Research Article

Design, Modeling, and Simulation of Two-Piece Trapezoidal Piezoelectric Devices for Sensing and Energy Harvesting

Nan Chen¹ and Vishwas Bedekar²

¹Department of Computational Science, Middle Tennessee State University, Murfreesboro 37132, TN, USA

²Department of Engineering Technology, Middle Tennessee State University, Murfreesboro 37132, TN, USA

Correspondence should be addressed to Vishwas Bedekar; vishwas.bedekar@mtsu.edu

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The objective of the research is to design a high power energy harvester device through a two-piece trapezoidal geometry approach. The performance of the composite two-piece trapezoidal piezoelectric PZT-PZN polycrystalline ceramic material is simulated using COMSOL Multiphysics. Results are analysed using the series configuration of a two-piece trapezoidal composite bimorph cantilever which vibrates at the first fundamental frequency. The two-piece trapezoidal composite beam designs resulted in a full-width half-maximum electric power bandwidth of 2.5 Hz while providing an electric power density of 16.81 mW/cm³ with a resistive load of 0.08 MΩ. The authors believe that these results could help design a piezoelectric energy harvester to provide local energy source which provides high electric power output.

1. Introduction

The recent microsensors and energy harvesters are made of smart materials, such as the piezoelectric materials, and magnetoelectric materials, as devices made of these materials convert the mechanical energy from surrounding ambient environment to the electric energy to provide the electric power to devices. However, magnetostrictive energy harvester not only requires the vibration in an environment but also demands the magnetic field in an environment. Piezoelectric energy harvester can provide low power at milliwatts level to the small electronic devices. Traditionally, these small electric devices were powered by batteries. When a battery is depleted, the old battery needs to be replaced with a new battery. The periodic battery replacements not only interrupt the operation of these devices but also pose risks when the devices are taken out of the service for battery placement. Therefore, these small electric devices call for a different power source other than the traditional chemical batteries. The piezoelectric material is one of the promising materials in harvesting the ambient mechanical vibration energy due to the high-voltage and milliwatt-level power

density output. There have been two different focuses to enhance the performance of a vibration energy harvester (VEH): the electric power [1, 2] and the bandwidth [2] of the VEHs. Zhang et al. claimed that the higher change rate of the section area of the VEH contributes to the higher power output of the VEH [2]. Zhang et al. reported that adding the external magnets and decreasing the distance between the VEH and the external magnets increase the bandwidth of the VEH [2]. Benasciutti et al. reported that the trapezoidal shaped VEHs produce more electric power density than the rectangular shaped VEHs due to the nonlinearly distributed stress of the trapezoidal VEH design [3]. However, Benasciutti et al. did not define the degree of the stress uniformity nor compare the bandwidth difference between the trapezoidal and the triangular shaped VEHs. Muthalif and Nordin reported that the electric output voltage and the mechanical resonance frequency increase when the width of the free end of the composite cantilever approaches to 0 mm because they suggested the uniform strain of the triangular VEH contributed to the higher voltage output [4], but neither the power density nor the power density bandwidth was investigated. Muthalif and Nordin proposed an

analytical solution for modes of the resonance frequency of the trapezoidal and the triangular composite piezoelectric cantilever beam [4]. Reilly et al. classified five broadband applications out of fourteen ambient vibrational applications [5], and the authors reported that the 50% of the electric power (2 mW/4 mW) bandwidth is 8 Hz (95–103 Hz) of their proposed composite trapezoidal piezoelectric VEHs [5]. Compared with Reilly et al.'s work, our first trapezoidal bimorph design reported 50% electric power (7.86 mW/15.71 mW) with a bandwidth of 2.5 Hz (19.5–22.0 Hz). This work addresses the goal on designing a two-piece trapezoidal composite bimorph piezoelectric energy harvester operating in the transverse mode with the aim to enhance the maximum real power density in an electric circuit with an optimal resistor, as well as the full-width half-maximum (FWHM) power density-mechanical vibration broadband performance, compared them with the results of the one-piece trapezoidal beam design based on the PZT-PZN-Scheme 4 polycrystalline ceramic piezoelectric material: $0.8 [\text{Pb} (\text{Zr}_{0.52} \text{Ti}_{0.48})\text{O}_3] - 0.2 [\text{Pb} (\text{Zn}_{1/3} \text{Nb}_{2/3})\text{O}_3]$ from our previous work on the one-piece trapezoidal beam model [6, 7]. The PZT-PZN-Scheme 4 material was reported because of its superior structural power density (0.1713 mW/cm^3 measured): 20.97% higher than that of PZT-ZNN-Scheme 2 (0.1416 mW/cm^3), 15.38% higher (49.5 mW/cm^3 measured) in the electric power output, and 31.13% higher (0.499 mW/cm^3 measured) in the piezoelement power density, as the “two-step sintering” method was used to reduce grain size and increase density, which leads to higher relative dielectric (ϵ_r is 1588, 2.95~4.23 times higher than those of PZT-ZNNs) and the piezoelectric property of PZN-PZN-Scheme 4 material (the charge constant in the thickness mode d_{33} is 400 pC/N, 2.4~2.6 times higher than that of PZT-ZNNs. The charge constant of PZT-PZN in the transverse mode d_{31} is 153.73 pC/N, 2.75~3.07 times higher than those of PZT-ZNNs) [8]. Not only are the power densities of the PZT-PZN-Scheme 4 higher than its PZT-ZNN competitors, but also is PZT-PZN-Scheme 4's quality factor Q significantly lower (78.7) than that of PZN-ZNN (780) which makes PZT-PZN-Scheme 4 a better material choice for a high power energy harvester as the PZT-PZN-Scheme 4 is a low-quality factor material [2]. Yuan et al. derived a formula for the voltage sensitivity of a triple-layer trapezoidal piezoelectric beam which is useful for sensor applications [9]. Many researchers reported that the electric power and voltages of a trapezoidal piezoelectric beam are higher than those of a rectangle beam, given the same volume of PZT [9, 10]. Benasciutti et al. studied the voltage and the electric power characteristics of a one-piece trapezoidal and the reversed one-piece trapezoidal shapes of the piezoelectric bimorphs. The maximum power generated was reported to be $650 \mu\text{W}$ at 50 Hz excitation frequency [10]. Yuan et al. reported that the rectangular piezoelectric beam's maximum electric power is about 8.6 mW at the operating frequency of 180 Hz, whereas the trapezoidal piezoelectric beam's maximum electric power is 24.2 mW at the operating frequency of 130 Hz [9]. Shachtele et al. proposed a two-ported model to describe a trapezoidal piezoelectric beam using an admittance matrix. The admittance matrix describes the linear relations between the electric charge Q , the tip deflection δ , and the voltage V and the force F

