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Research Article

Thermomechanical Fractional Model of Two Immiscible TEMHD

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We introduce a mathematical model of unsteady thermoelectric MHD flow and heat transfer of two immiscible fractional second-grade fluids, with thermal fractional parameters α_i and mechanical fractional parameters β_i, $i = 1, 2$. The Laplace transform with respect to time is used to obtain the solution in the transformed domain. The inversion of Laplace transform is obtained by using numerical method based on a Fourier-series expansion. The numerical results for temperature, velocity, and the stress distributions are represented graphically for different values of α_i and β_i. The graphs describe the fractional thermomechanical parameters effect on the case of two immiscible fluids and the case of a single fluid.

1. Introduction

In recent years, the requirement of modern technology has stimulated interest in flow and heat transfer studies which involve interaction of several phenomena such as heat exchangers, transport of heat or cooled fluids, chemical processing equipment, and microelectronic cooling [1]. The problem of flow and heat transfer was extensively investigated by many researchers with different hypothesis, in which the mentioned studies pertain to a single-fluid model [2–6].

Many problems relating to plasma physics, aeronautics, and geophysics and in petroleum industry and so forth involve multilayered fluid flow situations [7]. There has been some theoretical and experimental work in stratified laminar flow of two immiscible fluids in horizontal pipes. Singh et al. [8] studied the generalized Couette flow of two viscous incompressible immiscible fluids with heat transfer in the presence of heat source through two straight parallel horizontal walls. Umavathi et al. [9] investigated unsteady oscillatory flow and heat transfer in horizontal channel consisting of two viscous immiscible fluids with isothermal permeable walls. Malashetty et al. [10] examined the magnetoconvection of two immiscible fluids in vertical enclosure consisting of two conducting and nonconducting regions. Kamışlı and Öztop [11] examined the entropy generation in two immiscible incompressible fluid flows under the influence of pressure difference in thin slit of constant wall heat fluxes. Nikodijevic et al. [12] investigate the magnetohydrodynamic (MHD) Couette flow and heat transfer of two immiscible fluids in a parallel-plate channel in the presence of applied electric and inclined magnetic fields. Tigrine et al. [13] studied experimentally the Couette flow of two immiscible fluids between concentric spheres when the outer sphere is fixed and the inner one rotates.

Thermoelectric magnetohydrodynamics (TEMHD) theory was originally developed by Shercliff with direct application to a fusion environment [14]. The thermoelectric effect causes a current to develop between a liquid metal and a container wall when a temperature gradient is present along the interface between them [15–17].

Recently, the fractional derivatives have been found to be quite flexible in describing the behaviors of viscoelastic fluids and are studied by many mathematicians considering various motions of such fluids [18, 19]. In their studies, the constitutive equations for generalized non-Newtonian fluids are modified from the well-known fluid models by replacing

the time derivative of an integer order by the so-called Caputo fractional operator.

El-Shahed [20] studies the effect of transverse magnetic field on the unsteady flow of a generalized second-grade fluid through a porous medium. Jamil et al. [21] obtain an exact solution for the motion of fractionalized second-grade fluid due to oscillations of an infinite circular cylinder. Sherief et al. [22] derive fractional order theory of thermoelasticity using the methodology of fractional calculus. The effects of fractional derivative parameter of two media are discussed in [23]. Ezzat [24, 25] constructed a mathematical model of the TEMHD in the context of fractional heat conduction equation by using the Taylor series expansion of time fractional order developed by Jumarie [26]. Recently, Hamza et al. [27] established a new fractional theory of thermoelasticity associated with two relaxations.

In this work, we introduce a mathematical model of unsteady flow and heat transfer of two immiscible thermoelectric fluids with thermomechanical fractional parameters α_i and β_i, $i = 1, 2$. The thermal parameters α_i are due to the fractional heat conduction equation of TEMHD introduced by Ezzat [24]. The mechanical parameters β_i are due to the second-grade fluid in fractional form introduced by El-Shahed [20]. We will apply our model for the flow of two immiscible electrically conducting, incompressible, fractional viscoelastic second-grade fluids in the presence of a transverse magnetic field. The Laplace transform with respect to time is used. A numerical method based on a Fourier-series expansion is used for the inversion process. The numerical results for temperature, velocity, and the stress distributions are represented graphically for different values of α_i and β_i. The graphs describe the fractional thermomechanical parameters effect on the case of two immiscible fluids and the case of a single fluid.

2. Formulation of the Problem

Consider unsteady laminar stratified two immiscible fluid flows through nonconducting half space $y \geq 0$ in contact with an infinite plate that is rigidly fixed. Take the positive x-axis in the direction of the flow and the positive y-axis vertically downward perpendicular to the flow.

The regions $0 \leq y \leq h$ and $y \geq h$ are denoted as Region-1 and Region-2, respectively, and occupied by two immiscible electrically conducting incompressible thermoelectric generalized second-grade viscoelastic fluids with different viscosity μ_i, density ρ_i, specific heat at constant pressure c_{pi}, thermal conductivity κ_i, electrical conductivity σ_i, first normal stress modulus α_{1i}, thermal relaxation time τ_{oi}, Seebeck coefficient k_{oi}, and Peltier coefficient π_i, where the subscript $i = 1, 2$ represents the values of the parameters of the first fluid and second fluid, respectively.

A constant magnetic field of strength H_o is applied in the z-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. Due to the formulation of the problem all variables depend on y and t only.

The following assumptions are required.

(1) The stress tensor which is different from zero for generalized second-grade viscoelastic fluid is given by [20]

$$\tau_i = \mu_i \frac{\partial u_i}{\partial y} + \alpha_{1i} D_t^{\beta_i} \frac{\partial u_i}{\partial y}, \tag{1}$$

where τ_i is the stress tensor T_{xy} of the two fluids and D_t^{ζ} is Caputo fractional time derivative operator defined as [28]

$$D_t^{\zeta} f(t) = \begin{cases} \dfrac{1}{\Gamma(1-\zeta)} \displaystyle\int_0^t (t-p)^{-\zeta} f'(p)\, dp, & (0 < \zeta < 1) \\ \dfrac{df(t)}{dt}, & \zeta = 1, \end{cases} \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function.

(2) The modified Ohm and Fourier laws defined by Shercliff [14] for the two thermoelectric media are given by

$$\begin{aligned} \mathbf{q}_i &= -\kappa_i \nabla T_i + \pi_i \mathbf{J}_i, \\ \mathbf{J}_i &= \sigma_i \left(\mathbf{E} + \mathbf{V}_i \times \mathbf{B} - k_{oi} \nabla T_i\right), \end{aligned} \tag{3}$$

where $\mathbf{q}_i$, $\mathbf{J}_i$, and $\mathbf{V}_i$ are heat conduction vector, conduction current density vector, and velocity vector of the two fluids and $\mathbf{E}$ and $\mathbf{B}$ are the electric density and magnetic induction vectors, respectively.

(3) A mathematical model of fractional heat conduction equation by using the Taylor series expansion of time fractional order developed by Jumarie in the context of thermoelectric MHD [24] is

$$\left(1 + \frac{\tau_{oi}^{\alpha_i}}{\alpha_i!} D_t^{\alpha_i}\right) \mathbf{q}_i = -\kappa_i \nabla T_i + \pi_i \mathbf{J}_i, \tag{4}$$

where $D_t^{\alpha_i}$ is the Caputo fractional derivative operator of order α_i where $0 < \alpha_i \leq 1$, $i = 1, 2$.

(4) The magnetic induction has one constant nonvanishing component:

$$B_z = \mu_o H_o = B_o. \tag{5}$$

(5) The Lorentz force $\mathbf{F} = \mathbf{J} \times \mathbf{B}$ has only component in the x-direction for each fluid and is given by

$$F_{xi} = -\sigma_i B_o^2 u_i - \sigma_i k_{oi} B_o \frac{\partial T_i}{\partial y}. \tag{6}$$

Under these assumptions the governing equations for such flow will take the following form.

