Research Article

Flow Dynamics of Eyring–Powell Nanofluid on Porous Stretching Cylinder under Magnetic Field and Viscous Dissipation Effects

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The current paper scrutinized the flow dynamics of Eyring–Powell nanofluid on porous stretching cylinder under the effects of magnetic field and viscous dissipation by employing Cattaneo–Christov theory. In order to study impacts of thermophoretic force and Brownian motion, the two-phase (Buongiorno) model is considered. As a consequence, very nonlinear PDEs that govern flow problem were formulated, transformed into ODEs via relevant similarity variables, as well as tackled by utilizing R-K-45 integration scheme along with the shooting technique in the MATLAB R2018a software. Consequently, the numerical simulations reveal that Eyring–Powell fluid, curvature, velocity ratio parameters have the propensity to raise nanofluid velocity. Nanofluid temperature shows an increasing pattern with magnetic, curvature, dissipative heating, and thermophoresis parameters. Besides, Prandtl number, Eyring–Powell fluid, velocity ratio, thermal relaxation time, and porous parameters indicate the declining impact against the nanofluid temperature. Hence, the porous medium reasonably and successfully managed nanofluid temperature as well as the overall thermal system in terms of system cooling. The concentration profile gets fall down with escalating values of Schmidt number, magnetic, curvature, dissipative heating, thermophoresis, Brownian motion, and solutal relaxation time parameters. Moreover, coefficient of the skin friction gets rise for larger values of Eyring–Powell fluid, magnetic and curvature parameters however porous medium and velocity ratio parameters reveal the opposite trends on it. The magnetic, curvature, Eyring–Powell fluid, velocity ratio, and dissipative heating parameters indicate increasing impacts on both Nusselt \( Nu \) and Sherwood \( Sh \) numbers even though both \( Nu \) and \( Sh \) get cut down with the porous medium parameter. Moreover, an excellent and sound agreement was attained up on comparing coefficients of the skin friction for the current result against that of previously published literatures under some limiting cases.

1. Introduction

The great scientific breakthrough in fluid mechanics have made non-Newtonian fluids to be very important fluids as a result of their immense applications in biomedical, chemical engineering, and manufacturing industries including food processing, power engineering, petroleum production, paper manufacturing, glass sheet blowing, polymer solutions, and biological gels, etc. [1, 2]. From our real-life encounters, materials like blood, starch suspension, pharmaceuticals, toothpastes, shampoos, paints, cosmetics, butter, honey, etc. are excellent examples of non-Newtonian fluids. Such types of fluids exhibit a property of the shear thinning/thickening behaviors and frequently show the yielding stresses by which the shearing stresses are nonlinearly proportional to the deformation rates of strain resulting in so much complicated and complex mathematical analysis [3]. In this perspective, the well-known Navier–Stoke’s equations failed to express adequately the prominent characteristics of such fluids. Nowadays, hence, various flow models for the non-Newtonian fluids namely Carreau, Williamson, Maxwell, Micropolar, Casson, Jeffery, Eyring–Powell, etc. have been formulated. Among these fluids, Eyring–Powell [4] fluid model being formulated in 1994 through Eyring and Powell has obtained astonishing considerations because of the facts indicated as follows: first, the model
was obtained through liquids kinetic theories rather than the experiential study; second, in the case of higher and lower rates of shearing the model performs as that of the Newtonian fluids but as that of the non-Newtonian fluids in the case of modest rates of shearing.

Now a day, a number of research reports on Eyring–Powell fluid have been communicated. For instance, Ibrahim and Hindebu [5] analyzed MHD boundary layer flow of Eyring–Powell nanofluids using the Cattaneo–Christov heat-mass fluxes theories. The stretching cylinder prompted flow model equations were solved numerically via the Keller-Box technique and they announced that the Nusselt number was augmented with the Prandtl number, curvature parameter, thermal relaxation time, and Eyring–Powell fluid parameter. Meanwhile, Layek et al. [6] investigated the combined transport of heat and mass for unsteady, incompressible, viscous Eyring–Powell fluid along expanding/shrinking sheet with suction/injection, Dufour and Soret effects. According to their results, the fluid velocity is high for the Eyring–Powell fluid but Prandtl number and thermal radiation lessen the fluid temperature. Moreover, analysis of nonlinear stratified convection of Eyring–Powell fluid past a sheet, which is inclined and stretching with Cattaneo–Christov heat-mass flux model is presented by Jabeen et al. [7]. Their analysis revealed that the thermal stratification parameter and Cattaneo–Christov time relaxation dampen the distribution of fluid temperature. Salah [8] examined the flow and heat transfer of dissipative and chemically reacting MHD Eyring–Powell fluid past a sheet that stretches in an exponential manner under non-Fourier’s model. This examination revealed that the thermal relaxation time and the Eyring–Powell fluid parameter are inversely related to the temperature profile while the Eckert number indicates an increasing effect on the temperature profile. Later on, Akram et al. [9] presented the investigation of double diffusion effect on the flow of MHD Eyring–Powell nanofluid over the channel with not uniform property. Very recently, the analysis of MHD Eyring–Powell fluid convection through a sheet, which stretches in an exponential way was given by Naseem et al. [10]. Their analysis considered the Cattaneo–Christov model and hence, they concluded that both temperature field and thermal boundary layer thickness decline with the time relaxation parameter but both escalate with the Eckert number. It is also noticed that, fluid velocity for the Eyring–Powell is larger than that of the viscous fluid but this is the contrary scenario in the case of the fluid temperature, moreover, the magnetic field revealed the retarding effect on the velocity field but the enhancing effect on the temperature distribution.

