Research Article

Finite Element Modeling of Stress Behavior of FGM Nanoplates

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The mechanical response investigation of nanoplates especially the stress distribution plays a very important role in engineering practice, which is a condition to help test the durability as well as design and use the nanoplate structures most effectively. This pioneering paper uses the finite element method to simulate the stress field of FGM nanoplates based on the first-order shear deformation theory of Mindlin. The finite element formulations are derived by taking into account the effect of the nonlocal coefficient to analyze the mechanical response of nanometer-scale plates. This work presents the distribution of stress components in the xy -plane of plates with different boundary conditions. The numerical results also show clearly that the nonlocal coefficient has a significant influence on the deflection and stress of FGM nanoplates. These numerical results are very new and stunning which clearly show the position of the stress reaching the maximum value. This work is also the basis for scientists in testing the durability of FGM nanoplates.

1. Introduction

Functionally graded materials (FGMs) have been widely used in technical practice. These are a new type of material with many outstanding advantages such as good bearing capacity, abrasion resistance, and good working in a high-temperature environment. The reason given is that this type of material is made up of two or more component materials, mainly ceramics and metals, so FGMs carry the full range of features of ceramics and metals. Due to the wide applicability of FGM structures, scientists have been focusing on the mechanical behavior of structures made of such materials. Bui et al. [1] used the finite element method to analyze the static bending and free vibration of FGM plates in a thermal environment. For the static problem, the author only considered the stress distribution at the midpoint of FGM plates. Demirbas [2] used the three-dimensional elasticity theory to study the transient thermal residual stress analyses of one-dimensional FGM plates, in which the plate was clamped at all edges. Adineh and Kadkhodayan [3] studied the three-dimensional thermo-mechanical analysis of a 3D-FGM skew plate on an elastic foundation, where the stress response of structures was investigated, but only considering the stress distribution along with the plate thickness at the center point

and the corner point of plates. Do and his coworkers [4] used the third-order shear deformation theory and the finite element method to analyze the mechanical behavior of 2D-FGM plates; however, the authors just investigated the deflection and stress at one point of plates. Yahia et al. [5] explored the effect of homogenization models on stress analysis of functionally graded plates, where the stress distribution along the plate thickness was figured out by using the analytical solution, and only fully simply supported plates were explored.

Nowadays, with the development of science and technology, nanoscale structures made of FGM materials are used more and more in practice such as biological building blocks, solar cells, artificial structures, micro/nanosensors, micro/nano-electro-mechanical systems (MEMS and NEMS). The use of nanostructures is increasing in the exponential principle, so they have attracted a lot of researchers' attention. However, because of its nanosize, the calculations for FGM nanostructures are much different from the traditional structures; therefore, normally one has to take into account the effect of small-scale effects. The computation to determine bending behavior, as well as stress analysis of FGM nanoplates, has yielded some preliminary results. Hosseini et al. [6] researched the stress analysis of

rotating nanodisk made of functionally graded materials with variable thickness, and the results found out that the effects of thickness parameters are greater than the effect of graded-index. Shahriar and Akgöz [7] employed three-dimensional elasticity theory to investigate the static and dynamic behaviors of FGM nanoplates, and the authors focused on the variation of deflection along with the thickness of this structure. Ansari et al. [8] presented the bending analysis of nanoplates resting on elastic foundation based on the integral formulation of Eringen's nonlocal theory, where the authors were only interested in the maximum deflection according to the force effect direction for nanoplates with different boundary conditions. Babu and Patel [9] used the classical plate theory of Kirchhoff to explore the static bending, free vibration, and buckling of nanoplates with different boundary conditions, in which the static bending problem only gave the maximum deflection result of the plate under static load. Repka et al. [10] explored the bending of plates subjected to stationary transversal loading based on a moving finite elements method, and the results focused on vertical deflection w in the direction perpendicular to the plane of plates. Thai et al. [11] used modified isogeometric analysis (IGA) to analyze the free vibration and bending of nano FGM plates, where the calculation results for the static bending problem just stopped at the static deflection and did not mention the stresses. Ansari et al. [12] researched nonlinear static bending of functionally graded grapheneplatelet reinforced composite porous plates with arbitrary shape, where the survey results did not mention the stresses of this plate. Recently, there are some works [13–18] based on nonlocal elasticity theory to investigate the buckling and vibration responses of the nanostructures. And the works [19–21] flexibly used a variety of beam and plate theories to study bending, buckling, and dynamic response of the FGM structures.

Through the published data, one can see that most of these works mainly went into the stress and displacement behavior analysis of some points in the plate, usually the midpoint of the plate for plates with symmetrical boundary such as fully simply supported and fully clamped supported conditions. This makes it difficult to consider the overall stress distribution for the entire structure. However, from the stress distribution of the entire plate, it is of great help to test the strength and evaluate the failure position for the nanoplates as the basis for the design calculation and the most efficient use. Therefore, this work is the different and newest point compared with other published investigations. This work uses the finite element method to simulate the stress field of FGM nanoplates subjected to static loads. The finite element formulations are established based on the first-order shear deformation theory of Mindlin taking into consideration of small-scale effects. The numerical results show that the stress components reach maximum at different positions and depending on the boundary conditions of the plate, and these are new simulation results, which will greatly help in the work of evaluating the bearing capacity of FGM nanostructures.