The energy equation:

$$\rho_i c_{pi} \frac{\partial}{\partial t}\left(1+\frac{\tau_{oi}^{\alpha_i}}{\alpha_i!}D_t^{\alpha_i}\right)T_i = \left(\kappa_i+\sigma_i\pi_i k_{oi}\right)\frac{\partial^2 T_i}{\partial y^2}+\sigma_i\pi_i B_o\frac{\partial u_i}{\partial y}, \tag{7}$$

equation of motion:

$$\rho_i\left(\frac{\partial u_i}{\partial t}\right)=\mu_i\left(\frac{\partial^2 u_i}{\partial y^2}\right)+\alpha_{1i}D_t^{\beta_i}\frac{\partial^2 u_i}{\partial y^2}-\sigma_i B_o^2 u_i - \sigma_i k_{oi}B_o\frac{\partial T_i}{\partial y}, \tag{8}$$

the modified fractional Fourier law of heat conduction:

$$\left(1+\frac{\tau_{oi}^{\alpha_i}}{\alpha_i!}D_t^{\alpha_i}\right)q_i=-\left(\kappa_i+\sigma_i\pi_i k_{oi}\right)\frac{\partial T_i}{\partial y}-\sigma_i\pi_i B_o u_i, \tag{9}$$

the constitutive equation:

$$\tau_i=\mu_i\frac{\partial u_i}{\partial y}+\alpha_{1i}D_t^{\beta_i}\frac{\partial u_i}{\partial y}, \tag{10}$$

where u_i are the components of the fluids velocity, T_i are the temperature of the regions of the two fluids, and $D_t^{\alpha_i}$, $D_t^{\beta_i}$ are thermal and mechanical Caputo fractional time derivative operator of orders $(0<\alpha_i\le 1)$, $(0<\beta_i\le 1)$, $i=1,2$.

The thermal boundary and interface condition for the two fluids are written as

$$\begin{aligned} T_1(0)&=T_o H(t),\\ T_1(h)&=T_2(h),\\ q_1(h)&=q_2(h),\\ T_2&\longrightarrow 0 \quad \text{as } y\longrightarrow\infty, \end{aligned} \tag{11}$$

where $H(t)$ is the Heaviside unit step function.

The hydrodynamic boundary and interface condition for the two fluids are considered as

$$\begin{aligned} u_1(0)&=0,\\ u_1(h)&=u_2(h),\\ \tau_1(h)&=\tau_2(h),\\ u_2&\longrightarrow 0 \quad \text{as } y\longrightarrow\infty. \end{aligned} \tag{12}$$

Let us introduce dimensionless variables:

$$\begin{aligned} u_i^*&=\frac{u_i}{U},\\ y^*&=\frac{y}{h},\\ t^*&=\frac{\upsilon_1}{h^2}t,\\ \tau_{oi}^*&=\frac{\upsilon_1}{h^2}\tau_{oi},\\ \theta_i&=\frac{T_i-T_\infty}{T_o}\\ \tau_i^*&=\frac{h}{U\mu_1}\tau_i,\\ q_i^*&=\frac{h}{\kappa_1 T_o}q_i,\\ J_i^*&=\frac{J_i}{U\sigma_1 B_o},\\ \alpha_{1i}^*&=\left(\frac{h^2}{\upsilon_1}\right)^{1-\beta_i}\alpha_{1i}, \end{aligned} \tag{13}$$

where T_o is reference temperature defined as $T_o=T_1-T_2$.

Therefore, (7)–(10) are reduced to the nondimensional equations

$$\begin{aligned} &\rho c_p P_r\frac{\partial}{\partial t}\left(1+\frac{\tau_{oi}^{\alpha_i}}{\alpha_i!}D_t^{\alpha_i}\right)\theta_i = \left(\kappa+k_o^2\sigma Z T_o\right)\frac{\partial^2\theta_i}{\partial y^2}+\pi\sigma\Pi_o\frac{\partial u_i}{\partial y},\\ &\rho\frac{\partial u_i}{\partial t}=\left(\mu+\eta_i D_t^{\beta_i}\right)\frac{\partial^2 u_i}{\partial y^2}-\sigma M^2 u_i-\sigma k_o K_o\frac{\partial\theta_i}{\partial y},\\ &\left(1+\frac{\tau_{oi}^{\alpha_i}}{\alpha_i!}D_t^{\alpha_i}\right)q_i=-\left(\kappa+k_o^2\sigma Z T_o\right)\frac{\partial\theta_i}{\partial y}-\pi\sigma\Pi_o u_i,\\ &\tau_i=\left(\mu+\eta_i D_t^{\beta_i}\right)\frac{\partial u_i}{\partial y}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} P_r&=\frac{c_{p1}\mu_1}{\kappa_1},\\ M^2&=B_o^2h^2\frac{\sigma_1}{\mu_1},\\ \eta_i&=\frac{\alpha_{1i}}{h^2\rho_1},\\ Z T_o&=\frac{k_{o1}^2\sigma_1}{\kappa_1}T_o, \end{aligned}$$

$$\pi_i = k_{oi}T_o,$$
$$\Pi_o = \frac{\pi_1\sigma_1 B_o U h}{\kappa_1 T_o},$$
$$K_o = \frac{k_{o1}\sigma_1 h B_o T_o}{\mu_1 U},$$
$$\rho = \frac{\rho_i}{\rho_1},$$
$$\mu = \frac{\mu_i}{\mu_1},$$
$$\kappa = \frac{\kappa_i}{\kappa_1},$$
$$\sigma = \frac{\sigma_i}{\sigma_1},$$
$$c_p = \frac{c_{pi}}{c_{p1}},$$
$$k_o = \frac{k_{oi}}{k_{o1}},$$
$$\pi = \frac{\pi_i}{\pi_1}, \tag{15}$$

where P_r, M^2, η_i, and ZT_o are Prandtl number, Hartmann number, viscoelastic parameter, and dimensionless thermoelectric figure-of-merit [29], respectively, π_i is the first Thomson relation in thermoelectric medium [30], and ρ, μ, κ, σ, c_p, k_o, and π are the ratios of the materials parameter.

The thermal boundary and interface condition in nondimensional form become

$$\begin{aligned} &\theta_1(0) = H(t), \\ &\theta_1(1) = \theta_2(1), \\ &q_1(1) = q_2(1), \\ &\theta_2 \longrightarrow 0 \quad \text{as } y \longrightarrow \infty. \end{aligned} \tag{16}$$

The hydrodynamic boundary and interface condition in nondimensional form become

$$\begin{aligned} &u_1(0) = 0, \\ &u_1(1) = u_2(1), \\ &\tau_1(1) = \tau_2(1), \\ &u_2 \longrightarrow 0 \quad \text{as } y \longrightarrow \infty. \end{aligned} \tag{17}$$

Applying the Laplace transform for both sides of (14), defined by

$$\overline{F}(s) = \mathrm{L}[f(t)] = \int_0^\infty e^{-st} f(t)\,dt, \tag{18}$$

we get the following system of equations:

$$\frac{\partial^2\overline{\theta}_i}{\partial y^2} = a_i\overline{\theta}_i - b_i\frac{\partial\overline{u}_i}{\partial y}, \tag{19}$$

$$\frac{\partial^2\overline{u}_i}{\partial y^2} = c_i\overline{u}_i + g_i\frac{\partial\overline{\theta}_i}{\partial y}, \tag{20}$$

$$\overline{q}_i = -m_i\frac{\partial\overline{\theta}_i}{\partial y} - n_i\overline{u}_i, \tag{21}$$

$$\overline{\tau}_i = \ell_i\frac{\partial\overline{u}_i}{\partial y}, \tag{22}$$

where a_i, b_i, c_i, g_i, ℓ_i, m_i, and n_i are given in the Appendix. In obtaining the above equations we shall use the homogenous initial condition and the following relation [28]:

$$\int_0^\infty e^{-st} D_t^\zeta f(t)\,dt = s^\zeta \overline{F}(s) - \sum_{i=0}^{n-1} s^{\zeta-i-1} D_t^i f(t)\Big|_{t=0}, \quad (n-1<\zeta<n). \tag{23}$$

