Research Article

Effect of Variable Properties and Moving Heat Source on Magnetothermoelastic Problem under Fractional Order Thermoelasticity

Chunbao Xiong and Ying Guo

School of Civil Engineering, Tianjin University, Tianjin 300072, China

Correspondence should be addressed to Ying Guo; gytha_ying@tju.edu.cn

A one-dimensional generalized magnetothermoelastic problem of a thermoelastic rod with finite length is investigated in the context of the fractional order thermoelasticity. The rod with variable properties, which are temperature-dependent, is fixed at both ends and placed in an initial magnetic field, and the rod is subjected to a moving heat source along the axial direction. The governing equations of the problem in the fractional order thermoelasticity are formulated and solved by means of Laplace transform in tandem with its numerical inversion. The distributions of the nondimensional temperature, displacement, and stress in the rod are obtained and illustrated graphically. The effects of the temperature-dependent properties, the velocity of the moving heat source, the fractional order parameter, and so forth on the considered variables are concerned and discussed in detail, and the results show that they significantly influence the variations of the considered variables.

1. Introduction

The classical coupled thermoelasticity proposed by Biot [1] predicts an infinite speed for heat propagation, which is physically impossible. To eliminate such an inherent paradox, generalized thermoelastic theories such as Lord and Shulman's theory (L-S) [2] and Green and Lindsay's theory (G-L) [3] were developed in response. The L-S theory was the first description of generalized thermoelasticity in which a wave-type heat conduction law was postulated to replace the classical Fourier's law; this wave-type law is the same as that suggested by Catteneo [4] and Vernotte [5]. On the basis of these generalized models, many previous researchers have attempted to accurately capture thermomechanical behavior [6–11].

The generalized electromagnetothermoelastic problem in thermoelastic solids has attracted considerable research attention due to its extensive potential applications in diverse fields. Examples include understanding the effects of Earth's magnetic field on seismic waves, the damping of acoustic waves in magnetic fields, and the emissions of electromagnetic radiation from nuclear devices. He and Cao [12] investigated the magnetothermoelastic problem of a thin slim strip placed in a magnetic field and subjected to a moving plane of heat source. Sherief and Khader [13] studied wave propagation for a problem of an infinitely long, solid conducting circular cylinder with a traction-free lateral surface subjected to known surrounding temperatures in the presence of a uniform magnetic field in the axis direction. Abbas and Zenkour [14] presented the electromagnetothermoelastic analysis problem of an infinite functionally graded material hollow cylinder. Sarkar [15] investigated an electromagnetothermoelastic coupled problem for a half-space subjected to a thermal shock under different generalized thermoelastic theories. Pal et al. [16] dealt with the problem of magnetothermoelastic interactions in a rotating medium due to a periodically varying heat source in the context of generalized thermoelasticity. Singh and Chakraborty [17] studied the effects of the magnetic field and initial stress on the reflection of a plane magnetothermoelastic wave from the

boundary of a solid half-space. Said [18] solved a generalized magnetothermoelastic problem for a half-space with G-N theory.

Fractional calculus has been used successfully to modify many existing models of physical processes, especially in the field of heat conduction, diffusion, viscoelasticity, solids mechanics, control theory, and electricity [19–22]. Povstenko [23] proposed a quasistatic uncoupled theory of thermoelasticity which is based on the heat conduction equation with a time-fractional order derivative. In 2010, Youssef [24] introduced the Riemann-Liouville fractional integral operator into the generalized heat conduction and constructed the theory of fractional order generalized thermoelasticity. By employing this theory, Sarkar and Lahiri [25] concerned a two-dimensional generalized thermoelastic problem for a rotating elastic medium. Youssef [26] dealt with a two-temperature generalized thermoelastic medium subjected to a moving heat source in the context of fractional order generalized thermoelasticity. Yu et al. [27] formulated the fractional order generalized electromagnetothermoelastic theory and presented the effect of fractional order parameter. Youssef and Abbas [28] solved a one-dimensional problem of an elastic half-space in the context of fractional order generalized thermoelasticity. Song et al. [29] studied the vibration of microcantilevers during a photothermal process by coupling the theories of fractional order heat conduction and elastic waves. Abbas and Youssef [30] investigated a two-dimensional problem of a porous half-space with a traction-free surface and a constant heat flux with fractional order generalized thermoelasticity theory. Recently, a completely new fractional order generalized thermoelasticity theory was introduced by Sherief et al. [31]. Based on this theory, Kothari and Mukhopadhyay [32] solved an elastic half-space problem via Laplace transform and state-space method. Sherief and Abd El-Latief [33] investigated a half-space problem with varying extent of thermal conductivity. He and Guo [34] investigated a one-dimensional problem for a rod subjected to a moving heat source. Sherief and Abd El-Latief [35] investigated a one-dimensional thermal shock problem for a half-space in the context of the fractional order theory of thermoelasticity and later solved a two-dimensional problem for a traction-free half-space surface subjected to a heating in the context of the same theory [36]. Abbas [37] solved the problem of fractional order thermoelastic interaction for a material placed in a magnetic field and subjected to a moving plane of heat source. Sherief and Abd El-Latief [38] solved a one-dimensional problem with a spherical cavity subjected to a thermal shock with fractional order theory of thermoelasticity. Ma and He [39] dealt with a generalized piezoelectric-thermoelastic problem subjected to a moving heat source in the context of the fractional order theory of thermoelasticity.

To explore the effects of temperature-dependent properties on predicting the dynamic behavior of problems under generalized thermoelastic theories, Ezzat et al. [40] investigated a problem in which modulus of elasticity was dependent on temperature. Allam et al. [41] studied the electromagnetothermoelastic interactions in an infinite perfectly conducting body with a spherical cavity with G-N theory.

Xiong and Tian [42] investigated the magnetothermoelastic problem of a semi-infinite body with voids and temperature-dependent material properties placed in a transverse magnetic field. Abouelregal [43] solved a one-dimensional boundary value problem of a semi-infinite piezoelectric medium with temperature-dependent properties under the theory of fractional order. The problem of the generalized thermoelastic medium for three different theories under the effects of a gravitational field was investigated by Othman et al. [44]. Pal et al. [45] dealt with a thermoelastic problem of a cylindrical cavity subjected to time-dependent thermal and mechanical shocks in the context of fractional order generalized thermoelasticity with the L-S model and G-N model. The generalized magnetothermoelastic problem of an infinite homogeneous isotropic microstretch half-space with temperature-dependent material properties placed in a transverse magnetic field was investigated in the context of different generalized thermoelastic theories by Xiong and Tian [46]. Wang et al. [47] focused on a thermoelastic problem for an elastic medium with variable properties, which they constructed in the context of the fractional order heat conduction.