Nanofluids are new class of liquids that are engineered via homogeneous mixing of nanometer size (1 nm – 100 nm) solid particles and conventional base liquids namely oils, water, and ethyleneglycol. As far as the solid nanoparticles are concerned, metals, metallic-oxides, and carbides are often used to prepare nanofluids since they possess thermal conductivity larger than that of the common base fluids. Therefore, nanofluids own improved physico-thermal characteristics namely dynamic viscosity, transfer rate of heat, thermal conductivity, and hence they have huge applications in biomedicine like cancer therapy and nanodrug delivery as well as cooling process in the industries namely microelectronics cooling, cooling of microchips in computers, air-craft, vehicle cooling, nuclear reactors cooling, chillers, refrigerators, food processing, paper production, and many others [11]. Nanofluids were introduced and intensively studied by Choi [12] for the first time in 1995. After the pioneering work of Choi, a great deal of research has been explored for the convection of nanofluids over diverse geometries and various aspects in [13–21].

The porous media amalgamation with the nanofluids can further escalate the rates of heat energy transfers for many thermal system managements. Indeed, solid matrixes possessing the interconnected and networked pores/voided spaces through which fluids can flow are referred to as porous media [22]. Peculiar examples of natural porous media include rocks, sands, soils, biological tissues like bones, lungs, and kidneys whereas materials such as bread, sponges, cements, rubber, foams, and ceramics can be considered as man-made porous media. The enhanced thermo-physical properties of nanofluids may further improve because porous medium increases areas of the contacting surfaces among solids and fluid particles so that flow interruptions are increasing. Porous medium flow has many areas of applications including petroleum product filtration, underground water movement, geothermal extraction, crude oil extraction, storage of radioactive nuclear waste, heating and cooling in buildings, solar power collectors, biomedical sciences, and so on [23, 24]. The first description of transport phenomena via porous media was proposed in 1956 through Darcy Henry [25]. At the present time, following Darcy’s effortless work, fluid flows over a porous medium are attracting the attention of a prodigious scientific community including [26–32].

Magneto-hydro-dynamics (MHD) is the subject that focuses on the analysis of communal interactions among the imposed magnetic field and the fluid particles that also conduct electrically while in the state of motion such as electrolytes, metal fluids, plasma, and salt water. The phenomenon of MHD plays an outstanding role in medicine, technology, and engineering fields, like in metallurgy, treating of cancer tumors, reactors cooling, welding of plasmas, magnetic cells separation, solidification of magmas, gratings of optical, magnetic drug targeting, devices of astrophysics, measurements of blood flow, and so on [33, 34]. Based on these influencing attributes, currently a great deal of authors from the research and academic world is publishing articles consisting MHD flows under the different scenarios. To mention some, Maxwell nanofluid flow over the vertical sheet that stretches exponentially under second order slip effect, viscous dissipation, radiative heat flux as well as applied Lorentz force was analyzed by Abbas et al. [35] applying Buongiorno’s model. The mathematical analysis conducted for Eyring–Powell electrically conducting nanofluid motion created because of wedge surface elongating under the influences of radiative heat, generation of heat, and convectively imposed conditions on the boundaries was done through Raju et al. [36]. Moreover, Arif et al. [37] carried out a numerical study on the investigations of both mass and heat transfers regarding the motion of
Maxwell nanofluid through a heated sheet that also stretches under the impact of applied magnetic field. Similar MHD nanofluids flows are given in the relevant and up-to-date literature [38–46] and the references therein.

Furthermore, the dynamics of mutual influences of Viscous–Joule dissipations have many applications in the thermal transport process of nanofluids over the surfaces with stretching behaviors. The physical interpretation for the viscous-dissipation is that, it is just the rate of energy conversion per unit mass from its kinetic form to that of the thermal one. Concerning this issue, Narender et al. [47] performed a numerical study on heat and mass transport for chemically reacting MHD nanofluid with the influences of viscous-dissipation and radiative heat flux on a stretching sheet. As they declared, both the temperature profile and surface heat transfer rate enhance for flows with higher viscous dissipation. Later on, Abbas and Megahed [48] explored thermal radiation and viscous dissipation effects on the steady flow of Eyring–Powell fluid over a stratified a stretching sheet embedded in a porous medium. The numerical solution was obtained via the Chebyshev spectral method and they pointed out that the temperature profile escalates with the viscous dissipation and porous parameter.

