Mathematical Model and Numerical Simulation of Coalbed Methane Migration Considering the Adsorption Expansion Effect

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The influence of gas adsorption and desorption on the volumetric strain of coal was measured by a self-designed, fluid–solid coupling triaxial coal adsorption deformation experimental system. The experimental results show that coal deformation has a threshold value with an increase in gas content. Before the gas content reaches the threshold value, coal deformation is not obvious. When the gas content reaches the threshold value, the deformation will increase sharply. At the same time, coal volume strain changes with coal gas content in accordance with exponential law \( \varepsilon = \varepsilon_0 (e^{c - 1}) \). Based on the experimental results, considering the coupling effects of heat transfer, water seepage, coal and rock mass deformation, and coalbed methane desorption and seepage, a mathematical model of heat transfer–deformation–seepage coupling coalbed methane migration was established, and a numerical simulation study was carried out on the heat injection-enhanced coalbed methane mining project. The results show that (1) With continuous heat injection, the gas in the coal seam is rapidly desorbed, and the adsorbed gas content forms an elliptical funnel that extends from the extraction hole to the deep part of the coal body and takes the fracturing crack as the center to the upper and lower boundaries of the coal seam. At day 30, the adsorbed gas content in the whole drainage area has decreased to 0.1 m\(^3\)/t within 2m from the fracture zone. (2) On the basis of considering the gas adsorption expansion effect, the strain of the coal body decreases from roof to floor. With the increase in the heat injection time, the strain value will also increase, and the strain increase will decrease. When the heat injection is 30 days, the maximum strain at the roof is 0.015. The research results have important guiding significance for predicting coal rock deformation and determining gas extraction efficiency in the process of heat injection-enhanced coalbed methane extraction.

1. Introduction

A large number of domestic and foreign research data show that the coal gas disaster is the result of gas and coal solid under the joint action of stress and pore pressure [1–3]. The essence is that the gas adsorption equilibrium inside the coal body is destroyed, and then the adsorption expansion of the coal body and a series of changes such as coal temperature, pore pressure, and mechanical properties are induced. The core of the study on this problem is the thermo–fluid–solid coupling problem that occurs in the process of gas adsorption and desorption in the coal body.

In recent years, in order to meet the needs of coalbed methane extraction and coal mine safety, many scholars have done a lot of research work. Chinese famous scholar Zhou Shining [4] proposed a single seepage mathematical model of coalbed methane migration, Zhao Yangsheng [5] proposed a mathematical model of solid–gas coupling coalbed methane migration continuum considering coal and rock deformation. Zhu wancheng [6] studied the influence of temperature on gas content and coal porosity, and then proposed the gas migration equation considering the effect of temperature. Sun Keming [7] considered the influence of coal pore deformation, studied the coupling effect of high ground stress and pore pressure, stated that coalbed methane migration will show non-Darcy seepage characteristics, and established a gas–water, two-phase fluid–solid coupling model reflecting the flow of deep low permeability coal seam. Liu Jianjun [8], who firstly considered the influence of thermal effect caused by gas desorption on seepage...
field and coal deformation, established the heat–fluid–solid coupling seepage model of coalbed methane. Zhang [9] considered the effect of free-phase gas and adsorbed-phase gas on the mechanical deformation of coal body, and believed that the coupling of the two would change the pore volume, and the change in coal adsorption would lead to a change in the coal pore. The coupling gas flow and coal deformation processes were studied. Liang B., Sun KM [10, 11] studied the adsorption and desorption law of coalbed methane under non-isothermal conditions, and proposed the mathematical model and finite element numerical solution method of solid–fluid coupling of coal and coalbed methane under non-isothermal conditions. Zhu [12] established a coupling model of coal deformation, coalbed methane transmission and heat transmission, and studied the interaction between coal and gas under variable temperature conditions. Based on the gas migration model of homogeneous coal seam, Sun Peide [13] established a compressible migration model in heterogeneous coal seam based on power law. The above work is the continuous in-depth development of coalbed methane research, which undoubtedly has a greater role in promoting the theory of coalbed methane mining. There are three shortcomings in the above research results: Firstly, the effect of effective stress on coal deformation is limited to the deformation caused by pore pressure, while ignoring the solid deformation caused by coalbed methane adsorption and desorption. Therefore, it cannot truly reflect the change of coal caused by effective stress change. Second, the influence of temperature on coalbed methane migration is limited within 50 °C, which is not applicable to the theory of coalbed methane thermal recovery. Thirdly, in most research methods, the temperature field equation generally only considers heat conduction, not convection heat transfer, and the gas flow diffusion term caused by gas temperature is not considered in the coalbed methane seepage equation.