The body of this work is as follows: Section 2 gives a finite element solution for the bending problem of FGM nanoplates. Continuously, Section 3 reveals the verification study to show the accuracy and convergence of the proposed theory. Section 4 indicates the numerical and graphical investigation of FGM nanoplates, especially the stress field of FGM nanoplates. Finally, Section 5 summarizes some important conclusions on the numerical results of this work.

2. Finite Element Model of FGM Nanoplates in Static Bending Problem

Consider a rectangular FGM nanoplate (Figure 1) with length a , width b , and thickness h . The material properties are defined as follows [1, 22–24]:

$$R(z) = R_m + (R_c - R_m)V_c, \quad (1)$$

with R_c and R_m denote Young's modulus and Poisson's ratio of ceramic and metal, respectively. The volume portions of ceramic (V_c) and metal (V_m) are calculated as follows [1, 22]:

$$V_m + V_c = 1, \quad (2)$$

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^n,$$

where n is the volume fraction exponent.

3. Nonlocal Elasticity Theory

Based on the nonlocal theory of Eringen, the nonlocal stress tensor at any point [25]

$$\sigma_{ij} = \int_V \alpha(|\mathbf{x}' - \mathbf{x}|) \sigma_{ij}^L dV(\mathbf{x}'), \quad (3)$$

in which the Hookean stress tensor is calculated as

$$\sigma_{ij}^L = c_{ijkl} \epsilon_{kl}, \quad (4)$$

and $\alpha(|\mathbf{x}' - \mathbf{x}|)$ is the kernel function which is normalized over the whole body. This function can be obtained by combining the lattice dynamics with nonlocal results [25]. For instance, the kernel function for 2D problems has the following form:

$$\alpha(|\mathbf{x}|) = (2\pi l^2 \vartheta^2)^{-1} H_0\left(\frac{|\mathbf{x}|}{l\vartheta}\right), \quad \vartheta = \frac{e_0 a}{l}, \quad (5)$$

where H_0 is the improved Bessel function, a and l are internal and external characteristic lengths, respectively, and e_0 is material constant, which is found by the experiment. On the other hand, nonlocal elasticity is related to spatial integrals that present weighted averages of the contributions of the strain component of all points in the continuum body to the stress tensor at a point.

In the nonlocal linear elasticity, the equation of motion can be obtained from nonlocal balance law as follows:

FIGURE 1: A model of an FGM nanoplate.

$$\sigma_{ij,j} + f_i = 0, \quad (6)$$

in which i and j take the symbols x, y , and z and f_i is the parts of the body load [25]. Substituting equation (3) into equation (6), the integral form of nonlocal constitutive equation is possessed. It is seen that solving an integral equation is more complicated than a differential equation; therefore, Eringen [25, 26] proposed a differential form of the nonlocal constitutive equation as follows:

$$\sigma_{ij,j} + \Theta(f_i) = 0, \quad (7)$$

where the linear differential operator Θ is calculated as

$$\Theta = [1 - \psi^2 \nabla^2], \quad (8)$$

in which $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ is the Laplace operator, and $\psi^2 (nm^2) = (e_0 a)^2$ is the nonlocal coefficient, where a is an internal characteristic length and e_0 is a constant material, which is found by the experiment.

By substituting this operator into equation (3), the constitutive equation can be obtained as follows:

$$[1 - \psi^2 \nabla^2] \sigma = C \varepsilon. \quad (9)$$

In the general case, the nonlocality is naturally three-dimensional, rather than the two-dimensional simplification currently in nanostructures [27, 28]. When the feature size of plates is reduced to the nanoscale, classical mechanics will break down firstly at the thickness direction of plates due to the simple fact that plates' thickness dimensions are far smaller than their length dimension. This in turn implies that the nonlocal effect in the thickness is likely to play a dominant role in the contribution of size dependence and the cross section effect on the nonlocal stress, the bending moment, and the deflection [27]. Equation (9) is just an approximation; however, it is simpler and more convenient than the integral relation (3) to apply to various linear elasticity problems.

According to the first-order shear deformation theory of Mindlin, the displacement field at any points within the plate is expressed as follows [8, 29]:

$$\begin{cases} u(x, y, z) = u_0(x, y, 0) + z\phi_x, \\ v(x, y, z) = v_0(x, y, 0) + z\phi_y, \\ w(x, y, z) = w_0(x, y, 0), \end{cases} \quad (10)$$

in which u, v , and w are the displacements along the x -, y -, and z -directions; u_0, v_0 , and w_0 are the in-plane displacements at that point where $z=0$; and ϕ_x and ϕ_y are the rotations around Oy and Ox , respectively.

The strain components are calculated through the derivatives as follows:

$$\left\{ \begin{array}{l} \varepsilon = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} = \varepsilon_{xx}^0 + z\kappa_{xx} \\ \frac{\partial v_0}{\partial y} + z \frac{\partial \phi_y}{\partial y} = \varepsilon_{yy}^0 + z\kappa_{yy} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) = \gamma_{xy}^0 + z\kappa_{xy} \end{Bmatrix} = \underbrace{\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix}}_{\varepsilon_0} + z \underbrace{\begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}}_{\kappa} \\ \gamma = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w}{\partial x} + \phi_x \\ \frac{\partial w}{\partial y} + \phi_y \end{Bmatrix} \end{array} \right. \quad (11)$$