On the other hand, Joule heating (or Ohmic heating) refers to a process of creating excessive heat in nanofluids because of the applied magnetic field. Actually, Joule heating comprises important attributes in the food industry, bulbs, electrolysis, electric fuzes and heaters, conduction of oven, flashlights, etc. In line with this, Olkha and Dadheech [49] scrutinized the impact of Joule heating on the unsteady MHD slip flow of the Eyring–Powell fluid as well as the motile microorganisms across an inclined permeable stretching sheet embedded in porous channel with thermal radiation and thermal sink. Their results revealed that the magnetic and porosity parameters give rise in the thermal heat transfer rate. In addition, the flow and heat transfer characteristics examination of MHD nanofluid using Ag as solid nanoparticles suspended in water with combined dissipation of Viscous–Joule past a cylinder that stretches under the slip conditions at the boundary as well as injection/suction was given by Mishra and Kumar [50]. The numerical solutions were given via R-K-45 along the method of shooting. Thus, the Nusselt number falls down when the magnitudes of viscous dissipation and Joule heating parameters increase. Babu et al. [51] demonstrated the analysis of thermally radiating 2D MHD Eyring–Powell fluid flow over the surface that stretches under a joint impact of Viscous–Joule dissipations. They noticed that enhancing the viscous dissipation as well as the Joule heating parameters enhances the nanofluid temperature distribution. Indeed, a list of up-to-date and relevant references from the literature with regard to the joint influences of Viscous–Joule dissipations is reported by Ramesh et al. [52], Sadighi et al. [53], and Jayanthi and Niranjan [54].

Related literatures abovementioned motivated the current study and therefore, the current study investigates flow as well as mass and heat transport for Eyring–Powell nanofluid on the cylinder that stretches under the applied magnetic field. The survey of related literature established that no such problem has been studied regarding the jointed influences of Viscous–Joule heating caused by dynamic viscosity, applied magnetic field, and porous medium by employing the non-Fourier’s heat conduction model which is also known as Cattaneo–Christov heat and mass flux model. Moreover, same boundary conditions were not considered before. The solutions of the current study are given via R-K-45 integration scheme coupled by the method of shooting. As a consequence, parameter-dependent solutions for the velocity profile, temperature profile, concentration profile, wall shear stress, wall heat, and mass transfer rates were investigated as well as displayed through graphs. Moreover, an excellent and sound agreement was attained up on comparing results of the current numerical method coefficient of the skin frictions with the numerical solutions reported by formerly available literatures on behalf of various controlling cases. Therefore, it is a worthwhile attempt and the author believes that the current results are authentic and novel.

2. Problem Analysis and Mathematical Modeling

Consider a steady, laminar, and viscous two-dimensional MHD Eyring–Powell fluid past a porous cylinder with radius $R$ that stretches in the horizontal direction. Also, consider a polar cylindrical coordinate system $(x, r)$ as exhibited in Figure 1. The flow direction for the Eyring–Powell nanofluid is along $x$-axis as porous cylinder stretches linearly with the uniform velocity $u_w$.

Following Powell and Eyring [4], the shear stress for Eyring–Powell model can be written as follows:

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1}\left(\frac{1}{\beta} \frac{\partial u_i}{\partial r}\right) + \frac{1}{\rho} \sinh^{-1}\left(\frac{1}{\rho} \frac{\partial u_i}{\partial r}\right),$$

(1)

where $\tau_{ij}$ is the Cauchy stress tensor for Eyring–Powell fluid, $u$ is the axial fluid velocity, and $\mu$ is the dynamic viscosity whereas $\beta$ and $\rho$ are the rheological Eyring–Powell fluid model parameters. The Taylor’s series expansion of $\sinh^{-1}$ approximating to the second-order gives:
\[ \sinh^{-1} \left( \frac{1}{c} \frac{\partial u}{\partial r} \right) \approx \frac{1}{c} \frac{\partial u}{\partial r} + \frac{1}{3!} \left( \frac{1}{c} \frac{\partial u}{\partial r} \right)^3, \quad \frac{1}{c} \frac{\partial u}{\partial r} \ll 1. \] (2)

Thus, Equation (1) takes the form as follows:

\[ \tau_{ij} = \mu \frac{\partial u}{\partial r} + \frac{1}{\beta \varepsilon} \frac{1}{6\beta c^2} \left( \frac{\partial u}{\partial r} \right)^3. \] (3)

Moreover, it is assumed that a uniform magnetic field \( B_0 \) is imposed opposite to the flow direction. The uniform surface temperature \( T_w \) and surface concentration \( C_w \) are considered while \( T_\infty \) and \( C_\infty \) are temperature and concentration in the free stream, respectively. The induced magnetic field is considered as negligible when compared to the applied magnetic flux because of extremely small magnetic Reynolds number. Besides, consider a homogeneous and isotropic porous and two-phase Buongiorno model for nanofluids. Additionally, consider a homogeneous and isotropic porous and two-phase Buongiorno model for nanofluids in the energy and concentration equations. The zero net surface mass flux of the nanoparticles is considered. More importantly, instead of classical Fourier’s and Fick’s diffusion models, the frame indifferent Cattaneo–Christov diffusion model is considered.