at the tip [11]. Yet, the admittance is used to determine the internal inductance of a given trapezoidal bimorph in our research. Hosseini and Hamedei also reported improvement in energy harvesting using V-shaped piezoelectric bimorphs. It is shown that increasing the width W_2 (at free end) lowers the resonance frequency, which agrees with our results. It was also shown that the trapezoidal shape generates higher voltage compared to the simple rectangular cantilever beam bimorph which matches our findings in previous works [6, 12]. In this paper, we explore a unique design of 2-piece trapezoidal shaped piezoelectric bimorph to understand its effects on harvested power, power density, and bandwidth. The 2-piece trapezoidal geometry which is explained later has not been explored in the literature so far for energy harvesting applications. The material property values of PZT-PZN-Scheme 4 material are tabulated in Table 1.

2. Materials and Methods

The authors used the technique to convert the piezoelectric and the mechanical material property values from Table 1 to the compliance matrix (a tensor of 4th order) of the PZT-PZN-Scheme 4 piezoelectric material from our previous work [7].

The procedure used in seeking the maximum electric real power output is through finding the first resonance frequency of the two-piece piezoelectric beam; once the first resonance frequency of the beam is known through running eigenfrequency study on the model, then we vary the loading resistor to find the optimal resistance where the power reaches maximum: the external resistance was varied from 0.01 M Ω to 0.2 M Ω (an arbitrarily large range) with 0.01 M Ω resolution to match the internal impedance of the beam. 0.01 M Ω interval resolution is arbitrarily chosen to limit the computation time to a reasonable level. The optimal resistance is found when a peak of the electric power appears in the scanning range. When the optimal resistance is found for a given model, we then vary the vibrational frequency symmetrically around the first eigenfrequency to see how wide the vibration frequency can get from the peak power output to 50% of the peak power output.

The equivalent electric circuit is presented below in Figure 1. The piezoelectric energy harvester can be simply modeled as a series LRC electric circuit with an alternative voltage source V in a circuit, L represents the internal inductance, r represents the internal resistance (damping effect), and C represents the internal capacitance. The letter Z in the equivalent electric circuit represents the external impedance. When the external impedance Z matches the internal impedance Z_{int} of LRC circuit ($Z_{\text{ext}} = Z_{\text{int}} = \sqrt{r^2 + (X_c - X_L)^2}$, where X_c is the internal capacitive reactance the bimorph and X_L is the internal inductance of the bimorph, the electric power output reaches maximum (widely known as the rule of impedance matching). Due to the current plan of research, we only consider the external loading resistor Z . When the external load impedance Z_{ext} matches the internal impedance Z_{int} numerically ($Z_{\text{ext}} = Z_{\text{int}}$), the electric circuit delivers the

TABLE 1: PZT-PZN-Scheme 4 and brass material property names and values [8].

