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*Research Article*

# Uniaxial Bond Stress-Slip Relationship of Reinforcing Bars in Concrete

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This paper documents a study carried out on the estimation of the bond stress-slip relationship for reinforced concrete members under axial tension loading. An analytical model is proposed that utilizes the conventional bond stress-slip theories as well as the characteristics of deformed bar and concrete cross-sectional area. An equation for the estimation of the bond stress is formulated as the function of nondimensional factors (e.g., bond stress, slip, etc.). The validity, accuracy, and efficiency of the proposed model are established by comparing the analytical results with the experimental data and the JSCE design codes, as well as the analytical models given by Ikki et al. and Shima. The analytical results presented in this paper indicate that the proposed model can effectively estimate the bond stress-slip relationship of reinforced concrete members under axial tension loading.

## 1. Introduction

Bond of reinforcing bars to the surrounding concrete influences the behavior of reinforced concrete structures in many ways [1]. It can be a key element for the ultimate load-carrying capacity of reinforced concrete structures since it affects the anchorage of bars and the strength of lap slices. Moreover, the deformation capacity of the members, and hence the redistribution capacity in statically indeterminate structures, is directly influenced by the bond. For these reasons, it can be said that a fundamental issue for reinforced concrete structures is the bond between the reinforcing bars and the concrete [2].

One of the major drawbacks in reinforced concrete bond research is the absence of a generalized method for determining bond strength. This leads comparisons between various researches and test results on bond difficult. Most investigators have used pull-out tests that are commonly adopted in reinforced concrete bond studies [3–11]. In most research, not a great deal of attention has been focused on the embedment length and the stress state in the concrete. The embedment lengths and the stress states have, therefore, not coincided with those in real reinforced concrete members. Because this has not been recognized by many researchers, results often show a significant difference to real behavior.

In general, bond action in reinforced concrete members is represented by the bond stress-slip relationship. Many bond stress-slip relationships have been proposed and some of these have been formulated. These relationships have been used in the finite-element method (FEM) [12–14] and cracking [15, 16] analysis. However, most of the proposed relationships were derived from pull-out tests and differ from each other. The main limitation of the pull-out test is that it does not simulate the actual conditions in a reinforced concrete flexural member. Therefore, utilizing relationships derived from the pull-out test might be considered to contain inevitable problems. Consequently, it is necessary to establish a realistic bond stress-slip relationship that takes into account the actual conditions in a reinforced concrete flexural member, which is usually long embedment length and axial tension force at the cracking section.

This article presents an analytical model to estimate the bond stress-slip behavior of reinforced concrete members based on the axial tension test. The models suggested for the estimation of the bond stress-slip relationship in the JSCE code [17] and by Ikki et al. [18, 19] and Shima [20] are

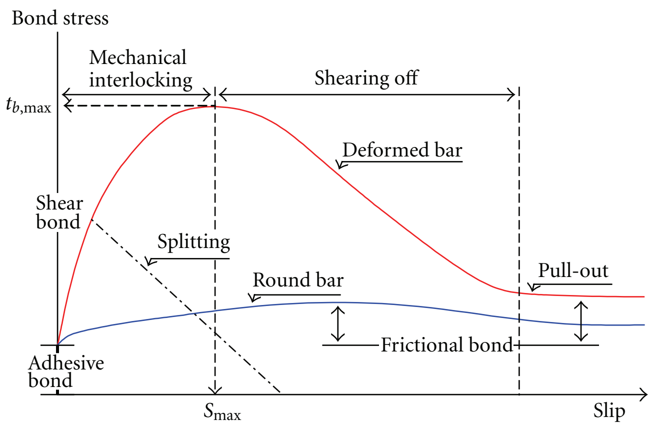


FIGURE 1: Bond stress-slip relationship.

extended in order to analyze the bond stress-slip behavior of reinforced concrete flexural members. An analytical equation is formulated as an exponential function of the relative rib area and the nondimensional slip. The validity, accuracy, and efficiency of the proposed model are established by comparing the results of the present study with the results obtained from the analytical and experimental studies. In addition, a parametric study was performed to evaluate the effects of the bond factors. The results of analysis presented in this paper indicate that the proposed model can be used effectively to estimate the bond stress-slip relationship.

## 2. Bond Mechanism

*2.1. Interaction between Reinforcing Bar and Concrete.* With the growth of bond researches, a reinforced concrete structure was known as a composite structure [21], whereby the slip occurs along the reinforcing bar under loading and the bond action is a function of the slip. To put it concretely, when the external force is progressively applied to a reinforced concrete member, interfacial stresses between the reinforcing bar and the concrete are created and the capacity of the interface to transmit stresses begins to weaken at certain load levels. These irreparable damages spread to the surrounding concrete. As a result of this process, the capacity of the interface to transmit stresses gradually deteriorates and a slip of both materials inevitably occurs.

As shown in Figure 1, the stress transfer mechanisms, which refer to the bond action, are usually expressed by the bond stress-slip relationship obtained from pull-out tests. The bond actions are comprised of an adhesive bond, a frictional bond, and a shear bond. In the case of deformed bars, the bond resistance capacity is mainly governed by the mechanical interlocking action.

*2.2. Relative Rib Area.* In fact, the bar geometry, and more specially the rib geometry, governs to a high degree the general bond behavior and determines the bond resistance [2]. In particular, as shown in Figure 2, the bar diameter ($d_s$), the rib height ($h_d$), and the rib spacing ($l_d$) are found to be the most important parameters [1]. In addition, deformed bars

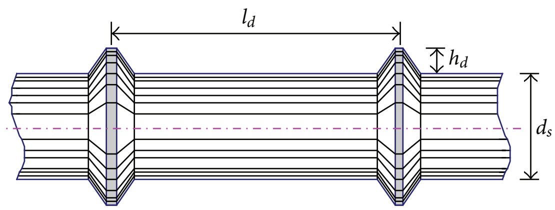


FIGURE 2: Details of deformed bar.

are currently mainly used for reinforcement in reinforced concrete structures. The best bond actions can be obtained by means of an appropriate combination of the 3 above mentioned parameters.

Rehm [22] showed that the relative rib area could be regarded as the ruling criterion for the bond action of deformed bars. The experimental evidence also proves that bond actions are relatively the same, provided that the relative rib areas are the same, and the rib face angle is greater than 30°. The generally accepted values ranging between 0.05–0.15 represent a good compromise for the relative rib area, in terms of ultimate bond strength and splitting ability, and so forth [1].

The relative rib area, the so-called bond index, is defined as

$$f_R = \frac{A_R}{\pi \times d_s \times l_d}, \tag{1}$$

where $A_R$: area of the projection of a single rib on the cross-section of a deformed bar.