Following the study by Christov [55], the Cattaneo–Christov heat-mass flux model instead of the Fourier’s and Fick’s diffusion models can be written as follows:

\[ \lambda_E \left( \frac{\partial q}{\partial t} + U \cdot \nabla q - q \nabla U + (\nabla \cdot U)q \right) + q = -k \nabla T, \] (4)

\[ \lambda_C \left( \frac{\partial f}{\partial t} + U \cdot \nabla f - f \nabla U + (\nabla \cdot U)f \right) + f = -D_b \nabla C, \] (5)

\[ U = (u, v) \] represent the fluid velocity vector, \( q \) is heat flux, \( J \) is mass flux, \( \lambda_E \) denotes the thermal relaxation time, \( \lambda_C \) denotes the solutal relaxation time, \( k \) is fluid thermal conductivity, and \( D_b \) stands for molecular mass diffusivity of the species. It is noteworthy that for \( \lambda_E = \lambda_C = 0 \), Equations (4) and (5) are simplified to the classical Fourier’s and Fick’s laws, respectively.

For steady incompressible fluid flow, Equations (4) and (5) reduce to:

\[ \lambda_E[U \cdot \nabla q - q \nabla U] + q = -k \nabla T, \] (6)

\[ \lambda_C[U \cdot \nabla J - J \nabla U] + J = -D_b \nabla C. \] (7)

Under the aforementioned considerations, the boundary-layer PDEs are formulated and hence written according to the following equations:

\[ \frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0, \] (8)

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \mu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + \frac{1}{\rho \beta c^2} \left( \frac{\partial u}{\partial r} \right)^3 + \frac{1}{r} \left( \frac{\partial u}{\partial r} \right)^2 - \frac{\sigma \rho \beta_b u}{\rho \beta_c} u^2, \] (9)

\[ \frac{\partial T}{\partial x} + \frac{\partial T}{\partial r} = \alpha_f \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{1}{\rho \beta c} \left( \mu + \frac{1}{\beta c} \right) \left( \frac{\partial u}{\partial r} \right)^2 - \frac{1}{6\beta c^2} \left( \frac{\partial u}{\partial r} \right)^4 + \frac{\sigma B_0^2}{\rho \beta_c} u^2, \] (10)

\[ \frac{\partial C}{\partial x} + \frac{\partial C}{\partial r} = D_b \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) + \frac{1}{\rho \beta c} \left( \frac{\partial u}{\partial r} \right)^2 \] (11)

where \( \rho_c \) designates specific heat capacity, \( D_T \) thermophoretic diffusion coefficient, \( D_f = \frac{(\rho_c p_f)}{\rho p_f} \) is heat capacity ratio, \( (\rho_c p)_f \) is nanoparticles heat capacity, and \( (\rho_c p)_f \) represents base fluid heat capacity.
Imposed boundary conditions are:

\[ u = u_w(x) = u_0 \left( \frac{x}{l} \right), \quad \nu = 0, \quad T = T_w(x) = T_\infty + T_0 \left( \frac{x}{l} \right), \quad D_B \frac{\partial C}{\partial r} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial r} = 0, \quad \text{for } r = R \text{ and } u = u_\infty, \nu = 0, \quad C = C_\infty, \]
\[ T = T_\infty \text{ as } r \to \infty. \]

(12)

To transform Equations (8)–(12) into ODEs the following dimensionless variables are introduced.

\[ \eta = \frac{r^2 - R^2}{2R} \sqrt{\frac{u_0}{\nu}}, \quad \psi = \sqrt{u_w \nu} \times \text{Re} (\eta), \quad u = u_w f'(\eta), \quad \nu = -\frac{R}{r} \sqrt{\frac{\nu u_0}{1}} f''(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\infty - C_\infty}. \]

(13)

Using Equation (13), Equation (8) is automatically satisfied while Equations (9)–(11) are transformed into the following ODEs:

\[ (1 + \delta)(1 + 2\gamma \eta) f'''' + f'''' - f'' + 2\gamma(1 + \delta) f'' - \frac{4}{3} \delta \lambda (1 + 2\eta) f''' - \delta \lambda(1 + 2\gamma \eta) f'' - (M + K) f' = 0, \]