Value		Name
k_{31}	0.32	Electro-mach coupling factor of PZT-PZN
d_{31}	153.73	Piezoelectric charge constant: pC/N
g_{31}	0.011	Piezoelectric voltage constant: Vm/N
k_p	0.59	The coupling coefficient of PZT-PZN
ϵ_r	1588	The dielectric constant of PZT-PZN
Q_m	78.7	Mechanical quality factor
ρ	7879	The density of PZT-PZN (kg/m ³)
ζ	0.017	Mechanical damping ratio
Temp	1100	Sintering temperature (°C)
T	3	Sintering time (hours)
M_t	0	Tip mass (g)
d_{33}	400	Piezoelectric charge constant: pC/N
g_{33}	0.028	Piezoelectric voltage constant: Vm/N
ρ_s	8800	The density of brass UNS C22000 (kg/m ³)
ν_s	0.307	Poisson's ratio of the brass layer
ϵ_{rs}	4500	The dielectric constant of the brass layer
Es	110	Young's modulus of the brass layer (GPa)
E	66	Young's modulus of PZT-PZN (GPa)

FIGURE 1: An equivalent LRC electric circuit of a piezoelectric bimorph energy harvester.

maximum electric power to the external load resistor Z_{ext} . In this case, when the external loading resistance matches the impedance of the magnetite of the internal impedance, the bimorph generates the maximum electric power, as the following equation shows:

$$Z = R_{loading} = \sqrt{r^2 + (X_L - X_C)^2}. \quad (1)$$

Taking the square on both sides of equation (1), we obtain equation (2), where f is the frequency of the oscillating electric signal in the equivalent electric circuit, which is presented in Figure 1:

$$R_{loading}^2 = +r^2 + \left(2\pi fL - \frac{1}{2\pi fC} \right)^2. \quad (2)$$

In equation (2), the internal resistance r , the internal inductance L , and the internal capacitance C of a bimorph are fixed values; therefore, the frequency f of the oscillating signal in the equivalent electric circuit will be affected by the value of the external loading resistor. The internal resistance r , the internal inductance L , and the internal capacitance C need to be found before modeling the electric characteristics of a piezoelectric bimorph cantilever beam. The internal resistance r can be found by applying Kirchhoff's voltage law

(KVL) and Ohm's law in a closed electric equivalent circuit as shown in Figure 1, as it is expressed in the following equation:

$$\epsilon = Ir + IR + IZ_C + IZ_L, \quad (3)$$

where ϵ is the electromotive force (EMF), I is the electric current, R is the external loading resistor, Z_C is the impedance of the capacitor, and Z_L is the impedance of the inductor in the closed electric equivalent circuit as shown in Figure 1. We can then take two measurements by using two different external loading resistors in the simulation and rewriting Kirchhoff's voltage law (KVL) and Ohm's law in a pair of the following equations:

$$U_1 + I_1 (X_L - X_C) = -rI_1 + \epsilon, \quad (4)$$

$$U_2 + I_2 (X_L - X_C) = -rI_2 + \epsilon, \quad (5)$$

where U_1 is the voltage across the external loading resistor R_1 , I_1 is the current going through the external resistor on the first measurement. U_2 is the voltage across a different external loading resistor R_2 , and I_2 is the current going through the external resistor R_2 in the second measurement. As U_1 , U_2 , I_1 , I_2 , X_L , and X_C are known parameters as they can be found in the simulations, we have two equations and two unknowns (r , ϵ), and the internal resistor r can be easily found by solving the pair of equations (4) and (5). Thus, the pair of equations (4) and (5) can be simplified to equation (6), where Z_1 is the vector sum of the impedance of the inductor and the capacitor in the first measurement with one external loading resistor R_1 , and Z_2 is the vector sum of the impedance of the inductor and the capacitor in the second measurement with one external loading resistor R_2 :

$$r = \frac{-(U_2 - U_1) + (I_2 |Z_2| - I_1 |Z_1|)}{I_2 - I_1}. \quad (6)$$

However, it would be impossible to find the internal resistor r of a given bimorph without knowing the values of the internal inductance L and the internal capacitance C of a bimorph in the closed electric equivalent circuit as shown in Figure 1. The inductance L and the capacitance C in the closed electric equivalent circuit can be found in the COMSOL simulation. The reactance of the equivalent electric circuit can be found by the following equations:

$$|Z_1| = |X_{L1} - X_{C1}| = \left| 2\pi f_1 L - \frac{1}{2\pi f_1 C} \right|, \quad (7)$$

$$|Z_2| = |X_{L2} - X_{C2}| = \left| 2\pi f_2 L - \frac{1}{2\pi f_2 C} \right|. \quad (8)$$

The frequencies f_1 and f_2 of the oscillating electric signal in two different resistive loading are different due to the loading resistance difference, which was expressed in equation (2); therefore, $|Z_1| \neq |Z_2|$. Equations (7) and (8) are substituted into equation (6), and the internal resistance r is expressed in the following equation:

$$r = \frac{-(U_2 - U_1) + (I_2 |2\pi f_2 L - 1/2\pi f_2 C| - I_1 |2\pi f_1 L - 1/2\pi f_1 C|)}{I_2 - I_1} \quad (9)$$

where the two distinctive oscillating frequencies f_1 and f_2 of the electric signal in two different resistive loadings can be found in simulations. The internal resistance r of the bimorph can be found by taking two different measurements as shown in equation (9). Equation (9) can be simplified to equation (10) once the inductance L and the capacitance C in the closed equivalent electric circuit are known through the simulation, as two oscillating frequencies f_1 and f_2 are set to the resonance frequencies of the equivalent electric circuit:

