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Research Article

Fracture Surface Fractal Characteristics of Alkali-Slag Concrete under Freeze-Thaw Cycles

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Fractal theory is introduced in fracture surface research of alkali-slag concrete (ASC) under freeze-thaw cycles; crack distribution of ASC fracture surface and freeze-thaw damage zone were calculated. Through fractal analysis of ASC sample fracture surfaces, relevance between section fractal dimension and fracture toughness and relationship between material composition and section fractal dimension are clarified. Results show that the specimen's cracks before freeze-thaw extend along force direction gently, and there are more twists and turns after freezing and thawing; the fractal dimension D also grows from 1.10 to 1.33. SEM internal microcracks' D of ASC internal microstructure after freezing and thawing is 1.37; 0 to 300 times ASC fractal dimension under freezing and thawing is between 2.10 and 2.23; with freeze-thaw times increasing, ASC fracture toughness decreases and fractal dimension increases, the fractal dimension and fracture toughness have a good linear relationship, and the fractal dimension can reflect the toughening effect of ASC. It is very feasible to evaluate ASC fracture behaviour under freezing and thawing with the fractal theory. Fractal dimension generally increases with activator solution-slag (A/S for short) or slag content. The greater the amount of A/S or slag content, the lower the dimension.

1. Introduction

ASC is a new green concrete material, attracting a lot of research. Meanwhile, using fractal theory in concrete material is becoming a hotspot in civil engineering. Fractal geometry is a branch of mathematics developed by a French-American scientist, Mandelbrot, in the 1970s [1]. Fractal theory is an effective way to describe material characteristics such as self-similar in micro-, meso-, and macrolevels and is an effective way to explore the physical and mechanical behaviour of different levels of the fine structure and macrofield performance. Through research on material microstructure changes, Hadjileontiadis and Douka [2] believe the fractal theory can be used to describe characterizations such as development of material cracks and depth; Mecholsky Jr. [3] considered that various fracture parameters of brittle materials can be represented by fractal dimension; He and Yang [4] introduced the pore structure of ASC using fractal theory; Yan et al. [5, 6] think that the fracture process of concrete also has fractal characteristics, its macroscopic mechanical properties degradation is associated with crack (i.e., damage) development, by means of fractal analysis, and concrete micro- and macroscale information can be grasped more reasonably and accurately.

The fractal research on fracture surface began in the mid-1980s. Saouma et al. [7] first studied the fractal nature of concrete fracture surface and found that the fractal dimension of concrete fracture surface changes from 1.10 to 1.20; Issa and Hammad [8] also investigated concrete fracture surface, obtaining similar results; Nagahama [9] established the unified relationship between fracture toughness and fractal dimension D based on numerous studies. Through experiments, Yan et al. found that using fractal dimension to reflect concrete fracture energy is feasible. With water-cement ratio increasing, the fractal dimension will also increase. By fitting the experimental data, Yan et al. got the relationship between fractal dimension and fracture energy [5] and calculated the fractal dimension of concrete fracture surface using laser method [10]; Issa et al. [11] found that a clear correlation exists between fractal dimension and fracture toughness: the tougher the material, the higher the fractal dimension. Vidya Sagar and Raghu Prasad [12] used singular fractal functions to generate stress-strain plots for quasi-brittle material like concrete and cement mortar. Fu et al. [13] studied the ASC damage model under freeze-thaw cycles; Šejnoha et al. [14] summarized the fracture toughness of concrete for tunnel lining; Cai et al. [15] studied the ASC antifreeze durability; Sun and Wu [16] investigated the chemical properties and frost resistance of fly ash inorganic mixture; Li et al. [17] studied fractal characteristics of concrete fracture surface before and after freezing and thawing and applied fractal theory to study freeze-thaw damage; results show that the fracture toughness and fractal dimension have a good correlation, so fractal theory and concrete fracture theory can be combined to evaluate the fracture behaviour of concrete under freeze-thaw cycles; Zhang [18] suggested that when concrete fracture surface fractal dimension increases, the fracture toughness will also increase and that the fracture surface roughness increases the material anticracking ability; Guo et al. concluded application of fractal theory in concrete material mechanical behaviour, distribution of aggregates, concrete fracture toughness and fracture energy, pore, concrete material acoustic emission, and many other aspects of application [19, 20]. Erdem and Blankson [21] determined quantitatively the fractal character and texture of fracture surfaces and investigated their influence on the fracture-related properties in different types of concrete. Konkol and Prokopski [22] studied fracture toughness of concrete with metakaolinite additive, drawing the conclusion that fracture toughness correlates with fractal dimension and water/binder ratio.

