Research Article

Fatigue Strength Evaluation for Remanufacturing Impeller of Centrifugal Compressor Based on Modified Grey Relational Model

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The fatigue strength, as the essential basis of residual life evaluation, is required to be obtained timely for remanufacturing. Since impeller damage is characterized with very-high-cycle fatigue (VHCF), it is difficult to directly test the strength data. The transformation method of multisource strength data is proposed to predict fatigue strength for impeller based on grey relational theory. The multisource strength data, as factor space, primarily include available existing experimental data and operating data, while the strength data of the remanufacturing impeller are taken as target data. The fatigue strength model of material and component are presented to analyze the influence factors of remanufacturing target strength. And similar material provides a theoretical basis for selecting reference data reasonably. Considering the correlation and difference between available data and target data, the grey relational function is established, and the correction function of the target residual is brought forward to reduce the transformation deviation. The entropy-weight theory is implemented to determine the different impacts of multisource data on target strength. A test case, predicting the unknown impeller fatigue strength with various impellers, is applied to validate the proposed transformation method, and the results show that the predicted strength data are consistent with the experimental data well.

1. Introduction

Remanufacturing is one of the effective ways to solve resource shortage and environmental protection problem restricting to manufacturing development. It can save 50% in cost, 60% in energy, and 70% in material compared to new products [1]. Remanufacturability assessment, aiming at estimating whether products or parts are worth to be remanufactured, is the first step towards remanufacturing. The key issue of remanufacturability assessment is to evaluate residual life of products for ensuring safety in service in the next life cycle, where the fatigue strength is the essential basis for residual life evaluation [2, 3].

Centrifugal compressor, as a kind of high value-added product, is widely used in petrochemical industry, natural gas, and so on. Since the centrifugal compressor is generally produced with one piece, it has diversity in material, structure, and working condition. Moreover, the service time of a centrifugal compressor is long, up to 20–30 years,

the fatigue damage typically belongs to high-cycle fatigue (HCF) or very high-cycle fatigue (VHCF) [4, 5]. The fatigue tests of HCF or VHCF for the centrifugal compressor impeller is very complicated and time-consuming. However, in order to reduce losses caused by compressor breakdown, the fatigue strength is required to be obtained timely for remanufacturability assessment.

Due to considerable divergence of the fatigue strength data for the centrifugal compressor impeller, it is difficult to directly obtain precise fatigue strength data based on existing theoretical models. Garwood and Nishijima et al. [6–8] found that, for low- or medium-strength steel with Vickers hardness (HV) less than 400, there is a linear relationship between rotating-bending or tension-compression fatigue strength (σ_w, MPa) and tensile strength (R_m) or Vickers hardness, as the fatigue fracture is mainly caused by surface damage. However, the linear relationship is not suitable to the material with higher hardness or tensile strength, which shows obvious divergence [5, 9]. The reason for this

phenomenon is that the damage mechanism of the centrifugal compressor impeller includes both surface damage and internal damage. Surface damage, the primary form of which is fatigue crack, mainly starts from the discontinuous part of geometry and material [6]. Internal damage under low stress is mainly caused by internal defects, such as nonmetallic inclusion and the GBF (granular bright facet) area [3]. Therefore, the data transformation, based on the available existing strength data, is a significant way to obtain target strength data for centrifugal compressor impeller remanufacturing.

The prerequisite of strength data transformation is to determine the impacts of material, structure, surface condition, and working condition on fatigue strength. In recent years, some researchers studied the influence of internal defect on fatigue strength and put forward several models based on micromechanics, such as the parallel layer model by Tanaka and Mura [10, 11] and the dislocation pileup model by Chang et al. [12, 13]. Among all models, the inclusion equivalent projected area model by Murakami et al. [14–16] is the most representative model. All these models provide theoretical basis to predict the fatigue strength of the centrifugal compressor, but it is quite difficult to transform the existing data to target data directly because high-strength steel is sensitive to small defects and nonmetallic inclusions [9] and the size and distribution of inclusion are random obviously [17–19].

For fatigue strength of centrifugal compressor impeller, the available existing experimental data and actual operating data are typically small-sample discrete data. The statistical deduction of small-sample discrete data is used to obtain the transformed data by multisource data acquirement and fusion. The common data fusion algorithm includes the grey relational method, Kalman filter method, adaptive weighted method, D-S evidence theory, and neural network theory [20–24]. These algorithms were mainly applied to signal analysis and fault diagnosis at first and recently used in the assessment of fatigue strength and residual life [25–27]. The fatigue strength of the centrifugal compressor impeller is influenced by multiple impact factors, such as design parameters, material, and structure. Since it is difficult to obtain all of the data on the impact factors, the existing available data of high-strength steel are often incomplete and scattered. Grey relational analysis [28, 29], assessing related data with the grey relational grade of sample data, has great advantage in mapping relationship analysis of incomplete and scattered data, but the problems of qualitative analysis and lack of precision still exist at the same time [30]. So, the grey relational theory must be modified in data transformation.

This paper proposes a data transformation method, according to available existing experimental data and actual operating data, to timely obtain fatigue strength data of the centrifugal compressor impeller. Firstly, impact factors are determined by analyzing the material and component fatigue strength characteristics of high-strength steel. Then, the transformation model of multisource strength data is presented based on the modified grey relational theory. The grey analysis function of factor space and target data is established; meanwhile, the residual modification on target data and the parameter correction and optimization ensure the rationality of transformation data.

2. Fatigue Strength Model

The prerequisite of establishing impact factor space and transforming fatigue strength data is to determine the impact factors of fatigue strength and the influence of each factor on fatigue strength. The fatigue strength models for material FV520B-I for the centrifugal compressor impeller are presented to analyze the impact factors of fatigue strength in two aspects: (1) the material fatigue strength model, which focuses on the influence of material hardness and yield strength; (2) the component fatigue strength model, which focuses on the influence of stress gradient, structure size, surface condition, working condition, etc.

2.1. Fatigue Strength Model of Material. Material fatigue strength is generally evaluated with material static performance. There are many impact factors on material fatigue strength; however, there is no an authoritative theory, with a quantitative and comprehensive analysis, containing all kinds of impact factors. Therefore, instead of all kinds of factors, the primary factors that have significant impacts on material fatigue strength include tensile strength, material hardness, and inclusion size [8, 14, 31].

For low-strength steel with Vickers hardness (HV) less than 400, there is the linear relationship between rotating-bending or tension-compression fatigue strength (σ_w, MPa) and material Vickers hardness [6–8]:

$$\sigma_w \approx 1.6 HV \pm 0.1 HV \, (HV \leq 400). \tag{1}$$

There is also a linear relationship between fatigue strength and tensile strength (σ_b, MPa) for structural steel with tensile strength less than 1200 MPa:

$$\sigma_w \approx 0.5 \sigma_b \, (\sigma_b \leq 1200 \, \text{MPa}). \tag{2}$$

The essence of fatigue strength is the microplastic deformation resistance of metal material, which is different from tensile strength in physical characteristics. Thus fatigue strength is only approximately estimated with tensile strength.