At this point, equation (9) is specifically rewritten in the following form:

$$\begin{aligned} [1 - \psi^2 \nabla^2] \sigma_{xx} &= \frac{E(z)}{1 - \nu(z)^2} \left[\frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} + \nu(z) \left(\frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} \right) \right], \\ [1 - \psi^2 \nabla^2] \sigma_{yy} &= \frac{E(z)}{1 - \nu(z)^2} \left[\frac{\partial v_0}{\partial y} + z \frac{\partial \phi_y}{\partial y} + \nu(z) \left(\frac{\partial v_0}{\partial y} + z \frac{\partial \phi_y}{\partial y} \right) \right], \end{aligned} \quad (12)$$

$$\begin{aligned} [1 - \psi^2 \nabla^2] \tau_{xy} &= \frac{E(z)}{2(1 + \nu(z))} \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) \right), \\ [1 - \psi^2 \nabla^2] \tau_{yz} &= \frac{E(z)}{2(1 + \nu(z))} \left(\frac{\partial w}{\partial x} + \phi_x \right), \\ [1 - \psi^2 \nabla^2] \tau_{xz} &= \frac{E(z)}{2(1 + \nu(z))} \left(\frac{\partial w}{\partial y} + \phi_y \right). \end{aligned} \quad (13)$$

The nonlocal normal forces, moments, and shear forces are calculated through integrals as follows:

From equations (6) and (7), the nonlocal moments and shear forces can be taken as follows:

$$\begin{aligned} [1 - \psi^2 \nabla^2] \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} &= \int_{-(h/2)}^{h/2} [1 - \psi^2 \nabla^2] \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} dz, \\ [1 - \psi^2 \nabla^2] \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} &= \int_{-(h/2)}^{h/2} z [1 - \psi^2 \nabla^2] \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} dz, \\ [1 - \psi^2 \nabla^2] \begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} &= \int_{-(h/2)}^{h/2} [1 - \psi^2 \nabla^2] \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz. \end{aligned} \quad (14)$$

$$\begin{aligned} [1 - \psi^2 \nabla^2] N_{xx} &= \int_{-(h/2)}^{h/2} \frac{E(z)}{1 - \nu(z)^2} \left[\frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} + \nu(z) \left(\frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} \right) \right] dz, \\ [1 - \psi^2 \nabla^2] N_{yy} &= \int_{-(h/2)}^{h/2} \frac{E(z)}{1 - \nu(z)^2} \left[\frac{\partial v_0}{\partial y} + z \frac{\partial \phi_y}{\partial y} + \nu(z) \left(\frac{\partial v_0}{\partial y} + z \frac{\partial \phi_y}{\partial y} \right) \right] dz, \end{aligned} \quad (15)$$

$$\begin{aligned} [1 - \psi^2 \nabla^2] N_{xy} &= \int_{-(h/2)}^{h/2} \frac{E(z)}{(1 - \nu(z)^2)} \left[\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) \right] dz, \\ [1 - \psi^2 \nabla^2] M_{xx} &= \int_{-(h/2)}^{h/2} \frac{zE(z)}{1 - \nu(z)^2} \left[\frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} + \nu(z) \left(\frac{\partial u_0}{\partial x} + z \frac{\partial \phi_x}{\partial x} \right) \right] dz, \\ [1 - \psi^2 \nabla^2] M_{yy} &= \int_{-(h/2)}^{h/2} \frac{zE(z)}{1 - \nu(z)^2} \left[\frac{\partial v_0}{\partial y} + z \frac{\partial \phi_y}{\partial y} + \nu(z) \left(\frac{\partial v_0}{\partial y} + z \frac{\partial \phi_y}{\partial y} \right) \right] dz, \end{aligned} \quad (16)$$

$$\begin{aligned} [1 - \psi^2 \nabla^2] M_{xy} &= \int_{-(h/2)}^{h/2} \frac{zE(z)}{(1 - \nu(z)^2)} \left[\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) \right] dz, \\ [1 - \psi^2 \nabla^2] Q_{xz} &= \int_{-(h/2)}^{h/2} \frac{E(z)}{2(1 + \nu(z))} \left(\frac{\partial w}{\partial x} + \phi_x \right) dz, \\ [1 - \psi^2 \nabla^2] Q_{yz} &= \int_{-(h/2)}^{h/2} \frac{E(z)}{2(1 + \nu(z))} \left(\frac{\partial w}{\partial y} + \phi_y \right) dz. \end{aligned} \quad (17)$$

Equations (15)–(17) can be written in the matrix form as follows:

$$\begin{aligned} \begin{Bmatrix} [1 - \psi^2 \nabla^2] N_{xx} \\ [1 - \psi^2 \nabla^2] N_{yy} \\ [1 - \psi^2 \nabla^2] N_{xy} \end{Bmatrix} &= \mathbf{A} \begin{Bmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \mathbf{B} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}, \\ \begin{Bmatrix} [1 - \psi^2 \nabla^2] M_{xx} \\ [1 - \psi^2 \nabla^2] M_{yy} \\ [1 - \psi^2 \nabla^2] M_{xy} \end{Bmatrix} &= \mathbf{B} \begin{Bmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \mathbf{D} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}, \\ \begin{Bmatrix} [1 - \psi^2 \nabla^2] Q_{xz} \\ [1 - \psi^2 \nabla^2] Q_{yz} \end{Bmatrix} &= \mathbf{D}_s \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}, \end{aligned} \quad (18)$$

where

$$\begin{aligned} \{\mathbf{A}, \mathbf{B}, \mathbf{D}\} &= \int_{-(h/2)}^{h/2} \frac{E(z)}{1 - \nu(z)^2} \begin{bmatrix} 1 & \nu(z) & 0 \\ \nu(z) & 1 & 0 \\ 0 & 0 & \frac{1 + \nu(z)}{2} \end{bmatrix} \{1, z, z^2\} dz, \\ \{\mathbf{D}_s\} &= \int_{-(h/2)}^{h/2} \frac{E(z)}{2(1 + \nu(z))} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} dz. \end{aligned} \quad (19)$$