Application of fractal theory in concrete provides a new way for concrete research, but there are few researches on the application of fracture theory for ASC antifrost durability. Measuring fracture surface fractal dimension is an important aspect of ASC fracture surface. It is necessary to obtain accurate fractal dimension before further exploring the relationship with other properties. The paper introduces fractal theory into fracture surface research of ASC under freeze-thaw cycles, as ASC surface crack and fracture surface graphics have good self-similar characteristics within a certain range; aimed at characteristics of two-dimensional digital image, crack distribution of ASC fracture surface and freeze-thaw damage zone are calculated. Through fractal analysis of ASC sample section, relationship between fracture surface fractal dimension and ASC fracture toughness and relationship between composition parameters and fractal dimension are clarified; fractal law of ASC specimen fracture surface and crack under freeze-thaw cycles is discussed.

2. Experiment Materials and Methods

2.1. Raw Materials and Specimen Preparation. The following material is used for making specimens: slag (specific surface is 410 m^2/kg and density is 2.86 g/cm^3), alkali-activator composed of NaOH and Na_2SiO_3 (modulus is 3.34), river sand with density of 2.63 g/cm^3 and cumulate density of 1.50 g/cm^3, and limestone rubble (5–20 mm, ~45%; 20–40 mm, ~55%) with density of 2.76 g/cm^3 and cumulate density of 1.59 g/cm^3. ASC mixture is mixed in a 60 L single horizontal axis mixer and the mixing process is as follows: sand and slag (30 s) → stone (30 s) → activator solution (120 s) [15]. The mix proportions of ASC and test results of mixture's workability are shown in Table 1. J. Lemaitre proposed that size of test-pieces applied to damage mechanics must reflect characteristics of materials. Thus, the test-pieces should be small enough so that they could avoid high gradient smooth and big enough so that they could reflect the average value of microprocess. Typical test-piece convenient for the study must satisfy these conditions. For concrete, size 100 mm × 100 mm × 400 mm is just convenient. GB/T 50082-2009 (standard test method for long-term performance and durability of ordinary concrete) and ASTM C666/C666 M-03 (standard test method for resistance of concrete to rapid freezing and thawing) also stipulate that size of test-piece for antifreeze experiment is 100 mm × 100 mm × 400 mm. Prismoid in the size of 100 mm × 100 mm × 400 mm is used in the freezing-and-thawing experiment and damage mechanics analysis. Three-point bending beam with incision is used in the tests of study of concrete fracture parameters; the specimen size is shown in Figure 1. In Figure 1, the length of the specimen L is 400 mm, the span length S is 300 mm, and the sectional height h and width t are all 100 mm, respectively, with initial notch-height ratio fixed at 0.4.

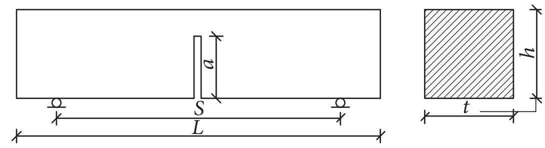

Figure 1: Fracture toughness test schema.

2.2. Test Methods. Dynamic elastic modulus and mass loss of 5 groups (6 test-pieces for each group) are tested after different times of freeze-thaw cycle in the freezing-and-thawing experiment and damage mechanics analysis. Besides, each mix proportion moulded another 13 groups (3 test-pieces for each group), testing the compression strength, flexural strength, and fracture toughness after different times of freeze-thaw cycle. The test-pieces were placed in 20°C environment for 1 week after being shaped up. We transferred them to standard curing room immediately until reaching the test age. Then they were demoulded and numbered. Fault test uses 810 MTS material testing machine of MTS companies; the maximum load is 250 kN. Clip-on stretch instrument of type 3541 was used in the tests meeting the standard of ASTM E1820, E813, and E399.