The mechanical properties of material FV520B used in the centrifugal compressor impeller are similar to high-strength steel (tensile strength $\sigma_b > 1200$ MPa). The above linear relationship between fatigue strength and tensile strength does not exist when tensile strength reaches up to 1250 MPa. Inclusion has been proved to be the key factor affecting fatigue strength of high-strength steel, and Murakami and Endo [32] proposed the expression of the impacts of steel matrix hardness HV and inclusion size $\sqrt{A_{in}}$ on material fatigue strength:

$$\sigma_w = \frac{1.56 \, (HV + 120)}{\left(\sqrt{A_{in}}\right)^{1/6}}. \tag{3}$$

The fatigue strength can also be determined by substituting inclusion size $\sqrt{A_{in}}$ with the size of GBF area $\sqrt{A_{GBF}}$ [33], and the size of GBF is formulated as follows:

$$\sqrt{A_{GBF}} = \left[\frac{2(HV + 120)}{\sigma_w} \right]^6. \tag{4}$$

The influence of stress ratio r and hydrogen on fatigue strength of high-strength steel is also significant. Considering the influence of stress ratio and hydrogen, the relationship between material hardness, inclusion size, and fatigue strength can be expressed as [34]

$$\sigma_w = 2.7 \frac{(HV + 120)^{15/16}}{\left(\sqrt{A_{in}} \right)^{3/16}} \left(\frac{1-r}{2} \right)^\alpha, \tag{5}$$

where α is the material parameter related to the material Vickers hardness, $\alpha = 0.226 + HV \times 10^{-4}$.

2.2. Material Similarity. Due to the great difference of fatigue property between different materials, the similar materials with similar fatigue strength are selected in data transformation, which ensures the effectiveness and accuracy of the transformed data. Generalized tangent modulus theory, estimating generality and similarity of different materials by the tangent modulus factor Φ_t, plays a great role in fatigue life prediction [35–37]. It can be used to determine fatigue property of unknown material according to the known material. Φ_t, a product factor of tangent modulus E_t, is used to express the equilibrium state of elastomer. The tangent modulus factor curve $\bar{\sigma} - \Phi_t$ can be expressed as

$$\Phi_t = \frac{1}{\bar{\sigma}} \frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{\overline{E}_t}{\bar{\sigma}} = \frac{1}{\bar{\sigma}} \frac{E_t}{E}. \tag{6}$$

Combined with *S-N* curve, the $\bar{n} - \Phi_t$ curve can be obtained by the relationship of $S = \bar{\sigma}$ and mean fatigue life $\bar{n} = N/10^6$ as well as the relationship of $S = \bar{\sigma}$ and Φ_t:

$$\bar{n} = \frac{N}{N_0} = \left\{ \left[\frac{1 - (\Phi_1 - \Phi_2)}{1 - (\Phi_t - \Phi_2)} \right]^{1/s} - 1 \right\}^{1/t}, \tag{7}$$

where $\bar{\sigma}$ and $\bar{n}$ are, respectively, the dimensionless values of S and N obtained from the *S-N* curve and $\bar{\sigma} - \Phi_t$ and $\bar{n} - \Phi_t$ curves are the dimensionless curve taking tangent modulus factor Φ_t as the parametric variable. Furthermore, the $\bar{\sigma} - \bar{n} - \Phi_t$ curve can be obtained by putting the two curves into a diagram. Figure 1 shows the $\bar{\sigma} - \bar{n} - \Phi_t$ curve for four kinds of material FV520B-I, X12Cr13-I, KMN-I, and FV520B-S. The $\bar{\sigma}(\Phi_t)$ curve is derived from the stress-strain curve of material, while the $\bar{n}(\Phi_t)$ curve is derived from the *S-N* curve of material. Thus, the $\bar{\sigma} - \bar{n} - \Phi_t$ curve reflects the working performance of material in the condition of static load and symmetrical cyclic load, respectively.

As shown in Figure 1, the above four kinds of materials have a high similarity for both $\bar{\sigma} - \Phi_t$ curve and $\bar{n} - \Phi_t$ curve, and especially when $\Phi_t \geq 1.4$, the curves for these material gradually tend to be unanimous. So the four kinds of material can be regarded as similar material, and their fatigue properties are similar. Therefore, the obtained fatigue strength data are more scientific and reliable using these materials.
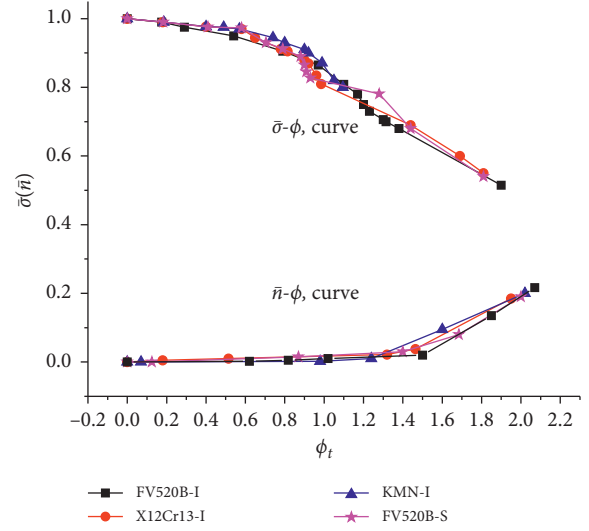

FIGURE 1: $\bar{\sigma} - \bar{n} - \Phi_t$ curve of these four materials.

2.3. Fatigue Strength of Components. Material fatigue strength is often obtained by experiments, and the component fatigue strength is obtained by transforming material fatigue strength, taking into account the impacts of structure shape, stress gradient, surface condition, and temperature The relationship between material fatigue strength and component fatigue strength can be expressed as [38]

$$\sigma_{af,C} = \sigma_{A,tsc} \cdot f_{tot,af}, \tag{8}$$

$$f_{tot,af} = \frac{1}{f_{ST,af}} \cdot \frac{\sqrt{f_{GR,af}^2 - 1.0 + f_{1,af}^2}}{f_{m,af}} \cdot f_{TI,af} \cdot f_{TP,af} \cdot f_{GS,af}, \tag{9}$$

where $\sigma_{af,C}$ is the endurance stress limit of the local component, $\sigma_{A,tsc}$ is the material alternating stress limit, $f_{tot,af}$ is the overall impact factor, $f_{ST,af}$ presents the statistical impact factor, $f_{GR,af}$ is the stress gradient impact factor, $f_{1,af}$ is the combined surface roughness and degree of forging impact factor, $f_{m,af}$ is the mean stress impact factor, $f_{TI,af}$ is the thermal impact factor, $f_{TP,af}$ is the technological parameter impact factor, and $f_{GS,af}$ is the general surface factor.