The thermal boundary and interface conditions become

$$\begin{aligned} &\overline{\theta}_1(0) = \frac{1}{s}, \\ &\overline{\theta}_1(1) = \overline{\theta}_2(1), \\ &m_1\frac{\partial\overline{\theta}_1}{\partial y} + n_1\overline{u}_1 = m_2\frac{\partial\overline{\theta}_2}{\partial y} + n_2\overline{u}_2, \quad y = 1, \\ &\overline{\theta}_2 \longrightarrow 0 \quad \text{as } y \longrightarrow \infty. \end{aligned} \tag{24}$$

The hydrodynamic boundary and interface conditions become

$$\begin{aligned} &\overline{u}_1(0) = 0, \\ &\overline{u}_1(1) = \overline{u}_2(1), \\ &\ell_1\frac{\partial\overline{u}_1}{\partial y} = \ell_2\frac{\partial\overline{u}_2}{\partial y}, \quad y = 1, \\ &\overline{u}_2 \longrightarrow 0 \quad \text{as } y \longrightarrow \infty. \end{aligned} \tag{25}$$

Now, we can rewrite (19) and (20) in the form of

$$\left(D^2 - a_i\right)\overline{\theta}_i = -b_i D\overline{u}_i, \tag{26}$$

$$\left(D^2 - c_i\right)\overline{u}_i = g_i D\overline{\theta}_i. \tag{27}$$

Eliminating $\overline{u}_i$, $i = 1, 2$, between (26) and (27), we obtain the following fourth-order differential equation satisfied by $\overline{\theta}_i$:

$$\left(D^4 - \left(a_i + c_i - b_i g_i\right)D^2 + a_i c_i\right)\overline{\theta}_i = 0. \tag{28}$$

Equation (28) can be factorized as

$$\begin{aligned}&\left(D^2-k_1^2\right)\left(D^2-k_2^2\right)\overline{\theta}_1=0, \quad \text{in Region-1},\\&\left(D^2-\lambda_1^2\right)\left(D^2-\lambda_2^2\right)\overline{\theta}_2=0, \quad \text{in Region-2},\end{aligned} \tag{29}$$

where k_1^2, k_2^2 and λ_1^2, λ_2^2 are the roots of the characteristic equations

$$\begin{aligned}&k^4-\left(a_1+c_1-b_1g_1\right)k^2+a_1c_1=0, \quad \text{in Region-1},\\&\lambda^4-\left(a_2+c_2-b_2g_2\right)\lambda^2+a_2c_2=0, \quad \text{in Region-2}.\end{aligned} \tag{30}$$

The general solution of (28) which is bounded at infinity is given by

$$\overline{\theta}_i(y,s)=\begin{cases}A_1e^{k_1y}+A_2e^{-k_1y}+A_3e^{k_2y}+A_4e^{-k_2y}, & i=1,\\ A_5e^{-\lambda_1y}+A_6e^{-\lambda_2y}, & i=2,\end{cases} \tag{31}$$

where the characteristic roots of Region-1 are $\pm k_1$, $\pm k_2$ while λ_1 and λ_2 are the roots with positive real parts of the characteristic equation of Region-2. These roots are given in the Appendix. The other roots in Region-2 are neglected in order to make the functions field bounded in this region. A_1–A_6 are parameters depending on s only.

By the same manner, we get

$$\overline{u}_i(y,s)=\begin{cases}B_1e^{k_1y}+B_2e^{-k_1y}+B_3e^{k_2y}+B_4e^{-k_2y}, & i=1,\\ B_5e^{-\lambda_1y}+B_6e^{-\lambda_2y}, & i=2,\end{cases} \tag{32}$$

where B_1–B_6 are parameters depending on s only.

From (31) and (32) into (26), we get

$$B_j=\begin{cases}(-1)^j\dfrac{\left(k_1^2-a_1\right)}{b_1k_1}A_j, & j=1,2,\\ (-1)^j\dfrac{\left(k_2^2-a_1\right)}{b_1k_2}A_j, & j=3,4,\\ \dfrac{\left(\lambda_{j-4}^2-a_2\right)}{b_2\lambda_{j-4}}A_j, & j=5,6.\end{cases} \tag{33}$$

We thus have the following.

Region-1

$$\begin{aligned}\overline{\theta}_1(y,s)&=A_1e^{k_1y}+A_2e^{-k_1y}+A_3e^{k_2y}+A_4e^{-k_2y},\\ \overline{u}_1(y,s)&=-\frac{\left(k_1^2-a_1\right)}{b_1k_1}\left(A_1e^{k_1y}-A_2e^{-k_1y}\right)\\&\quad-\frac{\left(k_2^2-a_1\right)}{b_1k_2}\left(A_3e^{k_2y}-A_4e^{-k_2y}\right).\end{aligned} \tag{34}$$

Region-2

$$\begin{aligned}\overline{\theta}_2(y,s)&=A_5e^{-\lambda_1y}+A_6e^{-\lambda_2y},\\ \overline{u}_2(y,s)&=\frac{\left(\lambda_1^2-a_2\right)}{b_2\lambda_1}A_5e^{-\lambda_1y}+\frac{\left(\lambda_2^2-a_2\right)}{b_2\lambda_2}A_6e^{-\lambda_2y}.\end{aligned} \tag{35}$$

In order to determine the unknown A_j, $j=1,2,\ldots,6$, we shall use (34) and (35) with boundary and interface conditions equations (24) and (25); we get

$$\begin{aligned}&A_1+A_2+A_3+A_4=\frac{1}{s},\\&A_1e^{k_1}+A_2e^{-k_1}+A_3e^{k_2}+A_4e^{-k_2}=A_5e^{-\lambda_1}\\&\quad+A_6e^{-\lambda_2},\\&\frac{\left(k_1^2-a_1\right)}{b_1k_1}\left(A_1-A_2\right)+\frac{\left(k_2^2-a_1\right)}{b_1k_2}\left(A_3-A_4\right)=0,\\&\frac{\left(k_1^2-a_1\right)}{b_1k_1}\left(A_2e^{-k_1}-A_1e^{k_1}\right)\\&\quad+\frac{\left(k_2^2-a_1\right)}{b_1k_2}\left(A_4e^{-k_2}-A_3e^{k_2}\right)\\&\quad=\frac{\left(\lambda_1^2-a_2\right)}{b_2\lambda_1}A_5e^{-\lambda_1}+\frac{\left(\lambda_2^2-a_2\right)}{b_2\lambda_2}A_6e^{-\lambda_2},\\&\frac{\ell_1\left(k_1^2-a_1\right)}{b_1}\left(A_1e^{k_1}+A_2e^{-k_1}\right)\\&\quad+\frac{\ell_1\left(k_2^2-a_1\right)}{b_1}\left(A_3e^{k_2}+A_4e^{-k_2}\right)\\&\quad=\frac{\ell_2\left(\lambda_1^2-a_2\right)}{b_2}A_5e^{-\lambda_1}+\frac{\ell_2\left(\lambda_2^2-a_2\right)}{b_2}A_6e^{-\lambda_2},\\&\left[m_1k_1-n_1\left(\frac{k_1^2-a_1}{b_1k_1}\right)\right]\left(A_1e^{k_1}-A_2e^{-k_1}\right)\\&\quad+\left[m_1k_2-n_1\left(\frac{k_2^2-a_1}{b_1k_2}\right)\right]\left(A_3e^{k_2}-A_4e^{-k_2}\right)\\&\quad=-\left[m_2\lambda_1-n_2\left(\frac{\lambda_1^2-a_2}{b_2\lambda_1}\right)\right]A_5e^{-\lambda_1}\\&\quad-\left[m_2\lambda_2-n_2\left(\frac{\lambda_2^2-a_2}{b_2\lambda_2}\right)\right]A_6e^{-\lambda_2}.\end{aligned} \tag{36}$$

Solving the above system of equations we get A_1–A_6; thus we have the solution in the Laplace transformed domain. To obtain the solution in the time domain, we can use

Table 1: The constants of the problem.