The fracture toughness is calculated according to the method in DL/T 5332-2005 (norm for fracture test of hydraulic concrete) [23–25]. As the specimen size used in the test is nonstandard (100 mm × 100 mm × 400 mm), the fracture toughness got in tests is corrected standard specimen (200 mm × 120 mm × 1000 mm), according to DL/T 5332-2005 (norm for fracture test of hydraulic concrete) as follows:

$$K_{IC}^{\text{sta}} = \left(\frac{V_{\text{sta}}}{V_{\text{non}}}\right)^{1/\alpha} \left(\frac{h_{\text{sta}}}{h_{\text{non}}}\right)^{1/2} K_{IC}^{\text{non}}, \quad (1)$$

Table 1: The mix proportions and workability of ASC.

Number	A1	A2	A3	A4	A5
Slag content (kg/m^3)	440	420	400	400	400
A/S (-)	0.56	0.56	0.58	0.56	0.54
Sand ratio (%)	34	34	34	34	34
Slump (mm)	205	197	193	172	130

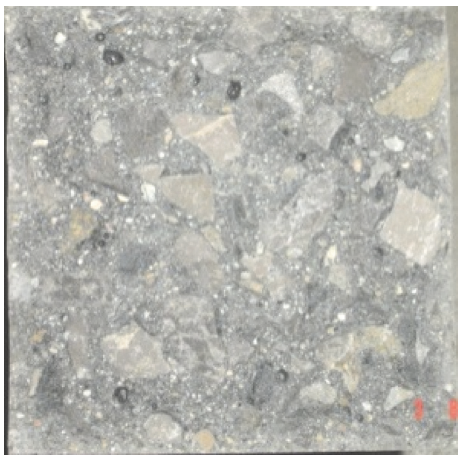

Figure 2: Prefreeze ASC fracture surface.

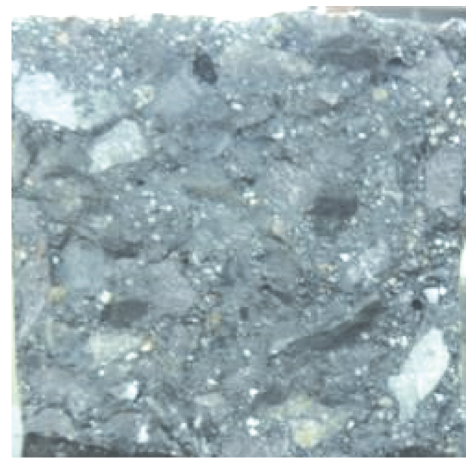

Figure 3: Postthaw ASC fracture surface.

where K_{IC}^{sta} and K_{IC}^{non} are, respectively, fracture toughness of standard and nonstandard method; h_{sta} and h_{non} are, respectively, height of specimen for standard (0.2 m) and nonstandard (0.1 m) method; V_{sta} and V_{non} are, respectively, volume of specimen for standard (0.024 m^3) and nonstandard (0.004 m^3) method; and α is Weibull parameter; it is generally advisable to take 7~13 for concrete, and 10 is taken in this paper [26].

2.3. Test Results. Tests results are shown in Table 2.

3. Prefreeze and Postthaw ASC Fracture Surface Fractal Analysis

3.1. Prefreeze and Postthaw ASC Fracture Surface. ASC fracture surfaces before and after freezing and thawing of concrete after fracture toughness test are shown in Figures 2 and 3. It can be seen from Figure 2 that fracture surface is relatively smooth before freezing and thawing, aggregate is mostly pulled off, and there are no significant cracks on the surface. It can be seen from Figure 3 that, due to internal damage after freezing and thawing, fracture toughness decreased severely, roughness of specimen and the fracture surface increased significantly, and the fracture surface aggregate declined completely. Because of the barrier of crack propagation in coarse aggregate, concrete fracture surface structure is very complex and has fractal characteristic. Therefore, fractal fracture surface is more suitable for the analysis of ASC after freezing and thawing.