The statistical impact is t assumed as a log-normal distribution of the strength parameters:

$$f_{ST,af} = \frac{\sigma_{af,90}}{\sigma_{af,i}}, \tag{10}$$

where $\sigma_{af,90}$ is the material alternating stress limit with a survival probability of 90% and $\sigma_{af,i}$ is the endurance stress limit value with the required survival probability i.

The stress gradient impact factor on endurance stress limit of the local component can be expressed as below:

$$f_{GR,af} = 1 + \frac{(\sigma_{A,b}/\sigma_{A,tsc}) - 1}{(2/b)^\nu} \cdot \chi'^\nu, \tag{11}$$

where χ' is the relative stress gradient, $\sigma_{A,b}$ is the material alternating stress limit for bending, $\sigma_{A,tsc}$ is the material alternating stress limit for tension or compression, ν is the

material parameter, and b is the diameter of the smooth sample under flexural load.

If the material sample has a mean roughness depth $R_{Z,s} = 1$, the respective surface condition of a component acts fully on the endurance stress limit:

$$f_{SR,af} = \frac{1 - 0.22\left(lgR_{t,C}\right)^{0.64} \cdot lg\sigma_{UTS} + 0.45 \cdot \left(lgR_{t,C}\right)^{0.53}}{1 - 0.22 \cdot \left(lgR_{t,M}\right)^{0.64} \cdot lg\sigma_{UTS} + 0.45 \cdot \left(lgR_{t,M}\right)^{0.53}}, \tag{12}$$

where $R_{t,M}$ is the maximum roughness depth of the material sample and $R_{t,C}$ is the maximum roughness depth of the component.

The mean stress impact on the endurance stress limit can be expressed as

$$f_{m,af} = \frac{\sigma_{A,tsc}}{\sigma_A\left(\sigma_m\right)}, \tag{13}$$

where $\sigma_{A,tsc}$ is the material alternating stress limit and $\sigma_A\left(\sigma_m\right)$ is the material alternating stress limit at the mean stress σ_m.

If the temperature is greater than 100°C, the appropriate impact factor is determined as

$$f_{TE,af} = 1 - a_{T,af} \cdot 10^{-3} \cdot \left(\frac{T}{°C} - 100\right), \tag{14}$$

where $a_{T,af}$ is the impact coefficient of temperature and T is the temperature in centigrade.

Taking into account the differing strengths of materials as a function of the effective diameter of the semi or the unfinished casting, the type of material and the technological treatment, e.g., tempering, the technological parameter impact factor can be calculated as

$$f_{TP,af} = \frac{1}{K_d\left(d_{eff,S}\right)} \cdot \min\left(K_d\left(d_{eff}\right), K_d\left(d_{eff,P}\right)\right), \tag{15}$$

where d_{eff} is the effective diameter of the semi or the unfinished casting, $d_{eff,P}$ represents d_{eff} up to which no technological parameter influence is considered, and $d_{eff,S}$ represents d_{eff} according to the respective material standard.

General surface factor $f_{GS,af}$ can be defined according to special surface properties, such as shot peening, rolling,

carburizing, nitriding, induction hardening, and flame hardening, and these impact factors are primarily based on professional experience.

2.4. Grey Relational Model of Multisource Strength Data considering Incomplete Factor.
The influence of design parameters, working conditions, and transmission modes on fatigue strength is smaller than material parameters and stress ratio, especially in the normal working mode. In this paper, design parameters, working conditions, and transmission modes are in the range of normal working parameters. Combined with the fatigue strength models above, the influence of these parameters on fatigue strength follows a roughly linear law.

Grey relational analysis for data with linear laws can be used to obtain a relatively precise result by considering the correlation of source data in factor space and establishing grey relational functions of target data and source data [28, 39]. However, it is difficult to obtain the strength data of the centrifugal compressor impellor, thus the source data are often incomplete. In addition, source data, formed and obtained in different periods, have obvious multisource characteristic. Thus, it is necessary to correct the transformation result acquired from limited data.

2.5. Grey Relational Grade for Fatigue Strength Data.
Based on the point relational coefficient model, the relational grade of fatigue strength can be determined, with taking the influence of HV, tensile strength, inclusion size, etc., on material fatigue strength, as well as the influence of structure shape, stress gradient, surface condition, mean stress, temperature, etc., on component fatigue strength into account.

Supposing the measured array $\mathbf{X}_0 = [x_0(1), x_0(2), \ldots, x_0(n)]$ is the characteristic array, and the reference array is denoted as

$$\mathbf{X}_i = [x_i(1), x_i(2), ..., x_i(n)]. \tag{16}$$

The relational coefficient $\alpha_i(k)$ between the characteristic array and the reference array is defined as [39]

$$\alpha_i(k) = \frac{\min_{i=1}^m \max_{k=1}^n \left|x_0(k) - x_i(k)\right| + \xi\max_{i=1}^m \max_{k=1}^n \left|x_0(k) - x_i(k)\right|}{\left|x_0(k) - x_i(k)\right| + \xi\max_{i=1}^m \max_{k=1}^n \left|x_0(k) - x_i(k)\right|}, \tag{17}$$

where ξ is the resolution coefficient and the value is in the range of 0 ~ 1.

Denote $\gamma\left(\mathbf{X}_0, \mathbf{X}_i\right)$ with the following formula:

$$\gamma\left(\mathbf{X}_0, \mathbf{X}_i\right) = \frac{1}{n}\sum_{k=1}^n \alpha_i(k). \tag{18}$$

If $\gamma\left(\mathbf{X}_0, \mathbf{X}_i\right)$ meets the axiom of normalization, integrality, duality, and proximity [40], $\gamma\left(\mathbf{X}_0, \mathbf{X}_i\right)$ is defined as the grey relational grade between $\mathbf{X}_0$ and $\mathbf{X}_i$.

The grey relational analysis is an approximate analysis method. Since the influence of each factor on fatigue strength follows an approximately linear law and source data are always incomplete, the calculation result has some deviation compared with the real data. It is necessary to reduce the deviation by residual error modification [29].

For any reference array $\mathbf{X}_i = [x_i(1), x_i(2), \ldots, x_i(n)]$, if $\mathbf{X}_0$ and $\mathbf{X}_i$ have an increment Δx_i, the variation of target value f_i is denoted with $\Delta f(k)$ and the variation of first data is $\Delta f(0)$. The modified residual error of target value is

$$\beta(\mathbf{X}_0, \mathbf{X}_i) = \varepsilon_i \Delta f(0), \qquad (19)$$

where

$$\varepsilon_i = \frac{1}{n} \sum_{k=1}^{n} \left(\frac{\Delta f(k)}{\Delta f(0)} \right). \qquad (20)$$

Furthermore, the modified transformation model based on grey relational grade can be established as follows:

$$f'_i = \gamma(\mathbf{X}_0, \mathbf{X}_i) f_i + \beta(\mathbf{X}_0, \mathbf{X}_i). \qquad (21)$$

In equation (21), since the original target value is modified with residual error considering individual difference of data in factor space, data deviation caused by data transformation can be decreased compared with the original target value.