$$r = \frac{-(U_2 - U_1)}{I_2 - I_1}, f = f_r. \quad (10)$$

The internal resistance r of the bimorph can be found by just taking two different voltage and current measurements ($U_1, U_2, I_1, I_2, U'_1, U'_2, I'_1, I'_2$) with two different external resistive loads as shown in equation (10), given the value of the inductance L and the capacitance C which are to be found in two separate simulations. The capacitance C is the ratio between the electric charge Q ($7.1028 \times 10^{-6} \text{ C}$, $8.3341 \times 10^{-6} \text{ C}$) on the upper surface of the bimorph and the voltage U (100 V for both designs) between the surfaces of the bimorph as shown in equation (11) which can be found by looking up the component of the capacitance matrixes. C_{d1} in the "Derived Values" section in a stationary study, where C_{d1} is the capacitance of the first trapezoidal bimorph beam design when the shorter width W_1 is 18 mm and the longer width W_2 is 40 mm. C_{d2} is the capacitance of the second trapezoidal bimorph beam design when the longer shorter W_1 is 18 mm and the longer width W_2 is 52 mm. The inductance L of the bimorph can be calculated by equation (13), where mef , Y_{11} is the admittance and $\text{mef} \cdot \omega$ is the angular frequency of the electric signal in the AC equivalent electric circuit, both of which can be found in the simulation:

$$C_{d1} = \frac{Q}{U} = \frac{7.1028 \times 10^{-6} \text{ C}}{100 \text{ V}} = 71028 \text{ pF},$$

$$L_{d1} = 776 \text{ H}, \quad (11)$$

$$f_{r1} \frac{1}{2\pi\sqrt{LC}} = 21.4 \text{ Hz},$$

$$C_{d2} = \frac{Q}{U} = \frac{8.3341 \times 10^{-6} \text{ C}}{100 \text{ V}} = 83341 \text{ pF},$$

$$L_{d2} = 643 \text{ H}, \quad (12)$$

$$f_{r2} \frac{1}{2\pi\sqrt{LC}} = 21.7 \text{ Hz},$$

$$\left(L = \text{imaginary} = \frac{1}{\text{mef} \cdot Y_{11} / \text{mef} \cdot i\omega} \right). \quad (13)$$

The internal resistances r_1 and r_2 are calculated by equation (10) under the resonance frequencies f_{r1} and f_{r2} of

the electric signal in the equivalent circuit by taking two different voltage and current measurements ($U_1, U_2, I_1, I_2, U'_1, U'_2, I'_1, I'_2$) with two different external resistive loads R_1 (0.01 M Ω) and R_2 (0.02 M Ω), which are arbitrarily chosen:

$$r_1 = \frac{-(U_2 - U_1)}{I_2 - I_1} = \frac{-(22.95 \text{ V} - 11.393 \text{ V})}{0.0011475 \text{ A} - 0.0011393 \text{ A}} = 1.4 \text{ M}\Omega, \quad (14)$$

$$r_2 = \frac{-(U'_2 - U'_1)}{I'_2 - I'_1} = \frac{-(31.537 \text{ V} - 17.844 \text{ V})}{0.0015768 \text{ A} - 0.0017844 \text{ A}} = 65 \text{ k}\Omega. \quad (15)$$

Thus far, the internal resistances (r_1 and r_2), the capacitance, and the inductance of two bimorphs can be obtained through calculation.

The values of the internal inductance L , the internal capacitance C , and the internal resistance r of the bimorph in an equivalent circuit together determine the value of the quality factor Q by equation (16) by using equations (10)–(13):

$$Q = \frac{1}{r} \sqrt{\frac{L}{C}}. \quad (16)$$

Alternatively, the quality factor of a structure can also be calculated by equation (17), which is also adopted by COMSOL. f is the resonance frequency of the vibration. Due to the mechanical damping, the complex value of resonance frequency has an imaginary part (complex number). For instance, the complex resonance frequency of the two-piece trapezoidal bimorph (in the first design, the length of the single plate is 60 mm, W_1 is 40 mm, and W_2 is 2 mm) $14.5 + 0.2i \text{ Hz}$ in a COMSOL eigenfrequency study. The quality factor is 28.645 can be obtained by the following equation:

$$Q = \frac{\text{abs}(f)}{2 * \text{imaginary}(f)}. \quad (17)$$

In such a way, we can plot the quality factor Q for all permutations of each bimorph geometry of two trapezoidal designs in Figures 2 and 3. The quality factor Q increases when the longer length W_2 increases for both trapezoidal designs. Yet, the shorter width W_1 does not have a significant effect on the quality factor Q .

