Research Article

Geometric Theorems and Application of the DOF Analysis of the Workpiece Based on the Constraint Normal Line

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Received 26 May 2021; Revised 21 July 2021; Accepted 16 August 2021; Published 1 September 2021

Academic Editor: Akbar Heidarzadeh

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Aiming at the problem of judging the degree of freedom (DOF) of the workpiece in the fixture by experience, it is difficult to adapt to the analysis of the DOF of some singular workpieces. The workpiece and the fixture are used as rigid bodies, and the workpiece is allowed to move in the plane or space under the constraints of the fixture positioning point, and a set of geometric theorems for judging the DOF and overconstraint of the workpiece can be derived according to the difference in the position of the instantaneous center of the workpiece speed. The judgment of the DOF and overconstraint of the workpiece is abstracted into rules with universal meaning, which effectively overcomes the limitations of existing methods. The research results show that (1) the DOF and overconstraint of the workpiece in the fixture depend entirely on the number of positioning normal lines of the workpiece and their geometric relationship; (2) the necessary and sufficient condition for limiting the DOF of rotation of the workpiece around a certain axis is that the workpiece has a pair of parallel normal lines in the vertical plane of the axis. Using geometric theorems to judge the DOF of the workpiece is more rigorous, simple, and intuitive, which is convenient for computer-aided judgment and the reasonable layout of the positioning points of the workpiece, which can effectively avoid the misjudgment of the DOF and unnecessary overpositioning when the complex workpiece is combined and positioned on different surfaces. Several examples are used to verify the accuracy of the method and correct unreasonable positioning schemes.

1. Introduction

The DOF of the workpiece in the fixture affects the machining accuracy and determines whether the fixture meets the working requirements. Based on this, the calculation of the DOF in the fixture design is the basic task, and many scholars at home and abroad have done in-depth research and exploration [1–4]. Wu [5] described the DOF limit of the workpiece positioning reference as the following four cases: translation between surfaces, translation inline direction, rotation between surfaces, and rotation inline direction, as well as presented a method for automatically extracting constraint DOF information from process requirements. Parag et al. [6] used the unified manufacturing resource model of the STEP-NC standard to represent the connection with the NC machining center. The main contribution is to allow universal modeling of fixtures and loading equipment,

as well as machining workpiece and process modeling, to analyze the DOF of workpieces on the fixture. Paulraj and Kumar [7] used a genetic algorithm (GA) to improve the positioning of the active clamps and workpieces in the fixture system and carried out a parametric design (APDL) based on Ansys software of finite element analysis. A contact force for clamping workpiece clamps is predicted in an elastic contact model. Hunter et al. [8] proposed a functional approach to formalize the fixture design process. When some information, such as part information, contact parameters, and position points, was inputted, a relatively reasonable positioning scheme could be given out, the aim of formalization of the fixture design process would be achieved, and the relationship between processing requirements and processing freedom and actual freedom would be analyzed. Huang et al. [9], based on the product master model idea, proposed a process model-driven