3.2. Fractal Characteristic Analysis of ASC Fracture Surface. ASC macroscopic or microscopic cracks can be thought to be present in two-dimensional form (not to consider a three-dimensional image), such as surface crack and internal injury. The two-dimensional image forms a curve after processing, on which we do this fractal study.

3.2.1. ASC Crack Fractal Digital Processing. With the development of information processing and computer technology, a large number of graphic images are obtained in the form of digital images or can be converted into a digital image. Fractal dimension is obtained by method of digital box dimension [27]. These digital images are different from traditional images carried by paper. Digital images are two-dimensional matrix represented by a series of binary digits (0 and l) after being sampled and quantized, which are with quantization and discrete features and have their characteristics in fractal dimension computing.

Digital images are formed by a series of pixels in a matrix form; pixels of different colors correspond to different values; for binary image (black and white bitmap) whose pixel is black or white, they are, respectively, represented as 1 or 0 in the matrix; then an image contains several submatrices containing 0 or 1. If the area concerned contains 1, mesh the whole matrix; the grid number can be counted, namely, the number of all nonzero submatrices; a descending sequence is constructed from

$$\varepsilon_k = \left\{ \frac{1}{c^{k-1}} \mid 0 < k < \left[\frac{\ln w}{\ln c} \right] + 1,\ k \in N \right\}. \quad (2)$$

Table 2: Results of experiments and fracture surface fractal dimension under freeze-thaw cycles.

Number	Freeze-thaw cycle times	Dimension D	Correlation coefficient R^2	Fracture toughness ($MPa \cdot m^{1/2}$)	Compressive strength (MPa)	Flexural strength (MPa)
A1	0	2.10	0.9624	6.65	67.4	7.06
	50	2.10	0.9432	5.51	67.1	6.86
	100	2.11	0.9527	4.84	64.7	6.71
	150	2.13	0.9365	4.19	64.0	6.36
	200	2.15	0.9255	3.36	62.0	6.00
	250	2.17	0.9736	2.74	59.2	5.58
	300	2.20	0.9718	2.41	55.5	5.10
A2	0	2.11	0.9367	6.79	67.6	7.15
	50	2.12	0.9284	6.18	67.3	6.94
	100	2.12	0.9273	5.42	64.9	6.80
	150	2.14	0.9198	4.70	64.2	6.44
	200	2.16	0.9324	3.77	62.2	6.08
	250	2.19	0.9563	3.08	59.4	5.65
	300	2.21	0.9497	2.70	55.7	5.16
A3	0	2.11	0.9564	6.84	67.8	7.24
	50	2.12	0.9275	6.36	67.5	7.03
	100	2.13	0.9389	5.57	65.1	6.88
	150	2.15	0.9567	4.45	64.3	6.52
	200	2.16	0.9657	3.72	62.2	6.15
	250	2.18	0.9481	3.03	59.5	5.67
	300	2.21	0.9219	2.66	55.9	5.15
A4	0	2.13	0.9728	8.55	68.0	7.74
	50	2.13	0.9658	7.95	67.7	7.52
	100	2.15	0.9290	6.96	65.3	7.36
	150	2.16	0.9309	5.57	64.6	6.97
	200	2.18	0.9482	4.65	62.5	6.58
	250	2.20	0.9193	3.80	59.7	6.07
	300	2.22	0.9571	3.32	55.9	5.58
A5	0	2.14	0.9674	10.75	69.9	7.91
	50	2.15	0.9329	10.25	69.6	7.68
	100	2.16	0.9198	9.03	67.2	7.52
	150	2.17	0.9437	7.50	66.4	7.13
	200	2.19	0.9522	6.17	64.3	6.72
	250	2.21	0.9346	5.14	61.5	6.25
	300	2.23	0.9633	4.27	57.6	5.77

The image is successively divided into c aliquots and is rounded (usually takes 2, 3, or 5); we use programming to convert digital images into binary image, extract digital matrix, and analyze and calculate the fractal. The calculation block diagram is shown in Figure 4.