To date, there are relatively few works devoted to the investigation of electromagnetothermoelastic problems involving heat source and temperature-dependent properties in the context of the fractional order theory of thermoelasticity. The present paper is devoted to our investigation of a generalized magnetothermoelastic medium with temperature-dependent properties subjected to a moving heat source in the fractional order theory, as proposed by Sherief et al. [31].

2. Basic Equations

The generalized magnetothermoelastic governing equations of an isotropic homogeneous conducting elastic medium under fractional order theory take the following forms:

$$\sigma_{ij,j} + (J \times B)_i = \rho \ddot{u}_i \tag{1}$$

$$B = \mu_0 H \tag{2}$$

$$J = \sigma_0 \left(E + \dot{u} \times B \right) \tag{3}$$

$$\sigma_{ij} = 2\mu e_{ij} + \left(\lambda e_{kk} - \gamma \theta \right) \delta_{ij} \tag{4}$$

$$e_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} \right) \tag{5}$$

$$q_{i,i} = -\rho T_0 \dot{\eta} + Q \tag{6}$$

$$\rho \eta = \gamma e_{kk} + \frac{\rho C_E}{T_0} \theta \tag{7}$$

$$\kappa_{ij} T_{,ij} = \left(1 + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \left(\rho C_E \dot{T} + T_0 \lambda_{ij} \dot{e}_{ij} - Q \right) \tag{8}$$

$$0 < \alpha \leq 1.$$

The fractional derivative is defined according to Sherief et al. [31]:

$$\frac{\partial^\alpha}{\partial t^\alpha} f(x,t) = \begin{cases} f(x,t) - f(x,0) & \alpha \longrightarrow 0 \\ I^{1-\alpha} \dfrac{\partial f(x,t)}{\partial t} & 0 < \alpha < 1 \\ \dfrac{\partial f(x,t)}{\partial t} & \alpha = 1, \end{cases} \quad (9)$$

where the Riemann-Liouville fractional integral I^α is introduced as a natural generalization of the well-known α-fold repeated integral $I^\alpha f(t)$ which is written in a convolution-type form as follows:

$$I^\alpha f(t) = \int_0^t \frac{1}{\Gamma(\alpha)} (t-s)^{\alpha-1} f(s) \, ds, \quad (10)$$

where $\Gamma(\alpha)$ is the well-known Gamma function and $f(t)$ is Lebesgue's integrable function. Equation (8) describes the process of heat conduction in the whole spectrum. The different values of the fractional parameter within the range $0 < \alpha \leq 1$ cover two cases of conductivity: ($0 < \alpha < 1$) for weak conductivity and $\alpha = 1$ for normal conductivity.

Here, we investigate the problem of a generalized magnetothermoelastic rod with temperature-dependent properties subjected to a moving heat source in the context of the fractional order theory of thermoelasticity. A magnetic field with constant intensity $H = (0, H_0, 0)$ acts perpendicularly to the axial direction of the rod, which is fixed at both ends and subjected to a moving heat source propagating along the x direction. The dimension along the x-axis is assumed to be much greater than those along the other two directions (y, z) orthogonal to the x-axis; thus, the problem can be treated as a one-dimensional problem.

For the one-dimensional problem, the components of the electromagnetic induction vector are given by

$$\begin{aligned} B_x = B_z &= 0, \\ B_y &= \mu_0 H_0, \end{aligned} \quad (11)$$

while the components of Lorentz force $F = J \times B$ in the motion equation (1) can be given by

$$\begin{aligned} F_x &= -\sigma_0 \mu_0^2 H_0^2, \\ F_y = F_z &= 0. \end{aligned} \quad (12)$$

For the one-dimensional problem, the components of the displacement are

$$\begin{aligned} u_x &= u(x,t), \\ u_y = u_z &= 0. \end{aligned} \quad (13)$$

The governing equations can be simplified as follows:

$$\sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma(T - T_0),$$

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \sigma_0 \mu_0^2 H_0^2 \frac{\partial u}{\partial t} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (14)$$

$$\kappa_{ij} \frac{\partial^2 T}{\partial x^2} = \left(1 + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha}\right) \left(\rho c_E \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial^2 u}{\partial x \partial t} - Q\right).$$

We consider a thermoelastic body of material with temperature-dependent properties in the following form [41]:

$$\begin{aligned} \lambda &= \lambda_0 f(T), \\ \mu &= \mu_0 f(T), \\ \kappa &= \kappa_0 f(T), \\ \gamma &= \gamma_0 f(T), \end{aligned} \quad (15)$$

where κ is the thermal conductivity, λ_0, μ_0, γ_0, and κ_0 are considered to be constants, and $f(T)$ is given in a nondimensional function of temperature. In the case of temperature-independent modulus of elasticity, $f(T) = 1$. We will assume the following [41]:

$$f(T) = 1 - \zeta T, \quad (16)$$

where ζ is the empirical material constant.

In generalized thermoelasticity, as well as in the coupled theory, only the infinitesimal temperature deviations from the reference temperature are considered. For linearity of the governing partial differential equations of the problem, we have to take into account the condition $|T - T_0|/T_0 \ll 1$, which gives the approximating function of $f(T)$ in the following form:

$$f(T) \approx 1 - \zeta T_0. \quad (17)$$

For convenience, the following nondimensional quantities are also introduced:

$$\begin{aligned} x^* &= c_0 \eta_0 x, \\ u^* &= c_0 \eta_0 u, \\ t^* &= c_0^2 \eta_0 t, \\ \tau_0{}^* &= c_0^2 \eta_0 \tau_0, \\ \theta^* &= \frac{T - T_0}{T_0}, \\ \sigma^* &= \frac{\sigma}{\mu}, \\ Q^* &= \frac{Q}{k T_0 c_0^2 \eta_0^2}, \\ c_0^2 &= \frac{\lambda + 2\mu}{\rho}, \\ \eta_0 &= \frac{\rho C_E}{k}. \end{aligned} \quad (18)$$

In terms of the nondimensional variables in (18), (14) take the following forms (dropping the asterisks for convenience):

$$\vartheta\sigma = \beta^2 \frac{\partial u}{\partial x} - b\theta$$

$$\frac{\partial^2 u}{\partial x^2} - \frac{b}{\beta^2}\frac{\partial \theta}{\partial x} = \vartheta\left(\frac{\partial^2 u}{\partial t^2} + \varepsilon\frac{\partial u}{\partial t}\right) \qquad (19)$$

$$\frac{\partial^2 \theta}{\partial x^2} = \left(1 + \frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\vartheta\frac{\partial \theta}{\partial t} + g\frac{\partial^2 u}{\partial x\partial t} - Q\right),$$

where

$$\vartheta = \frac{1}{1 - \zeta T_0},$$

$$\beta^2 = \frac{\lambda + 2\mu}{\mu},$$

$$b = \frac{\gamma T_0}{\mu}, \qquad (20)$$

$$g = \frac{\gamma}{\vartheta\rho C_E},$$

$$\varepsilon = \frac{\sigma_0\mu_0^2 H_0^2}{\eta_0(\lambda + 2\mu)}.$$

The rod is assumed to have reference temperature T_0 and homogeneous initial conditions:

$$u(x, 0) = \dot{u}(x, 0) = 0,$$
$$\theta(x, 0) = \dot{\theta}(x, 0) = 0. \qquad (21)$$

The rod is fixed at both ends with a nondimensional length l, so the boundary conditions are

$$u(0, t) = u(l, t) = 0,$$

$$\frac{\partial \theta(0, t)}{\partial x} = \frac{\partial \theta(l, t)}{\partial x} = 0. \qquad (22)$$

The heat source is moving along the x-axis with a constant velocity v, which can be described as follows:

$$Q = Q_0\delta(x - vt), \qquad (23)$$

where Q_0 is a constant and δ is the delta function.