$M^2 = 0.25$	$\Pi_o = 0.8$	$K_o = 2$	$ZT_o = 1$
$P_r = 0.73$	$\rho = 1.0033$	$\mu = 15$	$\sigma = 0.1$
$\kappa = 1$	$k_o = 1.005$	$c_p = 1$	$\pi = 1.009$
$\eta_1 = 0.856$	$\eta_2 = 0.905$	$\tau_{o1} = 0.015$	$\tau_{o2} = 0.1$

a numerical technique to invert the Laplace transform based on the Fourier expansion [31].

3. Numerical Results and Discussion

Two viscoelastic fluids were chosen for the purposes of numerical evaluations, namely, high density polyethylene (HDPE) and polypropylene (PP) [32–35]. The constants of the problem are shown in Table 1.

The computations were carried out for the case of two immiscible fluids (HDPE/PP) and the case of a single fluid (HDPE) on the whole region.

The temperature, the velocity, and the stress were calculated by using the numerical method of the inversion of the Laplace transform outlined above. The FORTRAN programming language was used on a personal computer. The accuracy maintained was 6 digits for the numerical program.

Figure 1 shows the temperature, the velocity, and the stress distributions for different values of time, namely, $t = 0.02, 0.03, 0.06, 0.15$, and 0.3, when all fractional parameters $\alpha_i = \beta_i$, $i = 1, 2$, are equal to one. Figures 2–5 show the temperature, the velocity, and the stress profiles for one value of time, namely, $t = 0.09$, and different values of the fractional parameters α_i, β_i, $i = 1, 2$. Figure 6 shows the thermal and mechanical effects of the fractional parameters on the two fluids.

In Figure 1, we observe that for the first two smaller values of time the temperature and stress have sharp jump (discontinuity) at the locations of the wave front while it disappears in the velocity profile. This is consistent with the physical observation that the effect of the second fluid does not appear for the small value of time while it appears for the larger time. For the larger two values of time the temperature vanishes for the case of two fluids before the case of one fluid; this is convenient with the physical phenomena of the incident waves and the reflected waves during the interface between the two fluids while the velocity and the stress have a larger value for the case of two fluids than the case of a single fluid.

Figures 2-3 show the temperature, the velocity, and the stress distributions for different values of thermal fractional parameters α_1 and α_2. These figures show that the fractional parameter α_1 has a noticeable effect on the temperature profile more than α_2, while the mechanical effects of α_1 are small on the velocity and the stress profiles and do not appear for α_2.

Figures 4-5 show the temperature, the velocity, and the stress distributions for different values of mechanical fractional parameters β_1 and β_2. These figures show that the fractional parameter β_1 has noticeable effects on the velocity and the stress profiles more than β_2, while the thermal effects of β_1 and β_2 do not appear.

Figure 6 describes the thermal effects of the fractional parameters α_1 and α_2 as shown in case (a) and the mechanical effects of β_1 and β_2 as shown in case (b).

Tables 2–7 show the temperature, the velocity, and the stress values for two values of time, namely, $t = 0.02$ and $t = 0.09$. For the smaller value of t, the function fields have a very small change for all values of the fractional parameters. This means that the effect of the presence of the second fluid disappears for small values of time, which is in agreement with the behavior of the function fields as in [23]. For the larger value of time the interaction between the two fluids is obvious in comparison with the absence of the second fluid.

Appendix

Consider

$$
\begin{aligned}
k_1 &= \frac{\sqrt{a_1 + c_1 - b_1 g_1 - \sqrt{(b_1 g_1 - a_1 - c_1)^2 - 4a_1 c_1}}}{\sqrt{2}}, \\
k_2 &= \frac{\sqrt{a_1 + c_1 - b_1 g_1 + \sqrt{(b_1 g_1 - a_1 - c_1)^2 - 4a_1 c_1}}}{\sqrt{2}}, \\
\lambda_1 &= \frac{\sqrt{a_2 + c_2 - b_2 g_2 - \sqrt{(b_2 g_2 - a_2 - c_2)^2 - 4a_2 c_2}}}{\sqrt{2}}, \\
\lambda_2 &= \frac{\sqrt{a_2 + c_2 - b_2 g_2 + \sqrt{(b_2 g_2 - a_2 - c_2)^2 - 4a_2 c_2}}}{\sqrt{2}}, \\
a_i &= \frac{\rho c_p P_r s}{\kappa + \sigma k_o^2 Z T_o} \left(1 + \frac{\tau_{oi}^{\alpha_i}}{\alpha_i!} s^{\alpha_i}\right), \\
b_i &= \frac{\sigma \pi \Pi_o}{\kappa + \sigma k_o^2 Z T_o}, \\
\ell_i &= \mu + \eta_i s^{\beta_i}, \\
c_i &= \frac{\rho s + \sigma M^2}{\ell_i}, \\
g_i &= \frac{\sigma k_o K_o}{\ell_i}, \\
m_i &= \frac{\kappa + \sigma k_o^2 Z T_o}{\left(1 + \left(\tau_{oi}^{\alpha_i}/\alpha_i!\right) s^{\alpha_i}\right)}, \\
n_i &= \frac{\sigma \pi \Pi_o}{\left(1 + \left(\tau_{oi}^{\alpha_i}/\alpha_i!\right) s^{\alpha_i}\right)}.
\end{aligned}
\tag{A.1}
$$