aeroengine parts' machining and fixture variant design method. By designing the clamping feature mapping algorithm for isomorphic parts, the design parameter linkage between the process model and the main model of the fixture variant design is realized, which satisfies the freedom requirements of the fixture and improves the response speed of the machine-fit fixture design to the design changes of aeroengine parts. Qin et al. [10, 11] argued that the positioning accuracy is consistent with the DOF of the process, which is the solution of the calculation equation of the DOF of the workpiece, and specified the overall number of conditions to accurately determine the DOF of the workpiece. In practical applications, the local factors that meet the accuracy of component positioning are accurately obtained, and based on this, the optimization planning algorithm for the positioning scheme is obtained. Then, they used the calculation method of homogeneous linear equations, combined with the constraint fixed point in the fixture to establish the parameters, and analyzed the optimal position characteristics of the workpiece. Lu et al. [12] improved the decomposition-based multiobjective evolutionary algorithm (MOEAD) based on the migration behavior of the Gaussian mutation, which satisfies a large number of combinations of positioning points in the fixture layout, and the algorithm has a strong search capability and makes the fixture's positioning of the workpiece more reasonable. Based on positioning rules of "N-2-1" and "3-2-1," Xing et al. [13] established two deviation analysis models of over-positioning and full release patterns by the difference in the release modes of the anchor point. Li et al. [14] proposed the assumption that the workpiece surface is composed of an anisotropic section, and the component of the workpiece is fixed to the workpiece through the point contact and simulated the calculation model of the DOF of the workpiece, which is determined by the value of the matrix rank. The surface of the workpiece is not completely on a plane. The constraint form of the fixed position device and the calculation formula for calculating the DOF of the workpiece after the fixture positioning are established. By calculating the rank of the established model matrix, the DOF of the workpiece is analyzed and judged. Wu et al. [15, 16] established the constraint model which can quantitatively analyze the DOF of the workpiece by rigid-body freedom analysis theory, and they developed a fixture constraint workpiece DOF analysis system using UG software. The positioning performance reference coefficient can be used to evaluate the performance of the fixture to constrain the position of the workpiece [17, 18] and to find the optimal position of each positioning point based on the coefficient. Based on the research of combined genetic algorithm [19, 20], the optimal positioning method of the workpiece based on the genetic material library is found, and the surface quality in the workpiece clamping is improved. Song and Rong [21] established a matrix for the position parameters, solved the rank of the matrix to analyze the positioning constraint of the workpiece, and obtained the DOF property of the constraint, which can accurately obtain the scheme to meet the machining requirements of the workpiece processing. Liang et al. [22] proposed a new analytical

method for the positioning of the workpiece, which firstly analyzed the constraint DOF of the workpiece by a single positioning element, then analyzed the constraint DOF of the workpiece by the positioning element combination, and finally comprehensively analyzed the constraint state of the workpiece by positioning elements. Yang et al. [23], based on the support vector regression (SVR) surrogate model and the elite nondominated sorting genetic algorithm (NSGA-II), proposed a new SMP fixture positioning and layout multiobjective optimization method. By using the ABAQUS™ Python scripting interface, a parameterized FEA model can be established. Taking the fixture positioning layout as a design variable, through the optimization of multiple objective functions, the best positioning plan for the workpiece is obtained. Qin et al. [24–26] established a positioning scheme kinematic model, which was based on the relationship between the workpiece position offset and the positioning source error, and proposed the optimal design criteria of the positioning scheme. The design process of fixtures is over when and only when a fixture achieves good fixturing performance.

The aforementioned research on the DOF of the workpiece plays an important role in the construction of the workpiece positioning plan and the optimization of the positioning plan. However, these methods require a deeper theoretical basis, and the methods have limitations. It is still difficult to adapt to the difficult positioning problems that are often encountered in machining, such as the positioning of complex surfaces and the DOF of the workpiece combined with different surfaces [27–29]. There are still two unsolved problems of the current method. Firstly, there is no accurate formula for the analysis and calculation of the DOF of the workpiece with virtual constraints, which brings difficulties to the fixture in the positioning design [18]. Secondly, the positioning of numerous irregular workpieces is common in engineering. Usually, the constraint DOF of the workpiece by the fixture in a certain direction is not in the same direction as the axis but any other direction in the space [30]. The existing method can barely adapt to this situation.

Figure 1 shows the positioning of an oblique axis on two short V-blocks. The two V-blocks limit the DOFs of the workpiece. Current positioning theory can barely solve this problem.

Figure 2(a) shows a workpiece positioning device. The workpiece is a cylinder, and three positioning points are set on its left, middle, and right sections, respectively. The difference between either two of the three points is 120° in the circumferential direction. In this case, the motion freedom of the workpiece on the YOZ plane is overpositioned, and the rotational DOF around any axis is not constrained. If, based on Figure 2(a), another positioning point is added to the right section (or any other section) as shown in Figure 2(b), then the overpositioning of the moving constraint in the plane is found by eliminating the constraint of the workpiece, and the rotational freedom of the workpiece in the Y- and Z-directions is constrained. Why is this happening? It is difficult to explain with the current positioning theory.