To investigate the quality factor Q , we need to understand the complex nature of the eigenfrequency. The reason for the imaginary component w_i of the angular frequency is that it determines if the amplitude ($e^{w_i t}$) of the oscillation of the VEH grows or shrinks in time, as equation (18) indicated. $A(t)$ is a general time-dependent damped oscillation function. $e^{-j\omega_r t}$ is a regular oscillation term, which can be further expanded by Euler's formula. The real part (w_r) of the angular frequency determines the physical oscillation frequency. The positive real part ($e^{w_i t}$) of the angular frequency indicates the amplitude of the oscillation that grows with time. The negative real part ($e^{w_i t}$) of the angular frequency indicates the amplitude of oscillation that shrinks with time [13]:

FIGURE 2: The quality factor of the first trapezoidal bimorph design. The structure loss factor is 0.025. The damping ratio is 0.017. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

FIGURE 3: The quality factor of the second trapezoidal bimorph design. The structure loss factor is 0.025. The damping ratio is 0.017. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

$$A(t) = e^{w_i t} e^{-j w_r t}, \quad (18)$$

$$w = w_i + j w_r.$$

$$\zeta = \frac{\text{imaginary}(f)}{\text{abs}(f)}. \quad (19)$$

The damping ratio ζ is defined by COMOSL in the following equation:

The damping ratio ζ (0.017) can be obtained by taking out the real part and the imaginary part out of the complex

eigenfrequency and plug them into equation (19). There is very little damping (ζ is 0.017) when the two-piece trapezoidal bimorph (in the first design, the length of the single plate is 60 mm, W_1 is 40 mm, and W_2 is 2 mm) vibrates under its first resonance frequency 14.5 Hz. The damping ratio or the structural loss factor (0.015) has no effect on the resonance frequency of the bimorphs. A high damping ratio ζ has a negative impact on the power output of a trapezoidal composite bimorph due to a high loss factor.

The quality factor Q and the resonance frequency f_r of the two-piece trapezoidal beam both contribute to the resonance width. The resonance bandwidth Δf of an oscillator can be defined by the full-width at half-maximum of its power at the resonance vibrational frequency. A piezoelectric energy harvester with a low-quality factor has a wider resonance bandwidth as the resonance bandwidth Δf (FWHM) is positively proportional to the resonance frequency f_r and is negatively proportional to the quality factor. The relation can be expressed in equation (20). f_r is the resonance frequency of the trapezoidal beam:

$$\Delta f = \frac{f_r}{Q}. \quad (20)$$

The resonance frequency f_r of a series electric circuit can be found when the capacitive reactance and the inductive reactance are equal ($X_c = X_L$ and $2\pi fL = 1/2\pi C$). Therefore, the resonance oscillating frequency f_r is commonly expressed by equation (21), where L is the inductance and C is the capacitance of a given bimorph:

$$f_r = \frac{1}{2\pi\sqrt{LC}}. \quad (21)$$

Equations (16) and (21) can be substituted into equation (20); therefore, the FWHM resonance bandwidth Δf can be expressed in a relation (22), where r is the internal resistance of the bimorph and L is the internal inductance of the equivalent electric circuit. The resistivity of the composite materials (PZTPZN, brass) and the internal inductance L of the bimorph both contribute to the length of the bandwidth of a bimorph, which can be expressed in the following equation:

$$\Delta f \sim \frac{r}{2\pi L}. \quad (22)$$

The bandwidth of the resonance frequency Δf is found once the quality factor Q and the resonance frequency f_r are known. As we can see from equations (20) and (22), a higher equivalent internal resistance r (series-connected bimorph) and/or a lower equivalent internal inductance L in an electrical circuit will contribute to a lower quality factor of a series LRC system, which ultimately leads to a wider resonance bandwidth Δf of a system. Connecting two unimorphs in series to make one bimorph helps to widen the bandwidth response as the series connection of bimorph has the higher total resistance and the lower total inductance compares it with that of the parallel connection, as the total internal resistance r and the total internal inductance L are expressed in equations (23) and (24). r_1 is the resistance of the upper PZTPZN layer. L_1 is the inductance of the upper PZTPZN

layer. r_2 is the resistance of the lower PZTPZN layer. L_2 is the inductance of the lower PZTPZN layer. r_3 is the resistance of the middle brass layer. Many researchers modeled the piezoelectric energy harvesters with similar equivalent LRC circuit [9, 14]:

$$r = r_1 + r_2 + r_3, \quad (23)$$

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}. \quad (24)$$

The resonance frequency can be expressed by the analytical computation and the eigenfrequency analysis. The analytical resonance frequency is calculated by Young's modulus E , the rotational momentum I (moment of inertia around the axis of rotation), the length of the beam L (60 mm), the mass of beam m , and tip mass M_t (0g), as explained in our previous work [7]. The analytical formula of the resonance vibrational frequency can be expressed in equation (25) for the transversal vibration. As the tip mass m increases, the resonance angular frequency ω decreases. k is the stiffness of the beam. Therefore, the tip mass is often used to fine-tune the resonance frequency of a beam:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{3EI/L^3}{(33/140)mL + M_t}}, \quad f = 2\pi\omega. \quad (25)$$