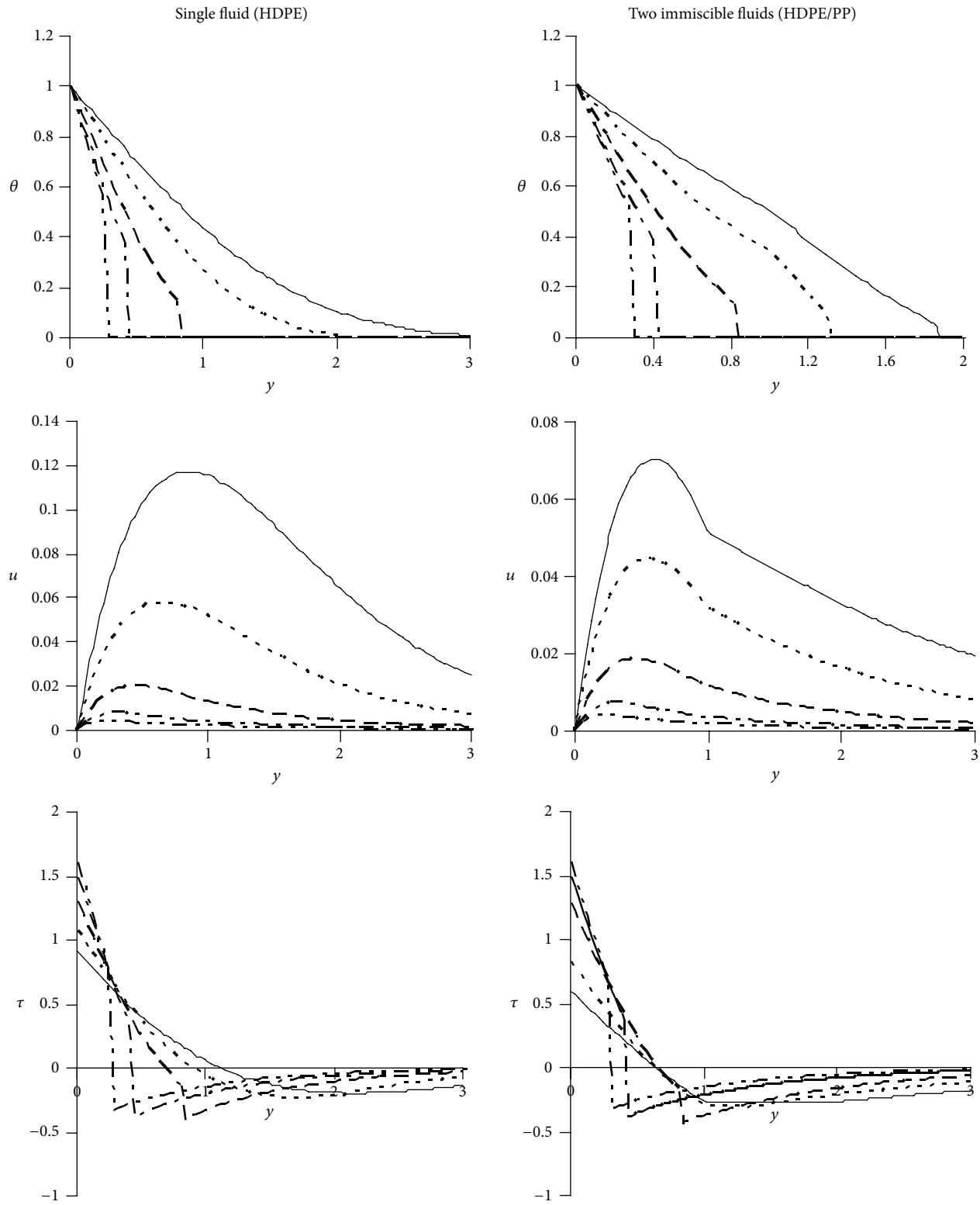

Figure 1: Temperature, velocity, and stress distribution for $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$ and $t = \{0.02$ (– · · –); 0.03 (– · –); 0.06 (– – –); 0.15 (· · ·); 0.3 (—)$\}$.

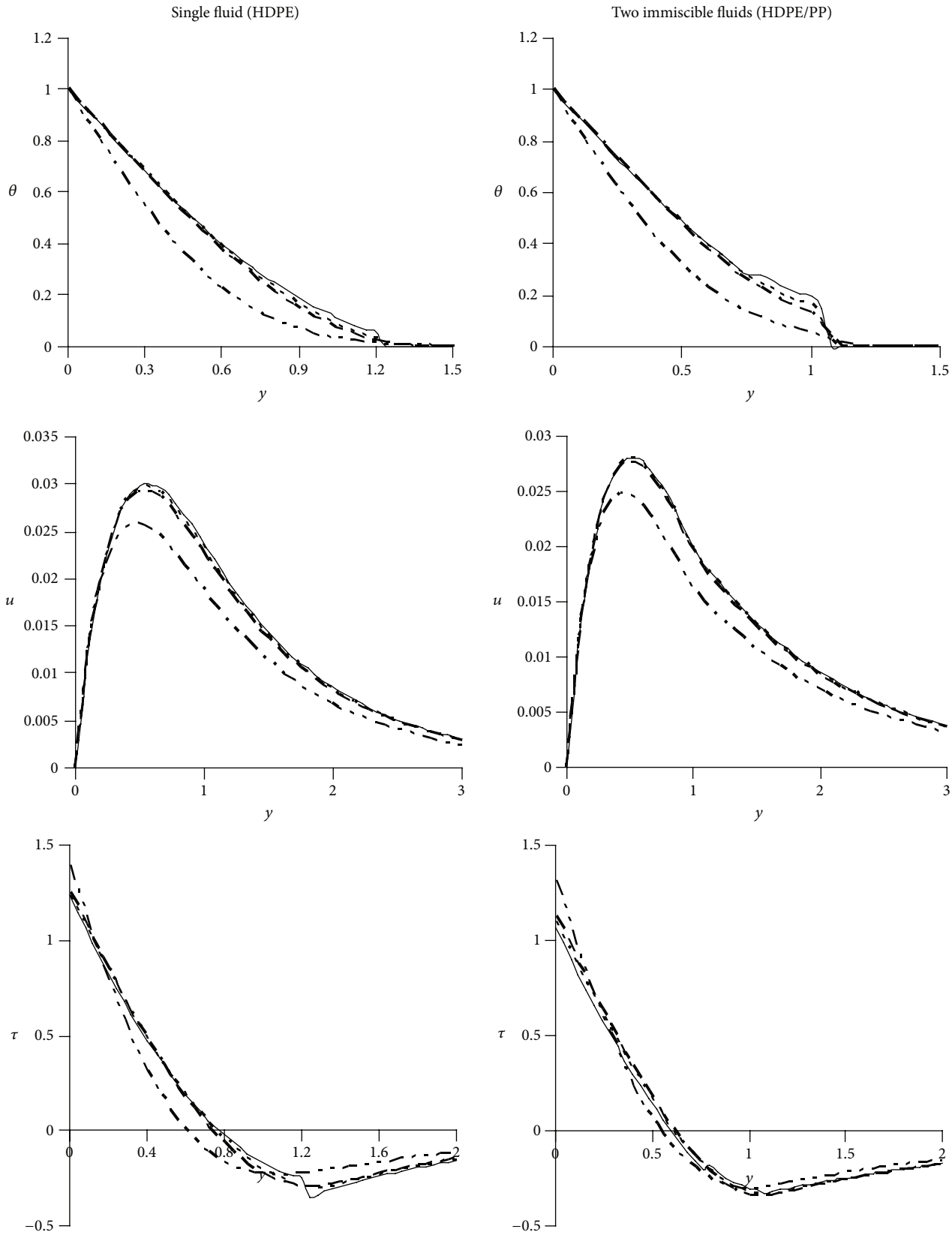

Figure 2: Temperature, velocity, and stress distribution for $\alpha_2 = \beta_1 = \beta_2 = 1$, $t = 0.09$, and $\alpha_1 = \{0.0\ (- \cdot \cdot -); 0.8\ (- - -); 0.9\ (\cdot \cdot \cdot); 1.0\ (—)\}$.