The strain energy of the nanoplate has the following form [30]:

$$\Pi = \frac{1}{2} \int_V \sigma_{ij} \epsilon_{ij} dV. \quad (20)$$

Substituting the stress and strain expressions into equation (20), one gets the following:

$$\Pi = \frac{1}{2} \int_S \left(N_{xx} \epsilon_{xx}^0 + N_{yy} \epsilon_{yy}^0 + N_{xy} \gamma_{xy}^0 + M_{xx} \kappa_{xx} + M_{yy} \kappa_{yy} + M_{xy} \kappa_{xy} + Q_{xz} \gamma_{xz} + Q_{yz} \gamma_{yz} \right) dS. \quad (21)$$

The variation of equation (21) has the following form:

$$\begin{aligned} \delta \Pi &= \int_S \left(N_{xx} \delta \epsilon_{xx}^0 + N_{yy} \delta \epsilon_{yy}^0 + N_{xy} \delta \gamma_{xy}^0 + \delta M_{xx} \delta \kappa_{xx} + M_{yy} \delta \kappa_{yy} + M_{xy} \delta \kappa_{xy} + Q_{xz} \delta \gamma_{xz} + Q_{yz} \delta \gamma_{yz} \right) dS - \int_S (q \delta w) dS \\ &= \int_S \left(N_{xx} \delta \left(\frac{\partial u_0}{\partial x} \right) + N_{yy} \delta \left(\frac{\partial v_0}{\partial y} \right) + N_{xy} \delta \left(\frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \right) + \delta M_{xx} \delta \left(\frac{\partial \phi_x}{\partial x} \right) + M_{yy} \delta \left(\frac{\partial \phi_y}{\partial y} \right) + M_{xy} \delta \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) + Q_{xz} \delta \left(\frac{\partial w}{\partial x} + \phi_x \right) + Q_{yz} \delta \left(\frac{\partial w}{\partial y} + \phi_y \right) \right) dS - (q \delta w) dS, \end{aligned} \quad (22)$$

in which q is the transverse distributed force.

After performing partial integral and setting the coefficients of δu_0 , δv_0 , $\delta \phi_x$, $\delta \phi_y$, and δw equal to zero, one gets the following:

$$\delta u_0: \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \quad (23)$$

$$\delta v_0: \frac{\partial N_{xy}}{\partial y} + \frac{\partial N_{yy}}{\partial x} = 0,$$

$$\begin{aligned} \delta \phi_x: \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_{xz} &= 0, \\ \delta \phi_y: \frac{\partial M_{xy}}{\partial y} + \frac{\partial M_{yy}}{\partial x} - Q_{yz} &= 0, \end{aligned} \quad (24)$$

$$\delta w: \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} = q. \quad (25)$$

Substituting equations (18)–(20) into equations (23)–(25), we have the following expressions:

$$\int_S \left(N_{xx} \delta \left(\frac{\partial u_0}{\partial x} \right) + N_{xy} \delta \left(\frac{\partial u_0}{\partial y} \right) \right) dS = 0,$$

$$\int_S \left(N_{xy} \delta \left(\frac{\partial v_0}{\partial y} \right) + N_{yy} \delta \left(\frac{\partial v_0}{\partial x} \right) \right) dS = 0,$$

$$\begin{aligned} \int_S \left(M_{xx} \delta \left(\frac{\partial \phi_x}{\partial x} \right) + M_{xy} \delta \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) \right. \\ \left. + M_{yy} \delta \left(\frac{\partial \phi_y}{\partial y} \right) - Q_{xz} \delta \phi_x - Q_{yz} \delta \phi_y \right) dS = 0, \end{aligned} \quad (26)$$

$$\int_S \left(Q_{xz} \delta \left(\frac{\partial w}{\partial y} \right) + Q_{yz} \delta \left(\frac{\partial w}{\partial x} \right) - \Theta q \right) dS$$

In this work, an m -node plate element is used, and each node has five degrees of freedom:

$$\mathbf{u}_e = \sum_{i=1}^m \{u_{0i}, v_{0i}, w_i, \phi_{xi}, \phi_{yi}\}^T. \quad (27)$$

Then, we obtain

$$\left\{ \begin{aligned} u_0 &= \sum_{i=1}^m \mathbf{N}_u \mathbf{u}_e, v_0 = \sum_{i=1}^m \mathbf{N}_v \mathbf{u}_e, w = \sum_{i=1}^m \mathbf{N}_w \mathbf{u}_e, \phi_x = \sum_{i=1}^m \mathbf{N}_{\phi_y} \mathbf{u}_e, \phi_y = \sum_{i=1}^m \mathbf{N}_{\phi_y} \mathbf{u}_e. \end{aligned} \right. \quad (28)$$