Yamao et al. [23] pointed out that bond stress-slip relationships in long embedment and short embedment lengths differ significantly, as explained in Figure 3. In addition, the maximum bond stress obtained from specimens with short embedment show a steady increase that coincides with the increase of the relative rib area.

Also, when the relative rib area increases, the maximum bond stress obtained from specimens with long embedment will have smaller values than that obtained from specimens with short embedment. However, even in cases where the embedment length is sufficient, an increase is shown in

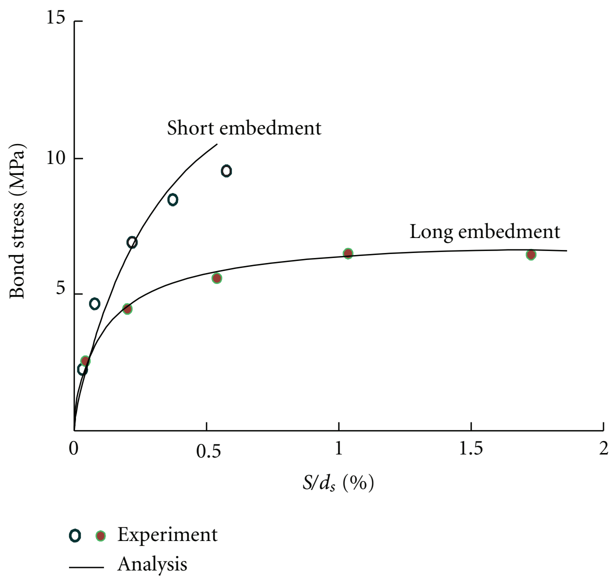


Figure 3: Bond stress-slip relationships of short embedment and long embedment.

proportion to the relative rib area, even though the maximum bond stress is smaller than that of cases where the embedment length is short. Hence, the effect of the relative rib area on the bond action in the case of long embedment length needs also be considered.

*2.3. Stress State by Boundary Conditions.* Generally, test methods used to measure bond stress and slip can be categorized into two methods, namely, pull-out tests and axial tension tests. The formulation of the bond-slip relationship has been developed for the following typical bond problems, as shown in Figure 4: (a) one side pull-out; (b) both sides pull-out. In these tests, the main limitation is that they do not simulate the actual conditions in a structural member. Due to the effect of the compressive force ($C$), concrete is in compression and the reinforcing bar is in tension. In reality, in the tensile zone of a reinforced concrete member, the concrete and reinforcing bar are both in tension. The presence of a lateral stress modifies the stress distribution shown in Figures 4(a) and 4(b) to a considerable degree.

In the axial tension tests shown in Figure 4(c), a monotonically increasing tensile force ($F$) is applied to the two protruding ends of the reinforcing bar embedded in the concrete prism. The distribution of tensile stresses induced in the reinforcing bar and concrete will be varied as shown in Figure 4(c). This distribution is very similar to that of the tension face of a reinforced concrete flexural member. It can, therefore, be considered that utilizing the bond-slip relationships obtained from the axial tension tests in the analysis is more realistic than utilizing those obtained from the pull-out tests.

## 3. Bond Models

*3.1. Existing Models.* The bond behaviors of reinforced concrete members have been studied extensively in the last century. A number of bond stress-slip relationships and corresponding bond models have been proposed in the last three decades. A number of popular bond models are introduced in the following paragraphs.

In 1957, Rehm [22] carried out pull-out tests with specimens that have a short embedment. He also proposed the following bond model, which is a function of slip, and comparatively analyzed the bond stress distribution with the proposed model as

$$\tau_b = c_1 S^a + c_2 S, \tag{2}$$

where $\tau_b$: bond stress, $S$: slip, $c_1$, $c_2$, and $a$: experimentally obtained values.

In 1967, based on the pull-out tests and axial tension tests, Mugurama and Morita [24, 25] proposed the bond model as

$$\tau_b = \tau_{b,\max} \times \exp\left[\frac{\ln\{(\exp -1)S/S_{\max} + 1\}}{(\exp -1)S/S_{\max} + 1}\right], \tag{3}$$

where $\tau_{b,\max}$: maximum bond stress determined by experiment, $S_{\max}$: slip at the maximum bond stress determined by experiment.

They also estimated the bond behavior under three different boundary conditions using (3). Results from the above analysis concurred with the experimental results. However, in this model, it needs to be assumed that the slip corresponding to the maximum bond stress and the maximum bond stress obtained from the long specimens under axial tension is significantly smaller than that of the short pull-out specimens.

In 1987, Shima [20] reported the following bond models that can predict the tension stiffening effect and that can be used under any boundary condition. Equation (4) were proposed for the bar with a long embedment and for the bar with a short embedment, respectively, while considering the strain effect in accordance with the boundary conditions. This model is beneficial when the concrete is under the compressive stress condition at the reinforcing bar level and can be applied to the analysis of the behavior of a reinforced concrete member without any experiment in order to determine experimental factors. For this reason, this model is the representative bond model that is currently being used in the analysis and design of reinforced concrete members as

$$\begin{aligned} \tau_b &= 0.9 f_c'^{2/3} \times \left[1 - \exp\left\{-40\left(\frac{S}{d_s}\right)^{0.6}\right\}\right], \\ \tau_b &= 0.73 f_c'^{2/3} \times \left[\ln\left\{1 + 5000\left(\frac{S}{d_s}\right)\right\}\right]^3 \times \frac{1}{1 + 10^5 \times \varepsilon_s}, \end{aligned} \tag{4}$$

where $f_c'$: compressive concrete strength, $d_s$: bar diameter, $\varepsilon_s$: strain of the bar.

However, (4) was obtained from the experimental results of a pull-out test for an anchored bar embedded in the footing of a reinforced concrete pier or column. The applicability of this equation may, therefore, be limited because restrained cracks occurred in the specimens. In addition, the

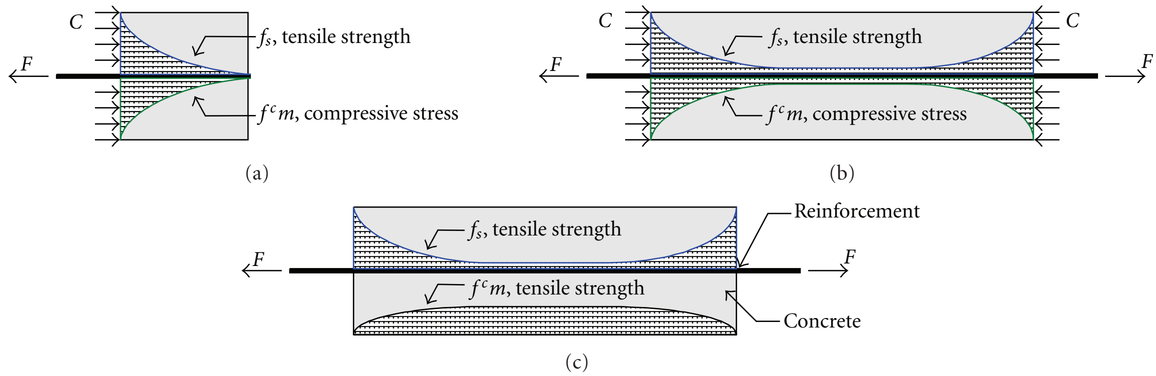


FIGURE 4: Typical stress distributions for (a) one side pull-out, (b) both sides pull-out, and (c) axial tension.

bond behavior must be influenced by many factors such as the concrete cover, the amount of confining reinforcement, or the axial force of the column.