The principal axis of the two-piece trapezoidal bimorph designs is along the edge of the fixed end of the beam during vibration. Let us name that the fixed end of the beam in the x -direction. The rotational momentum I_x at the center of the two-piece bimorph can be calculated by the definition of the rotational momentum of a rigid body in equation (26):

$$I_x = \int r^2 dm, \quad (26)$$

where m_i is the mass of an infinitesimally small volume in the two-piece trapezoidal bimorph domain. r is the distance from that region to the axis. dm can be calculated by finding the product of the density of the composite materials ρ and the infinitesimally small volume dv :

$$dm = \rho dv. \quad (27)$$

The infinitesimally small volume dv is the product of the surface area dA of that small region and the thickness dt of that small region on the bimorph domain:

$$dv = dA dt. \quad (28)$$

The surface area dA of the small region is the product of the width dw and the height dl :

$$dA = dw dt. \quad (29)$$

Equation (28) is substituted into equation (27). Therefore, equation (26) is rewritten into equation (30):

$$dm = \rho dA dt = \rho dw dl dt. \quad (30)$$

Equation (30) is substituted into equation (26). Therefore, equation (26) can be rewritten to the integral formula in

equation (31), where w_1 is the shorter width of the bimorph and w_2 is the longer width of the bimorph:

$$I_x = 2 \int_{w_1}^{w_2} \int_0^L \int_0^t \int_0^L r^2 \rho dw dl dt dr. \quad (31)$$

The multiplier “2” in the equation is accounted for the “two”-piece composite bimorph in equation (31). Equation (31) can be simplified to equation (32), which is used to approximate the rotational momentum of a composite bimorph with the rotational axis along the x -axis through the centroid of the beam. The density of PZT-PZN and the density of brass are similar numerically and the thickness of the brass layer is very thin (0.05 mm). t is the total thickness of the composite bimorph (0.65 mm). Therefore, equation (32) is derived to approximate the rotational momentum of any trapezoidal shaped composite bimorph beam with a thin substrate along the center principle axis:

$$I_x = 2t(W_2 - W_1) \frac{L^4}{3}. \quad (32)$$

The rotational momentum I_x along the fixed edge in the x -direction can be obtained by applying the Parallel Axis Theorem in equation (32), where I_x is the rotational momentum with the rotational axis along the x -axis through the centroid of the composite trapezoidal bimorph. M is the total mass of the trapezoidal. d is the distance of the translation from the original axis to the new axis, which is the length of the single trapezoidal plate L (60 mm):

$$I'_x = I_x + Md^2. \quad (33)$$

Equation (32) is substituted into equation (33). The rotational momentum I'_x along the fixed edge in the x -direction can be obtained as follows:

$$I_x = 2t\rho(W_2 - W_1) \frac{L^4}{3} + ML^2. \quad (34)$$

Thus far, we can calculate the rotational momentum of any one end-free and one end-fixed composite trapezoidal bimorph cantilever with a thin substrate in the middle like a sandwiched structure by using equation (34). For example, equation (34) can be used to calculate the rotational momentum of a composite trapezoidal bimorph cantilever with a longer width W_2 (60 mm), a shorter width W_1 (2 mm), the length of L (60 mm), and total thickness of 0.65 mm. The density ρ of the PZT-PZN is 7869 kg/m^3 . The rotational momentum is $6.85 \times 10^{-5} \text{ kg}\cdot\text{m}^2$. Table 2 shows the resonance frequency when the widths of the bimorph change.

For the first design of the two-piece trapezoidal beam (Figure 4(a)), the volume of the two-piece trapezoidal beam ranges from 1.638 cm^3 to 3.042 cm^3 as the widths W_1 and W_2 grow. The real electric power density of the beam reaches a maximum of 19.595 mW/cm^3 (38.044 mW) when the shorter width W_1 is 2 mm and the longer width W_2 is 56 mm. The beam vibrates at the first resonance frequency at 13.7 Hz with the optimal resistor of $0.14 \text{ M}\Omega$ connected in the series configuration. The maximum full-width half-maximum (FWHM) bandwidth of the real electric power density is found by iterating 30 geometries and scanning the

TABLE 2: Resonance frequency when the widths of the bimorph change.

Width W_1	Width W_2	Thickness $2 T_p + T_s$	Resonance f_r
2 mm	40 mm	0.65 mm	14.5 Hz
6 mm	40 mm	0.65 mm	17.4 Hz
10 mm	40 mm	0.65 mm	18.7 Hz
14 mm	40 mm	0.65 mm	19.5 Hz
18 mm	40 mm	0.65 mm	20.0 Hz
2 mm	44 mm	0.65 mm	14.3 Hz
6 mm	44 mm	0.65 mm	17.1 Hz
10 mm	44 mm	0.65 mm	18.5 Hz
14 mm	44 mm	0.65 mm	19.4 Hz
18 mm. . .	44 mm. . .	0.65 mm. . .	19.9 Hz. . .

Note: the length of the beam is 60 mm (the first trapezoidal design). The variable widths, thickness, and the first real resonance frequency of the two-piece trapezoidal composite beams with no tip mass (two out of six iterations of W_1 and W_2 incremental pattern, the total of ten permutations of W_1 and W_2 out of 30 geometry permutations). The imaginary part of the resonance f_r is ignored as the imaginary part indicates the damping of the structure.

vibration frequency around the first resonance frequency f_r of 20.0 Hz of the two-piece trapezoidal beam (W_1 40 mm and W_2 18 mm) with an optimal resistor: the scan ranges from $0.9 f_r$ (18.0 Hz, 0.1 Hz interval) to $1.1 f_r$ (22.0 Hz, 0.1 Hz interval) with an optimal resistor of $0.08 \text{ M}\Omega$. The beam vibrates at the minimum half-real electric power density of 3.6327 mW/cm^3 (15.17 mW power) when vibrating between 19.5 Hz and 22.0 Hz; it reaches peak real power 15.17 mW at a frequency close to first resonance frequency at 20.767 Hz; therefore, the real power density FWHM bandwidth of the two-piece trapezoidal beam is 2.5 Hz. The voltage-frequency plot and power-frequency plot are shown in Figure 5.