(14)

\[ \frac{1 + 2\gamma \eta}{Pr} \theta'' + \frac{2\gamma}{Pr} \theta' + f \theta' - \alpha(t^2 \theta'' + ff' \theta') + Nb(1 + 2\gamma \eta) \theta' \phi' + Nt(1 + 2\gamma \eta) \theta' + Ec \frac{\phi''}{(1 + 2\gamma \eta)} \left[ (1 + \delta)(1 + 2\eta) - \frac{1}{3} \delta \lambda f'' ight] \]

\[ + Ec(M + K) f'' = 0, \]

(15)

\[ (1 + 2\gamma \eta) \phi'' + 2\gamma \phi' + Sc f \phi' - Sc \alpha_c (f^2 \phi'' + ff' \phi') + \frac{Nt}{Nb} \left[ (1 + 2\gamma \eta) \theta'' + 2\gamma \theta' \right] = 0, \]

(16)

where \( \gamma = 1/R \sqrt{\nu / u_w} \) is the curvature parameter, \( \delta = 1/\mu \beta c \) is the Eyring–Powell fluid parameter, \( \lambda = u_0 \nu x^2 / 2c^2 l \nu \) is the fluid parameter, \( Pr = \nu / \alpha f \) is the Prandtl number, \( at = \lambda \nu / \rho \) is the magnetic parameter, \( Pr = \nu / \alpha_f \) is the Magnetic number, \( at = \lambda \nu / \rho \) is the magnetic parameter, \( Nb = \beta c D_B (C_w - C_\infty) / \nu \) is the Brownian motion parameter, \( Nt = \beta c D_T (T_w - T_\infty) / \nu T_\infty \) is the thermophoresis parameter, \( Sc = \nu / D_B \) is the Schmidt number, \( ac = \lambda c u_0 / l \) is the solutal relaxation time parameter, \( K = \mu / u_0 \rho l \) is porous parameter, and \( Ec = u_w / c_p (T_w - T_\infty) \) is the Eckert number (or viscous dissipation parameter).

The imposed boundary conditions in Equation (12) become:

\[ f'(0) = 1, f(0) = 0, \theta(0) = 1, \quad Nt \theta'(0) + Nb \phi'(0) = 0, \quad f''(\infty) = A, \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \]

(17)

where \( A = u_\infty / u_w \) is the velocity ratio.

Engineering and industrial interest quantities are the skin friction coefficient, the Nusselt number and the Sherwood number. The skin friction coefficient \( C_f \) can be written as follows:

\[ C_f = \frac{2 \tau_w}{u_w^2}, \]

(18)

However, \( \tau_w \) is the wall shear stress and it can be obtained from the Cauchy stress tensor for the Eyring–Powell fluid as follows:
\( \tau_w = [\tau_{ij}]_{r=R} = \left[ \mu \frac{\partial u}{\partial r} + \frac{1}{\beta c} \frac{\partial u}{\partial r} - \frac{1}{\beta \theta c^3} \left( \frac{\partial u}{\partial r} \right)^3 \right]_{r=R}. \) (19)

The nondimensional coefficient of skin friction \( C_f \) in Equation (18) takes the form:

\[ \frac{1}{2} C_f \sqrt{Re_x} = (1 + \delta) f''(0) - \frac{1}{3} \delta f'''(0). \] (20)

The wall heat transfer rate or local Nusselt number \( Nu \) is defined as follows:

\[ Nu = \frac{xq_w}{k(T_w - T_\infty)}, \] (21)

where \( q_w = -\frac{k(\partial T)}{\partial r}_{r=R} \) represents wall heat-flux. Thus, the nondimensional \( Nu \) takes the following form:

\[ \frac{Nu}{\sqrt{Re_x}} = -\theta'(0). \] (22)

Similarly, the wall mass transfer rate or local Sherwood number \( Sh \) is defined as follows:

\[ Sh = \frac{xJ_w}{D_h(C_w - C_\infty)}, \] (23)

where \( J_w = -\frac{D_h(C_w - C_\infty)}{r=R} \) designates wall mass-flux and thus, the nondimensional form of \( Sh \) takes the following form:

\[ \frac{Sh}{\sqrt{Re_x}} = \theta - \phi'(0). \] (24)

3. Method of Numerical Solutions

The closed form solutions for the highly nonlinear governing Equations (14)–(16) with boundary conditions Equation (17) are not possible. Hence, this highly nonlinear governing Equations (14)–(17) are numerically solved by using the fourth–fifth order Runge–Kutta–Fehlberg integration scheme in the MATLAB R2018a software. The initial conditions were guessed by utilizing the shooting technique until the boundary conditions were satisfied. Also, in the numerical calculations the maximum step size \( \Delta \eta = 0.01 \). The criteria used for convergence is the variation in the dimensionless velocity, temperature, and concentration should be less than \( 10^{-7} \) between any two consecutive iterations. The asymptotic boundary conditions Equation (17) were approximated by using \( \eta_{max} = 12, \theta(12) = 0 = \phi(12) \). Therefore, to transform the governing BVPs Equations (14)–(17) into the first order IVP, the new variables are defined as follows:

\[
\begin{align*}
\dot{f} &= y_1, \\
\dot{f}' &= y_2, \\
\dot{f}'' &= y_3, \\
\dot{f}''' &= y_4, \\
\dot{\theta} &= y_5, \\
\dot{\theta}' &= y_6, \\
\dot{\theta}'' &= y_7, \\
\dot{\phi} &= y_8, \\
\dot{\phi}' &= y_9, \\
\dot{\phi}'' &= y_{10}.
\end{align*}
\] (25)

Using Equation (18) into Equations (14)–(16) yields the following system of first order IVPs.