Concrete aggregate interface shape is very irregular, whose surface of microscopic and macroscopic crack is uneven. The crack path is irregular, showing fractal characteristic. We can use fractal geometry to describe the crack irregularities and crack surface roughness [28]. In the macro- and microcracks of concrete fractal analysis, we need to process and analyze photos taken from the optical microscope or scanning electron microscope and convert graphic images into digital images. We can realize the transformation with modern digital image processing technology. Since the digital images captured are usually saved as matrix through sampling and quantization, we can extract useful information by processing. Figure 5 shows the cracks of ASC under freeze-thaw cycles, and Figure 6 shows the binary graph calculated by the method above.

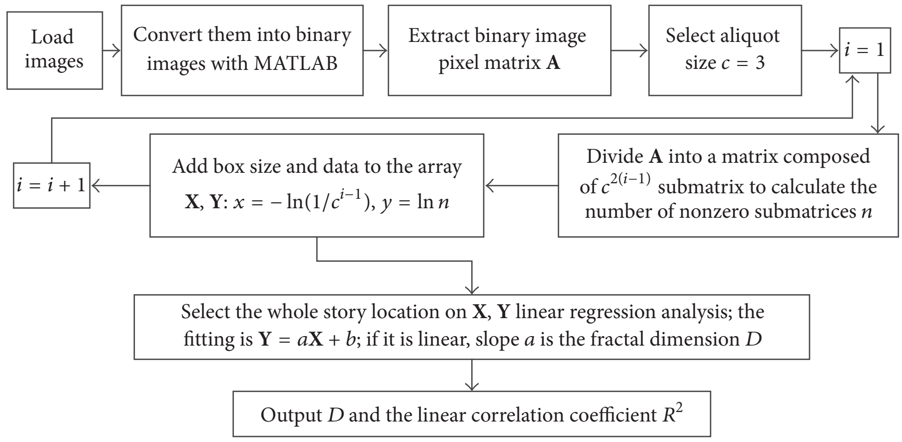

Figure 4: Fractal dimension calculation process.

Figure 5: ASC cracking figure.

Figure 6: ASC cracking binary image.

3.2.2. Prefreeze and Postthaw ASC Macroscopic Crack Fracture Surface Fractal Analysis. Figures 7 and 8 show macroscopic cracks left after fracture toughness test before and after freezing and thawing; it can be seen through contrast that the specimen cracks before freeze-thaw extend along the direction of the force gently, and the curve has more twists and turns after freezing and thawing. Figures 9 and 10 are simplified binary diagrams of concrete fracture path around freezing and thawing. By calculating, the prefreeze fractal dimension D is 1.10 and correlation coefficient R^2 is 0.9823; postthaw fractal dimension D is 1.33 and correlation coefficient R^2 is 0.9863. The results show that fractal theory is applicative in the study of ASC's crack surface.

3.3. Postthaw ASC Microcracks Fractal Analysis. Figure 11 is an ASC internal microstructure SEM photograph after freezing and thawing; from the photos you can clearly see the internal microcracks. After treatment, we get the internal microcracks binary image (Figure 12). By calculation using the method mentioned in this paper, fractal dimension D is 1.37 and correlation coefficient R^2 is 0.9820.

We did fractal calculations to fracture toughness of groups A1~A5 after 0, 50, 100, 150, 200, 250, and 300 freeze-thaw cycles. The measured fractal dimensions are shown in

Figure 7: Prefreeze cracking path.

Figure 8: Postthaw cracking path.

Figure 9: Prefreeze cracking path binary image.

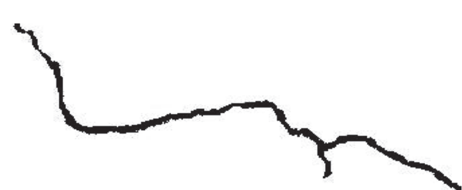

Figure 10: Postthaw cracking path binary image.