For the second design of the two-piece trapezoidal beam (Figure 4(b)), the volume of the two-piece trapezoidal beam ranges from 1.638 cm^3 to 3.042 cm^3 as the widths W_1 and W_2 grow. The real power density of the beam has a maximum of 43.52 mW/cm^3 (97.136 mW), when the shorter width of the bimorph W_1 is 2 mm and the longer width of the bimorph W_2 is 60 mm. The beam vibrates at the first resonance frequency f_r at 15.1 Hz with an optimal resistor of $0.07 \text{ M}\Omega$ connected in the series. The maximum full-width half-maximum (FWHM) bandwidth of the real electric power density is revealed by iterating 30 geometries and scanning the vibration frequency around the first resonance frequency f_r of 15.1 Hz of the two-piece trapezoidal beam (W_1 52 mm and W_2 18 mm) with an optimal resistor: the scan ranges from $0.9 f_r$ (13.6 Hz) to $1.1 f_r$ (16.6 Hz) with an optimal resistor of $0.04 \text{ M}\Omega$. The beam has the minimum half-real electric power density of 8.408 mW/cm^3 (42.376 mW power) when vibrating between 19.5 Hz and 20.6 Hz; it reaches peak real power 42.376 mW at a frequency close to first resonance frequency at 20.0 Hz; therefore, the real electric power density FWHM bandwidth of the two-piece trapezoidal beam is 1.1 Hz. The voltage-frequency plot and power-frequency are shown in Figure 6.

Although the FWHM bandwidth of the one-piece trapezoidal composite beams shows its results in the designing of broadband energy harvesters in the previous work

FIGURE 4: The top-down view of the two-piece piezoelectric trapezoidal beam designs. (a) Subplot of the first two-piece trapezoidal beam design, and (b) subplot of the second two-piece trapezoidal beam design. W_1 is the shorter width, W_2 is the longer width, and L is the length of one single plate.

FIGURE 5: Electric power output vs. vibration frequency of the first trapezoidal bimorph two-piece design (a). Voltage vs. vibration frequency of the first trapezoidal bimorph two-piece design (b). W_1 is 40 mm, and W_2 is 18 mm. The full-width half-maximum is 2.5 Hz. The quality factor is 28. The structure loss factor is 0.025. The damping ratio is 0.017. The optimal resistance is 0.08 M Ω . The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

[6], the two-piece trapezoidal beam design has the potential to increase the energy harvester's power output and the frequency bandwidth. The authors proposed two new two-piece trapezoidal beam designs which are shown in Figure 7.

For the first design, the average resonance frequency of thirty permutations of the two-piece trapezoidal beams is 17.6 Hz. For the second design, the mean resonance frequency of 30 permutations of the two-piece trapezoidal beams is 17.9 Hz. T_p is the thickness of one PZT-PZN layer. T_s is the thickness of the UNS C22000 Brass layer. $2T_p + T_s$ is the total thickness of the composite bimorph beam as shown in Figure 8.

3. Results and Discussion

For both trapezoidal composite bimorph cantilever beam designs, the resonance frequency increases as the shorter width W_1 of the beam increases as the pattern is shown in Figures 9 and 10. Among thirty permutations of geometries for each two-piece bimorph design, the average resonance frequency of the first trapezoidal beam design is 17.6 Hz, and the average resonance frequency of the second trapezoidal beam design is 17.9 Hz. It indicates that the first trapezoidal beam design is suitable for harvesting lower vibration frequency applications, while the second trapezoidal beam

FIGURE 6: Electric power output vs. vibration frequency of the second trapezoidal bimorph two-piece design (a). Voltage vs. vibration frequency of the second trapezoidal bimorph two-piece design (b). W_1 is 40 mm and W_2 is 18 mm. The full-width half-maximum is 2.5 Hz. The quality factor is 28. The structure loss factor is 0.025. The damping ratio is 0.017. The optimal resistance is 0.08 M Ω . The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

FIGURE 7: The displacement of two different two-piece trapezoidal piezoelectric bimorph beam which vibrates at its first resonance frequency. (a) The left subplot is the stationary displacement (without deformation) of the first design. (b) The right subplot is the stationary displacement (without deformation) of the second design. The shorter width W_1 is 18 mm, and the longer width W_2 is 40 mm for both two bimorph designs. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

FIGURE 8: The thickness view of the composite two-piece trapezoidal beam's upper and lower PZT-PZN-Scheme 4 layer thickness for both two bimorph designs. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

design is suitable for harvesting higher vibration frequency applications. From Figures 9 and 10, we can also see the first resonance frequency of the two-piece trapezoidal composite beam decreases linearly as the longer width W_2 increases from 40 mm to 60 mm due to the increase in the mass of the beam.