In this case, the prime represents the derivative with respect to \( \eta \). The transformed boundary conditions in Equation (17) are written as follows:
where $u_1, u_2$, and $u_3$ are the suitable values of the initial guesses for $f'(0), \theta'(0)$ and $\phi(0)$, respectively. Besides, $y_1(\infty) \to A, y_2(\infty) \to 0, y_3(\infty) \to 0, y_4(\infty) \to 0$. The suitable values of the unknown initial conditions $u_1, u_2$, and $u_3$ are iteratively estimated until the solutions satisfy the boundary conditions at $\eta = \infty$ whereas R-K-45 is used to solve later on at $\eta = 0$.

4. Results and Discussions

4.1. Flow Field: Velocity Profile. Figure 2(a) is a double graph displaying the velocity profile against the magnetic parameter $M$ and Eyring–Powell fluid parameter $\delta$. It is noticed that the momentum boundary layer thickness and the velocity profile considerably increase as the size of $\delta$ increases because large value of $\delta$ indicates lower fluid viscosity so that it flows easily. Thus, as $\delta$ increases, the velocity profile also increases considerably. Therefore, velocity for the Eyring–Powell fluid exceeds that of viscous Newtonian fluid. Reverse to this, the momentum boundary layer thickness and the velocity profile decrease with increasing values of $M$. Physically, large value of magnetic parameter $M$ corresponds to large resisting force that is called Lorentz resistance force, which dampens the fluid flow. Thus, as $M$ increases, both the fluid velocity and the momentum boundary layer thickness decrease. Impact of the curvature parameter $\gamma$ on fluid velocity is portrayed in Figure 2(b). Accordingly, both momentum boundary layer thickness and fluid velocity escalate when magnitude of $\gamma$ rises. This can be justifiable because as $\gamma$ increases the radius of the cylinder decreases which results in slender cylinder so that the contact surface area of the cylinder with the fluid decreases.

Thus, the surface of the cylinder accounts lower friction force on the movement of the fluid, which leads to an increase of fluid velocity.

Figures 3(a) and 3(b) illustrate the effects of the porous parameter $K$ and the velocity ratio parameter $A$, respectively, on the velocity profile. Fluid velocity and thickness of the corresponding momentum boundary layer decrease as the value of the porous parameter $K$ increases (Figure 3(a)). The argument behind this result is that bigger values of $K$ physically indicates stronger resisting forces on the motion of the fluid that leads to the reduction of the fluid velocity. Hence, the inclusion of porous medium slows down the flow and a deceleration in the fluid velocity is observed. Figure 3(b) also illustrates that both fluid velocity and thickness of the corresponding momentum boundary layer rise...
with increasing size of the velocity ratio parameter $A$ because physically, velocity ratio speeds up fluid motion that in turn boosts the velocity profile.

4.2. The Temperature and Concentration Profiles. Both the thickness of thermal boundary layer and fluid temperature enhance for larger values of the magnetic parameter $M$ as depicted in Figure 4(a). Physically, when $M$ increases the Lorentzian resistant force increases and the friction between fluids layers which produces more heat leading to the rise of temperature profile. However, Figure 4(b) reveals the reverse situation for the nanoparticles concentration because of the
effect of cross diffusion. That is, small increase in temperature may cause small decrease in the nanoparticles concentration. Besides, as indicated in Figure 4(a), the thickness of thermal boundary layer and fluid temperature decline for bigger values of Eyring–Powell fluid parameter $\delta$ because bigger values in $\delta$ implies less viscous fluid which results in the lessening of friction among fluid particles and hence temperature distribution in the fluid falls down. Therefore, temperature of viscous Newtonian fluid is bigger as compared to temperature of non-Newtonian fluid past the porous cylinder that stretches. Although, Figure 4(b) demonstrates the reverse result for the case of the nanoparticles concentration.
Temperature profile is increasing significantly with the curvature parameter $\gamma$ as exhibited in Figure 5(a). The physical interpretation can be, larger values of $\gamma$ results in the decrement of radius of the cylinder resulting in the reduction of the friction force so that the velocity of the nanofluid rises as elaborated in Figure 2(b). Thus, the enhanced fluid velocity results in the escalation of the nanofluid kinetic energy, which in turn augments the heat energy of the nanofluid. Consequently, the nanofluid temperature increases as the curvature parameter $\gamma$ increases. Additional insight into the effect of the same parameter on the concentration profile can be noticed from Figure 5(b). This figure reveals that the concentration profile significantly decreases with the increasing values of $\gamma$. 