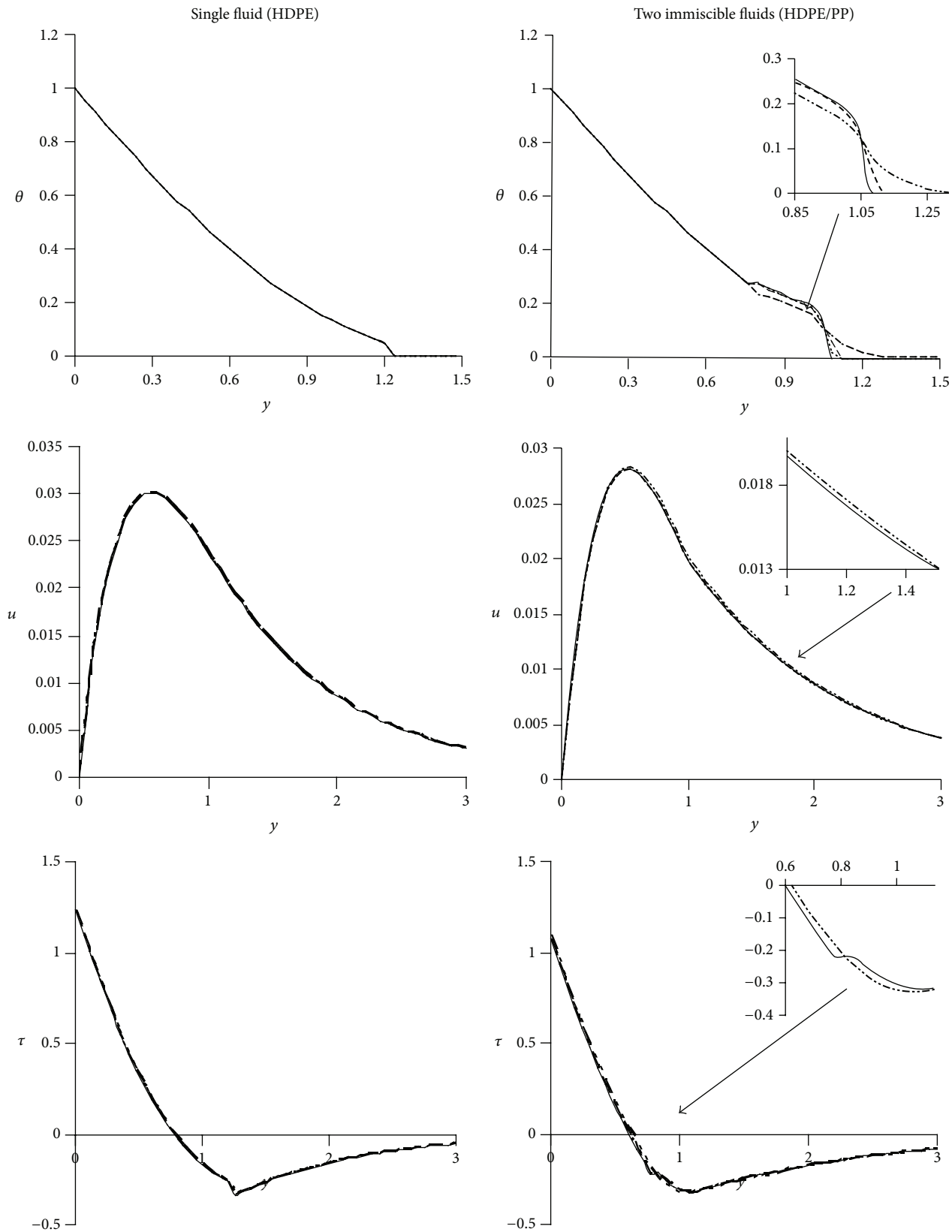

Figure 3: Temperature, velocity, and stress distribution for $\alpha_1 = \beta_1 = \beta_2 = 1$, $t = 0.09$, and $\alpha_2 = \{0.0\ (- \cdot \cdot -); 0.8\ (- - -); 0.9\ (\cdot \cdot \cdot); 1.0\ (—)\}$.

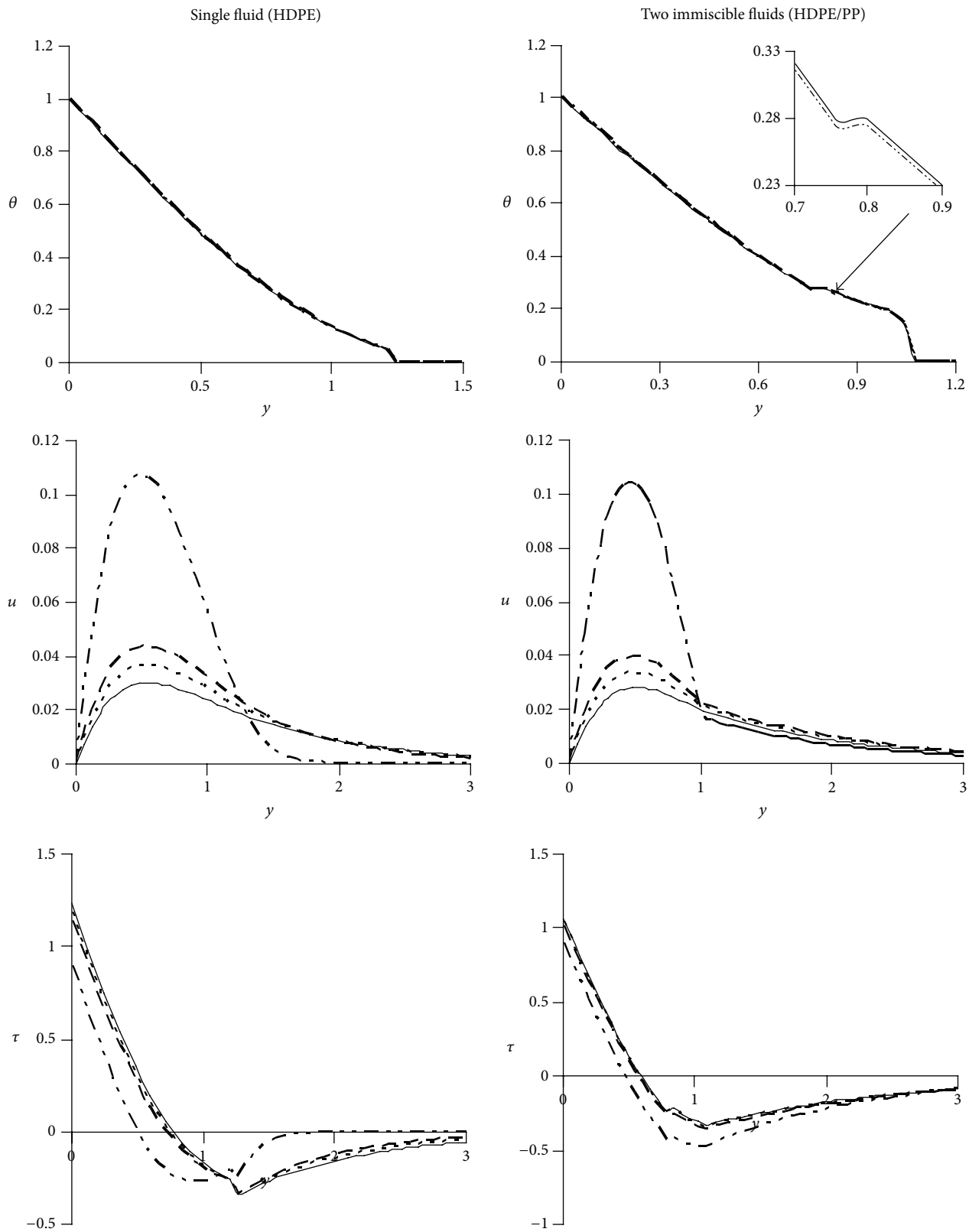

Figure 4: Temperature, velocity, and stress distribution for $\alpha_1 = \alpha_2 = \beta_2 = 1$, $t = 0.09$, and $\beta_1 = \{0.0\ (- \cdot \cdot -); 0.8\ (- - -); 0.9\ (\cdot \cdot \cdot); 1.0\ (—)\}$.