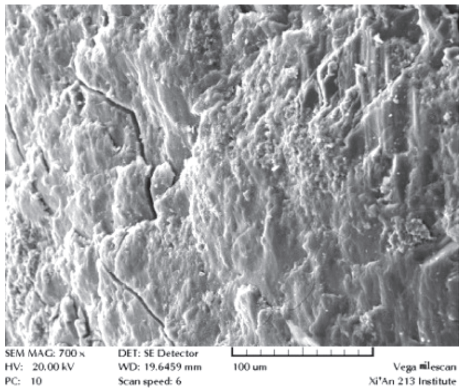

Figure 11: ASC internal microcracks under freeze-thaw cycles.

Figure 12: ASC internal microcracks binary image under freeze-thaw cycles.

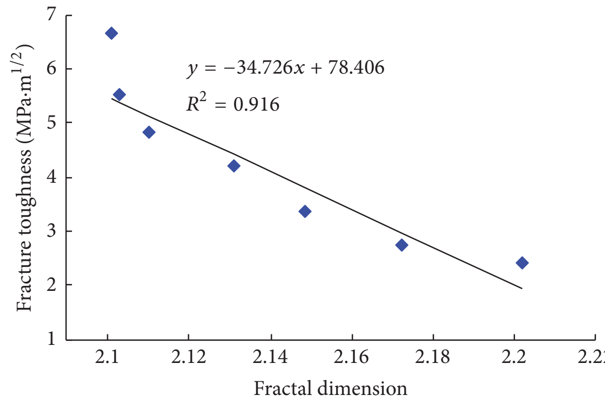

Figure 13: A1 fracture toughness versus fractal dimension.

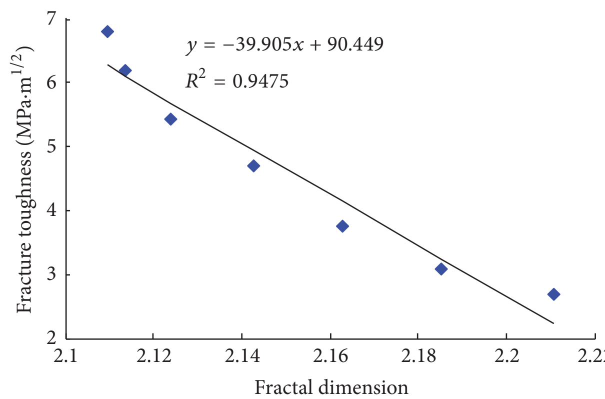

Figure 14: A2 fracture toughness versus fractal dimension.

Table 2. From fractal dimension linear regression in Table 2, each group of ASC fractal dimension is visible. It is feasible to evaluate ASC fracture behaviour under freeze-thaw cycles using fractal theory.

4. ASC Characteristics and Fracture Surface Fractal Dimension Correlation Analysis

4.1. ASC Fracture Toughness and Fractal Dimension Correlation Analysis. Section of concrete is not a plane, with projections and depressions; these features reflect the effect of fracture energy dissipation and microstructure damage during concrete fracture. With different degrees of freezing and thawing, the actual surface area of the fracture surface and roughness are different, showing different fracture toughness and different fractal dimension. Through the fractal study on fracture surface, relationship between fractal dimension and fractal parameter can be established. Figures 13–17 show tested and fitted values of fracture toughness and fractal dimension under 0~300 freeze-thaw cycles. It can be seen that the value is between 2.10 and 2.23. With freeze-thaw cycle time increasing, ASC fracture toughness falls and fractal dimension increases; there is a good linear relationship. Five groups' ASC correlation coefficients are above 0.9. This indicates that fractal dimension changes reflect changes in mechanical properties of ASC.

With the good linear correlation between fractal dimension and fracture toughness, when raw material and mix proportion are invariant, we can deduce ASC fracture toughness with the fractal dimension of fracture surface after freeze-thaw cycles. Fractal theory and fracture mechanics can be combined, characterizing ASC fracture behaviour after freezing and thawing damage. Fractal theory to study ASC under freezing and thawing is reasonable and has some practical engineering value.

4.2. Relationship between Fractal Dimension of Fracture Surface and Material Composition. ASC fracture characteristic is mainly influenced by factors such as sol ratio, the amount of slag, and raw nature of the materials, and the fractal dimension changes with material composition. To study the impact of ASC fracture surface on mechanical properties, influence of A/S and the amount of slag on fractal dimension are studied.

