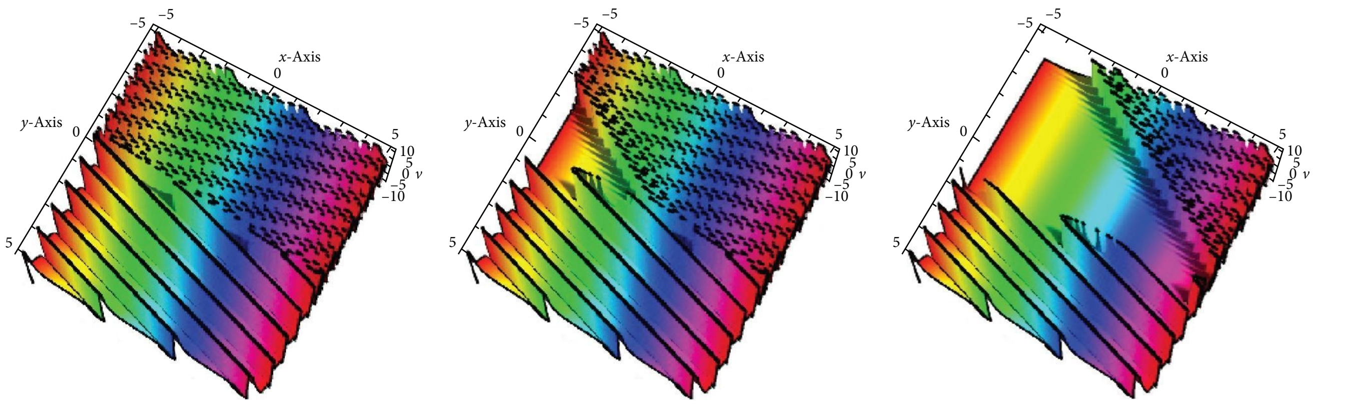


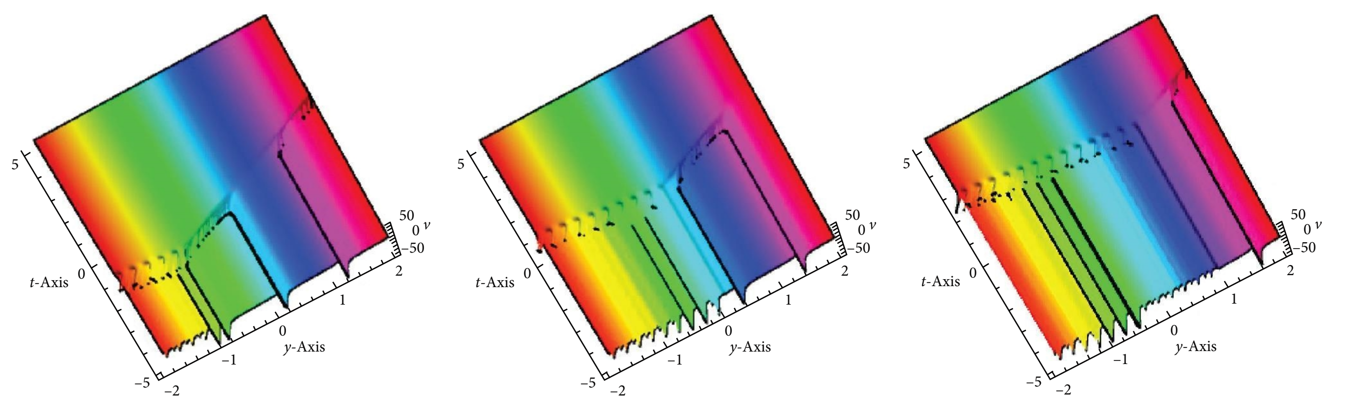
FIGURE 3: Corresponding contour plots (a–c) of Figure 2.