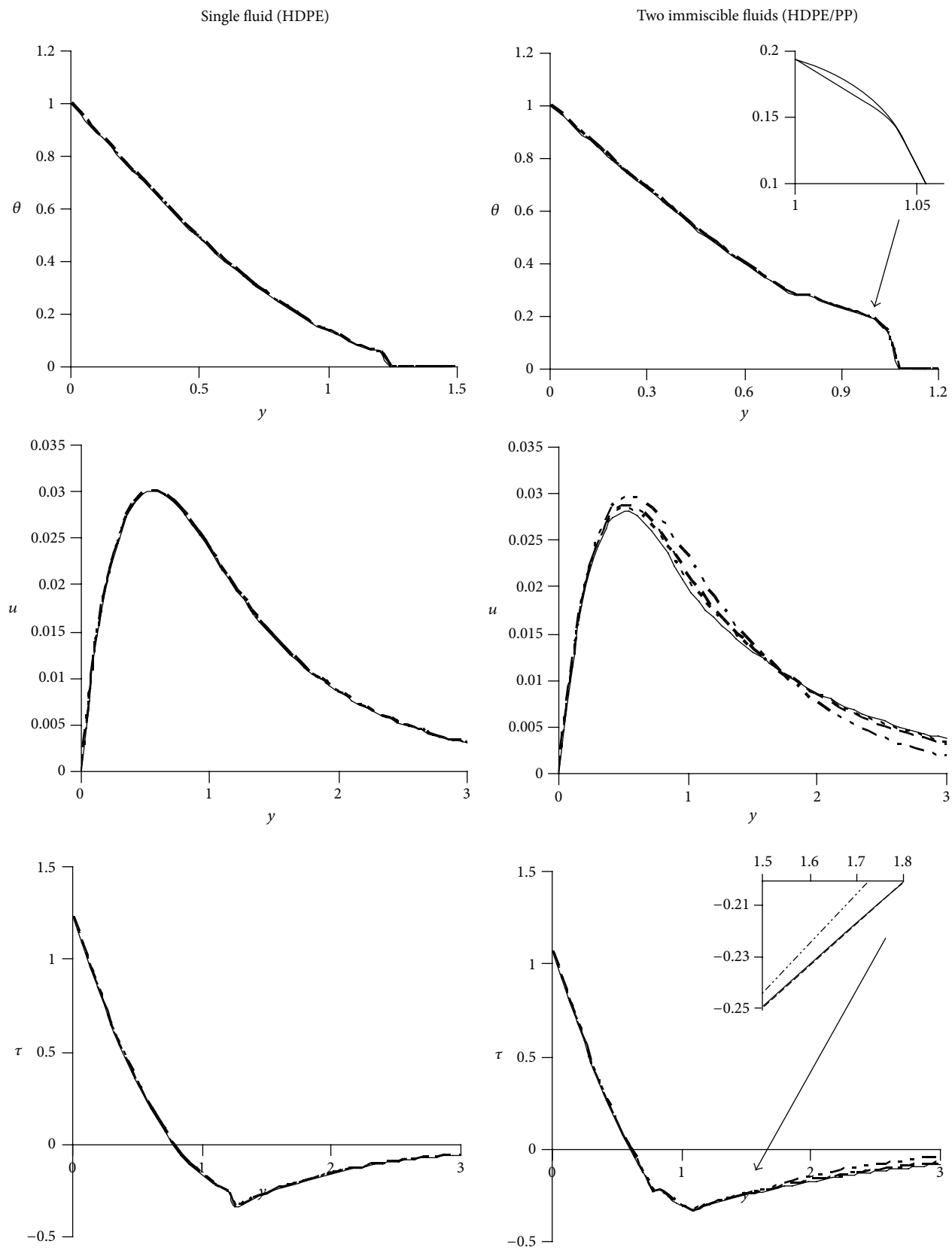

Figure 5: Temperature, velocity, and stress distribution for $\alpha_1 = \alpha_2 = \beta_1 = 1$, $t = 0.09$, and $\beta_2 = \{0.0\ (- \cdot \cdot -); 0.8\ (- - -); 0.9\ (\cdot \cdot \cdot); 1.0\ (—)\}$.

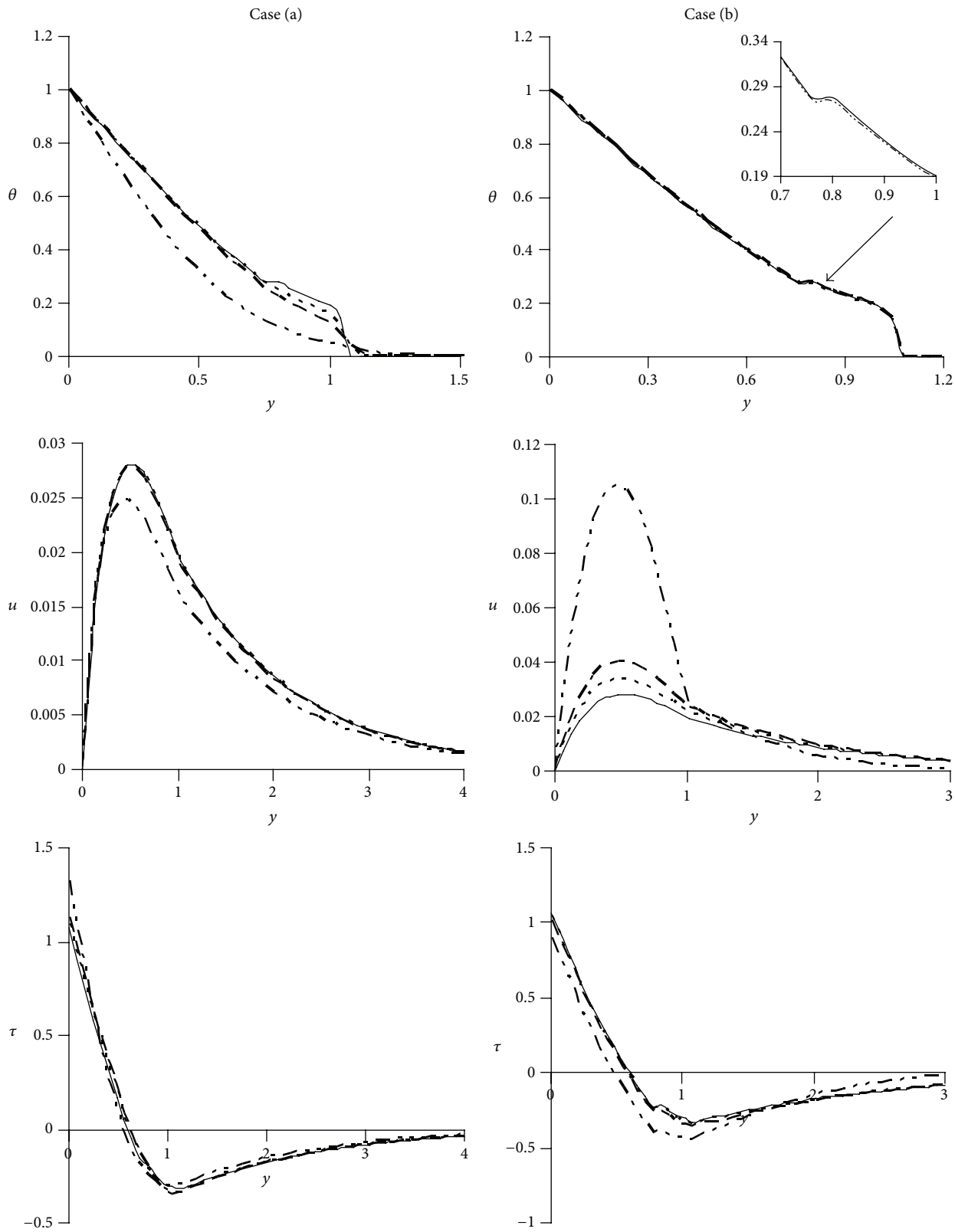

Figure 6: Temperature, velocity, and stress distribution for HDPE/PP, $t = 0.09$, and case (a) $\beta_1 = \beta_2 = 1$ and $\alpha_1 = \alpha_2 = \{0.0\ (- \cdots -); 0.8\ (- - -); 0.9\ (\cdots); 1.0\ (—)\}$ and case (b) $\alpha_1 = \alpha_2 = 1$ and $\beta_1 = \beta_2 = \{0.0\ (- \cdots -); 0.8\ (- - -); 0.9\ (\cdots); 1.0\ (—)\}$.

Table 2: Temperature values of HDPE and HDPE/PP fluids for different α_1 when $\alpha_2 = \beta_1 = \beta_2 = 1$; $t = 0.02, 0.09$.

y	θ when $\alpha_1 = 0.0$		θ when $\alpha_1 = 0.9$		θ when $\alpha_1 = 1.0$	
	HDPE	HDPE/PP	HDPE	HDPE/PP	HDPE	HDPE/PP
			$t = 0.02$			
0.0	1.000001	1.000001	1.000001	1.000001	1.000001	1.000001
0.4	0.087479	0.087478	0.0000007	0.00000008	0.0000007	0.00000008
0.8	0.000621	0.000611	0.0000005	0.00000006	0.0000005	0.00000005
1.2	0.0000007	0.000000	0.0000003	0.000000	0.0000003	0.000000
1.6	0.0000005	0.000000	0.0000002	0.000000	0.0000002	0.000000
2.0	0.0000003	0.000000	0.000000	0.000000	0.000000	0.000000
			$t = 0.09$			
0.0	1.000001	1.000001	1.000001	1.000001	1.000001	1.000001
0.4	0.420681	0.421193	0.579084	0.578967	0.582487	0.582363
0.8	0.106871	0.112543	0.234755	0.249912	0.250953	0.279957
1.2	0.015363	0.002334	0.030723	0.000204	0.054408	0.0000208
1.6	0.001059	0.0000008	0.0000311	0.0000009	0.000000	0.000000
2.0	0.0000085	0.00000005	0.000000	0.000000	0.000000	0.000000

Table 3: Temperature values of HDPE and HDPE/PP fluids for different β_1 when $\alpha_1 = \alpha_2 = \beta_2 = 1$; $t = 0.02, 0.09$.