While taking into consideration all the above-mentioned problems, Ikki et al. [18, 19] performed axial tension tests which were evaluated by using the following bond model, modified from (4) given by Shima [20]. Generally, (5) was comparatively accurate at predicting the bond-slip relationship of test specimens. However, the maximum bond stress corresponding to the slip cannot be calculated by the following (5) as

$$\tau_b = k_{sf} \times k_d \times 0.9 f_c'^{2/3} \times \left\{ 1 - \exp\left[ -40 \left( \frac{S}{d_s} \right)^{0.6} \right] \right\}, \quad (5)$$

where $k_{sf}$: coefficient of the concrete stress condition (1.0 when the concrete is compressed, 0.7 when the concrete is tensioned), $k_d$: coefficient of the bar direction during concrete casting (1.0 for a vertically cast reinforcing bar, 0.9 for a horizontally cast reinforcing bar).

A review of earlier literature indicates that most of the bond models were obtained from pull-out tests and, therefore, might not be suitable for reinforced concrete flexural members. Furthermore, the maximum bond stress corresponding to the slip cannot be calculated. This is very important because the maximum bond stress is a main factor in the determination of crack spacing, anchorage strength, and the strength of lap slices. Therefore, it is not possible to accurately calculate the maximum crack width and the lengths of anchorage and lap slices using the bond models proposed by the previously mentioned researchers.

It should be noted that the models given by Ikki et al. [18, 19] and Shima [20], among the aforementioned models, were compared with the proposed model. This is because some coefficients of the models given by Rehm [22] and Mugurama and Morita [24, 25] must be decided experimentally.

*3.2. Newly Proposed Model.* In order to formulate a bond model which works successfully under axial boundary conditions as shown in Figure 4(c), the following factors are considered to express the bond stress. In research carried out by Shima [20], it is discovered that the bond strength increases lineally according to the compressive concrete strength ($f_c'^{2/3}$). Therefore, the effect of the compressive concrete strength is considered by using a nondimensional bond stress ($\tau_b / f_c'^{2/3}$). Also, the effect of the bar diameter is considered by using a nondimensional slip ($S/d_s$). This is because the slip is proportional to the bar diameter [23, 26]. These nondimensional indexes were used to formulate the bond stress. In addition, the relative rib area was introduced in order to consider the effects of the rib geometry of the deformed bars. With some modification of the Ikki et al. [18, 19] model, the bond model is proposed as

$$\tau_b = k \times f_c'^{\,2/3} \times \left\{ 1 - \exp\left[ -4500 \left( \frac{S}{d_s} \right)^{1.45} \right] \right\}^{0.5} \times \exp\left[ -5 \left( \frac{S}{d_s} \right) + 5.5 f_R^{\,0.9} \right], \quad (6)$$

where $k$: coefficient that accounts for the effects of the proposed model on bond stress, $f_R$: relative rib area.

From the position of the reinforcing bar during casting, and depending on whether or not a stirrup is used, $k$ can be classified as

$$k = 0.2 \times \exp\left\{ \left[ -4.5 + 55 (3.06 f_R - 0.24) \right] \times \frac{100}{A_c} \right\} \quad \text{(vertically cast bar)},$$

$$k = 0.2 \times k_{sh} \times \exp\left\{ \left[ -4.5 + 55 (3.06 f_R - 0.24) \right] \times \frac{100}{A_c} \right\} \quad \text{(vertically cast bar with stirrups)},$$

$$k = 0.2 \times \exp\left\{ \left[ -4.5 + 55 f_R \right] \times \frac{100}{A_c} \right\} \quad \text{(horizontally cast bar)},$$

$$k = 0.2 \times k_{sh} \times \exp\left\{ \left[ -4.5 + 55 f_R \right] \times \frac{100}{A_c} \right\} \quad \text{(horizontally cast bar with stirrups)}, \quad (7)$$

where $A_c$: cross sectional area of the concrete, $k_{sh}$: coefficient which expresses the effect of the stirrups (1.0 for a vertically

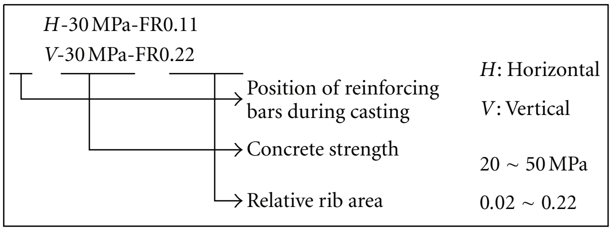


Figure 5: Key to specimen numbering system.

cast stirrup, 0.85 for a horizontally cast stirrup), $f_R$: relative rib area.

Although the Equations $k$ are divided into a horizontally cast bar and a vertically cast bar, the change in bond capacity due to the position of the bar during casting was insignificant. This is because the depth of the concrete below a horizontal bar of reinforced concrete members was usually smaller than 30 cm. As shown in research carried out by Rehm [22], bars that are horizontally cast 30 cm above the bottom of the specimen show an inferior bond behavior compared to bars that are cast in a horizontal position marginally above the bottom of the specimen.

In addition, the reciprocal value of the cross-sectional area of concrete ($100/A_c$) has the same meaning as the average stress of the corresponding cross sectional area of concrete falling under the unit load. In this study, it is believed that the size effects of the cross section need to be considered in accordance with the cross sectional area of concrete, which is actually loaded.

As reflected in the $k_{sh}$, the effect of lateral confinement on bond behavior or bars in structural members should be considered, since the degree of confinement of a bar affects both the magnitude of the ultimate bond stress and the bond stress-slip relationship. Ikki et al. [19] reported significant results that showed that the bond stress was slightly higher in specimens that did not have stirrups than it was in specimens that did have stirrups. Even though Ikki et al. [19] may not have determined the fundamental cause for the previously described phenomena, it is assumed that the bond-slip relationship does not rely on the in-plane restraint (which is the perpendicular reinforcing bar axis), until splitting cracks occur. This is because splitting cracks had not occurred within the range for measuring the maximum bond stress.

Furthermore, the bond stress at any location along a reinforcing bar is proportional to the slope of the steel stress distribution curve at that point. The bond stress can be obtained at any point by using the bond model proposed by many researchers. However, the bond model is very sensitive, since the slope of the steel stress distribution curve is a differential term. Hence, the dispersion of the bond stress test data is currently unavoidable.