The heat–fluid–solid coupling theory of coalbed methane migration reveals the mechanism of gas desorption, seepage, and volume expansion in coal under different temperatures. In recent years, some scholars through a large number of laboratory studies found that the process of gas adsorption was accompanied by the expansion of coal deformation. At the same time, the adsorption deformation of coal is related to many factors such as temperature, stress, water content, and gas content, and the deformation is nonlinear with various factors [14–16]. The expansion of gas adsorption will directly lead to a change in the pore fracture volume in coal and then change the pore pressure. The pore pressure determines the effective stress in coal, which will directly affect the gas migration process in coal. Based on the above research results, it can be seen that the gas adsorption/desorption process, the seepage process, and the coupling process of gas and coal seam and temperature will present new characteristics compared to the coal adsorption expansion deformation. Zhao Yangsheng considered that the coal seam is a fractured medium composed of matrix rock blocks and fissures, and used the modified effective stress law to reflect the influence of pore pressure change caused by coalbed methane desorption on coal deformation. A mathematical model of coalbed methane migration in a fractured medium was proposed, and the numerical simulation of coalbed methane extraction by drilling and coalbed methane emission law in coal mining face were carried out. The model considered the effect of free gas on pore pressure and did not consider the effect of adsorbed gas deformation. Gu and Chalaturnyk [14] established a new porosity and permeability model. The permeability of fractured and matrix rock blocks as equivalent continuous elastic media and anisotropic coal seams is analyzed. It is considered that gas desorption/adsorption can cause coal matrix shrinkage/expansion, considering the influence of temperature change and coal rock parameters. Although the model takes into account the influence of adsorption expansion on coal deformation, it takes a linear treatment on the relationship between coal deformation and gas pressure, ignoring the nonlinear characteristics between deformation and gas pressure, which has certain limitations.

Therefore, in this paper, the relationship between coal adsorption expansion deformation and gas content is obtained by laboratory experiments. Based on the experimental results, a more comprehensive mathematical model of deformation–seepage–heat transfer coupling coalbed methane migration is established by comprehensively considering the coupling effect of coal and rock adsorption expansion deformation, heat transfer, water seepage, and coalbed methane desorption. With the help of this model, the numerical simulation of thermal injection-enhanced coalbed methane mining project in Tunlan Coal Mine of Shanxi Coking Coal Group is carried out by Fortran language, and the gas distribution, migration, and deformation law of coal during thermal injection and gas extraction are obtained.

2. Test Scheme

2.1. Sample Preparation. The coal samples required for the test were taken from the 2#coal seam of Tunlan Coal Mine and the 3#coal seam of Guandi Coal Mine under Xishan Coal and Electricity Group of China. Among them, Tunlan coal mine 2#coal belongs to coking coal, and Guandi coal mine 3#coal belongs to lean coal. In order to avoid the damage of the coal body caused by coal mining process, the samples required for the test were taken from the underground non-mining area, and then transported to the laboratory with preservative film package. Considering the low strength of the coal body itself and the large sample size required for the test, a sand wire cutter was used to process the sample into a 100 × 100 × 20 mm cuboid sample (the axial direction is perpendicular to the bedding plane), ensuring that the error of the sample is not more than 0.03 mm, the unevenness of the end face is less than 0.05 mm, and the perpendicularity of the adjacent end face is not more than 0.25°. Some processed samples are shown in Figure 1.

2.2. Experimental Device and Method. The three-axis stress state coal gas adsorption deformation test using the self-developed, flow-solid-coupled coal adsorption deformation
three-axis test system studied the different ground stress and pore pressure coupling with the effect of gas adsorption on coal deformation law, equipment layout, and principles as shown in Figure 2. The system mainly includes five sub-systems of the adsorption deformation reactor, the shaft pressure loading system, the peripressure loading system, the pore pressure system, and the thermostatic water bath system, which can apply a maximum shaft pressure of 30 MPa and a perimeter pressure of 10 MPa. Among them, the shaft pressure is the applied stress through the hydraulic cylinder, and the surrounding pressure uses high-pressure water to apply stress through the rubber sleeve.

In order to study the influence of gas adsorption on the volume deformation of coal in situ, the simulated coal is buried at 360m ~ 600m depth, the applied axial pressure is 9 MPa and 12 MPa, respectively, and the confining pressure is 5 MPa. In order to apply confining pressure on the coal sample and avoid high-pressure water entering the coal sample affecting gas adsorption, the coal sample is first put into the rubber sleeve and then placed into the reactor before the experiment, and the air tightness of the reactor is checked and vacuumized. Then, control the axial pressure and the surrounding rock loading device to load slowly to the target pressure value in sequence. After the pressure is stable, gas is introduced into the reactor, and the axial and lateral deformation are recorded simultaneously. In order to ensure that the coal sample can fully absorb gas, each group of experiments shall be carried out for 8h. After the coal deformation is fully completed, the volume deformation of the coal sample shall be recorded. Then increase the axial pressure to 12 MPa and repeat the above steps to complete the adsorption deformation test under axial pressure.

3. Experimental Results and Analysis

Figure 3 shows the variation curves of volumetric strain with methane content of different coal samples under 9MPa and 12 MPa axial pressure, respectively. Since the experimental results obtained from various specimens from the same origin are not different, only the results of Guandi 1# and Tunlan 1# experiments are listed here for analysis and discussion.