FIGURE 1: Oblique axis positioned on the V-block. 1, V-block; 2, workpiece.

FIGURE 2: Three-section positioning of a cylinder. (a) Positioning point on each of the three sections. (b) Add another positioning point to the right section. 1, Workpiece; 2, supporting bolt; 3, pedestal.

The study found that the DOF of the workpiece fixed on the processing table depends on the number of positioning normal lines generated at the positioning point and the geometric relationship of the normal lines. A simple, intuitive, and universal analysis method is proposed for various plane-combined positioning, which can accurately calculate the DOF of the workpiece in the fixture positioning. Based on the positioning constraints of the fixture to the workpiece, this paper proposes the concepts of the constraint positioning normal and the constraint positioning plane and proposes a set of geometric theorems for accurately calculating the DOF of the workpiece. This method can realize the accurate calculation of the DOF of the workpiece and provide a basis for rationally arranging the positioning points of the workpiece.

2. Analysis and Verification of the Normal Line Geometry Theorem of Workpiece Freedom

2.1. Position Normal Line and Normal Plane

- (1) Position constraint normal line: it is the normal line that indicates the workpiece pointing to the constraint direction at the fixture position point. In Figure 3, the workpiece is positioned in the fixture, and the position constraint normal line Z at position O is the normal line generated by the constraint point of the workpiece.

- (2) Fixed position normal line: the normal lines are specified in uppercase letters. If there are multiple normal lines of the same type in the same direction, we can add a different subscript to the alphabet to make a distinction.

For example, F_1 and F_2 are parallel in geometric relations. The normal line points to the constraint direction of the workpiece from the position of the constraint point. Because the workpiece has six degrees of freedom according to the space, it is affected by six normal lines at most.

- (3) Constraint normal plane: if several normals of the workpiece act on the same plane in the space, it is called the constraint normal plane.
- (4) Representation of the normal plane: the normal plane is represented by Greek letters. If two normal planes are parallel, they are written in the same letter. For example, β and β_1 represent two parallel normal planes. In Figure 4(a), the plane formed by the intersection point of the constraint normal lines M and N with the workpiece can be represented by $\beta(MN)$. In Figure 4(b), the plane composed of the parallel constraint normals L and L_1 on workpiece 1 can be expressed as $\alpha(LL_1)$. In Figure 4(c), there are three constraint normal lines represented by L , M , and N , respectively, which jointly act on workpiece 1. They are in different positions in the same plane and have

FIGURE 3: Positioning normal line of the workpiece. 1, Workpiece; 2, fixture.

FIGURE 4: Several distributions of the normal line of workpieces. (a) Intersection and equivalence of two normal lines. (b) Two parallel normal lines. (c) Three normal lines intersect with one another. 1, Workpiece; 2, fixture; 3, supporting bolt; 4, V-block.

at least two intersections with each other. $\gamma(LMN)$ can be used to represent the normal plane where the normal lines are L , M , and N .

When the workpiece is positioned normally, the distribution of its normal lines in a plane can only be one of the three cases.

2.2. Geometric Relation Theory for Determining the Workpiece Constraint and Its Demonstration

Theorem 1. *The workpiece is constrained by a single normal line, and the freedom of movement towards the normal line direction is constrained.*

Proof. In Figure 3, the workpiece is installed in the fixture, and the contact point O can be simplified as a constraint normal line Z . The workpiece has the freedom of movement away from the plane of the fixture, and we assume that the relative velocity of the workpiece in the fixture at contact point O is V_r . If the relative velocity V_r is projected on the dividing velocity of the normal line, the projection is zero, indicating that the dividing velocity of the workpiece relative to the fixture in the constraint normal line direction is zero, which is the normal line Z direction. Therefore, the

translation freedom of the workpiece along the Z -direction of the constraint normal line is constrained, it can be expressed as $\vec{Z}$, but the workpiece can rotate around any point in the normal plane. $\square$