2.6. Weight Calculation of Multisource Strength Data.
Generally speaking, the factors in factor space and target data have the same parameter name but have the difference in material, construction, or working condition data. Due to the multisource data in factor space, it is necessary to weigh the influence of different types of data on transformation result, to distinguish the influence grade of each factor. Entropy-weight theory has an advantage on simple calculation and little subjective interference to calculate weight coefficient [41, 42].

Based on the comprehensive evaluation method [43, 44], the entropy-weight coefficient decision matrix of strength data for centrifugal compressor is established to express the correlation among three kinds of factors that include working condition, design parameter, and construction parameter. Thus, the decision matrix is

$$\mathbf{Q}_{m \times n} = \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \vdots & \vdots \\ q_{m1} & \cdots & q_{mn} \end{pmatrix}. \qquad (22)$$

In $m \times n$ matrix, the number of influence factors are n and each factor is assessed m times independently. The treated decision matrix $\mathbf{Q}_{m \times n}$ reduces subjective interference to some extent.

Define X_{ij} as the membership degree of the jth factor, and $Q_M = \max(q_{ij})$, $Q_m = \min(q_{ij})$, $i = 1, 2, \ldots, m$. Then the membership degree of each factor in $\mathbf{Q}_{m \times n}$ is

$$X_{ij} = \begin{cases} \dfrac{Q_{ij} - Q_m}{Q_M - Q_m}, & \text{positive factors}, \\[3mm] \dfrac{Q_M - Q_{ij}}{Q_M - Q_m}, & \text{negative factors}. \end{cases} \qquad (23)$$

Normalizing each factor of the membership degree:

$$x_{ij} = \frac{X_{ij}}{\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij}}. \qquad (24)$$

Thus, the decision matrix of membership degree $\mathbf{x}_{m \times n}$ will be established. The entropy value of the jth factor is

$$e_j = \frac{-1}{\ln m} \sum_{i=1}^{m} y_{ij} \ln y_{ij}. \qquad (25)$$

To avoid the failure of logarithm in equation (25) due to possible zero element in decision matrix $\mathbf{x}_{m \times n}$, y_{ij} is expressed as

$$y_{ij} = \frac{(1 + x_{ij})}{\sum_{i=1}^{m}(1 + x_{ij})}. \qquad (26)$$

Finally, the weight value ω_j, which reflects the influence of jth factor on strength data, can be calculated with

$$\omega_j = \frac{1 - e_j}{n - \sum_{j=1}^{n} e_j}. \qquad (27)$$

Data transformation is determined not only by the geometric correlation of factor space but also by the logic correlation between element of factor space and target value, which is consistent with perceptual knowledge. It provides the mathematical basis for the following multidata transformation. In this circumstance, the strength data could be transformed equivalently by the modified grey relational model, as long as the space factor is established and the element of space is quantified.

3. Transformation of Multisource Strength Data of Centrifugal Compressor Impeller

The strength of the centrifugal compressor impeller can be determined by transforming the multisource data, including design parameter, working condition, material, and structure. The difference of the known strength data and unknown data of the centrifugal compressor impeller can be divided into the following types: (1) the same material and structure, but different working conditions, (2) the same structure and working condition, but different materials, (3) the similar material, structure, and working condition, but different design parameters, (4) the similar material and blade structure, but different structure of the compressor impeller, and (5) the comprehensive differences involving many impact factors.

3.1. Parametric Treatment of Each Impact Factor.
The influence of the stress ratio on fatigue strength is discussed with material parameters, and these parameters have the largest influence on fatigue strength of the compressor impeller. The difference between these parameters for different materials is significant, which cannot be transformed according to approximate grey relational grade. Therefore, the fatigue strength of target material is obtained by analogically scaling tensile strength and Vickers hardness of material in factor space based on equation (5). The scaling factor κ_{Mi}, used as the whole scaling factor of multisource data transformation, is formulated as follows:

$$\kappa_{\text{Mi}} = \frac{\sigma_{bi}}{\sigma_{b0}} \left(\frac{\text{HV}_0 + 120}{\text{HV}_i + 120} \right)^{16/15} \cdot \frac{(1 - r_0)^{\alpha_0}}{(1 - r_i)^{\alpha_i}} \cdot 2^{\alpha_i - \alpha_0}, \qquad (28)$$

where $\alpha = 0.226 + \text{HV} \times 10^{-4}$.

The design parameters and quantifiable working condition parameters related to fatigue strength of the compressor impeller, such as blade inlet width, blade inlet diameter, blade thickness, and other typical impeller blade structure parameters, can be determined according to equations (8)–(15). Assuming that other parameters are constant, the influence of these parameters on the fatigue strength conforms to a linear law approximately. Bringing source data into factor space, the corresponding grey relational grade can be obtained by equations (17)–(21).

There are some unquantifiable working condition factors, including lubrication condition, wear degree, and cleanness of fluid, also affecting fatigue strength of the compressor impeller. And it is difficult to directly qualify these factors. Thus, the evaluation mechanism of "good" or "bad" is used to quantify these factors. The rating scales are shown in Table 1.

The impeller transmission mode affects the fatigue strength, and the actual carrying capacity is not the same even if the same impeller is in different levels. Therefore, the impeller transmisson mode, including the impeller position level n, blade form N, impeller form M, rotor form P, and other qualitative parameters, should be quantified by the evaluation mechanism as shown in Table 1 and bring them into factor space in data transformation.

3.2. Comprehensive Transformation Model of Multisource Strength Data.

In the above factors, the material factor acted as the independent correction factor κ_{Mi}, while the other three kinds of factors have different influence on data translation in different conditions. Therefore, the weights of the three kinds of impacts factors on strength data translation should be taken into account for coordinating these factors, and the general translation model is

$$S_i' = \kappa_{\text{Mi}}\left(\omega_{\text{Wi}}S_{\text{Wi}} + \omega_{\text{Pi}}S_{\text{Pi}} + \omega_{\text{Si}}S_{\text{Si}}\right), \qquad (29)$$

where S_i' is the predicted fatigue strength data of the unknown centrifugal compressor impeller, κ_{Mi} is the material correction factor, S_{Wi} is the transformation data of working condition, S_{Pi} is the transformation data of the design parameter, S_{Si} is the transformation data of the transmission mode, ω_{Wi} is the influence weight of the working condition, ω_{Pi} is the influence weight of design parameters, and ω_{Si} is the influence weight of the transmission mode.