*3.3. Parametric Study.* In order to estimate the various factors of the proposed model, a parametric study was performed by transmuting nondimensional slip ($S/d_s$) to 1 ~ 4%, relative rib area ($f_R$) to 0.02 ~ 0.22, compressive concrete strength ($f_c'$) to 20 ~ 50 MPa, and the reciprocal of the cross sectional area of concrete ($100/A_c$) to 0.111%, 0.0625%, and 0.04%, respectively. Each specimen is identified by a numbering system containing a code to help identify the characteristics of that specimen. The specimen numbering system is explained in Figure 5.

Figures 6 and 7 demonstrate the effect of the aforementioned factors on the bond stress-slip relationship for the specimens with a 0.0625% reciprocal of the cross-sectional area of concrete and 30 MPa and 40 MPa $f_c'$ compressive concrete strength, respectively. Due to the scope of this paper, not all figures are shown.

When using deformed bar, the uniform variation of the bond stress noted for a general range of the relative rib area ($f_R$) was 0.10 ~ 0.15. However, a very high variation was seen at 0.16 and above. Based on the above results, the proposed model in (6) was varied by a fixed ratio for the nondimensional slip ($S/d_s$) and the compressive concrete strength ($f_c'$) with small sensitivity for the relative rib area ($f_R$) at a general range of 0.10 ~ 0.15. However, sensitive variation was verified when the size of the concrete cross-section is relatively small. Nevertheless, such effects would not be large because the size of the cross-section in a real application is usually sufficient.

Rehm [27] reported that the upper limit of bond resistance capacity is controlled through shear failure in the case of $f_R = h_d l_d > 0.15$. This fact was unexpectedly proved through the parametric study for the general range imposed on the value $f_R$ by the relevant standards or by approval documents. The results of the parametric study verify that the parameters considered in the proposed model appropriately simulate the bond behavior of a reinforced concrete member under axial tension boundary conditions.

## 4. Experimental Program

*4.1. Specimen and Experimental Conditions.* For verification of the proposed model, the test results performed by Ikki et al.[18, 19] were used. The experimental conditions and properties of each specimen are shown in Table 1.

The specimens for the axial tension tests are made up of two series with different properties, namely, the T-Series and the D-Series. The concrete cover of all specimens is 40 mm, regardless of the concrete cross-section. In the T-Series, the square section was used, with the exception of specimen TV0-50, which was used in the circular section. The length of each specimen was 764 mm as shown in Figure 8. Concrete was cast in a horizontal direction perpendicular to the embedded reinforcing bars. However, specimens TV0-45 and TV0-45 were cast in a vertical direction parallel to the embedded reinforcing bars. In addition, a stirrup was arranged only in specimens TH-30 and TH-45. The tensile reinforcing bar was embedded at the center of the square and circular sections. Here, D19 bar was used as the tensile reinforcing bar for all specimens, while the Young's modulus of the bar is $E_s = 181.3$ GPa, and the yield stress is $f_y = 357.8$ MPa. The test details of specimen TH1-45 are shown in Figure 8. In the D-Series, two types of circular sections

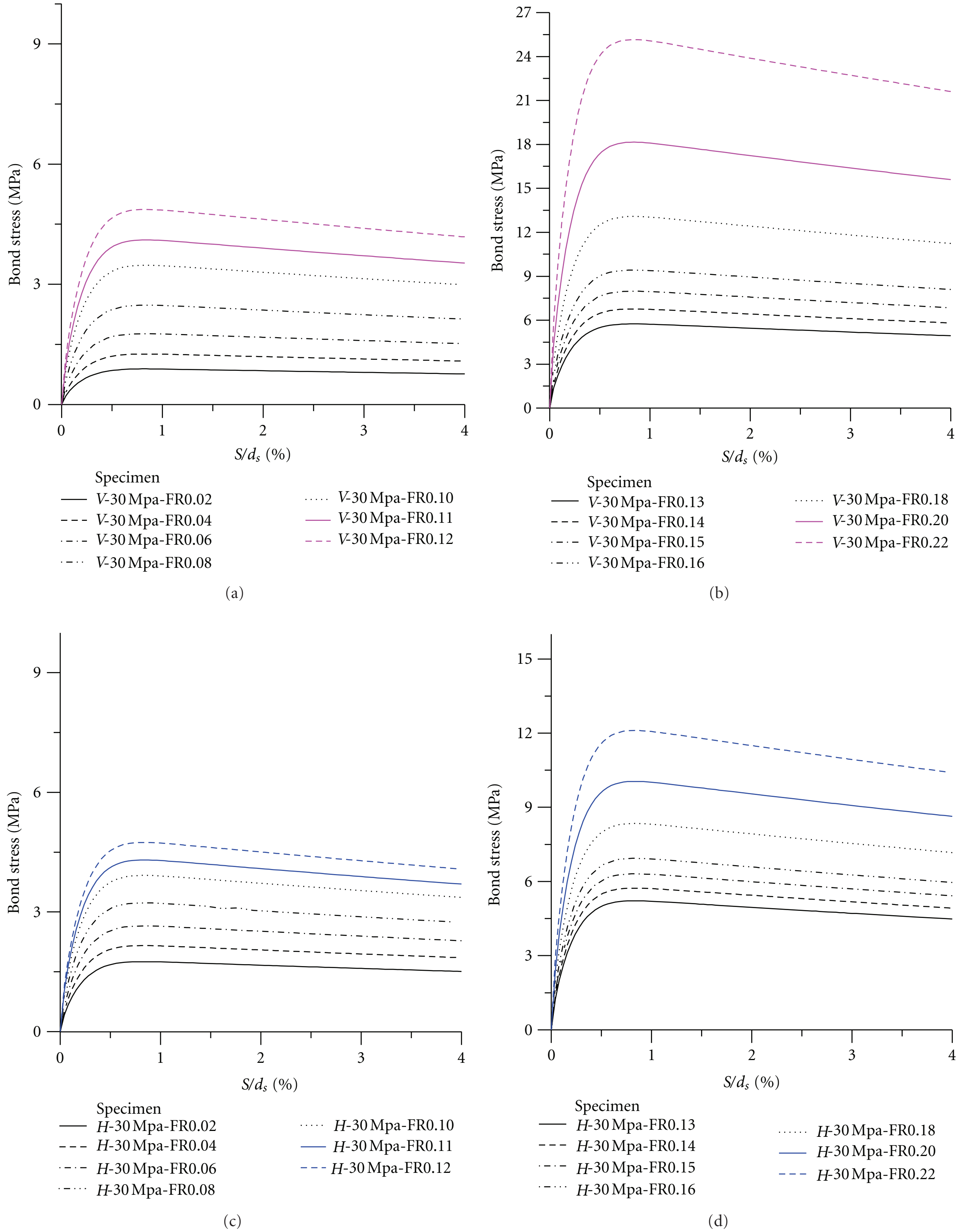


Figure 6: Effects of bond factors ($f_c' = 30$ MPa, $100/A_c = 0.0625\%$).