Theorem 2. *If the two constraint normal lines acting on different constraint points intersect, the translation freedom of the workpiece in the normal plane is limited and can be replaced by any two intersecting constraint normal lines at the intersection.*

Proof. In Figure 4(a), the workpiece is constrained at points a and b in the fixture, and the two equivalent constraint normal lines M and N on the constraint normal plane $\alpha(MN)$ intersect at point O . The workpiece and fixture are regarded as a rigid body, and the workpiece can move in the positioning normal plane $\alpha(MN)$. We assume that the relative velocity of the workpiece at the two contacting points, a and b , is V_{r1} and V_{r2} , respectively. Considering that the instantaneous center of the rigid body is the intersection of two vertical lines of two-point velocities, the rotation center of the workpiece motion is the intersection O of the constrained normal lines M and N in the space, the constrained workpiece can only rotate around the instantaneous center O , and the workpiece cannot move in any direction on the

plane relative to the intersection O . Hence, (1) the intersection normal line constrains the motion freedom of the workpiece in any direction. (2) The function of two intersecting constraint normal lines can be expressed as any two nonoverlapping intersecting normal lines at the intersection point O , such as normal lines P and T . $\square$

Theorem 3. *If the two constraint normals acting on the workpiece are parallel but not collinear, the freedom of movement and rotation of the workpiece in the constraint plane are limited.*

Proof. In Figure 4(b), the two constraint normal lines L and L_1 generated at the fixed positions m and n of the workpiece are parallel, and the positioning mode is a special one, as shown in Figure 4(a). The two normal lines intersect at infinity. Two results can be concluded from Theorem 2. They are as follows: (1) the translation freedom of the workpiece is limited in the normal L direction; (2) the instantaneous center of the workpiece is at infinity in this case. Thus, there is no rotation center in the constraint plane; that is, the rotational freedom of the workpiece in the constraint positioning plane $\alpha(LL_1)$ is constrained. It can be expressed as $\hat{\alpha}(LL_1)$. $\square$

Theorem 4. *If three noncoplanar constraint normal lines intersect at a point O , the translation freedom of the workpiece in the three-dimensional space is constrained and can also be equivalent to three arbitrary noncoplanar intersection normals at point O .*

Proof. The workpiece is fixed on the fixture, and the movement of the workpiece in the fixture is constrained by the intersection of three constraint normals which are not on the same plane at a point O . Given that the rotation center of the rigid body must be in the same direction with the perpendicular line (normal line) of the velocity, the rotation center of the workpiece must be the point of intersection O of the three normal lines. At this moment, the workpiece can only rotate around the fixed point O , and the workpiece cannot move in any direction relative to point O . Therefore, 1) the workpiece is subject to three noncoplanar constraint normal lines, which constraint its freedom of translation in space 2). The constraint effect of three noncoplanar constraint normal lines at intersection O is the same as that of any three noncoplanar constraint normal lines at intersection O . $\square$

Theorem 5. *The DOF of the workpiece is the constraint generated by the total DOF minus all normal lines.*

Each constraint normal line can only restrict one DOF, and the normal line sets of different geometric relations between the workpieces restrict the DOF. As shown in Figure 4(c), the three constraint points of the workpiece produce the normal lines L , M , and N that are not intersected at a point, forming the constraint normal plane $\alpha(LMN)$. It can be seen from Theorem 3 that, as shown in Figure 5, the normal lines L_1 and N intersecting in the normal plane jointly limit the translation freedom of the workpiece in any two

FIGURE 5: Equivalence and a set of normal lines. 1, Supporting bolt; 2, workpiece; 3, V-block.

directions along the constraint normal plane. Under the action of normal line L and equivalent normal L_1 , the rotational freedom (LL_1N) of the workpiece in the normal plane is constrained. Therefore, in a positioning normal plane, if three normal lines intersect at least two intersection points, the translation freedom of the workpiece in any two directions and the rotation freedom of the workpiece in the constrained normal plane are constrained, and the three freedoms in the constrained normal plane are constrained.