Applying the Laplace transform defined by

$$L[f(t)] = \overline{f}(p) = \int_0^\infty e^{-pt} f(t)\, dt \quad \text{Re}(p) > 0 \qquad (24)$$

to (19) with (21), we obtain

$$\vartheta\overline{\sigma} = \beta^2 \frac{d\overline{u}}{dx} - b\overline{\theta}, \qquad (25)$$

$$\frac{d^2\overline{u}}{dx^2} - \frac{b}{\beta^2}\frac{d\overline{\theta}}{dx} = \vartheta p(p + \varepsilon)\overline{u}, \qquad (26)$$

$$\frac{d^2\overline{\theta}}{dx^2} = \left(1 + \frac{\tau_0^\alpha}{\alpha!}p^\alpha\right)\left(p\vartheta\overline{\theta} + gp\frac{d\overline{u}}{dx} - \omega\vartheta e^{-(p/v)x}\right), \qquad (27)$$

where

$$\omega = \frac{Q_0}{v}. \qquad (28)$$

By applying the Laplace transform, the boundary conditions in (21) can be expressed as follows:

$$\overline{u}(0, p) = \overline{u}(l, p) = 0,$$

$$\frac{d\overline{\theta}(0, p)}{dx} = \frac{d\overline{\theta}(l, p)}{dx} = 0. \qquad (29)$$

3. Solutions in the Laplace Domain

Eliminating $\overline{\theta}$ between (26) and (27), we obtain the following equation satisfied by $\overline{u}$:

$$\frac{d^4\overline{u}}{dx^4} - m_1\frac{d^2\overline{u}}{dx^2} + m_2\overline{u} = m_3 e^{-(p/v)x}, \qquad (30)$$

where

$$m_1 = \left(\vartheta + \frac{gb}{\beta^2}\right)\left(1 + \frac{\tau_0^\alpha}{\alpha!}p^\alpha\right)p + \vartheta p(p + \varepsilon),$$

$$m_2 = \left(1 + \frac{\tau_0^\alpha}{\alpha!}p^\alpha\right)\vartheta^2 p^2(p + \varepsilon), \qquad (31)$$

$$m_3 = \frac{b\omega p\vartheta(1 + (\tau_0^\alpha/\alpha!)p^\alpha)}{\beta^2 v}.$$

The general solution of (30) is

$$\overline{u} = C_1 e^{-k_1 x} + C_2 e^{k_1 x} + C_3 e^{-k_2 x} + C_4 e^{k_2 x} + C_5 e^{-(p/v)x}, \qquad (32)$$

where C_i $(i = 1, 2, 3, 4)$ are parameters depending on p to be determined from the boundary conditions, and

$$C_5 = \frac{m_3}{\left[(p/v)^4 - m_1(p/v)^2 + m_2\right]}, \qquad (33)$$

where k_1 and k_2 are the roots of the following characteristic equation:

$$k^4 - m_1 k^2 + m_2 = 0, \qquad (34)$$

and k_1, k_2 are given by

$$k_1 = \sqrt{\frac{m_1 + \sqrt{m_1^2 - 4m_2}}{2}},$$

$$k_2 = \sqrt{\frac{m_1 - \sqrt{m_1^2 - 4m_2}}{2}}. \qquad (35)$$

Similarly, eliminating $\overline{u}$ between (26) and (27), we obtain

$$\frac{d^4\overline{\theta}}{dx^4} - m_1\frac{d^2\overline{\theta}}{dx^2} + m_2\overline{\theta} = m_4 e^{-(p/v)x}, \qquad (36)$$

where

$$m_4 = \omega\vartheta\left(1 + \frac{\tau_0^\alpha}{\alpha!}p^\alpha\right)\left[\vartheta p\left(p + \varepsilon\right) - \frac{p^2}{v^2}\right]. \tag{37}$$

The general solution of (36) is

$$\overline{\theta} = C_{11}e^{-k_1 x} + C_{22}e^{k_1 x} + C_{33}e^{-k_2 x} + C_{44}e^{k_2 x}$$
$$+ C_{55}e^{-(p/v)x}, \tag{38}$$

where the parameters of C_{ii} $(i = 1, 2, 3, 4)$ are dependent on p.

Substituting $\overline{u}$ from (32) and $\overline{\theta}$ from (38) into (26), the following relationships become clear:

$$C_{11} = -\frac{\beta^2\left[k_1^2 - \vartheta p\left(p + \varepsilon\right)\right]}{bk_1}C_1,$$

$$C_{22} = \frac{\beta^2\left[k_1^2 - \vartheta p\left(p + \varepsilon\right)\right]}{bk_1}C_2,$$

$$C_{33} = -\frac{\beta^2\left[k_2^2 - \vartheta p\left(p + \varepsilon\right)\right]}{bk_2}C_3, \tag{39}$$

$$C_{44} = \frac{\beta^2\left[k_2^2 - \vartheta p\left(p + \varepsilon\right)\right]}{bk_2}C_4,$$

$$C_{55} = \frac{\beta^2\left[v^2\vartheta\left(p + \varepsilon\right) - p\right]}{bv}C_5.$$

To obtain the parameters of C_i $(i = 1, 2, 3, 4)$ and C_{ii} $(i = 1, 2, 3, 4)$, (32) and (38) are substituted into the equation of boundary conditions as follows:

$$C_1 + C_2 + C_3 + C_4 = -C_5,$$

$$C_1 e^{-k_1 l} + C_2 e^{k_1 l} + C_3 e^{-k_2 l} + C_4 e^{k_2 l} = -C_5 e^{-(p/v)l},$$

$$-C_{11}k_1 + C_{22}k_1 - C_{33}k_2 + C_{44}k_2 = \left(\frac{p}{v}\right)C_{55}, \tag{40}$$

$$-C_{11}k_1 e^{-k_1 l} + C_{22}k_1 e^{k_1 l} - C_{33}k_2 e^{-k_2 l} + C_{44}k_2 e^{k_2 l}$$
$$= \left(\frac{p}{v}\right)C_{55}e^{-(p/v)l}.$$