Substituting the expressions in equation (28) into equations (25)–(28), the static equilibrium equation of the plate is written in the form of finite element formulation as follows:

$$\mathbf{K}_e \mathbf{u}_e = \mathbf{F}_e,$$

$$\mathbf{K}_e = \int_S (\mathbf{B}_1 \mathbf{A} \mathbf{B}_1 + \mathbf{B}_1 \mathbf{B} \mathbf{B}_2 + \mathbf{B}_2 \mathbf{B} \mathbf{B}_1 + \mathbf{B}_2 \mathbf{D} \mathbf{B}_2 + \mathbf{B}_3 \mathbf{D}_s \mathbf{B}_3) dS, \quad (29)$$

where

$$\mathbf{F}_e = \int_S [1 - \psi^2 \nabla^2] \mathbf{N}_w \mathbf{q} dS, \quad (30)$$

with

$$\begin{aligned}
\mathbf{B}_1 &= \sum_{i=1}^m \begin{bmatrix} \frac{\partial N_u}{\partial x} \\ \frac{\partial N_v}{\partial y} \\ \frac{\partial N_u}{\partial y} + \frac{\partial N_v}{\partial x} \end{bmatrix}, \\
\mathbf{B}_2 &= \sum_{i=1}^m \begin{bmatrix} \frac{\partial N_{\phi_x}}{\partial x} \\ \frac{\partial N_{\phi_y}}{\partial y} \\ \frac{\partial N_{\phi_x}}{\partial y} + \frac{\partial N_{\phi_y}}{\partial x} \end{bmatrix}, \\
\mathbf{B}_3 &= \sum_{i=1}^m \begin{bmatrix} \frac{\partial N_w}{\partial x} + N_{\phi_x} \\ \frac{\partial N_w}{\partial y} + N_{\phi_y} \end{bmatrix}, \\
\mathbf{N}_w &= \sum_{i=1}^m [N_w].
\end{aligned} \tag{31}$$

So, from equation (36), one can get displacements of the plate, and then substituting these displacements into equations (12) and (13), we obtain the stress field of FGM nanoplates.

Boundary conditions are expressed as follows:

- (i) The plate is simply supported at $x=0$ and $x=a$:

$$\begin{aligned}
u_0 \\
v_0 \\
w \\
\phi_y
\end{aligned} \tag{32}$$

- (ii) The plate is simply supported $y=0$ and $y=b$:

$$\begin{aligned}
u_0 &= 0, \\
v_0 &= 0, \\
w &= 0, \\
\phi_x &= 0.
\end{aligned} \tag{33}$$

- (iii) One edge is clamped:

$$\begin{aligned}
u_0 &= 0, \\
v_0 &= 0, \\
w &= 0, \\
\phi_x &= 0, \\
\phi_y &= 0.
\end{aligned} \tag{34}$$

Herein, some acronyms are explained as follows: simply supported edge—S, clamped edge—C, and free edge—F. Several boundary conditions of the plate used in this paper are presented in Figure 2.

4. Verification Study

In this section, numerical results of displacement and stress of FGM plate and nanoplate are compared with those of published data to confirm the reliability of the calculation theory.

Example 1. Consider an FGM plate with material properties $E_c = 380$ GPa, $E_m = 70$ GPa, $\nu_c = \nu_m = 0.3$, $a/b = 1$, and $a/h = 10$. The plate is fully simply supported and subjected to uniformly distributed load q_0 and sinusoidal distributed load as follows:

$$q = q_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right). \tag{35}$$

Nondimensional displacement and stress are normalized as follows:

$$\begin{aligned}
\hat{w} &= \frac{10E_c h^3}{q_0 a^4} w \left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right), \\
\hat{\sigma}_x &= \frac{10h}{q_0 a} \sigma_x \left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right).
\end{aligned} \tag{36}$$

Tables 1 and 2 present the results of displacement and stress of FGM plate obtained from this approach, the finite element method [31], and the analytical method [30] in which this work uses several mesh sizes, and it can be seen that with the 800-element mesh size, the accurate data can be reached. Therefore, for all upcoming investigations, this mesh is employed.

Example 2. Consider a square FGM plate with geometrical and material properties $a/b = 1$, $a/h = 10$, $E_c = 380$ GPa, $E_m = 70$ GPa, and $\nu_c = \nu_m = 0.3$. The plate is fully simply supported and subjected to bisinusoidal load as presented in equation (35). Nondimensional stresses are calculated as the following formulas:

$$\left\{ \begin{aligned} (\sigma_x^{**}, \sigma_y^{**}) &= \left(\frac{\sigma_x}{q_0} \left(\frac{a}{2}, \frac{b}{2}, z \right), \frac{\sigma_y}{q_0} \left(\frac{a}{2}, \frac{b}{2}, z \right) \right), \\ \tau_{xy}^{**} &= \frac{\tau_{xy}(0, 0, z)}{q_0}. \end{aligned} \right. \tag{37}$$

FIGURE 2: Boundary conditions of nano FGM plates.

TABLE 1: Nondimensional displacement $\hat{w} = (10E_ch^3/q_0a^4)w((a/2), (b/2))$ of square FGM plates ($a/h = 10$) subjected to uniformly distributed load and sinusoidal distributed load.