2.3. Criteria for Determining Overpositioning of the Workpiece. Each normal line acting on the workpiece will form a constraint, which will restrict the freedom of movement or rotation of the workpiece. According to the normal set theorem, too much positioning will cause the number of constrained normals on the workpiece to exceed the limit value of its DOF, which will cause the same DOF of the workpiece to be repeatedly constrained. When two or more normal lines of the workpiece restrict the same DOF, the workpiece is overpositioned. The judgment of overpositioning of the workpiece will directly affect the accurate analysis of the DOF of the workpiece on the fixture.

In fixture positioning, if the total DOF of the workpiece in the positioning space is less than the number of constraint normal lines generated on the workpiece, then there must be excessive positioning on the constraint of the workpiece. For example, the normal line number in a straight line is limited to 1, the normal line number on a plane is limited to 3, and the normal line number in the space is limited to 6. If the overconstrained normal is an intersection normal, then the translation DOF is overpositioned. If the overconstrained normal line is a parallel normal line, the rotational DOF in the normal plane is overpositioned.

- (1) Determination of overpositioning on a straight line: if n ($n > 1$) constraint normals on a workpiece coincide with a straight line, the workpiece has $(n - 1)$ overconstraints
- (2) Determination of overpositioning on a plane: if the workpiece has n ($n \geq 3$) normals on the constraint plane and the n ($n \geq 3$) normal lines intersect at a point in the plane or are parallel to each other, then the workpiece has $(n - 2)$ overconstraints
- (3) Determination of overpositioning in the space: if the number of constraint normal lines positioned on the workpiece is greater than six, four or more normal lines intersect at one point, four or more normal lines are parallel to one another, four or more normal lines do not intersect with one another, and either two are noncoplanar, then the workpiece is overconstrained

2.4. Necessary and Sufficient Conditions of Constraining the Rotational DOF of the Workpiece. Theorem 2 proves that the normal line parallelism generated by the workpiece positioned in the fixture was the necessary and sufficient condition to limit the rotational freedom of the workpiece in the constraint normal plane. The following proves the necessary condition to constrain the rotational DOF of the workpiece.

Theorem 1 proves that the normal line generated by a single constraint can only limit the translation freedom of the workpiece moving along the normal line direction. Here, to constrain the rotational freedom of the workpiece in the plane, two normal lines must jointly act on the workpiece, and the geometric relationship between two normal lines can only be one of the following four scenarios. The impact on the DOF of the workpiece is as follows:

- (1) Two normal lines coincide: the two constraint normal lines have the same constraint effect on the workpiece, and they are overconstrained along the normal line direction
- (2) Two normal lines intersect: the intersection normal line can be replaced by two intersecting normal lines in any direction, which can restrict the translation freedom of the workpiece in any two directions in the plane, but the rotational freedom is not restricted
- (3) Two normal lines are parallel: a pair of parallel normal lines in the constrained normal plane can constrain the translation freedom of the workpiece in the normal direction and the rotation freedom in the normal plane
- (4) Two normal lines are noncoplanar: according to the set theorem, two noncoplanar normal lines restrict two translational degrees of freedom of the workpiece along the normal line direction in the space

A single normal constrains the freedom of movement in the normal direction, the intersecting normal constrains the freedom of movement in the plane, and the spatial noncoplanar normal constrains the freedom of movement in the normal direction of the space. Only when parallel constrained normals appear in a plane or space can the

rotational freedom of the workpiece be constrained. Therefore, in the constraint normal plane, the workpiece has two parallel constraint normal lines which are a necessary condition to limit its rotational freedom.