Solving (40), we obtain C_i $(i = 1, 2, 3, 4)$ as follows:

$$C_1 = \frac{\left(k_2^2 - p^2/v^2\right)\left(e^{k_1 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}C_5,$$

$$C_2 = -\frac{\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}C_5,$$

$$\tag{41}$$

$$C_3 = -\frac{\left(k_1^2 - p^2/v^2\right)\left(e^{k_2 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)}C_5,$$

$$C_4 = \frac{\left(k_1^2 - p^2/v^2\right)\left(e^{-k_2 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)}C_5.$$

Substituting (41) into (32), we obtain

$$\overline{u} = \frac{\left(k_2^2 - p^2/v^2\right)\left(e^{k_1 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}C_5 e^{-k_1 x}$$

$$- \frac{\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}C_5 e^{k_1 x}$$

$$- \frac{\left(k_1^2 - p^2/v^2\right)\left(e^{k_2 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)}C_5 e^{-k_2 x} \tag{42}$$

$$+ \frac{\left(k_1^2 - p^2/v^2\right)\left(e^{-k_2 l} - e^{-(p/v)l}\right)}{\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)}C_5 e^{k_2 x}$$

$$+ C_5 e^{-(p/v)x}.$$

The relationship between C_i and C_{ii} in (39) gives

$$C_{11} = -\frac{\beta^2\left[k_1^2 - \vartheta p\left(p + \varepsilon\right)\right]\left(k_2^2 - p^2/v^2\right)\left(e^{k_1 l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}$$
$$\cdot C_5,$$

$$C_{22}$$
$$= -\frac{\beta^2\left[k_1^2 - \vartheta p\left(p + \varepsilon\right)\right]\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1 l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}$$
$$\cdot C_5,$$

$$C_{33} = \frac{\beta^2\left[k_2^2 - \vartheta p\left(p + \varepsilon\right)\right]\left(k_1^2 - p^2/v^2\right)\left(e^{k_2 l} - e^{-(p/v)l}\right)}{bk_2\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)} \tag{43}$$
$$\cdot C_5,$$

$$C_{44} = \frac{\beta^2\left[k_2^2 - \vartheta p\left(p + \varepsilon\right)\right]\left(k_1^2 - p^2/v^2\right)\left(e^{-k_2 l} - e^{-(p/v)l}\right)}{bk_2\left(k_1^2 - k_2^2\right)\left(e^{k_2 l} - e^{-k_2 l}\right)}$$
$$\cdot C_5,$$

$$C_{55} = \frac{\beta^2\left[v^2\vartheta\left(p + \varepsilon\right) - p\right]}{bv}C_5.$$

Substituting (43) into (38), we obtain

$$\overline{\theta} = -\frac{\beta^2\left[k_1^2 - \vartheta p\left(p + \varepsilon\right)\right]\left(k_2^2 - p^2/v^2\right)\left(e^{k_1 l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}$$

$$\cdot C_5 e^{-k_1 x}$$

$$- \frac{\beta^2\left[k_1^2 - \vartheta p\left(p + \varepsilon\right)\right]\left(k_2^2 - p^2/v^2\right)\left(e^{-k_1 l} - e^{-(p/v)l}\right)}{bk_1\left(k_1^2 - k_2^2\right)\left(e^{k_1 l} - e^{-k_1 l}\right)}$$

$$\cdot C_5 e^{k_1 x}$$

$$+ \frac{\beta^2 \left[k_2{}^2 - \vartheta p \left(p + \varepsilon\right)\right] \left(k_1{}^2 - p^2/v^2\right) \left(e^{k_2 l} - e^{-(p/v)l}\right)}{bk_2 \left(k_1{}^2 - k_2{}^2\right) \left(e^{k_2 l} - e^{-k_2 l}\right)}$$

$$\cdot C_5 e^{-k_2 x}$$

$$+ \frac{\beta^2 \left[k_2{}^2 - \vartheta p \left(p + \varepsilon\right)\right] \left(k_1{}^2 - p^2/v^2\right) \left(e^{-k_2 l} - e^{-(p/v)l}\right)}{bk_2 \left(k_1{}^2 - k_2{}^2\right) \left(e^{k_2 l} - e^{-k_2 l}\right)}$$

$$\cdot C_5 e^{k_2 x} + \frac{\beta^2 \left[v^2 \vartheta \left(p + \varepsilon\right) - p\right]}{bv} C_5 e^{-(p/v)x}. \tag{44}$$

Substituting (42) and (44) into (25) yields

$$\bar{\sigma} = -\frac{\beta^2 p \left(p + \varepsilon\right) \left(k_2{}^2 - p^2/v^2\right) \left(e^{k_1 l} - e^{-(p/v)l}\right)}{k_1 \left(k_1{}^2 - k_2{}^2\right) \left(e^{k_1 l} - e^{-k_1 l}\right)}$$

$$\cdot C_5 e^{-k_1 x}$$

$$- \frac{\beta^2 p \left(p + \varepsilon\right) \left(k_2{}^2 - p^2/v^2\right) \left(e^{-k_1 l} - e^{-(p/v)l}\right)}{k_1 \left(k_1{}^2 - k_2{}^2\right) \left(e^{k_1 l} - e^{-k_1 l}\right)}$$

$$\cdot C_5 e^{k_1 x}$$

$$+ \frac{\beta^2 p \left(p + \varepsilon\right) \left(k_1{}^2 - p^2/v^2\right) \left(e^{k_2 l} - e^{-(p/v)l}\right)}{k_2 \left(k_1{}^2 - k_2{}^2\right) \left(e^{k_2 l} - e^{-k_2 l}\right)} \tag{45}$$

$$\cdot C_5 e^{-k_2 x}$$

$$+ \frac{\beta^2 p \left(p + \varepsilon\right) \left(k_1{}^2 - p^2/v^2\right) \left(e^{-k_2 l} - e^{-(p/v)l}\right)}{k_2 \left(k_1{}^2 - k_2{}^2\right) \left(e^{k_2 l} - e^{-k_2 l}\right)}$$

$$\cdot C_5 e^{k_2 x} - \beta^2 v \left(p + \varepsilon\right) C_5 e^{-(p/v)x}.$$