4. Case Study

4.1. Fatigue Strength Prediction for Impeller.

It is unlike to obtain failure strength data of impeller directly due to the shortage of failure data. Therefore, it should select the exciting source data with similar centrifugal compressor impeller to transform equivalently. The No. 5 and the predicted No. 0 impellers are used in the centrifugal coal gas compressor. The No. 1 and No. 4 impellers are used in the centrifugal ethylene compressor, and the No. 2 and No. 3 impellers are used in the centrifugal air compressor. The appearance of No. 3 and No. 5 impellers is shown in Figure 2. The five reference impellers in this paper have similar material of FV520B-I, X12Cr13-I,

TABLE 1: Rating scales for unquantifiable working condition factors.

Parameter name	Symbol	Good	Bad
Lubrication condition	L	1	5
Wear degree	W	1	5
Cleanness of fluid	C	1	5

KMN-I, and FV520B-S, respectively, as well similar structure and working condition, with predicted impeller. Meanwhile, these five reference impellers have similar stress distribution with the predicted No. 0 impeller, just like Figure 3. The reference impellers data are listed in Tables 2 and 3.

The four kinds of material can be regarded as similar material, and their fatigue properties are similar. Meanwhile, these six impellers have roughly similar structure, shape, and operating mode, so they have similar stress distribution and fatigue damage positions. Therefore, it is a reasonable assumption that all the impellers have a similar fatigue damage mechanism, and similar nondimensional fatigue damage versus the nondimensional number of cycles.

In order to show multisource data transformation with the method presented above, the No. 1 reference impeller is exampled as follows.

4.2. Transformation of Multisource Design Parameter Data.

The factor space of design parameters for the No. 0 compressor impeller is $\mathbf{X}_0 = [38.9\ 357\ 2.5\ 73\ 33\ 9.12\ 434.5]$, and the factor space for the No. 1 impeller is $\mathbf{X}_1 = [29\ 225\ 2.8\ 134.5\ 30\ 9\ 325]$. If $\xi = 0.5$, according to equations (17) and (18), the relational grade of two factor spaces is

$$\gamma_{\text{P}}\left(\mathbf{X}_0, \mathbf{X}_1\right) = \frac{1}{n}\sum_{k=1}^{n}\alpha_i(k) = 0.7155. \qquad (30)$$

If the same proportion of increment $\Delta(x_i) = 0.1x_i$ is added to each element of $\mathbf{X}_0$ and $\mathbf{X}_1$, the residual error for fatigue strength data affected by design parameters is $\beta_{\text{P}}(\mathbf{X}_0, \mathbf{X}_1) = 97.3\ \text{MPa}$ according to equations (19)–(21).

Thus, the transformed fatigue strength for the No. 1 impeller in the condition of multisource design parameters is

$$S_{P1}' = \gamma_{\text{P}}\left(\mathbf{X}_0, \mathbf{X}_1\right)S_1 + \beta_{\text{P}}\left(\mathbf{X}_0, \mathbf{X}_1\right) = 570.9\ \text{MPa}. \qquad (31)$$

4.3. Transformation of Multisource Structure Parameter Data.

According to Table 3, the factor space of the structure for the No. 0 compressor impeller and No. 1 compressor impeller is

$$\begin{cases} \mathbf{X}_{\text{S}}(0) = \left(n_i, N_i, F_i, W_{Ri}\right) = (1\ 1\ 2\ 1), \\ \mathbf{X}_{\text{S}}(1) = \left(n_1, N_1, F_1, W_{R1}\right) = (1\ 1\ 2\ 1). \end{cases} \qquad (32)$$

The relational grade of two factor spaces is $\gamma_{\text{S}}(\mathbf{X}_0, \mathbf{X}_1) = 1$, and the transformed fatigue strength is 662 MPa.

4.4. Transformation of Multisource Working Condition Data.

The working conditions, such as lubrication, wear, and cleanliness, are classified and are evaluated by the
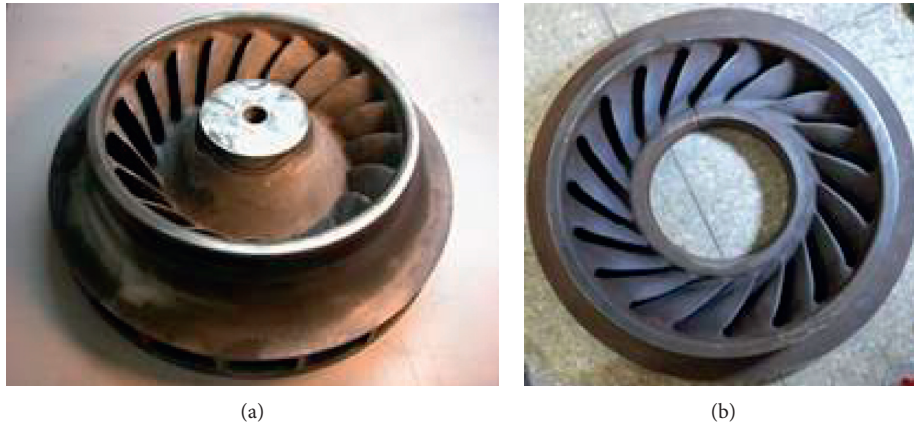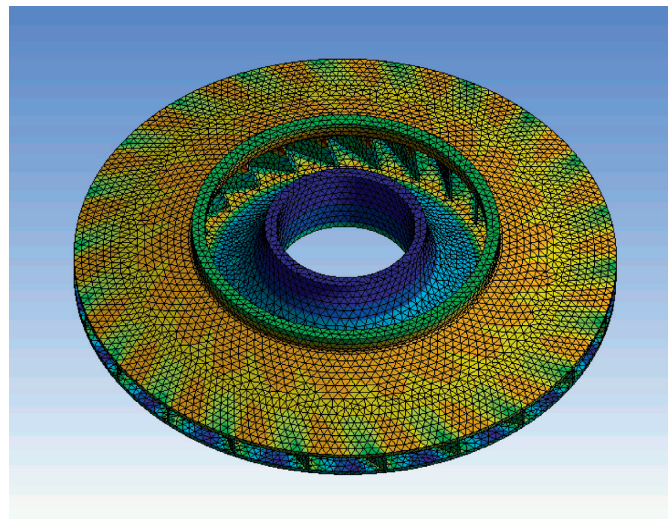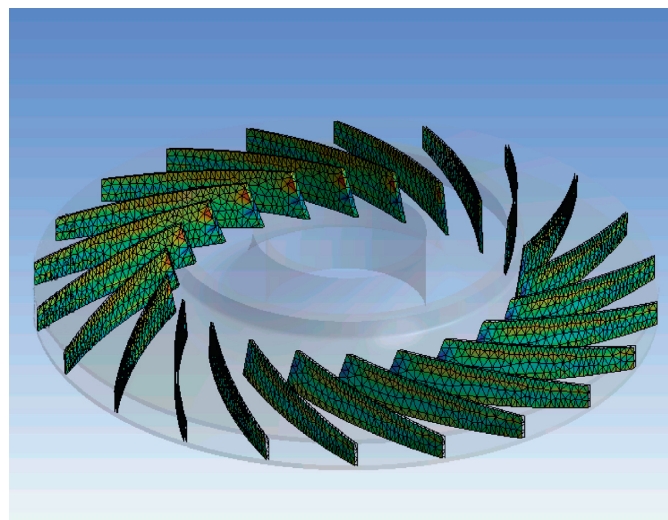
(a)

(b)

FIGURE 2: No. 3 (a) and No. 5 (b) reference impellers.