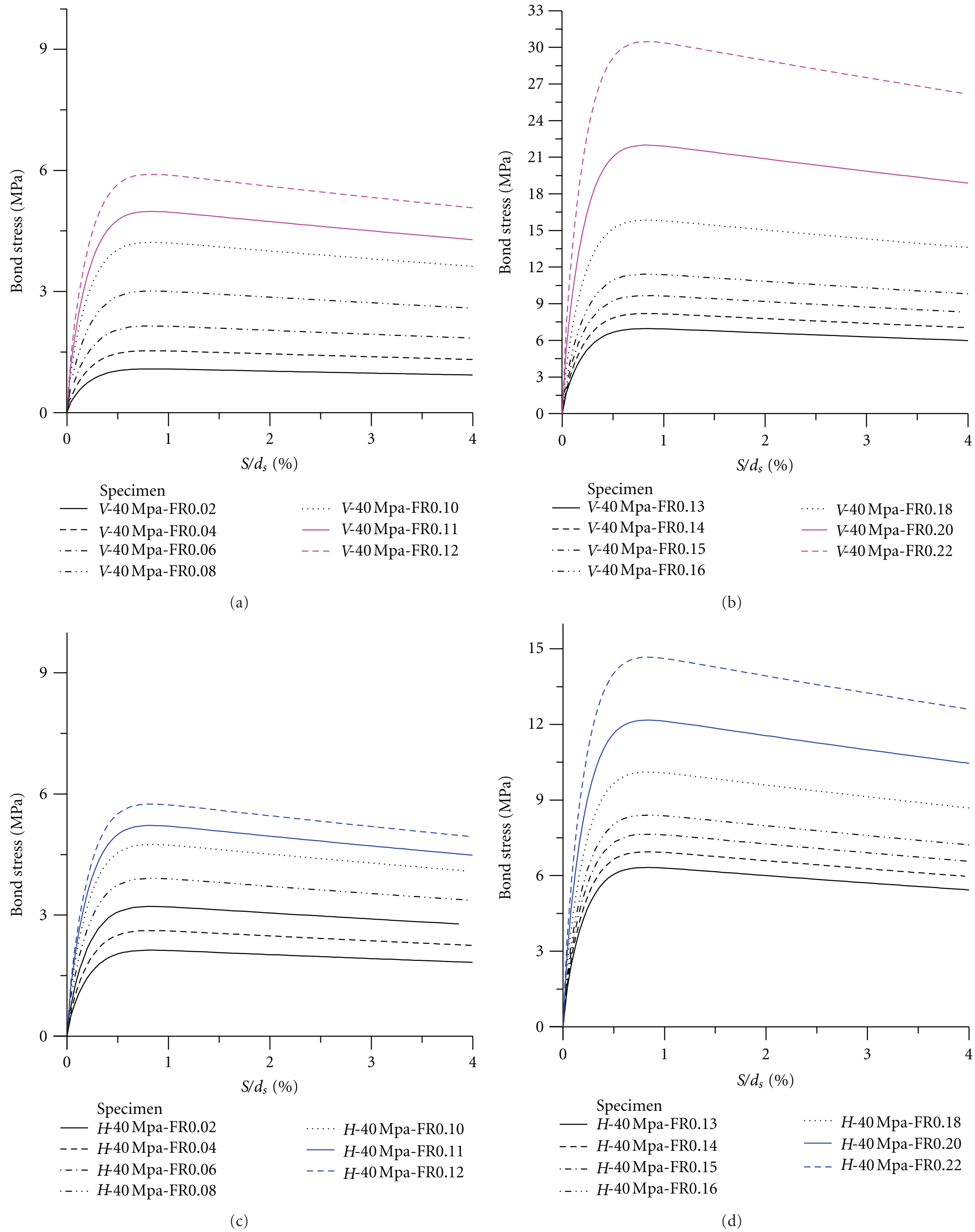


FIGURE 7: Effects of bond factors ($f_c' = 40$ MPa, $100/A_c = 0.0625\%$).

Table 1: Properties of test specimen.

| Type (Series) (1) | Specimen (2) | Direction of bar on casting (3) | Existence of stirrup (D6) (4) | Sectional property [mm] (5) |
|---|---|---|---|---|
| T-Series | TH1-30 | Horizontal | Yes | $300 \times 300$ |
| | TH1-45 | Horizontal | Yes | $450 \times 450$ |
| | TH0-30 | Horizontal | No | $300 \times 300$ |
| | TH0-45 | Horizontal | No | $450 \times 450$ |
| | TV0-45 | Vertical | No | $450 \times 450$ |
| | TV0-50 | Vertical | No | $\phi$ 500 mm |
| D-Series | D-30 | Vertical | No | $\phi$ 300 mm |
| | D-45 | Vertical | No | $\phi$ 450 mm |

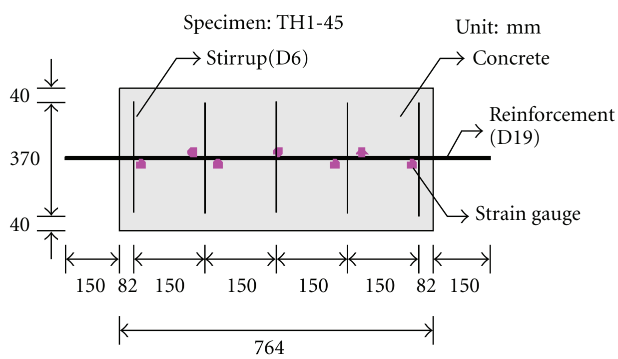


Figure 8: Sectional properties of the specimen TH1-45 and positions of strain gauges.

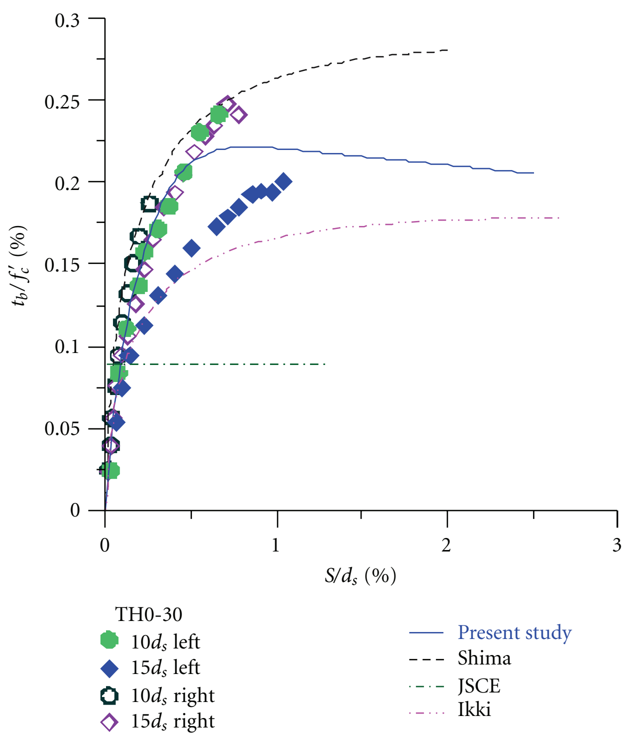


Figure 9: Bond stress-slip relationship of TH0-30 specimen.