4.2.1. Relationship between D and A/S under Freeze-Thaw Cycles. Figure 18 shows the diagram fractal dimension

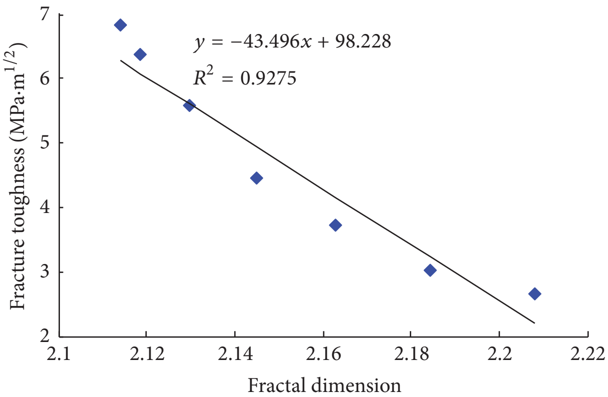

FIGURE 15: A3 fracture toughness versus fractal dimension.

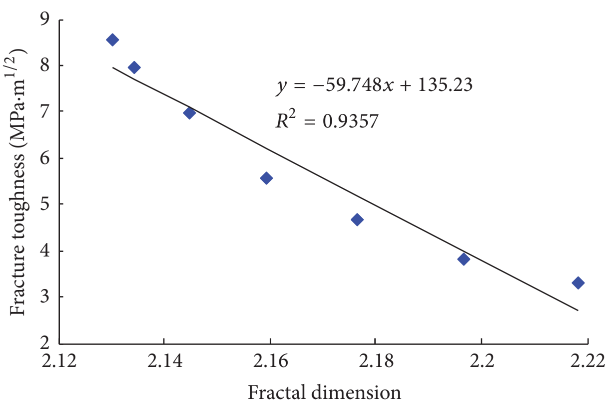

FIGURE 16: A4 fracture toughness versus fractal dimension.

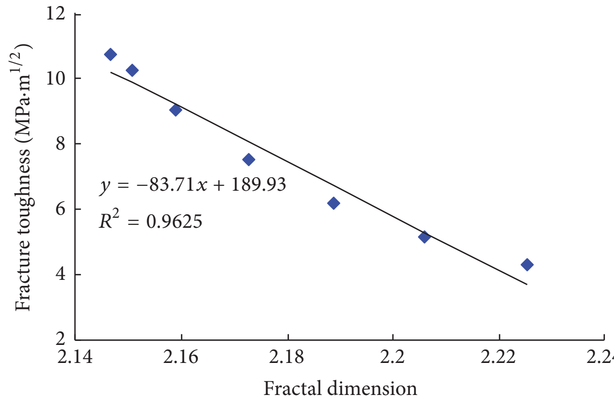

FIGURE 17: A5 fracture toughness versus fractal dimension.

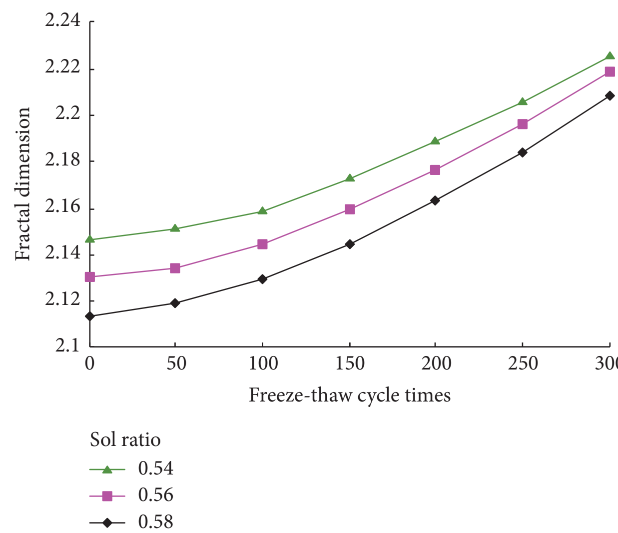

FIGURE 18: Relationship between A/S and dimension.