(a)

(b)

FIGURE 3: (a) The stress distribution of the No. 0 impeller and (b) its blade.

TABLE 2: Design parameters of the reference impeller.

Parameter name		No. 0	No. 1	No. 2	No. 3	No. 4	No. 5
Width of blade inlet	b_1 (mm)	38.9	29	72	57	45	59
Diameter of blade inlet	D_1 (mm)	357	225	480	340	245	324
Blade thickness	t (mm)	2.5	2.8	3.8	6	5.6	5
Velocity of blade inlet	c_1 (m/s)	73	134.5	155.5	89.5	81.2	120
Installation angle of blade inlet	β_{1A} (°)	33	30	25	25	20	28
Slope of wheel cover	θ (°)	9.12	9	7.5	0	6	9.1
Curvature radius of blade	R (mm)	434.5	325	576	413	397	512

TABLE 3: Parameter of working condition, transmission mode, and material property of the reference impeller.

Parameter name		No. 0	No. 1	No. 2	No. 3	No. 4	No. 5
Input speed	n_0 (r/min)	8877	22500	6667	5035	11200	8200
Inlet pressure	P_{in} (kg/cm^2)	1.02	1.025	1.1005	0.955	1.008	1.048
Inlet temperature	t_{in} (°C)	27	21	30	37	29	32
Outlet pressure	P_{out} (kg/cm^2)	1.78	2.954	3.533	3.793	2.223	1.898
Lubrication condition	L	1	1	1	1	1	1
Wear degree	W	0	1	0	0	0	1
Cleanness of fluid	C	1	1	2	3	3	2
Position of impeller	N	1	1	1	1	1	1
Impeller form	N	1	1	2	1	1	1
Blade form	F	2	2	2	3	2	3
Rotor form	W_R	1	1	1	3	1	2
Stress ratio	R	0.82	0.72	0.85	0.77	0.81	0.79
Material	M	FV520B-I	FV520B-I	X12Cr13-I	KMN-I	FV520B-S	FV520B-S
Tensile strength	σ_b	1078	1078	735	931	931	887
Vickers hardness	HV	370	370	280	320	320	300
Fatigue strength value	S (MPa)	—	662	445	529	476	579

professional engineers in factory according to Table 1. The factor space is established as follows.

$$\begin{cases} \mathbf{X}_W(0) = \left(P_{in0}, P_{out0}, n_{00}, t_{in0}, L_0, W_0, C_0\right) = (1.02, 1.78, 8877, 27, 1, 0, 1), \\ \mathbf{X}_W(1) = \left(P_{in1}, P_{out1}, n_{01}, t_{in1}, L_1, W_1, C_1\right) = (1.025, 2.954, 22500, 21, 1, 1, 1). \end{cases} \tag{33}$$

The relational grade of two factor spaces is $\gamma_W(\mathbf{X}_0, \mathbf{X}_1) = 0.7325$, and the residual error of fatigue strength data affected by working conditions is $\beta_W(\mathbf{X}_0, \mathbf{X}_1) = 75$ MPa. So the transformed fatigue strength value is

$$S'_{W1} = \gamma_W(\mathbf{X}_0, \mathbf{X}_1)S_1 + \beta_W(\mathbf{X}_0, \mathbf{X}_1) = 559.1 \text{ MPa.} \tag{34}$$

4.5. Transformation of Multisource Material Parameter Data.
According to equation (28), the scaling coefficient of data transformation $\kappa_{Mi} = 0.89$.

4.6. Weight Calculation of Multisource Data.
Table 4 confirms the weight of each element in factor space. It is available to confirm the influence of each subfactor on impeller fatigue strength by grading.

According to Table 4 and judging one by one, based on the comprehensive evaluation method, the decision matrix of entropy weight is

$$\mathbf{Q}_{m \times n} = \begin{bmatrix} 3.3 & 2.6 & 4.5 \\ 3.2 & 2.4 & 4.1 \\ 4.2 & 2.5 & 4.8 \\ 3.6 & 2.1 & 4.3 \end{bmatrix}. \tag{35}$$

According to the above decision matrix, the membership degree matrix $x_{m \times n}$ can be obtained by equations (23) and (27) and the weight value of the corresponding three impact factors is $\omega_{Pi} = 0.345$, $\omega_{Si} = 0.267$, and $\omega_{Wi} = 0.363$ respectively.

Taking the weight value into equation (29), the fatigue strength value for the No. 1 impeller after transformation is

$$S'_1 = 0.89 \times (0.345 \times 570.9 + 0.267 \times 662 + 0.363 \times 559.1)$$
$$= 513 \text{ MPa.} \tag{36}$$

Similarly, the equivalent data of the fatigue strength for the else four impellers can also be obtained as shown in Table 5.

TABLE 4: Factor set, subfactor set, and the rating scale.

Factor sets		Subfactor sets	Score
	u_{11}	Width of blade inlet	4
	u_{12}	Diameter of blade inlet	3
	u_{13}	Installation angle of blade inlet	1
Parameter (U_1)	u_{14}	Curvature radius of blade	1
	u_{15}	Blade thickness	5
	u_{16}	Velocity of blade inlet	3
	u_{17}	Slope of wheel cover	1
	U_{21}	Inlet pressure	2
Working condition	U_{22}	Outlet pressure	3
(U_2)	U_{23}	Input speed	4
	U_{24}	Inlet temperature	2
	u_{31}	Lubrication condition	2
Working condition	u_{32}	Wear degree	2
(U_3)	u_{33}	Cleanness of fluid	1

TABLE 5: Equivalent data for fatigue strength after transformation.