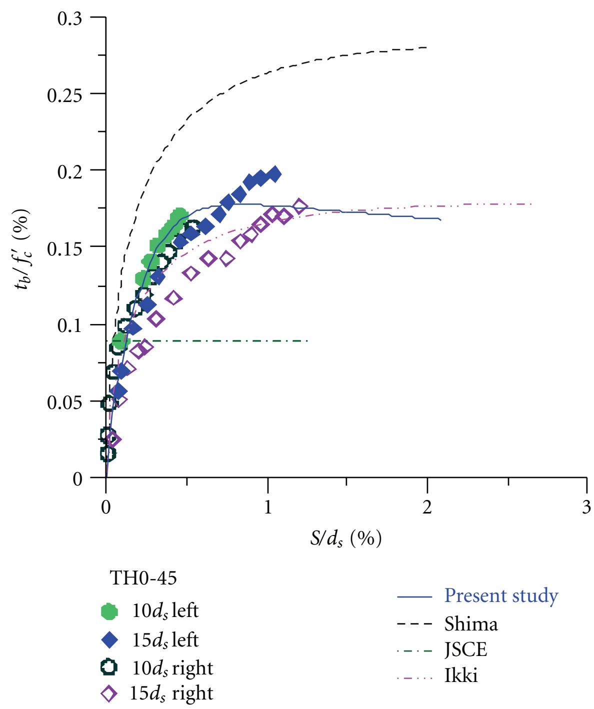


Figure 10: Bond stress-slip relationship of TH0-45 specimen.

were used, with diameters of 30 cm and 45 cm, respectively. The length of each specimen was 964 mm. Concrete was cast in a vertical direction. No stirrup was arranged and the tensile reinforcing bar was also embedded at the center of the circular section. Here, D16 bar was used as the tensile reinforcing bar, with the Young's modulus of the bar $E_s$ = 192 GPa, and the yield stress $f_y$ = 390 MPa.

*4.2. Determination of Bond Stress and Slip.* To obtain the bond stress and slip from the measured strain, the method proposed by Yamao et al. [23] was utilized. At any point, the bond stress derived from an equilibrium condition can be written as

$$\tau_b = \frac{d_s}{4}\frac{df_s}{dx} = \frac{d_s E_s}{4}\frac{d\varepsilon_s}{dx}, \tag{8}$$

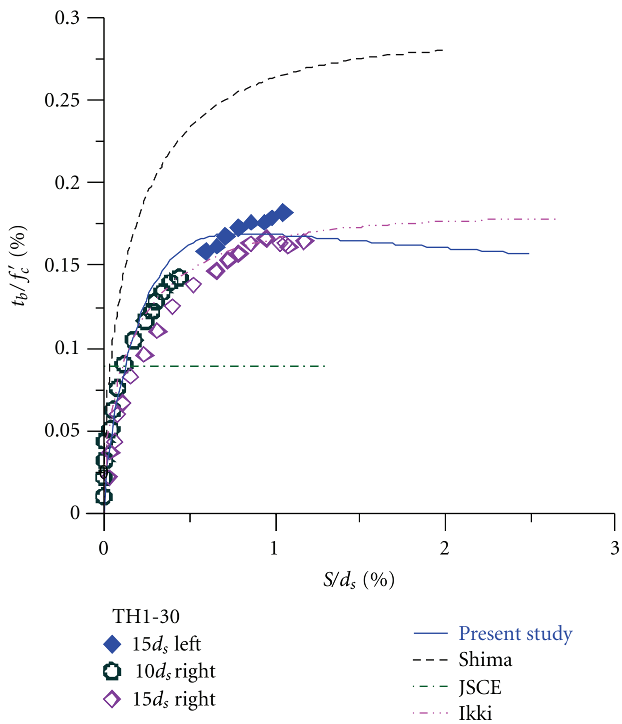


FIGURE 11: Bond stress-slip relationship of TH1-30 specimen.

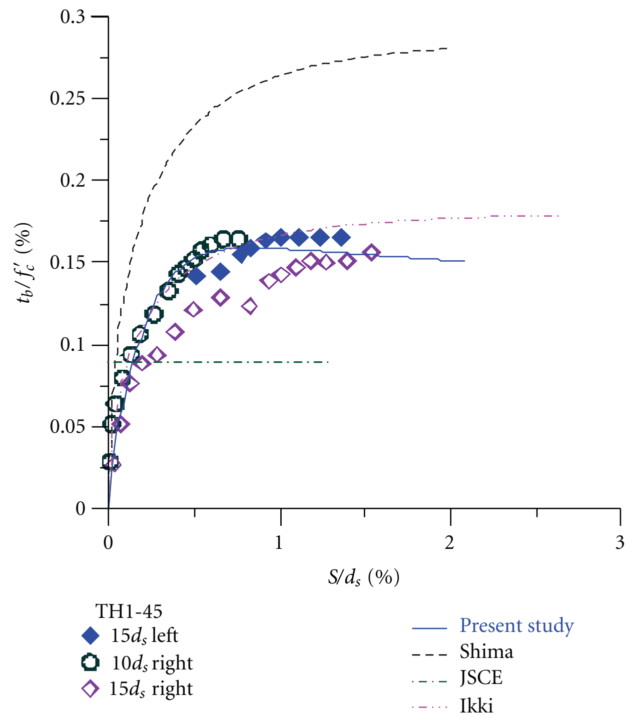


FIGURE 12: Bond stress-slip relationship of TH1-45 specimen.

where $E_s$: Young's modulus of the bar, $d\varepsilon_s/dx$: slope of the strain distribution curve.

The function of strain $\varepsilon_s = \varepsilon(x)$ is assumed as the strain distribution curve obtained by connecting every three neighboring point with a second degree polynomial function. Here, if the three neighboring points are set as $(x_{i-1}, \varepsilon_{i-1})$, $(x_i, \varepsilon_i)$, and $(x_{i+1}, \varepsilon_{i+1})$, the strain curve can be expressed and the obtained strain curve shows the strain distribution from point $(i-1)$ to point $(i)$ as

$$\varepsilon_s = \varepsilon(x) = a_i + b_i x + c_i x^2, \tag{9}$$

where $a_i$, $b_i$, $c_i$: numbers, called coefficients.