4. Numerical Inversion of the Transforms

In order to determine the nondimensional temperature, nondimensional displacement, and nondimensional stress in the rod, we need to invert the parameters of $\bar{\theta}$, $\bar{u}$, and $\bar{\sigma}$ from the Laplace domain. Unfortunately, the obtained solutions in the Laplace domain are too complicated to be inverted analytically; thus, a feasible numerical method, that is, the Riemann-sum approximation method, is used to complete the inversion. Accordingly, any function $\bar{f}(x, p)$ in the Laplace domain can be inverted to the time domain as follows [48]:

$$f(x, t)$$

$$= \frac{e^{\beta t}}{t} \left[\frac{1}{2} \bar{f}(x, \beta) + \mathrm{Re} \sum_{n=1}^{N} \bar{f}\left(x, \beta + \frac{in\pi}{t}\right) (-1)^n \right], \tag{46}$$

where Re is the real part and i is the imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of β satisfies the relation $\beta t \approx 4.7$ [48].
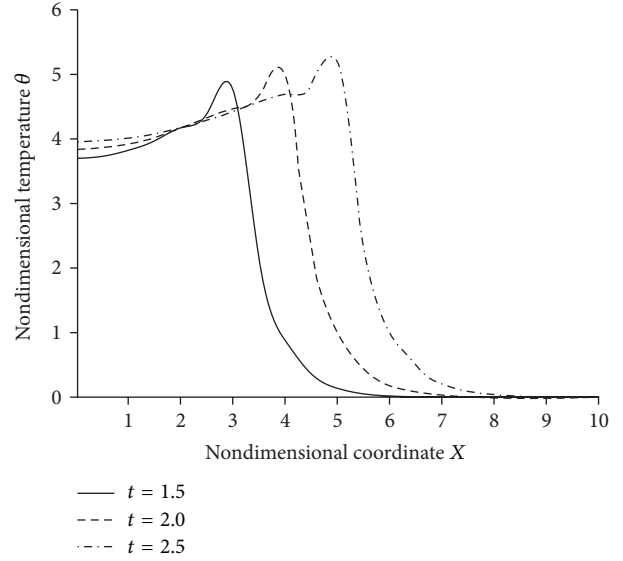

FIGURE 1: Nondimensional temperature distribution for different t values.

— $t = 1.5$
- - - $t = 2.0$
- · - · $t = 2.5$

5. Numerical Results and Discussions

In terms of the Riemann-sum approximation defined in (46), numerical Laplace inversion is implemented to obtain the nondimensional temperature, displacement, and stress in the rod in the time domain. For the purposes of simulation, the thermoelastic material is specified as copper and the parameters are

$$\lambda = 7.76 \times 10^{10}\,\mathrm{Nm^{-2}},$$

$$\mu = 3.86 \times 10^{10}\,\mathrm{Nm^{-2}},$$

$$\rho = 8954\,\mathrm{kgm^{-3}},$$

$$\alpha_t = 1.78 \times 10^{-5}\,\mathrm{K^{-1}},$$

$$C_E = 383.1\,\mathrm{JKg^{-1}\,K^{-1}}, \tag{47}$$

$$\kappa = 386\,\mathrm{Wm^{-1}\,K^{-1}},$$

$$T_0 = 293\,\mathrm{K},$$

$$\zeta = 0.0005\,\mathrm{K^{-1}}.$$

The other constants are

$$Q_0 = 10,$$

$$\tau_0 = 0.05, \tag{48}$$

$$l = 10.$$

Numerical calculation is carried out for the following five cases.

In Case 1, we investigate the nondimensional temperature, displacement, and stress varying with time as shown in Figures 1–3 with the moving heat source velocity, fractional order parameter, value of temperature-dependent properties,
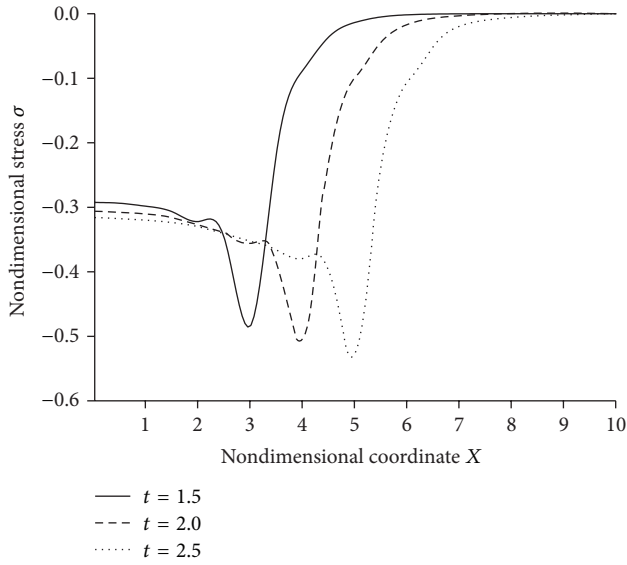
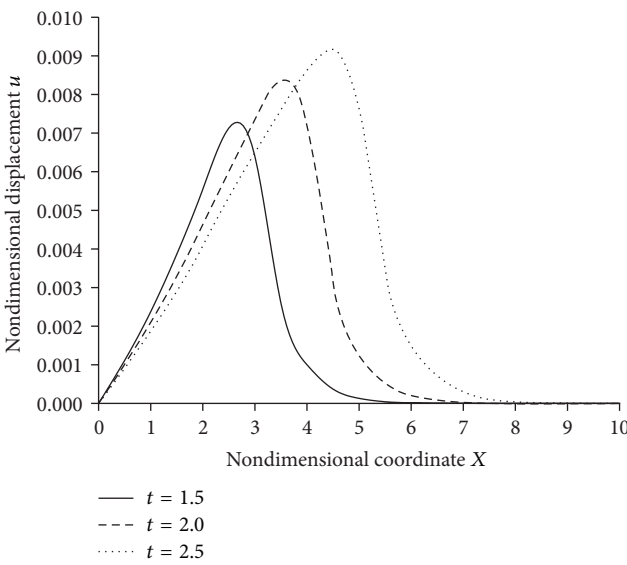FIGURE 2: Nondimensional stress distribution for different t values.

FIGURE 4: Nondimensional temperature distribution for different v values.
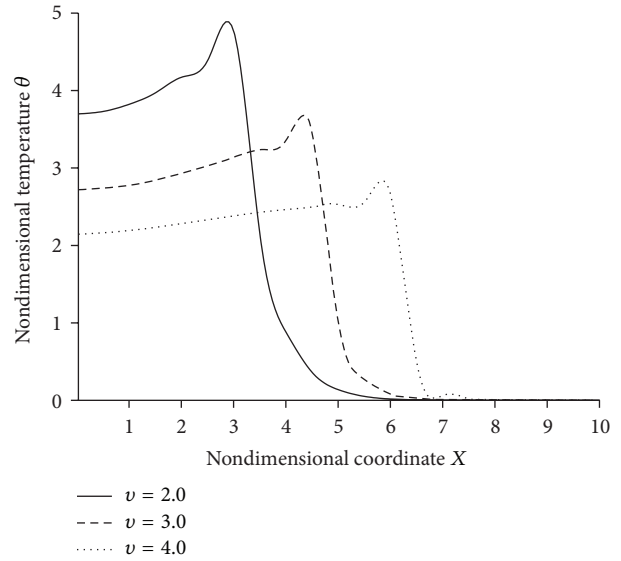