Impeller number	No. 1	No. 2	No. 3	No. 4	No. 5
Equivalent data (MPa)	513	472	622	507	598

4.7. Effectiveness Discussion for the Transformed Fatigue Strength Data.

The transformed fatigue strength data with the proposed method in this paper and the theoretical strength data of centrifugal compressor impeller are presented as below:

$$\begin{cases} S'_1 = 513, \ S'_2 = 472, \ S'_3 = 622, \ S'_4 = 507, \ S'_5 = 598, \\ S_1 = 624, \ S_2 = 420, \ S_3 = 592, \ S_4 = 467, \ S_5 = 505. \end{cases} \tag{37}$$

Taking the distribution of theoretical fatigue strength data as reference, the effectiveness of the transformation data can be discussed by judging whether the transformation data and theoretical data obey the approximate population distribution. According to hypothesis testing rules, compatibility and consistency for the transformation data and theoretical data are solved, that the mean value of strength and the variance of strength are tested by t-test and F-test, respectively. The critical value is $t_{0.05}(8)$ in the condition of significant level $\alpha = 0.1$. The test statistics is calculated as follows:

$$|t| = \frac{542.4 - 521.6}{75.43 \times \sqrt{1/5 + 1/5}} = 0.4360 \leq t_{0.05}(8) = 1.8595. \tag{38}$$

Similarly,

$$F_{0.95}(4, 4) = 0.1565 \leq F = \frac{4125.3}{7255.29} \tag{39}$$

$$= 0.5686 \leq F_{0.05}(4, 4) = 6.39.$$

There is no significant difference between the transformation data and theoretical data in the mean value and the variance. Therefore, these two sets of data are consistent and approximately obey the same population distribution. It shows that the transformation data have certain credibility
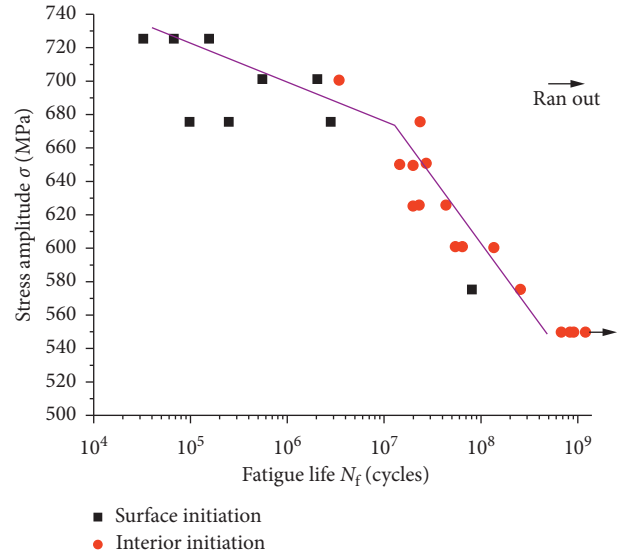

FIGURE 4: Experimental result of fatigue property for material FV520B-I.

and the transformed strength can be regarded as the fatigue strength of the No. 0 impeller.

In Figure 3, the maximum stress of the impeller, as well as the most likely to fatigue damage position, is located near the blade root connecting with wheel cover. Since the tensile strength of material FV520B-I for the No. 0 centrifugal compressor impeller is close to the high-strength steel up to 1100 MPa, ultrasonic fatigue testing is applied to verify the failure stress state of this position and test the rationality of transformation strength data. The S-N curve of FV520B-I is shown in Figure 4.

In Figure 4, there is no fatigue limit at 10^7 cycles and the fatigue strength is about 550 MPa corresponding to the stress amplitude at $4 \times 10^8 {\sim} 10^9$ cycles. By comparing the transformed fatigue strength data obtained by the proposed method in this paper and the material strength data of the centrifugal compressor impeller, we see that the maximum difference in the 5 studied cases is 15%, which meets the requirements of safety margin (100% or 150%) according to codes. Therefore, it shows that the transformed strength data are effective.

5. Conclusions

To obtain the fatigue strength of the centrifugal compressor impeller quickly, a fatigue strength data transformation model based on modified grey relational theory is established, combining with the impact of tensile strength, structure size, mean stress, temperature, etc., on fatigue strength. This model has the following characteristics:

(1) The grey relational model is established to analyze the impact of each factor on fatigue strength, and it is modified with residual error to reduce the deviation by source strength data incompletion.

(2) The multisource strength data transformation model and the decision matrix of factor space based on entropy-weight theory are presented, to evaluate the

impact of multisource data, such as structure parameter, working condition, and transmission mode, on fatigue strength comprehensively.

(3) Taking the influence of nonquantitative parameters into account, the working condition and transmission mode are quantified with the related standard. By analyzing the influences of quantitative and nonquantitative parameters comprehensively, the transformed strength data are more scientific.

The very-high-cycle fatigue test of FV520B-I is applied to verify the rationality of the fatigue strength transformation model, which agrees with multisource data transformation result well; therefore, the above model and method are effective.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Acknowledgments

References

[1] B. Xu, S. Y. Dong, S. Zhu et al., "Prospects and developing of remanufacture forming technology," *Journal of Mechanical Engineering*, vol. 48, no. 15, pp. 96–105, 2012.

[2] X. Zhang, X. F. Chen, B. Li et al., "Review of life prediction for mechanical major equipments," *Journal of Mechanical Engineering*, vol. 47, no. 11, pp. 100–116, 2011, in Chinese.

[3] Y. Murakam, T. Nomoto, and T. Ueda, "Factors influencing the mechanism of superlong fatigue failure in steels," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 22, no. 7, pp. 581–590, 1999.

[4] R. Rajasekaran and D. Nowell, "Fretting fatigue in dovetail blade roots: experiment and analysis," *Tribology International*, vol. 39, no. 10, pp. 1277–1285, 2006.

[5] S. Sivaprasad, N. Narasaiah, S. K. Das et al., "Investigation on the failure of air compressor," *Engineering Failure Analysis*, vol. 17, no. 1, pp. 150–157, 2010.

[6] M. F. Garwood, *Interpretation of Tests and Correlation with Service*, American Society for Metals, Cleveland, Ohio, USA, 1951.

[7] S. Nishijima, "Statistical analysis of fatigue test data," *Journal of the Society of Materials Science, Japan*, vol. 29, no. 316, pp. 24–29, 1980, in Japanese.

[8] Y. Furuya and S. Matsuoka, "Improvement of gigacycle fatigue properties by modified ausforming in 1600 and 2000 MPa-class low-alloy steels," *Metallurgical and Materials Transactions A*, vol. 33, no. 11, pp. 3421–3431, 2002.

[9] C. Y. Chen, *Fatigue and Fracture*, Huazhong University of Science and Technology Press, Wuhan, China, 1st edition, 2002.

[10] K. Tanaka and T. Mura, "A dislocation model for fatigue crack initiation," *Journal of Applied Mechanics*, vol. 48, no. 1, pp. 97–103, 1981.

[11] K. Tanaka and T. Mura, "A theory of fatigue crack initiation at inclusions," *Metallurgical Transactions A*, vol. 13, no. 1, pp. 117–123, 1982.

[12] R. Chang, W. L. Morris, and O. Buck, "Fatigue crack nucleation at intermetallic particles in alloys—a dislocation pile-up model," *Scripta Metallurgica*, vol. 13, no. 3, pp. 191–194, 1979.

[13] W. L. Morris and M. R. James, "Statistical aspects of fatigue crack nucleation from particles," *Metallurgical Transactions A*, vol. 11, no. 5, pp. 850-851, 1980.

[14] Y. Murakami, S. Kodama, and S. Konuma, "Quantitative evaluation of effects of non-metallic inclusions on fatigue strength of high strength steels. I: basic fatigue mechanism and evaluation of correlation between the fatigue fracture stress and the size and location of non-metallic inclusions," *International Journal of Fatigue*, vol. 11, no. 5, pp. 291–298, 1989.