Source		N				
		0	1	2	4	∞
<i>Uniform load</i>						
[31]		0.4666	0.9290	1.1952	1.3908	0.4666
[30]		0.4665	0.9287	1.1940	1.3890	0.4665
This work	200 elements	0.4666	0.9376	1.1995	1.3869	0.4666
	512 elements	0.4666	0.9376	1.1995	1.3869	0.4666
	800 elements	0.4666	0.9376	1.1995	1.3870	0.4666
	1800 elements	0.4666	0.9376	1.1995	1.3870	0.4666
<i>Bisinusoidal load</i>						
[31]		0.2961	0.5891	0.7582	0.8831	0.2961
[30]		0.2960	0.5889	0.7573	0.8819	0.2960
This work	200 elements	0.2960	0.5945	0.7606	0.8799	0.2960
	512 elements	0.2960	0.5945	0.7606	0.8800	0.2960
	800 elements	0.2960	0.5945	0.7606	0.8800	0.2960
	1800 elements	0.2960	0.5945	0.7606	0.8800	0.2960

TABLE 2: Nondimensional stress $\hat{\sigma}_x = (10h/q_0a)\sigma_x((a/2), (b/2), (h/2))$ of square FGM plates ($a/h = 10$) subjected to uniformly distributed load and sinusoidal distributed load.

Source		N				
		0	1	2	4	∞
<i>Uniform load</i>						
[31]		2.8688	4.4303	5.1689	5.8035	2.8678
[30]		2.8932	4.4745	5.2296	5.8915	2.8932
This work	200 elements	2.9638	4.6289	5.4120	6.1146	2.9638
	512 elements	2.9265	4.5706	5.3438	6.03763	2.9265
	800 elements	2.9178	4.5570	5.3279	6.0196	2.9178
	1800 elements	2.9091	4.5435	5.3121	6.0017	2.9091
<i>Bisinusoidal load</i>						
[31]		1.9679	3.0389	3.5456	3.9813	1.9679
[30]		1.9955	3.0870	3.6094	4.0693	1.9955
This work	200 elements	2.0480	3.1986	3.7397	4.2252	0.2960
	512 elements	2.0166	3.1495	3.6823	4.1604	2.0166
	800 elements	2.0091	3.1379	3.6687	4.1450	2.0091
	1800 elements	2.0017	3.1262	3.6551	4.1296	2.0017

Figure 3 presents the stress distribution by the thickness direction with different volume fraction exponent n of this work and the exact solution of Hiroiyuki [32]. The comparison results in Figure 3 show that the stresses are nonlinearly distributed according to the plate thickness, and these stresses reach the maximum value at the top of the plate (full of ceramic). Also from Figure 3, one can see that for the case of $n=0$ (homogeneous material), the stress is linearly distributed. However, when $n=1$, the material properties vary smoothly from one surface to another one of the plates, so the stress also varies smoothly from one side to the other side of the plate following a nonlinear curve. This represents a complete difference from homogeneous materials and laminated composites.

Example 3. Finally, the comparison of the maximum deflection of a nanoplate is presented, and this plate has $a/b=1$, $a/h=10$, Young's modulus $E=30$ GPa, and Poisson's ratio $\nu=0.3$. The plate is fully simply supported and subjected to uniformly distributed load q_0 . The nondimensional deflection is defined as follows:

$$\bar{w} = w_c \frac{1000E_c h^3}{12(1-\nu^2)q_0 a^4}. \quad (38)$$

The numerical results of this work compared with those of analytical solution [33] are listed in Table 3. It can be seen that the data have a good agreement with two b/a ratios.

Through all the comparison examples above, it can be seen that the established theory has the necessary accuracy.

5. Numerical Results

In this section, an FMG nanoplate is considered, in which the dimensions are length a , width b , thickness h , and mechanical properties as follows:

Ceramic ZrO_2 : $E_c = 168$ GPa and $\nu_c = 0.3$

Metal Al: $E_m = 70$ GPa and $\nu_m = 0.3$

As Example 2 in the above section has considered, for the plate with the mechanical property varying in the thickness direction, the maximum stress position would have the coordinate $z=h/2$, so the following investigations focus on the analysis of stress components at this position, and these are completely novel explorations, which have never mentioned by any previous studies.

The plate is subjected to uniformly distributed load q_0 and bisinusoidal load as shown in equation (35), where the survey quantities are expressed as follows:

$$\begin{aligned} w^* &= w \frac{100E_c h^3}{q_0 a^4}, \\ \sigma_x^* &= \frac{10h}{q_0 a} \sigma_x \left(x, y, \frac{h}{2} \right), \\ \sigma_y^* &= \frac{10h}{q_0 a} \sigma_y \left(x, y, \frac{h}{2} \right), \\ \tau_{xy}^* &= \frac{10h}{q_0 a} \tau_{xy} \left(x, y, \frac{h}{2} \right), \\ \tau_{xz}^* &= \frac{10h}{q_0 a} \tau_{xz} \left(x, y, \frac{h}{2} \right). \end{aligned} \quad (39)$$

The boundary conditions are described as follows:

- (i) Fully simply supported plate: SSSS
- (ii) Fully clamped plate: CCCC
- (iii) The plate with two opposite edges are clamped and the other edges are free: CFCF
- (iv) The plate with one edge is clamped and the other edges are free: CFFF

5.1. Effect of Boundary Conditions on the Stress Distribution of FGM Nanoplate. In order to clearly see the positions with maximum values of stress components, this work calculates and draws the stress distribution surface of FGM nanoplate in (x, y) coordinates with different boundary conditions. The numerical results are presented in Figures 4–6 and Tables 4 and 5.