FIGURE 3: Nondimensional displacement distribution for different t values.
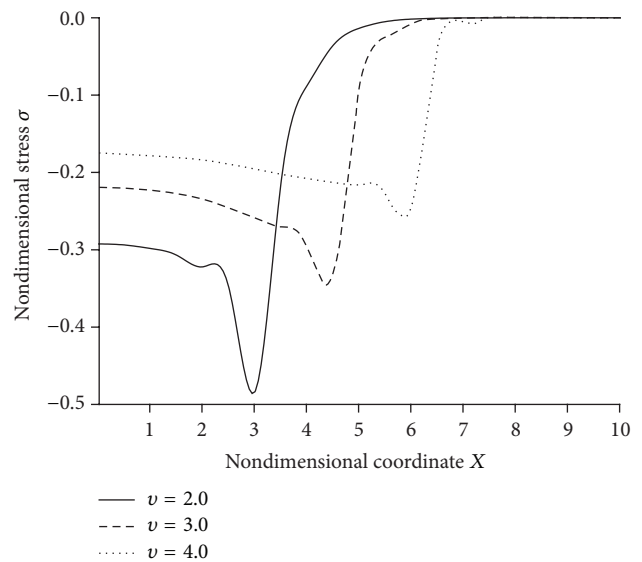
FIGURE 5: Nondimensional stress distribution for different v values.

and magnetic field set as constants (i.e., $v = 2.0$, $\alpha = 0.25$, $\vartheta = 0.5$, $\varepsilon = 5$). In Case 2, the considered variables varying with different moving heat source velocity are investigated when $t = 1.5$, $\alpha = 0.25$, $\vartheta = 0.5$, and $\varepsilon = 5$ (Figures 4–6). In Case 3, we investigate how the considered variables vary with different magnetic fields when $t = 1.5$, $v = 2.0$, $\vartheta = 0.5$, and $\alpha = 0.25$ (Figures 7–9). Case 4 involves the considered variables varying with different temperature-dependent properties when $t = 1.5$, $v = 2.0$, $\alpha = 0.25$, and $\varepsilon = 5$ (Figures 10–12), and, in Case 5, we investigate how the considered variables vary with different fractional order parameters when $t = 1.5$, $v = 2.0$, $\vartheta = 0.5$, and, $\varepsilon = 5$ (Figures 13–15).

In Figures 1–3 (Case 1), the solid line, dash line, and dot line refer to $t = 1.5$, $t = 2.0$, and $t = 2.5$, respectively. Figure 1 shows that the nondimensional temperature increases as the time increases. At location $x = vt$, the heat source releases its maximum energy which leads to a peak value. As shown in Figure 2, the nondimensional stress in the rod is compressive. Due to the fixed ends, thermal expansion deformation is restrained in both ends and leads to the occurrence of compressive thermal stress in the rod. The absolute value of stress increases with the passage of time. As shown in Figure 3, the nondimensional displacement also increases as time progresses. Due to the applied moving heat source, the rod undergoes thermal expansion deformation. With the passage of time, the heat disturbed region enlarges so that thermal expansion deformation evolves along the rod.
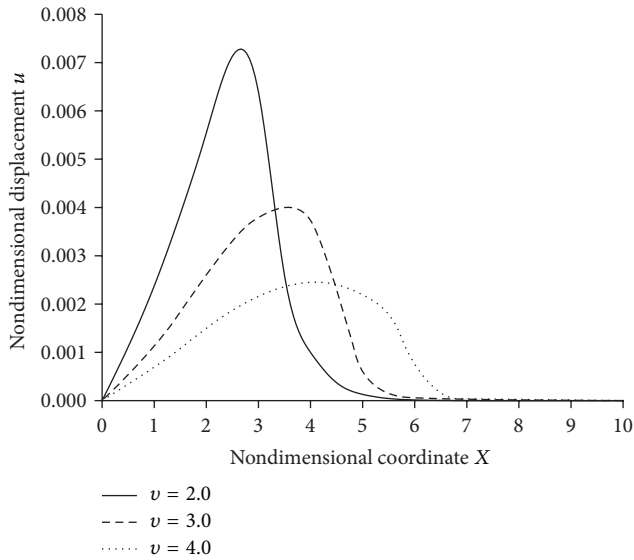
FIGURE 6: Nondimensional displacement distribution for different υ values.
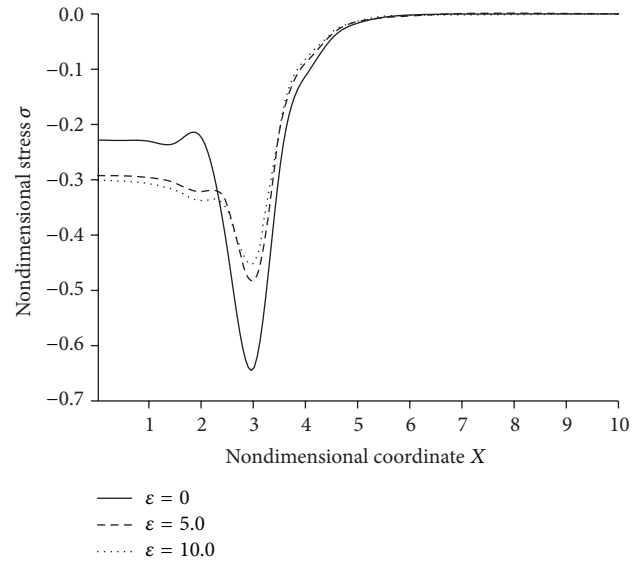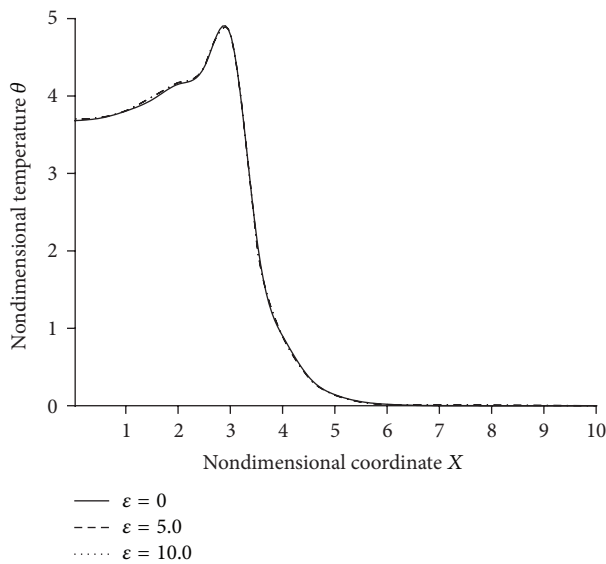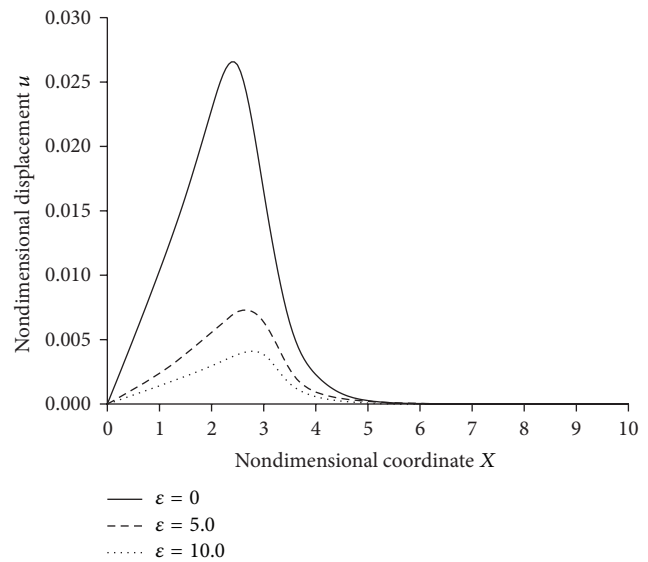