[15] Y. Murakami and H. Usuki, "Quantitative evaluation of effects of non-metallic inclusions on fatigue strength of high strength steels. II: fatigue limit evaluation based on statistics for extreme values of inclusion size," *International Journal of Fatigue*, vol. 11, no. 5, pp. 299–307, 1989.

[16] Y. Murakami, C. Sakae, and K. Ichimaru, "Three-dimensional fracture mechanics analysis of pit ormation mechanism under lubricated rolling-sliding contact loading," *Tribology Transactions*, vol. 37, no. 3, pp. 445–454, 1994.

[17] S. Beretta and Y. Murakami, "Statistical analysis of defects for fatigue strength prediction and quality control of materials," *Fatigue*, vol. 21, no. 9, pp. 1049–1065, 1998.

[18] C. W. Anderson, G. Shi, H. V. Atkinson et al., "Interrelationship between statistical methods for estimating the size of the maximum inclusion in clean steels," *Acta Materialia*, vol. 51, no. 8, pp. 2331–2334, 2003.

[19] G. Shi, H. V. Atkinson, C. M. Sellars et al., "Computer simulation of the estimation of the size of the maximum inclusion in clean steels by the generalized Pareto distribution method," *Acta Materialia*, vol. 49, no. 10, pp. 1813–1820, 2001.

[20] G. Niu, B. S. Yang, and M. Pecht, "Development of an optimized condition-based maintenance system by data fusion and reliability-centered maintenance," *Reliability Engineering & System Safety*, vol. 95, no. 7, pp. 786–796, 2010.

[21] J. Cuadrado, D. Dopico, A. Barreiro et al., "Real-time state observers based on multibody models and the extended Kalman filter," *Journal of Mechanical Science and Technology*, vol. 23, no. 4, pp. 894–900, 2009.

[22] S. S. Gao, Y. M. Zhong, and B. Shirinzadeh, "Random weighting estimation for fusion of multi-dimensional position data," *Information Sciences*, vol. 180, no. 24, pp. 4999–5007, 2010.

[23] O. Basir and X. H. Yuan, "Engine fault diagnosis based on multi-sensor information fusion using Dempster-Shafer evidence theory," *Information Fusion*, vol. 8, no. 4, pp. 379–386, 2007.

[24] G. Niu, S. S. Lee, B. S. Yang et al., "Decision fusion system for fault diagnosis of elevator traction machine," *Journal of Mechanical Science and Technology*, vol. 22, no. 1, pp. 85–95, 2008.

[25] H. C. Kuo and L. J. Wu, "Prediction of heat-affected zone using Grey theory," *Journal of Materials Processing Technology*, vol. 120, no. 1, pp. 151–168, 2002.

[26] L. Mao, Q. S. Zuo, G. L. Liu et al., "Residual life prediction of three-way catalytic converter by using non-equidistance grey forecasting model," *Journal of Central South University*, vol. 29, no. 3, pp. 1351–1354, 2012.

[27] D. Xu, Y. C. Xu, X. Chen et al., "Residual fatigue life prediction based on grey model and EMD," *Journal of Vibration Engineering*, vol. 24, no. 1, pp. 104–110, 2011, in Chinese.

[28] J. L. Deng, "Control problems of grey systems," *Systems & Control Letters*, vol. 1, no. 5, pp. 288–294, 1982.

[29] K. W. David, "Grey system and grey relational model," *ACM SIGICE Bulletin*, vol. 20, no. 2, pp. 2–9, 1994.

[30] K. W. David and J. L. Deng, "Contrasting grey system theory to probability and fuzzy," *ACM SIGICE Bulletin*, vol. 20, no. 3, pp. 3–9, 1995.

[31] Q. Y. Wang, C. Bathias, N. Kawagoishi et al., "Effect of inclusion on subsurface crack initiation and gigacycle fatigue strength," *International Journal of Fatigue*, vol. 24, no. 12, pp. 1269–1274, 2002.

[32] Y. Murakami and M. Endo, "Effects of defects, inclusions and inhomogeneities on fatigue strength," *International Journal of Fatigue*, vol. 16, no. 3, pp. 163–182, 1994.

[33] Y. Murakami, "Metal fatigue: effects of small defects and nonmetallic inclusions," *Chromatographia*, vol. 70, no. 7, pp. 1197–1200, 2002.

[34] Q. Y. Wang, J. Y. Berard, S. Rathery et al., "High-cycle fatigue crack initiation and propagation behaviour of high-strength spring steel wires," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 22, no. 8, pp. 673–677, 1999.

[35] P. L. Luo, "The substance and significance of combined theory of strength and stability," *Journal of Mechanical Strength*, vol. 8, no. 2, pp. 56–60, 1986, in Chinese.

[36] P. L. Luo, K. L. Liu, and H. Y. Luo, "Application of combined theory of strength and stability to fracture mechanics," in *Proceedings of the 6th International Offshore and Polar Engineering Conference*, pp. 26–31, Springer, Los Angeles, CA, USA, May 1996.

[37] P. L. Luo, H. W. Luo, and F. S. Tong, "The influence of prebuckling deformations and stresses on the buckling of the spherical shell," *International Journal of Offshore and Polar Engineering*, vol. 1, no. 4, pp. 284–292, 1991.

[38] E. Haibach, *FKM Guideline-Analytical Strength Assessment of Components in Mechanical Engineering*, VDMA-Verlag, Frankfurt, Germany, 5th edition, 2003.

[39] J. L. Deng, *The Primary Methods of Grey System Theory*, Huazhong University of Science Press, Wuhan, China, 1st edition, 1987.

[40] S. Liu and J. Y. L. Forrest, *Grey Systems: Theory and Applications*, Springer-Verlag Berlin Heidelberg, Berlin, Germany, 1st edition, 2011.

[41] D. L. Mon, C. H. Cheng, and J. C. Lin, "Evaluating weapon system using fuzzy analytic hierarchy process-based on entropy weight," *Fuzzy Sets and Systems*, vol. 62, no. 2, pp. 127–134, 1994.

[42] Y. B. Du, H. J. Cao, F. Liu et al., "Evaluation of machine tool remanufacturing scheme based on entropy weight and AHP," *Computer Integrated Manufacturing*, vol. 17, no. 1, pp. 84–88, 2011.

[43] J. E. Yang, M. J. Hwang, T. Y. Sung et al., "Application of genetic algorithm for reliability allocation in nuclear power plants," *Reliability Engineering & System Safety*, vol. 65, no. 3, pp. 229–238, 1999.

[44] J. Ma, Z. P. Fan, and L. H. Huang, "A subjective and objective integrated approach to determine attribute weights," *European Journal of Operational Research*, vol. 112, no. 2, pp. 397–404, 1999.