The dynamic properties to Equation (20) are described in Figures 1–3. Figure 1 shows the interaction between two periodic soliton solutions for different values of t. Figure 2 shows the interaction between two periodic soliton solutions for different values of x. Figure 3 is the corresponding contour plots of Figure 2. The dynamic properties to Equation (21) are shown in Figures 4–6. Other solutions can be discussed in the same way but we will not repeat them here.



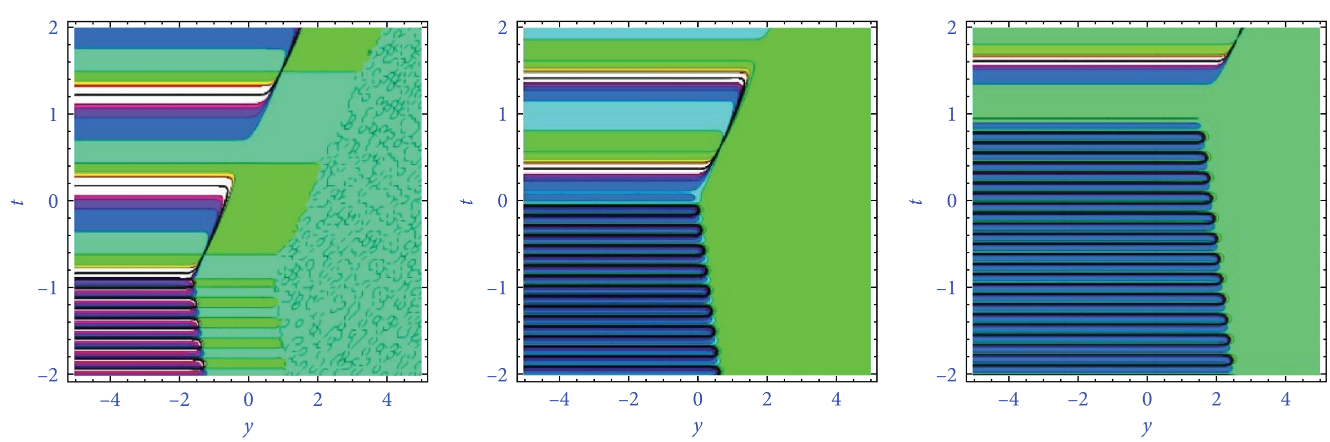
(a) (b) (c)

FIGURE 4: Solution (21) when (a) $t = -1$, (b) $t = 0$, and (c) $t = 1$.



(a) (b) (c)

FIGURE 5: Solution (21) when (a) $x = -10$, (b) $x = 0$, and (c) $x = 10$.



(a) (b) (c)

FIGURE 6: Corresponding contour plots (a–c) of Figure 5.

4. Conclusion

Periodic soliton solution is one of the most important soliton solutions. It is usually obtained by using the three-wave method. Many scholars at home and abroad have conducted relevant research and obtained many good conclusions. With the development of computer [24–50], the three-wave method has been modified continuously to obtain more and different types of periodic soliton solutions of NLPDES. In this paper, a modified three-wave method is used to obtain 14 different two-periodic soliton solutions to the Ito equation, none of which have been seen in other literature. The modified three-wave method contains more arbitrary parameters than the traditional three-wave method, which can obtain more forms of accurate solutions and include more different physical structures. We analyze and demonstrate these results using some 3D and contour graphs. Although, the expression of this method is very complicated, it is direct and effective. In particular, with the help of symbolic computing software, we can apply this method to many other NLPDES. In the future, we will combine neural network algorithms to consider more solutions to the $(2+1)$-dimensional Ito equation. Best of all, the calculation code for the paper is quite complex. If readers need the calculation source program for the paper, they can contact the corresponding author of the paper via email, and we will provide it for free.

Data Availability

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Ethical Approval

The authors state that this research complies with the ethical standards. This research does not involve either human participants or animals.

Consent

All authors have given consent to participate in the study.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this article.

Authors' Contributions

All authors contributed to writing-original draft, methodology, software, formal analysis, and funding acquisition. All authors read and approved the final version of manuscript for publication.

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