FIGURE 7: Nondimensional temperature distribution for different ε values.

FIGURE 8: Nondimensional stress distribution for different ε values.

FIGURE 9: Nondimensional displacement distribution for different ε values.

In Figures 4–6 (Case 2), the solid line, dash line, and dot line refer to $\upsilon = 2.0$, $\upsilon = 3.0$, and $\upsilon = 4.0$, respectively. As shown in Figure 4, the nondimensional temperature decreases with the increasing of the moving heat resource velocity. In a given period, the energy that the heat source can release is constant; however, the intensity of the released energy per unit length decreases as the source speed increases. It is also clear in Figures 5 and 6 that the magnitudes of the nondimensional stress and displacement decrease as the moving heat source velocity increases, which is the result of reduction in heat energy intensity per unit length at greater velocities.

In Figures 7–9 (Case 3), the solid line, dash line, and dot line refer to $\varepsilon = 0$, $\varepsilon = 5.0$, and $\varepsilon = 10.0$, respectively.

In Figure 7, the three lines overlap because the magnetic field has no effect on the temperature. As shown in Figure 8, the absolute value of stress increases as the ε value increases prior to the point where the curves intersect; then, after the intersection, the absolute value of stress decreases as the applied magnetic field value increases. Figure 9 for Case 3 shows that nondimensional displacement decreases as the magnetic field increases. The largest displacement value is in the case of $\varepsilon = 0$, namely, in the absence of the magnetic field. This indicates that the magnetic field acts to damp the thermal expansion deformation of the rod.

In Figures 10–12 (Case 4), the solid line, dash line, and dot line refer to $\vartheta = 0.5$, $\vartheta = 0.75$, and $\vartheta = 1.0$, respectively. As shown in Figure 10, the temperature increases as the ϑ
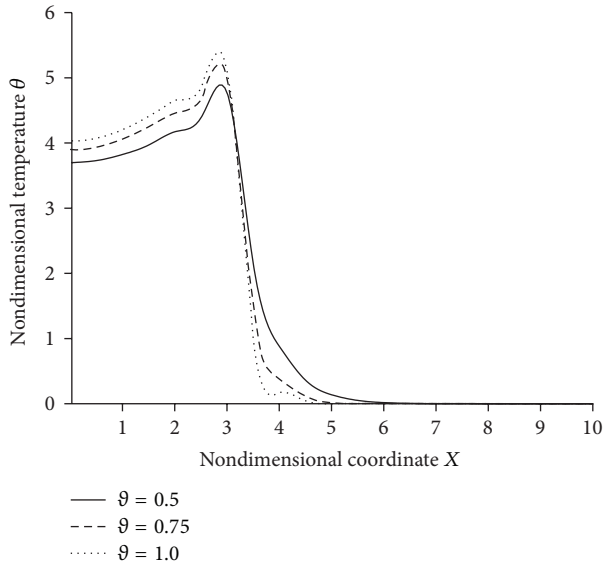
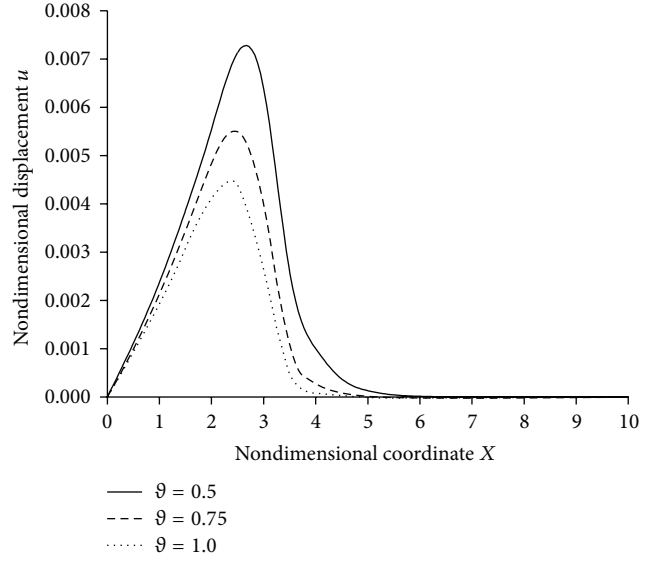FIGURE 10: Nondimensional temperature distribution for different ϑ values.

FIGURE 11: Nondimensional stress distribution for different ϑ values.

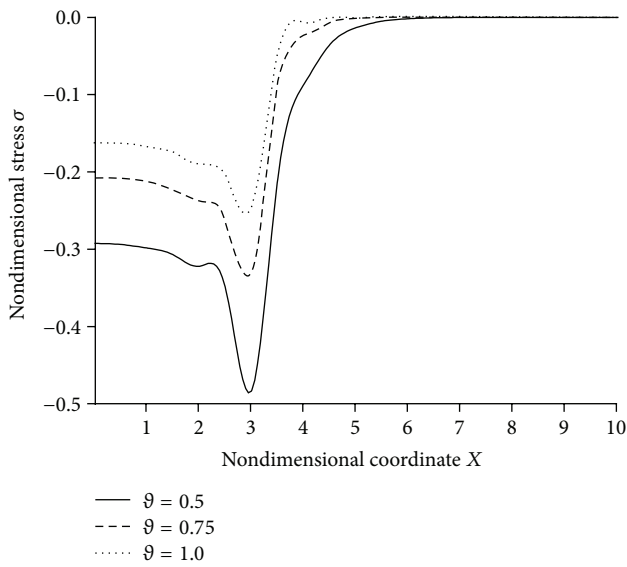FIGURE 12: Nondimensional displacement distribution for different ϑ values.
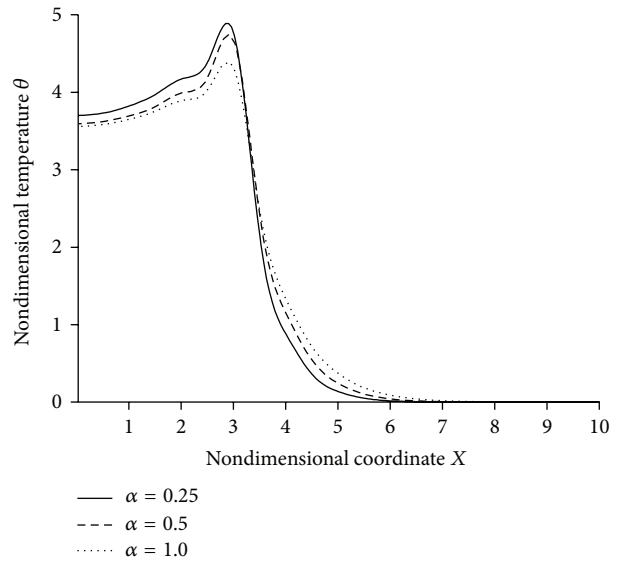