From the numerical data, some discussions are given as follows:

- (i) The stresses σ_x^* and σ_y^* obtain the maximum values at the center point of the SSSS plate and zero at the boundaries. In contrast, for the CCCC plate, these stresses obtain the maximum values at the boundaries and then they decrease gradually to zero and increase at the center point. Thus, the values of these stresses are nonzero and relatively large at the center of the plate.
- (ii) The stresses τ_{xy}^* , τ_{xz}^* , and τ_{yz}^* have symmetrical values across the lines $x=a/2$ and $y=b/2$ and get zero values in the center point of the plate. The maximum values of the stress τ_{xy}^* are located in four corners of the SSSS plate, while they are situated in some distance from the boundary. For both SSSS and CCCC plates, the maximum values of the stresses τ_{xz}^* and τ_{yz}^* are located in the middle point of each edge. The magnitudes of the maximum values τ_{xz}^* and τ_{yz}^* for both plates are not as

FIGURE 3: The comparison of the stress distribution of FGM subjected to bisinusoidal load: (a) $\tau_{xy}^{**}(0, 0, z)$; (b) $\sigma_x^{**}((a/2), (b/2), z)$; (c) $\sigma_y^{**}((a/2), (b/2), z)$.

TABLE 3: Nondimensional deflection of fully simply supported homologous nanoplate subjected to uniformly distributed load.

ψ	$b/a = 1$		$b/a = 2$	
	[33]	This work	[33]	This work
0	4.1853	4.2732	0.7169	0.7144
0.5	4.5607	4.6416	0.8767	0.8567
1	4.9362	5.0100	1.0364	0.9991
1.5	5.3116	5.3784	1.1961	1.1415
2	5.6871	5.7468	1.3558	1.2838
2.5	6.0625	6.1152	1.5155	1.4262
3	6.4380	6.4836	1.6752	1.5685

distinctly different as the difference of the maximum value τ_{xy}^{**} .

- (iii) For the CFFF plate, the maximum values of all stresses σ_x^* , σ_y^* , τ_{xy}^* , τ_{xz}^* , and τ_{yz}^* are greater than

those of the SSSS, CCCC, and CFCF plates. For all cases of the CCCC plate, there is a position that the stresses σ_x^* , σ_y^* , τ_{xy}^* , τ_{xz}^* , and τ_{yz}^* are zero; however, these positions are different for each particular

FIGURE 4: The stress distribution surface (xy -plane) of FGM nanoplate subjected to uniformly distributed load q_0 , $n = 1$, $a/h = 10$, $a/b = 1$, and $\psi = 1$.

FIGURE 5: The stress distribution surface (xy -plane) of FGM nanoplate subjected to uniformly distributed load q_0 , $n = 1$, $a/h = 10$, $a/b = 1$, and $\psi = 1$.

FIGURE 6: The stress distribution surface (xy -plane) of FGM nanoplate subjected to sinusoidally distributed load $n = 1$, $a/h = 10$, $a/b = 1$, and $\psi = 1$.

TABLE 4: The maximum values of deflection and stress of FGM nanoplate subjected to uniformly distributed load q_0 , $n = 1$, $a/h = 10$, $a/b = 1$, and $\psi = 1$.

	Boundary condition			
	SSSS	CCCC	CFCF	CFFF
$ w^* _{\max}$	8.204	2.448	4.934	208.730
$ \sigma_x^* _{\max}$	42.885	38.942	71.300	418.320
$ \sigma_y^* _{\max}$	42.885	38.942	21.120	125.500
$ \tau_{xy}^* _{\max}$	42.906	6.131	24.540	115.070
$ \tau_{xz}^* _{\max}$	5.743	7.0311	9.629	64.537
$ \tau_{yz}^* _{\max}$	5.743	7.0311	2.802	11.585

TABLE 5: The maximum values of deflection and stress of FGM nanoplate subjected to sinusoidally distributed load q_0 , $n = 1$, $a/h = 10$, $a/b = 1$, and $\psi = 1$.

	Boundary condition			
	SSSS	CCCC	CFCF	CFFF
$ w^* _{\max}$	6.144	2.402	3.912	95.890
$ \sigma_x^* _{\max}$	36.239	33.932	51.379	216.270
$ \sigma_y^* _{\max}$	36.239	33.844	16.741	64.882
$ \tau_{xy}^* _{\max}$	19.506	6.112	9.305	51.342
$ \tau_{xz}^* _{\max}$	1.964	3.160	4.433	31.929
$ \tau_{yz}^* _{\max}$	0.745	1.715	4.652	6.429

FIGURE 7: An FGM nanoplate with a circle hole.

plate. The calculation results also show that the maximum stresses of all stress components for CFCF and CFFF plates are located in the middle points of clamped edges and around the corners of the plates. This demonstrates that plates under these boundary conditions are susceptible to damage at the clamped edges, and these positions should be preferred to test for their strength during working processes.

- (iv) For the same uniformly distributed load q_0 , the maximum displacement and stress of the plate under uniformly distributed load are much greater than those of the sinusoidally distributed load.
- (v) The numerical results also confirm that the position with the largest deflection $w_{\max}$ does not coincide with the position with the maximum values of the stress, except for the case of the SSSS plate.

5.2. The Stress Distribution of FGM Nanoplate with a Circle Hole. Consider a square FGM nanoplate with a circle hole (the radius of the circle hole $R = a/10$) in the center of the plate as shown in Figure 7. The plate is under a uniformly distributed load q_0 .

5.2.1. Effect of the Hole. Figure 8 presents the stress distribution in the longitudinal directions of edges for the SSSS and CCCC plates. Table 6 shows the maximum deflection and stress of plates with different boundary conditions. The numerical results indicate that the CCCC plate with and without a hole has the same position as the maximum stress. However, for the SSSS plate with and without a hole, the maximum stresses σ_x^* and σ_y^* appear in the rear area of the hole. The maximum values of deflection and stress of the CFFF plate are much higher than those of other boundary conditions.