By substituting (9) into (8), $\tau_{bi}(x)$ can be written as:

$$\tau_{bi} = \frac{E_s d_s (b_i + 2c_i x)}{4}, \tag{10}$$

where $\tau_{bi}$: bond stress corresponding to point $i$.

In the axial tension test, the slip at any point could be obtained by the integration of the strain from the certain point $(i-1)$ with zero slip to the point concerned $i$ as

$$S_i(x) = S_{i-1}(x) + \int_{x_{i-1}}^{x_i} \left(a_i + b_i x + c_i x^2\right) dx. \tag{11}$$

## 5. Validation of the Proposed Model

All the test specimens are calculated by using the proposed model and the results obtained are compared with the experimental data given by Ikki et al. [18, 19] and analytical solutions provided by eminent researchers [18–20] and the JSCE code [17]. For notational convenience, the analytical solutions have been designated according to the surname of the first authors (Ikki, Shima) and of design codes (JSCE). For example, Shima represents the solution provided by Hiroshi Shima. Also, the solution obtained from the proposed model is denoted by the present study.

The following equation is the concrete design bond strength proposed by the JSCE code [17] as

$$f_{bdj} = 0.28 f_c'^{2/3} \left(f_{bdj} \le 3.2\ \text{MPa}\right). \tag{12}$$

In Figures 9–16, analytical solutions are shown compared to the experimental data and the proposed model at different locations along a reinforcing bar. From these figures, it is verified that the bond stress-slip relationship in a specimen is the same, independent of the locations along a reinforcing bar. However, the bond stress in the data obtained from the right sides of specimens D-30 and D-45 shown in Figures 15 and 16 is underestimated compared to that of the left sides. For this reason, Ikki and Kiyomiya [18, 19] explained that the local hardness of the concrete layer in contact with the ribs of indentation had decreased due to the bleeding of the concrete.

The important basic factor for the analysis of deformational behavior and the calculation of crack width in reinforced concrete members is the constitutive law for reinforcing bar and concrete. The constitutive law of bond action between reinforcing bar and concrete is also important for FEM analysis. Consequently, the JSCE model [17] is always consistently conservative.

The Shima [20] model yields an over-predicted overall bond-slip behavior for all specimens, which is not desired.

Table 2: Comparison results for the maximum bond stress and the associated slip.

| Method Type | Data of Experiment | | Present Study | | Ikki | Shima | JSCE |
|---|---|---|---|---|---|---|---|
| | $S/d_s$ [%] | $\tau_{b,\max}/f_c'$ | $S/d_s$ [%] | $\tau_{b,\max}/f_c'$ | $\tau_{b,1\%}/f_c'$ | $\tau_{b,1\%}/f_c'$ | $\tau_{bdj}/f_c'$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| TH1-30 | 1.013 | 0.189 | 0.852 | 0.169 | 0.166 | 0.264 | 0.0891 |
| TH1-45 | 0.813 | 0.179 | 0.845 | 0.159 | 0.166 | 0.264 | 0.0891 |
| TH0-30 | 0.834 | 0.230 | 0.857 | 0.221 | 0.166 | 0.264 | 0.0891 |
| TH0-45 | 1.050 | 0.197 | 0.850 | 0.178 | 0.166 | 0.264 | 0.0891 |
| TV0-45 | 0.847 | 0.197 | 0.865 | 0.201 | 0.185 | 0.264 | 0.0891 |
| TV0-50 | 0.832 | 0.196 | 0.809 | 0.190 | 0.185 | 0.264 | 0.0891 |
| D-30 | 0.771 | 0.199 | 0.831 | 0.205 | 0.207 | 0.295 | 0.0999 |
| D-45 | 0.930 | 0.180 | 0.850 | 0.172 | 0.207 | 0.295 | 0.0999 |
| Mean | 0.886 | 0.196 | 0.845 | 0.187 | 0.181 | 0.272 | 0.092 |

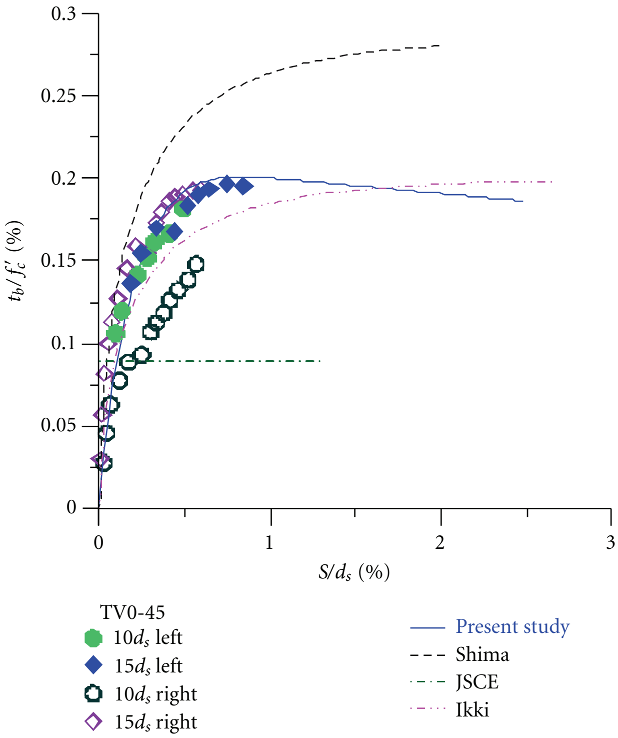


Figure 13: Bond stress-slip relationship of TV0-45 specimen.

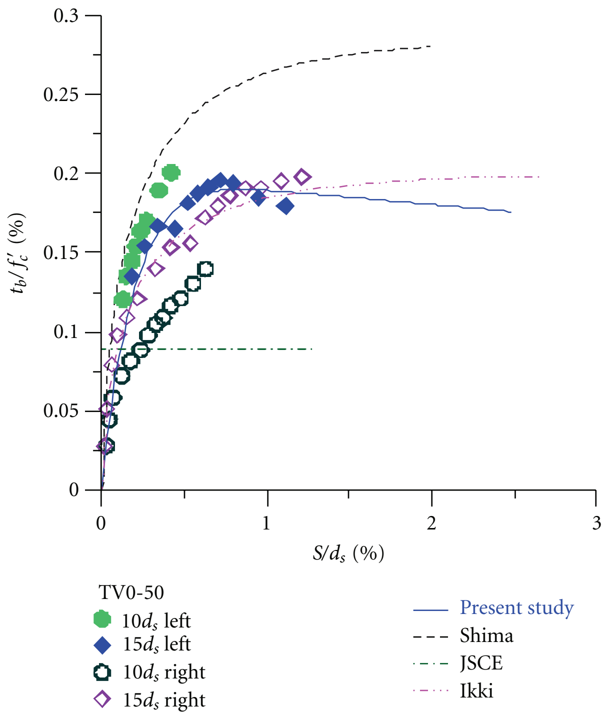


Figure 14: Bond stress-slip relationship of TV0-50 specimen.