3. Results and Discussion

3.1. Application Analysis

3.1.1. Positioning Analysis of the Oblique Axis in the V-Block. Figure 1 shows the positioning of the oblique axis on the fixture. Four normal lines are generated by the action of two V-shaped blocks, which intersect at two points O_1 and O_2 . Combined with Theorem 2, the constraint normal line can be replaced by four normal lines Z_1 , Y_1 , Z_2 , and P_2 . They constrain the four DOFs of the workpiece, that is, the DOF of movement in the normal lines $\vec{Z}$, $\vec{Y}$, and $\vec{P}$, and the DOF of rotation in the normal plane $\hat{\alpha}(Z_1Z_2)$. If another V-block is added, as shown in Figure 6, then the two other rotational DOFs are constrained, and all DOFs of the workpiece in the space are constrained.

3.1.2. Positioning Analysis of the Cylindrical Section. The positioning analysis of Figure 2(a) is shown in Figure 7(a). The parallel normal lines M , N , and L in the YOZ plane can only constrain the translational DOFs parallel to the YOZ plane. However, there are only two translational DOFs in the plane that are parallel to the YOZ plane. Therefore, there must be a normal line resulting in the displacement overpositioning of the workpiece.

Compared with Figure 7(a), a bearing is added to the right section in Figure 7(b), and the normal line is P . These conditions make the constraint on the normal lines M and P of the right section at the intersection O to be expressed as two normal lines L_1 and N_1 parallel to the normal lines of L and N . The normal lines M and P in the right section can be equivalent to the normal lines L_1 and N_1 , which are parallel to the normal lines L and N at intersection O so that the normal lines constrain four DOFs, $\vec{Z}$, $\vec{Y}$, $\hat{\alpha}(LL_1)$, and $\hat{\alpha}(NN_1)$, respectively.

3.1.3. Positioning Analysis of the Noncentering Shaft. The noncentering shaft in Figure 8 is positioned on the two top holes, and its positioning is analyzed. The fixture has three intersecting constraint normal lines at two positioning points O and O_1 . Combined with Theorem 2, the normal lines X , Y , and Z can be represented at position O and X_1 , Y_1 , and Z_1 at position O_1 . These normal lines form three positioning normal planes $\alpha(XX_1)$, $\beta(YY_1)$, and $\alpha(ZZ_1)$.

Here, $\alpha(XX_1)$ constrains $\vec{X}$ and $\hat{\alpha}(XX_1)$; $\beta(YY_1)$ constrains $\vec{Y}$ and $\beta(YY_1)$; and $\alpha(ZZ_1)$ constrains $\vec{Z}$ and $\hat{\alpha}(ZZ_1)$. It seems that the workpiece is completely positioned, but the two positioning normal planes $\alpha(YY_1)$ and $\alpha(ZZ_1)$ are in the same plane YOZ (or $Y_1O_1Z_1$), which restricts the same rotational freedom. Hence, the rotational freedom in the plane YOZ is repeatedly constrained. The right top, becoming the revolving top, results in the removal of the normal line Y_1 , and then the overpositioning is eliminated.

FIGURE 6: Positioning analysis of the oblique axis with three V-blocks' bearing. 1, V-block; 2, workpiece.

FIGURE 7: Positioning analysis of the cylindrical section. (a) Three-point positioning. (b) Four-point positioning. 1, Workpiece; 2, supporting bolt; 3, pedestal.

FIGURE 8: Positioning analysis of the noncentering shaft. 1, Left top hole; 2, eccentric shaft; 3, right top hole.

Moreover, the workpiece has rotational freedom around OO_1 .