y	θ when $\beta_1 = 0.0$		θ when $\beta_1 = 0.9$		θ when $\beta_1 = 1.0$	
	HDPE	HDPE/PP	HDPE	HDPE/PP	HDPE	HDPE/PP
			$t = 0.02$			
0.0	1.000001	1.000001	1.000001	1.000001	1.000001	1.000001
0.4	0.000031	0.000030	0.000001	0.000001	0.000001	0.000001
0.8	0.000001	0.000001	0.0000008	0.0000007	0.0000008	0.0000007
1.2	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1.6	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
			$t = 0.09$			
0.0	1.000001	1.000001	1.000001	1.000001	1.000001	1.000001
0.4	0.581747	0.581597	0.582525	0.582342	0.582487	0.582363
0.8	0.246487	0.274262	0.250745	0.279630	0.250953	0.279957
1.2	0.051544	0.0000035	0.054211	0.0000240	0.054408	0.0000208
1.6	0.0000203	0.000000	0.0000426	0.0000010	0.0000332	0.000000
2.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 4: Velocity values of HDPE and HDPE/PP fluids for different α_1 when $\alpha_2 = \beta_1 = \beta_2 = 1$; $t = 0.02, 0.09$.

y	u when $\alpha_1 = 0.0$		u when $\alpha_1 = 0.9$		u when $\alpha_1 = 1.0$	
	HDPE	HDPE/PP	HDPE	HDPE/PP	HDPE	HDPE/PP
			$t = 0.02$			
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.6	0.002939	0.003013	0.002620	0.002688	0.002616	0.002684
1.2	0.001571	0.001570	0.001397	0.001398	0.001395	0.001396
1.8	0.000838	0.000868	0.000745	0.000769	0.000744	0.000768
2.4	0.000447	0.000479	0.000397	0.000423	0.000397	0.000422
3.0	0.000238	0.000265	0.000212	0.000233	0.000211	0.000232
			$t = 0.09$			
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.6	0.025095	0.023630	0.029555	0.027560	0.029917	0.027718
1.2	0.015306	0.013589	0.018988	0.016542	0.019510	0.016752
1.8	0.008268	0.008287	0.010202	0.010043	0.010472	0.010187
2.4	0.004441	0.005030	0.005475	0.006068	0.005620	0.006164
3.0	0.002385	0.003040	0.002938	0.003651	0.003016	0.003712

Table 5: Velocity values of HDPE and HDPE/PP fluids for different β_1 when $\alpha_1 = \alpha_2 = \beta_2 = 1$; $t = 0.02, 0.09$.

y	u when $\beta_1 = 0.0$		u when $\beta_1 = 0.9$		u when $\beta_1 = 1.0$	
	HDPE	HDPE/PP	HDPE	HDPE/PP	HDPE	HDPE/PP
			$t = 0.02$			
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.6	0.002372	0.002243	0.003562	0.003525	0.002616	0.002684
1.2	0.000000	0.000001	0.001608	0.001491	0.001395	0.001396
1.8	0.000000	0.000000	0.000725	0.000820	0.000744	0.000768
2.4	0.000000	0.000000	0.000327	0.000451	0.000397	0.000422
3.0	0.000000	0.000000	0.000147	0.000248	0.000211	0.000232
			$t = 0.09$			
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.6	0.103453	0.096869	0.036271	0.032740	0.029917	0.027718
1.2	0.029575	0.014052	0.022237	0.017948	0.019510	0.016752
1.8	0.000684	0.008051	0.010770	0.010925	0.010472	0.010187
2.4	0.000007	0.004603	0.005210	0.006615	0.005620	0.006164
3.0	0.000000	0.002627	0.002518	0.000000	0.003016	0.000000

Table 6: Stress values of HDPE and HDPE/PP fluids for different α_1 when $\alpha_2 = \beta_1 = \beta_2 = 1$; $t = 0.02, 0.09$.

y	τ when $\alpha_1 = 0.0$		τ when $\alpha_1 = 0.9$		τ when $\alpha_1 = 1.0$	
	HDPE	HDPE/PP	HDPE	HDPE/PP	HDPE	HDPE/PP
			$t = 0.02$			
0.0	1.662574	1.648661	1.631704	1.617246	1.623545	1.608846
0.6	−0.19194	−0.201456	−0.225428	−0.234513	−0.22988	−0.23904
1.2	−0.11365	−0.123767	−0.120332	−0.129629	−0.12270	−0.13202
1.8	−0.06069	−0.069907	−0.064232	−0.072580	−0.06549	−0.07385
2.4	−0.032418	−0.039437	−0.034286	−0.040603	−0.034957	−0.041275
3.0	−0.017314	−0.022221	−0.018302	−0.022696	−0.018659	−0.023051
			$t = 0.09$			
0.0	1.392190	1.323119	1.241577	1.100666	1.231832	1.070555
0.6	−0.00054	−0.079583	0.183463	0.013853	0.190674	−0.003849
1.2	−0.22762	−0.277199	−0.29069	−0.315591	−0.25326	−0.303997
1.8	−0.13970	−0.180148	−0.19048	−0.207686	−0.19573	−0.202145
2.4	−0.07573	−0.115562	−0.10268	−0.134565	−0.10550	−0.132031
3.0	−0.04088	−0.073419	−0.05534	−0.086082	−0.05687	−0.084999

Table 7: Stress values of HDPE and HDPE/PP fluids for different β_1 when $\alpha_1 = \alpha_2 = \beta_2 = 1$; $t = 0.02, 0.09$.

y	τ when $\beta_1 = 0.0$		τ when $\beta_1 = 0.9$		τ when $\beta_1 = 1.0$	
	HDPE	HDPE/PP	HDPE	HDPE/PP	HDPE	HDPE/PP
			$t = 0.02$			
0.0	0.938168	0.931532	1.553433	1.530803	1.623545	1.608846
0.6	−0.051838	−0.048786	−0.246749	−0.264793	−0.229881	−0.239039
1.2	−0.000012	−0.000402	−0.114039	−0.141048	−0.122702	−0.132024
1.8	0.000000	−0.000216	−0.052630	−0.078897	−0.065493	−0.073849
2.4	0.000000	−0.000116	−0.024256	−0.044096	−0.034957	−0.041275
3.0	0.000000	−0.000062	−0.011164	−0.024627	−0.018659	−0.023051
			$t = 0.09$			
0.0	0.903874	0.900399	1.182936	1.043413	1.231832	1.070555
0.6	−0.12195	−0.195861	0.155454	−0.024652	0.190674	−0.003849
1.2	−0.21473	−0.435217	−0.25726	−0.318253	−0.25326	−0.303997
1.8	−0.00902	−0.264756	−0.18232	−0.212666	−0.19573	−0.202145
2.4	−0.00013	−0.159753	−0.09039	−0.139431	−0.10551	−0.132031
3.0	−0.00001	−0.095721	−0.04474	−0.090033	−0.05687	−0.084999

Nomenclature

B: Magnetic induction
c_{pi}: Specific heat at constant pressure of region-i
D_t^{ζ}: Caputo fractional time derivative operator of order $0 < \zeta \leq 1$
E: Electric density
F: Lorentz force
H_o: Constant magnetic field
$H(t)$: Heaviside unit step function
J: Conduction current density vector
κ_i: Thermal conductivity of region-i
k_{oi}: Seebeck coefficient of region-i
M^2: Hartmann number
P_r: Prandtl number
$\mathbf{q}_i$: Heat conduction vector of region-i
t: Time
T_i: Temperature of the fluid in region-i
T_∞: Temperature of the fluid away from the plate
T_o: Reference temperature $T_o = T_1 - T_2$
ZT: Dimensionless figure-of-merit
α_i: Thermal fractional parameters of region-i
α_{1i}: Material modulus of region-i
β_i: Mechanical fractional parameters of region-i
μ_i: Viscosity of region-i
ρ_i: Density of region-i
σ_i: Electrical conductivity of region-i
η_i: Dimensionless viscoelastic parameter of region-i
υ_1: Kinematic viscosity in Region-1 (μ_1/ρ_1)
π_i: Peltier coefficient of region-i
τ_{oi}: Thermal relaxation time of region-i
τ_i: Stress tensor T_{xy} of region-i.

Subscripts

$i = 1$: Fluid in Region-1
$i = 2$: Fluid in Region-2.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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