FIGURE 13: Nondimensional temperature distribution for different α values.

value increases before the three curves intersect, at which point it decreases as the temperature-dependent properties increase. Figures 11 and 12 show where the displacement and absolute value of stress decrease as the temperature-dependent properties increase.

Figures 13–15 (Case 5) show the variations of the nondimensional temperature, stress, and displacement, respectively, which demonstrate the effects of the fractional order parameter on the variations of the considered variables. To this end, a series of values of the fractional order parameter α within the region $(0, 1]$ are tested through numerical calculation. As representation of the effect of α, three typical values of α, $\alpha = 0.25$, $\alpha = 0.5$, and $\alpha = 1.0$, are considered. Figures 13–15 show that temperature, displacement, and

absolute stress value all decrease as the fractional order value increases prior to the intersection of the three curves. After the intersection, however, all considered variables increase with the increasing of the fractional order parameter.

6. Conclusions

A generalized magnetothermoelastic rod with temperature-dependent properties subjected to a moving heat source is investigated in the fractional order theory. The results provided the following most notable conclusions:
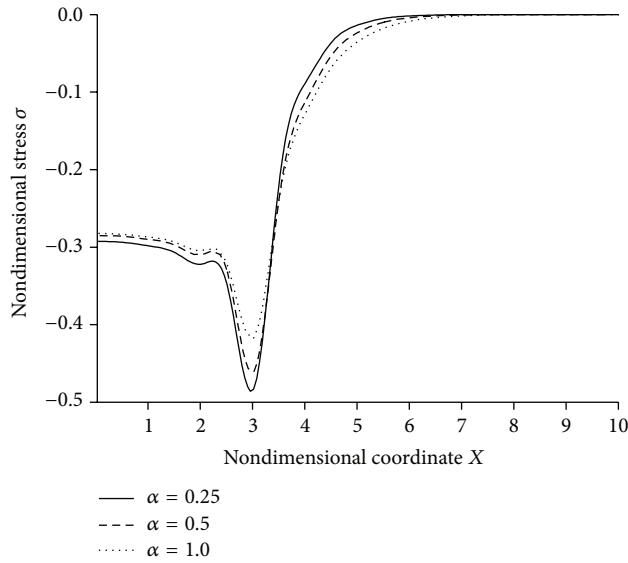
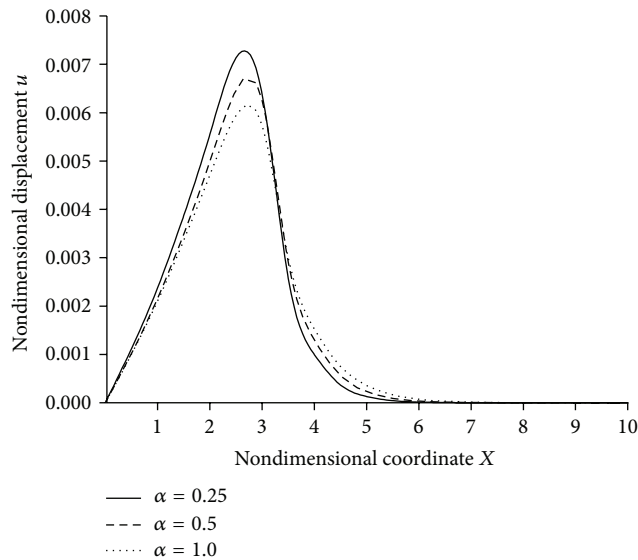FIGURE 14: Nondimensional stress distribution for different α values.

FIGURE 15: Nondimensional displacement distribution for different α values.

(1) The effects of the fractional parameter α on all the considered variables are very significant, as clearly evidenced by the peak values of the curves.

(2) The magnetic field also has a significant effect on the solution of the nondimensional displacement and stress but barely influences the variation in nondimensional temperature.

(3) The temperature-dependent properties play a significant role on all distributions.

As a final remark, the results presented in this paper should prove useful for researchers in material sciences, designers of new materials, low-temperature physics researchers, and those working to further develop the theory of thermoelasticity with fractional calculus. The introduction of fractional calculus and variable temperature-dependent modulus to the generalized thermoelastic rod may provide a more realistic model for these studies.

Nomenclature

J: Current density vector
U: Displacement vector
B: Magnetic induction vector
σ_{ij}: The components of stress tensor
e_{ij}: The components of strain tensor
e_{kk}: Cubic dilation
u_i: The components of displacement vector
θ: $\theta = T - T_0$
T: Absolute temperature of the medium
T_0: Reference temperature
κ_{ij}: The coefficient of thermal conductivity
τ_0: Thermal relaxation time
ρ: Mass density
C_E: Specific heat at constant strain
λ, μ: Lame's constants
α_t: Linear thermal expansion coefficient
Q: The strength of the applied heat source per unit mass
γ: $(3\lambda + 2\mu)\alpha_t$
η: The entropy density
q_i: The components of heat flux vector
v: Velocity of the moving heat source
α: Constant parameter such that $0 < \alpha \leq 1$.
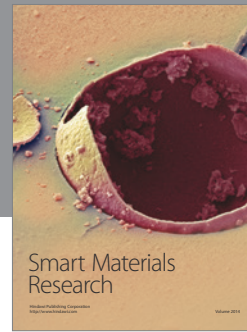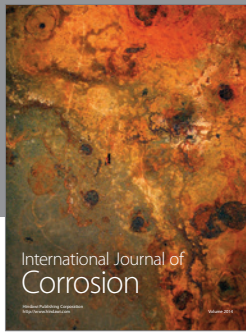
Competing Interests

The authors declare that they have no competing interests.

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