3.1.4. Positioning Analysis of the Oblique Incision Connecting the Rod. In Figure 9, the oblique incision connecting the rod is positioned on a supporting plate and two V-blocks, and both sides are machined. The three normal lines of the supporting plate that supports the workpiece are L_1 , L_2 , and L_3 ; the normal lines of V-block 1 can be equivalent to Y and Z , and the normal lines of V-block 2 can be equivalent to Y_1 and Z_1 . The positioning normal planes are $\alpha(ZZ_1)$, $\alpha(L_1L_3)$, $\beta(YY_1)$, and $\gamma(L_1L_2)$, respectively. Furthermore, $\alpha(ZZ_1)$

constrains $\vec{Z}$ and $\hat{\alpha}(ZZ_1)$; $\beta(YY_1)$ constrains $\vec{Y}$ and $\hat{\beta}(YY_1)$; $\alpha(L_1L_3)$ constrains $\vec{L}$ and $\hat{\alpha}(L_1L_3)$; and $\gamma(L_1L_2)$ constrains $\vec{\gamma}(L_1L_2)$. As the positioning normal planes $\alpha(L_1L_3)$ and $\alpha(ZZ_1)$ are parallel and constrain the same rotational DOF, they result in rotation overpositioning of the workpiece. In theory, removing any of the four normal lines, L_1 , L_3 , Z , and Z_1 , can eliminate overpositioning.

However, to ensure that the milling plane is symmetrical about the center position of the connecting rod, the constraint normal line corresponding to the V-shaped block should not be removed. Normal lines L_1 and L_3 can be combined into one, whereby adding a floating supporting block to the supporting plate can eliminate overpositioning, as shown in Figure 10.

3.2. Discussion. The six-point positioning principle of the workpiece is the main theoretical basis of this article, but the principle does not accurately describe what geometric conditions must be met by the six points to be able to fully position the workpiece nor does it explore which attributes of the constraint points are related to the DOF. And the geometric conditions of each constraint point will cause problems such as positioning. In response to the above problems, this paper proposes five geometric theorems for judging the DOF of the workpiece, including the single normal line theorem, the two intersecting normal lines' theorem, the two parallel normal lines' theorem, the three

FIGURE 9: Overpositioning of the connecting rod. 1, Supporting plate; 2, V-shaped block 1; 3, connecting rod; 4, V-shaped block 2.

FIGURE 10: Adding the floating support to eliminate the overpositioning of the connecting rod. 1, Supporting plate; 2, V-shaped block 1; 3, workpiece; 4, V-shaped block 2; 5, floating supporting block.

intersecting normal lines' theorem, and the normal lines' set theorem, and establishes the judgment. The geometric theorem of the workpiece overconstraint clarifies the number and nature of the DOF that can be constrained by each point or a combination of several points and refines the abstract six-point positioning principle.

The guiding significance of the "Geometric Theorem for Judging Workpiece Freedom Constraint" proposed in the article for the analysis of the degree of freedom of workpiece positioning is as follows: the DOF and overconstraint of the workpiece in the fixture depend entirely on the number of positioning normal lines of the workpiece and its geometric relationship. The method of using normal lines to analyze the DOF of workpiece positioning is very ingenious. Compared with various optimization positioning methods such as positioning matrix and genetic algorithm, it is simpler and more intuitive. I hope that, with the deepening of future research, this theory can be used in theoretical learning. And promotion will have more advantages.

4. Conclusions

- (1) The number of positioning normal lines of the workpiece in the fixture and their geometric relationship determine the DOF and overconstraint of the workpiece in the fixture
- (2) The parallel normal lines of the workpiece are a sufficient and necessary condition to limit the DOF of the rotation of the workpiece in this plane

- (3) The geometric theorem is easy to analyze the DOF and overconstraint in the singular positioning of the workpiece, and it can also provide a theoretical basis for the reasonable layout of the positioning point of the workpiece in the fixture
- (4) Using the geometric theorem to analyze and judge the DOF and overconstraint of the workpiece is simple and intuitive, more universal, and suitable for computer-aided judgment of the DOF of the workpiece

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

This research was supported by the Committee of National Natural Science Foundation of China (Project no. 51975395) and the Provincial Special Fund for Coordinative Innovation Center of Taiyuan Heavy Machinery Equipment. The authors thank the financial support.

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