\[ \delta = 0.2, M = 0.2, \gamma = 0.6, \lambda = 0.1, K = 0.5, Pr = 1.2, Ec = 1, Nt = 0.1, Nb = 0.1, Sc = 1.3, at = 0.1, ac = 0.1 \]
As revealed in Figure 6(a), both the thickness of thermal boundary layer and fluid temperature escalate remarkably as the viscous dissipation factor (or Eckert number $Ec$) rises. The physical interpretation for this result is that the increment of $Ec$ accumulates heat energy in the nanofluid because of heating caused by friction. Therefore, both temperature profile as well as the thickness of thermal boundary layer escalate as magnitude of $Ec$ rises. The influence of the viscous dissipation parameter $Ec$ on the concentration profile is reversed because of the cross-diffusion effect (Figure 6(b)). Figure 6(a) also displays the impact of the porous parameter $K$ on the nanofluid temperature. Rising the values of $K$ result in the lowering of the nanofluid temperature because the nanofluid velocity declines for larger values of $K$ which in turn lowers the kinetic energy and thus, less heat energy is generated in the flow regime that leads to a low-nanofluid temperature.
Therefore, the porous medium reasonably and successfully managed nano fluid temperature as well as the overall thermal system in terms of system cooling, however, Figure 6(b) presented an opposite effect for the nanoparticles concentration.

Figure 7(a) illustrates that the temperature profile has shown a retarding pattern as the value of velocity ratio parameter $A$ enhances whereas the concentration profile has indicated an opposite scenario as demonstrated in Figure 7(b).

The influences of thermophoresis $Nt$ as well as Prandtl number $Pr$ against the fluid temperature are demonstrated in Figure 8(a). It can be witnessed from the figure that the nano fluid temperature falls down strongly with rising $Pr$. Physically, large value of $Pr$ represents a smaller thermal conductivity and hence there is a weak heat energy diffusion which results in a significant fall down of temperature profile. Besides, from the same figure, one can notice that
nano fluid temperature enhances for increasing magnitudes of thermophoresis parameter \( Nt \). In fact, thermophoresis is a process by which nanoparticles are moving out toward the cold region from the hot one and hence greater value of \( Nt \) corresponds to the tougher thermophoresis forces that favor the hot nanoparticles movement toward the cold fluid resulting in higher temperature distribution throughout the boundary layer flow. The influences of \( Pr \) and \( Nt \) on the concentration profile are also portrayed in Figure 8(b) where the scenarios are reversed because of the cross-diffusion effect.

Figure 9(a) portrayed the thickness of thermal boundary layer and temperature field against the Cattaneo–Christov time relaxation \( \alpha t \). Consequently, an increment of \( \alpha t \)
indicates a decreasing pattern for the temperature profile. In physical sense, as $\alpha t$ increases, the nanofluid acquires additional more periods for transferring heat energy into the surrounding and thus, nanofluid temperature falls down. Therefore, a falling down of nanofluid temperature is observed when a magnitude of $\alpha t$ rises. Indeed, for the mixed convection of nanofluid over a porous cylinder the Cattaneo–Christov model produces more heat when compared to the heat conduction laws of Fourier ($\alpha t = 0$). Therefore, the Cattaneo–Christov heat flux model is favorable in regulating large heat flux situations when compared to the heat conduction laws of Fourier. Similarly, Figure 9(b) reveals variation of the

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**Figure 15:** (a) Nusselt number with varying $Nt$ and (b) Sherwood number with varying $Nt$.

**Figure 16:** Nusselt number with varying (a) $Pr$ and (b) $\alpha t$. 
concentration profile with assorted values of solutal relaxation time parameter $\alpha t$. The nanoparticle concentration shows a decreasing behavior with the escalating values of $\alpha t$.

Figure 10(a) illustrates that the concentration profile drops in response to a rise in the Schmidt number $Sc$. Physically, big value of $Sc$ indicates a small mass diffusion in the flow regime and thus nanoparticles concentration drops when a size of $Sc$ increases. Moreover, Figure 10(b) displayed impacts of Brownian motion factor $Nb$ against nanoparticles concentration. Both the thickness of concentration boundary layer and nanoparticles concentration decline with the escalating sizes of $Nb$. This is the case since large value of $Nb$ shows an enhanced nanoparticles collision as well as random movements that in turn declines the nanoparticles concentration throughout flow regime.

4.3. Skin Friction Coefficient, Nusselt Number, and Sherwood Number. Figures 11(a) and 11(b) are presented to visualize the effects of velocity ratio parameter $A$, porous parameter $K$, curvature parameter, and Eyring–Powell fluid parameter $\delta$ on coefficient of skin friction $C_f$ where $M$ (magnetic parameter) is used for scaling the horizontal axis. From Figure 11(a) it is clearly examined that $C_f$ gets cut down with increasing values of $K$ and $A$. However, $C_f$ gets rise up for increasing values of $\gamma$ and $\delta$ as elucidated in Figure 11(b).