Moreover, the model predicted the maximum bond stress of $\tau_{b,\,1\%}/f_c'$ as significantly more than the experimental values for all test specimens. This trend can also be seen in Table 2. This model is quite erroneous when used for predicting the bond stress-slip behavior for the axial tension members. This may be due to the fact that the model was developed based on the pull-out tests.

The Ikki et al. [18, 19] model, in general, predicts comparatively closer to the experimental values for all the test specimens. For the square specimens, this model slightly underestimated the maximum bond stress $\tau_{b,1\%}/f_c'$, yet the predictions were very close to the test values. Conversely, this model slightly overestimated the circular specimens. As for the Shima [20] model discussed above, this model also did not give the maximum bond stress and the associated slip.

The model proposed in this study is quite accurate at predicting the bond stress-slip relationship of test specimens and its accuracy is not affected by changes in test variables. This is important since the test variables change rapidly among different types of reinforcing bars and various concrete strengths, and so forth. Also, the benefit of this model is that the maximum bond stress corresponding to the associated slip can be obtained, which is a core factor in the estimation crack spacing.

It should be noted that the peak value of the nondimensional bond stress curve $\tau_b/f_c'^{2/3}$, compared to the nondimensional slip index curve $S/d_s$, is of greater significance when identifying the maximum crack-width-related aspects of reinforced concrete members. That is, the maximum bond stress is a main factor in the estimation of crack spacing, so

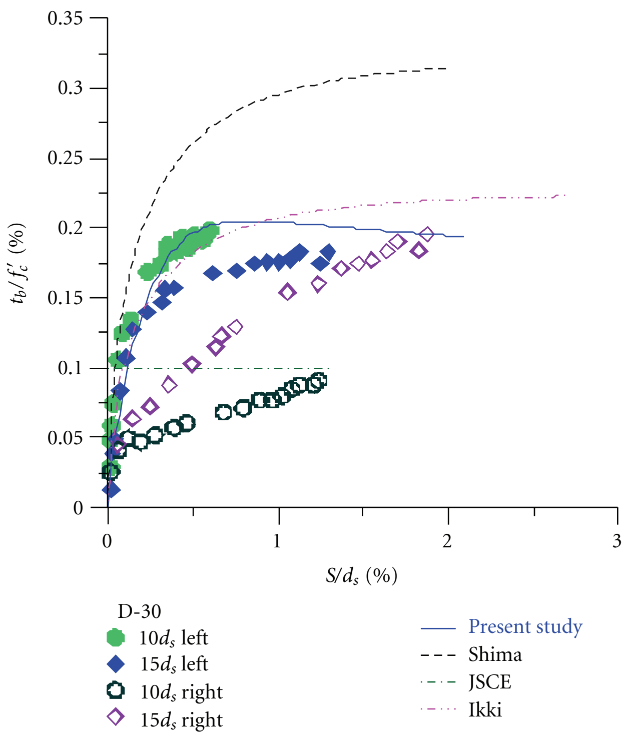


Figure 15: Bond stress-slip relationship of D-30 specimen.

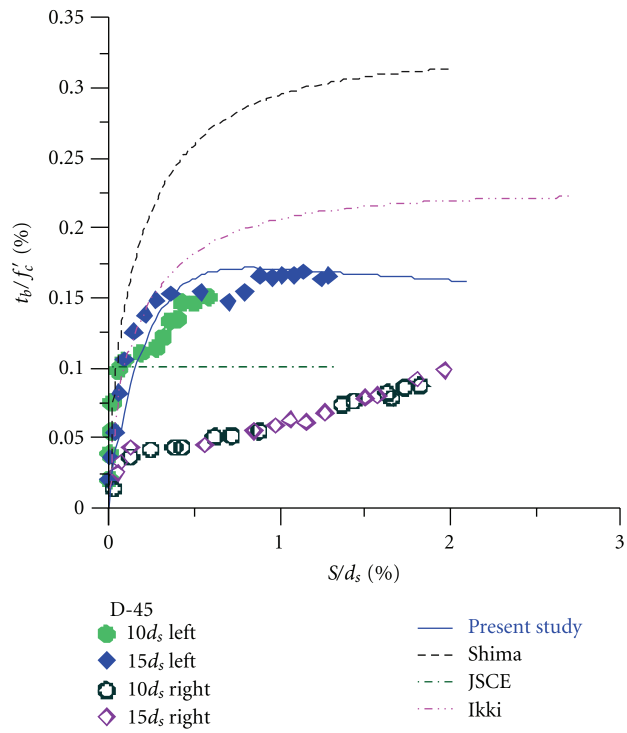


Figure 16: Bond stress-slip relationship of D-45 specimen.

direct calculation of the maximum crack width is possible from the estimation of the maximum bond stress.

Furthermore, in order to consider the predictive assessment in respect of the analytical solutions including the proposed model, the calculations were carried out for the nondimensional maximum bond stress and the corresponding nondimensional slip for all specimens. The comparison results can be seen in Table 2.

However, the bond stress $\tau_{b,1\%}$ when $S/d_s = 1\%$ was used, since the estimation of the maximum bond stress by using the formula proposed by Ikki et al. [18, 19] and Shima [20] was difficult. Based on the comparison results, the model proposed by the author showed that the mean values for the nondimensional maximum bond stress and the corresponding nondimensional slip were 0.845 and 0.187, respectively. These values are the nearest values for the experimental values. Consequently, the model proposed in this paper is the most accurate model for estimating the bond behavior of axial tension members.

## 6. Conclusion

In this paper, an analytical model was proposed to estimate the bond stress-slip relationship for reinforced concrete members under axial tension loadings. By extending the conventional bond stress-slip theory and considering the relative rib area of deformed bars and the concrete cross-sectional area, an equation to compute the bond stress was formulated as a function of the nondimensional factors. To estimate the various factors of the proposed model, a parametric study has been performed on each of the bond factors. Also, based on the results of the parametric study, the factors considered in the proposed model appropriately simulate the bond behavior of reinforced concrete members under axial tension boundary conditions.

The results of the analysis presented in this paper indicate that the proposed model can be used to effectively estimate the bond stress-slip relationships of reinforced concrete flexural members. The results compared well with the experimental data and validation was obtained.

Although the current design codes and the bond models proposed by different researchers provide design criteria for the bond behaviors of reinforced concrete members, these criteria are not well defined and there is no concurrence among the different variables affecting the bond behaviors. Therefore, the proposed model could be of benefit to structural engineers when analyzing and designing reinforced concrete members under bending.

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