The maximum real electric power density is defined by the unit real electric energy consumed in an electric circuit. Both two-piece trapezoidal designs revealed two different patterns as shown in Figures 11 and 12. For the second bimorph beam design, the electric power density (y -axis) increases as the shorter width W_1 increases (x -axis). However, the electric power density (y -axis) increases as the

FIGURE 9: The first resonance frequency vs. 30 permutations of the two-piece trapezoidal piezoelectric composite bimorph cantilever beam (first design). The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

FIGURE 10: The first resonance frequency of 30 permutations of the two-piece trapezoidal piezoelectric composite bimorph beam (second design). The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

FIGURE 11: The maximum electrical power density of 30 various geometries of the two-piece trapezoidal composite piezoelectric bimorph beam (first design) in a closed circuit with the optimal resistor load. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

FIGURE 12: The maximum electrical power density of 30 various geometries of the trapezoidal two-piece composite piezoelectric bimorph beam (second design), in a closed circuit with the optimal resistor load. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

TABLE 3: FWHM bandwidth, the min./max. real power density of various trapezoidal bimorph beams [6, 8]. The one-piece trapezoidal bimorph design has the same dimension of the two-piece trapezoidal bimorph for the comparison.

Design	Max. FWHM bandwidth	Min. structural power density	Max. structural power density
1 st trapezoidal (one-piece)	2.9 Hz	5.18 mW/cm ³	10.37 mW/cm ³
2 nd trapezoidal (one-piece)	5.6 Hz	2.11 mW/cm ³	4.22 mW/cm ³
1 st trapezoidal (two-piece)	2.5 Hz	3.63 mW/cm ³	7.26 mW/cm ³
2 nd trapezoidal (two-piece)	1.1 Hz	8.40 mW/cm³	16.81 mW/cm³

FIGURE 13: Real electrical power density FWHM bandwidth of 30 geometries of the two-piece trapezoidal composite piezoelectric bimorph beam (first design) in a closed circuit with corresponding optimal resistor load fixed. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

longer width W_2 increases (x -axis) in Figure 12. In order words, for the second trapezoidal bimorph design, the electric power density increases when the longer width of the beam increases at the fixed and free end; the electric power density increases when the shorter width of the beam decreases at the midpoint; the length of the two-piece trapezoidal beams is fixed at 60 mm, and the total thickness T of the composite bimorph cantilever is 0.65 mm. The maximum electric real power densities are compared between both two-piece trapezoidal piezoelectric bimorph composite beam designs in Table 3.

The full-width half-maximum bandwidth of the real electrical power is evaluated on both two-piece trapezoidal beam designs. The length L of both two-piece trapezoidal beams is fixed at 60 mm. The total composite thickness T is 0.65 mm. The full-width half-maximum bandwidth of the real electrical power is found by scanning the vibrational frequency near the first resonance frequency with an optimal resistor.

For the first two-piece trapezoidal design, the mean value of the real electric power FWHM density bandwidth of thirty

geometry permutations is 17.6 Hz. The maximum FWHM bandwidth is 2.5 Hz (minimum power density is 3.63 mW/cm³, the maximum power density is 7.27 mW/cm³, W_1 is 40 mm, W_2 is 18 mm, the structural volume is 2.262 cm³, and the output electric power is 7.27 mW) as they are shown in Figure 13.

For the second two-piece trapezoidal design, the average real electric power FWHM density of thirty geometry permutations is 17.9 Hz. The maximum FWHM bandwidth is 1.1 Hz (the minimum power density is 18.66 mW/cm³, the maximum power density is 37.32 mW/cm³, W_1 is 18 mm, W_2 is 52 mm, the structural volume is 2.73 cm³, and the electric power is 94.04 mW), and the electric power density increases FWHM bandwidth when the shorter width W_1 at the center of the bimorph as they are shown in Figure 14.

The comparison between the two-piece and the one-piece trapezoidal beam designs is tabulated in Table 3. The one-piece trapezoidal bimorph design has the same dimension (length, width, and thickness) of the two-piece trapezoidal bimorph for the comparison. The two-piece

FIGURE 14: Real electrical power density FWHM bandwidth of 30 various geometries of the trapezoidal composite piezoelectric bimorph beam (second design) in a closed circuit with an optimal resistor load fixed. The length of the beam L is 60 mm, the thickness of T_{piezo} is 0.3 mm, and the thickness of the brass layer T_s is 0.05 mm.

trapezoidal beam (second design) has the maximum structure electric power density of 16.81 mW/cm^3 and the highest minimum structure electric power density of 8.41 mW/cm^3 . The two-piece trapezoidal beam (first design) has a broader FWHM power frequency bandwidth of 2.5 Hz.

4. Conclusions

In this paper, the authors conclude that newer geometry and design can result in enhanced power density of bimorph piezoelectric energy harvester. The bandwidth of the harvester was found to be decreased with 2-piece trapezoidal design; however, the minimum structure power density increased by 131% to 8.40 mW/cm^3 from 5.18 mW/cm^3 as well as the maximum structural power density improved by 131.5% from 10.37 mW/cm^3 to 16.81 mW/cm^3 . Authors believe these results would help researchers design high power, high sensitivity bimorph harvesters that can provide onboard battery solutions to sensor networks.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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