Escalating the magnitudes of $\gamma$ (curvature parameter) as well as $\delta$ (Eyring–Powell fluid parameter) results in the enhancement of both $Nu$ (Nusselt number) and $Sh$ (Sherwood number) as illustrated in Figures 12(a) and 12(b), respectively, with scaled values of the magnetic parameter $M$. Furthermore, $Nu$ and $Sh$ are rising with the velocity ratio parameter $A$ but both are falling with the porous parameter $K$ as displayed in Figures 13(a) and 13(b), respectively. The Eckert number $Ec$ is directly related to both $Nu$ and $Sh$ (Figures 14(a) and 14(b)). The thermophoresis parameter $Nt$ has revealed a retarding effect on $Nu$ but a rising effect on $Sh$ as demonstrated in Figures 15(a) and 15(b), respectively. In addition, Figures 16(a) and 16(b) displayed that the Nusselt number $Nu$ increases with the increasing values of Prandtl number $Pr$ and thermal relaxation time $\alpha t$. Furthermore, Schmidt number $Sc$ indicates a retarding influence on $Sh$ while solutal relaxation time $\alpha t$ shows a rising impact on $Sh$ as portrayed in Figures 17(a) and 17(b), respectively.

In order to validate the accuracy of the current numerical technique, comparison is done in the case of the skin friction coefficients for the present results with some limited conditions against that of already available literatures. Thus, an excellent as well as a sound agreement is attained up on comparing with Ibrahim and Hindebu [5] Layek et al. [6] as witnessed in Table 1.

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TABLE 1: Comparison of $-1/2C_f\sqrt{Re}$ in the case of varying $\lambda$ values for $Pr = 1.7, Sc = 1.0, Ec = 0.1, A = 0.1, K = 1, Nt = 0.1, Nb = 0.1, Sc = 1.3, at = 0.1, \alpha t = 0.1, \lambda = 0.6$.
5. Conclusions

The current study aims to scrutinize the flow dynamics of Eyring–Powell nanofluid on porous stretching cylinder under the effects of magnetic field and viscous dissipation by employing Cattaneo–Christov theory. In order to study impacts of thermophoretic force and Brownian motion, the two-phase (Buongiorno) model is considered. As a consequence, very nonlinear PDEs that govern flow problem were formulated, transformed into ODEs via relevant similarity variables, as well as tackled by utilizing R-K-45 integration scheme along with the shooting technique in the MATLAB R2018a software. Hence, the effects of pertinent embedded thermo-physical parameters on velocity, temperature, and concentration profiles as well as on the skin friction coefficient, the wall heat transfer and mass transfer rates are investigated and displayed through graphs. Based on the results and discussions made under the previous section, the major findings of the current investigation are summarized as follows:

(i) The Eyring–Powell fluid parameter $\delta$, curvature parameter $\gamma$, and velocity ratio parameter $A$ have a propensity to raise the momentum boundary layer thickness and the velocity profile.

(ii) The thickness of thermal boundary layer as well as temperature profile show rising trend as the magnitudes of $M$ (magnetic parameter), $\gamma$ (curvature parameter), viscous dissipation factor (or Eckert number $Ec$), and thermophoresis parameter $Nt$ enhance.

(iii) The Eyring–Powell fluid parameter $\delta$, Prandtl number $Pr$, porous parameter $K$, velocity ratio parameter $A$, and thermal relaxation time $at$ indicate a retarding effect on the temperature profile and thermal boundary layer thickness.

(iv) The concentration profile gets fall down with rising values of the magnetic parameter $M$, curvature parameter $\gamma$, Eckert number $Ec$ and thermophoresis parameter $Nt$, Brownian motion parameter $Nb$, Schmidt number $Sc$, and solutal relaxation time $ac$.

(v) The skin friction coefficient $C_f$ escalates for increasing values of the Eyring–Powell fluid parameter $\delta$, $M$ (magnetic parameter), and $\gamma$ (curvature parameter) while it falls down when both $K$ (porous parameter) as well as $A$ (velocity ratio parameter) rise.

(vi) The curvature parameter $\gamma$, magnetic parameter $M$, Eyring–Powell fluid parameter $\delta$, velocity ratio parameter $A$, and Eckert number $Ec$ revealed an escalating pattern against both Nusselt number $Nu$ and Sherwood number $Sh$ while both get cut down with the porous parameter $K$.

(vii) The Nusselt number $Nu$ increases with increasing values of Prandtl number $Pr$ and thermal relaxation time $at$.

(viii) The Sherwood number $Sh$ indicates a decreasing pattern with rising sizes for $Sc$ (Schmidt number) while $ac$ (solutal relaxation time) shows an increasing effects on Sherwood number $Sh$.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The author declares that there is no conflicts of interest.

References