FIGURE 8: The stress distribution surface along the edges of the plate subjected to uniformly distributed load, $n = 1$, $a/h = 10$, $a/b = 1$, and $\psi = 1$.

TABLE 6: Maximum deflection and stress of FGM nanoplate with a hole ($R = a/10$) subjected to uniformly distributed load, $n = 1$, $a/h = 10$, $a/b = 1$, and $\psi = 1$.

	Boundary condition			
	SSSS	CCCC	CFCF	CFFF
$ w^* _{\max}$	8.761	2.223	4.961	211.323
$ \sigma_x^* _{\max}$	64.686	36.990	76.070	404.569
$ \sigma_y^* _{\max}$	64.686	36.990	22.821	121.371
$ \tau_{xy}^* _{\max}$	44.475	6.147	26.410	119.865
$ \tau_{xz}^* _{\max}$	5.520	6.726	9.490	63.296
$ \tau_{yz}^* _{\max}$	5.520	6.726	4.117	52.180

TABLE 7: The deflection $|w^*|_{\max}$ of fully simply supported FGM nanoplate, $a/h = 10$, and $a/b = 1$.

ψ	N					
	0	0.5	2	3	5	10
0	5.153	6.804	8.566	8.933	9.364	10.072
0.5	5.496	7.258	9.136	9.526	9.986	10.740
1.0	5.839	7.712	9.706	10.120	10.607	11.407
1.5	6.182	8.165	10.276	10.713	11.228	12.075
2.0	6.525	8.619	10.846	11.307	11.849	12.743
2.5	6.868	9.073	11.416	11.900	12.470	13.410
3.0	7.210	9.527	11.986	12.494	13.091	14.078

TABLE 8: Maximum stresses at $z = h/2$ of fully simply supported FGM nanoplate subjected to uniformly distributed load, $a/h = 10$, and $a/b = 1$.

	n	ψ					
		0	0.5	1.0	1.5	2.0	2.5
$ \sigma_x^* _{\max}$	0	48.298	49.056	49.814	50.593	51.350	52.106
	0.5	57.740	58.645	59.550	60.456	61.361	62.267
	2	67.767	68.828	69.890	70.951	72.013	73.074
	10	85.703	87.042	88.381	89.720	91.059	92.398
$ \sigma_y^* _{\max}$	0	48.298	49.056	49.814	50.503	51.350	52.126
	0.5	57.740	58.585	59.550	60.456	61.361	62.267
	2	67.767	68.828	69.890	70.951	72.013	73.074
	10	85.703	87.042	88.381	89.720	91.059	92.398
$ \tau_{xy}^* _{\max}$	0	25.456	27.260	34.260	41.260	48.259	55.259
	0.5	30.401	32.564	40.926	49.287	57.649	66.010
	2	35.696	38.228	48.403	57.859	67.675	77.490
	10	45.185	48.368	60.787	73.207	85.626	98.045
$ \tau_{xz}^* _{\max}$	0	3.893	3.894	3.895	3.896	3.897	3.898
	0.5	4.852	4.853	4.854	4.855	4.856	4.857
	2	6.401	6.402	6.403	6.404	6.405	6.406
	10	8.344	8.345	8.346	8.347	8.348	8.349
$ \tau_{yz}^* _{\max}$	0	3.893	3.894	3.895	3.896	3.897	3.898
	0.5	4.852	4.853	4.854	4.855	4.856	4.857
	2	6.401	6.402	6.403	6.404	6.405	6.406
	10	8.344	8.345	8.346	8.347	8.348	8.349

5.2.2. *Effect of Volume Fraction Exponent n .* Consider a fully simply supported FGM nanoplate subjected to uniformly distributed load. Let the volume fraction exponent n varies in a range of 0 to 10, and the maximum value of deflection $|w^*|_{\max}$ and stress components changing as a function of n and ψ are shown in Tables 7 and 8. It can be seen that when

the volume fraction exponent n increases, both maximum deflection and stress increase. In addition, when ψ increases, the distance between molecules increases, the plate also becomes softer, and therefore the maximum deflection and stress increase.

6. Conclusions

This paper presents new numerical results of stress distributions along the edges of FGM nanoplates. The finite element formulations are derived from the first-order shear deformation theory of Mindlin; the proposed theory and mathematical model are verified by comparing the numerical data with those of published results. Some new main points of this work are concluded as follows:

- Boundary condition and the shape of the plate have a strong effect on the maximum value of stress as well as the positions of maximum stress. For the plate without a hole, the stresses σ_x^* and σ_y^* obtain the maximum values at the center point of the SSSS plate. In contrast, for the CCCC plate, these stresses obtain the maximum values at the boundaries. Nevertheless, for the plate with a hole, the CCCC plate has the same position as the maximum stress; for the SSSS plate, the maximum stresses σ_x^* and σ_y^* appear in the rear area of the hole. This is an important suggestion for scientists when considering the strengths of FGM nanoplates to pay close attention to the positions with maximum stress.
- The fewer degree of freedom of the structure is constrained; the greater the maximum stress value, the maximum stress position usually appears around clamped regions.
- When increasing the value of ψ , for any values of the volume fraction exponent n , deflection and stress are increased due to the increase in the distance between molecules.

Data Availability

The data used to support the findings of this study are included in the article.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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