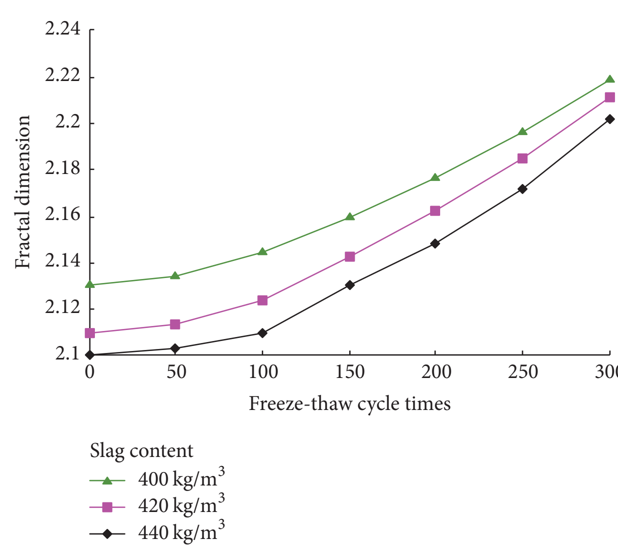

FIGURE 19: Relationship between slag content and dimension.

changing with sol ratio; as can be seen, D decreases with sol ratio. The main reason is when other conditions are certain, density inclement increases with lower sol ratio; then the strength of ACS material increases, but the fragility also increases unfortunately. Then, compressing after freeze-thaw cycles generates internal cracks more easily, which can be reflected from increase of fractal dimension of fracture surface.

4.2.2. Relationship between D and Slag Content under Freeze-Thaw Cycles. Figure 19 shows the diagram fractal of dimension changing with slag content. As can be seen, D decreases with slag content; with more slag, D is smaller. Under the condition that A/S and other mix proportions are equal, more slag in unit volume concrete causes less coarse aggregate content, joint surface in cement is less, and the cement is more compacted undoubtedly. Higher degree compaction signifies less inner crack; then D becomes smaller.

5. Conclusions

In this paper, the fractal analysis of ASC fracture surfaces is preliminary studied; aimed at the two-dimensional digital image characteristics, through digital fractal process of ASC crack, macro- and microcracks fractal analysis before and after freezing and thawing is done, revealing relationship between fracture toughness and fractal dimension and relationship between fracture surface fractal dimension and material composition. ASC fracture surface fractal characteristics are studied before and after freeze-thaw cycles; the results are as follows:

(1) By calculating fractal dimension D of fracture surface before and after freeze-thaw cycles for ASC using the method mentioned in this paper, all correlation coefficients are above 0.9. The results show that fractal theory is applicative in the study of ASC's crack surface.

(2) By fracture surface macro- and microcrack fractal analysis before and after freeze-thaw cycles, the law of crack fractal dimension is established. After comparing macroscopic cracks left by fracture toughness tests before and after freezing and thawing, it can be seen that specimen crack before freeze-thaw extends along the direction of the force gently and shows more twists and turns after freezing and thawing; the fractal dimension D also increases from 1.10 to 1.33; SEM internal microcracks' D is 1.37 after freezing and thawing.

(3) The fractal dimension under freeze-thaw cycles is given; fractal dimension of 0 to 300 times freeze-thaw cycles is between 2.10 and 2.23.

(4) Relationship between ASC fracture surface fractal dimension and fracture toughness is established. With freeze-thaw times increasing, ASC fracture toughness decreases and fractal dimension increases; the fractal dimension and fracture toughness have a good linear relationship. The fractal dimension can reflect the toughening effect of ASC; it is feasible to evaluate ASC fracture behaviour with the fractal theory; the ASC fracture surface fractal dimension after freezing and thawing can be applied to deduce its fracture toughness.

(5) The relationship between ASC fracture surface fractal dimension and material composition is established. The fractal dimension shows a substantially monotonous growth trend with A/S or slag content; the bigger the A/S or slag content is, the lower the fractal dimension tends to be.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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