Through the analysis of the volume deformation curve of each coal sample with gas content, it is found that the volume strain of the coal sample presents the law of exponential distribution with the change in gas content. For the same coal sample, under the condition of the same gas content, the smaller the axial pressure, the greater the volume deformation of the coal sample because when the pore pressure and gas content are certain, the smaller the axial pressure acting on the coal, the smaller the effective stress in the sample and the greater the coal deformation caused by free and adsorbed gas [17, 18]. Through comprehensive comparison of Figure 3(a) and 3(b), it is found that there are differences in the response characteristics of different types of coal samples to the deformation caused by gas content. Taking the axial pressure of 12 MPa as an example, for the Guandi and Tunlan coal samples, the increase in gas content in coal has little effect on the coal volume deformation before the gas content reaches 27 m$^3$/t coal and 23 m$^3$/t coal, respectively. When the gas content in the two coal samples exceeds 27 m$^3$/t coal and 23 m$^3$/t coal, coal deformation will increase sharply with an increase in gas content. It shows that there is a threshold value for coal deformation with the increase in gas content [19]. When the gas content is lower than the threshold value, the increase in gas content will not cause the obvious deformation of coal. When the gas content reaches the threshold value, the volume deformation will increase sharply. At this time, the gas adsorption and desorption process will change the coal volume. The mechanism of the nonlinear change of coal strain with the increase in gas content is that when the external stress conditions are certain, coal deformation is jointly affected by the pore pressure produced by free gas and the expansion deformation produced by adsorbed gas [20, 21]. Under low gas conditions, coal deformation is mainly caused by the change in effective stress under different pore pressure with an increase in gas content; the increase in pore pressure will promote the transformation from...
free gas to adsorbed gas, resulting in the nonlinear increase of the coal body strain.

Under the action of external stress, the coal seam with zero CBM content is assumed to have an initial strain value \( \varepsilon_0 \); the experimental data are fitted exponentially, and the fitting results are shown in Table 1.

The error analysis shows that the error of the fitting result is very small. Therefore, it is considered that the change in coal volume strain with coal gas content conforms to the exponential law.

\[
\varepsilon = \varepsilon_0 e^{\varepsilon C - 1}.
\]  
(1)

It can be seen that the deformation of the gas-bearing coal body is closely related to the in situ stress of the coal body and the gas content in coal body. The research results can be used to calculate the coal deformation caused by gas adsorption and desorption in coal when analyzing and calculating coal seam gas drainage.

### 4. Heat–Fluid–Solid Mathematical Model

#### 4.1. Basic Assumptions.

The heat–fluid–solid coupling mathematical model for enhancing coalbed methane extraction in the coal body by heat injection involves not only the adsorption and desorption of coal and coalbed methane and the change of the solid stress field, but also the change in coal physical parameters such as permeability coefficient and adsorption constant and further affect the deformation characteristics of coal [22]. In order to make the model better reflect these essential laws, the following basic assumptions are proposed.

1. As the temperature rises, the coal seam gas changes from the adsorption state to the free state instantly.

2. Coalbed methane, water, and coal solid skeleton instantly reach local thermal equilibrium.

3. Free coal seam gas can be regarded as ideal gas

\[
q_{df} = n \rho_g = n \frac{P_g M}{RT}.
\]  
(2)

The variation of the adsorbed gas content with temperature rise follows the Langmuir adsorption equation.

\[
q_{xf} = \frac{abp_g}{1 + bp_g \rho_0}\rho_0.
\]  
(3)

The variation of the adsorption coefficients \( a \) and \( b \) with temperature is referred to Zhao D experimental results [23].

\[
a = a_0 e^{-\alpha T};
\]
\[
b = b_0 e^{-\beta T}.
\]  
(4)

The total coalbed methane content in coal is as follows:

\[
C = q_{df} + q_{xf} = n \frac{P_g M}{RT} + \frac{abp_g}{1 + bp_g} \rho_0.
\]  
(5)

4. Volume strain caused by gas adsorption by the coal body conforms to the exponential law:

\[
\varepsilon = \varepsilon_0 e^{\varepsilon C - 1}.
\]  
(6)

5. The seepage law of coalbed methane and water in coal seams conforms to the linear Darcy’s law on the micro-pressure gradient:

\[
\Delta q_i = K q \Delta P_{i,j}.
\]  
(7)
The entire interval conforms to the following formula:

\[ q_i = K_{ij}P_{j'} \]  

(8)

And, \( K_{ij} = K(\Theta, p) \), i.e., the permeability coefficient is a function of stress and pore pressure [24].

\[ k = k_0p^{-\eta} \exp[-b(\Theta - 3\alpha p)]. \]  

(9)

(6) The pressure of water and coalbed methane in the whole seepage field is always consistent, i.e., the influence of surface tension is not considered at the gas-liquid interface, and the pressure of coalbed methane is the same as that of water.

\[ P_g = P_w. \]  

(10)

(7) Coalbed methane and water are always saturated in the pores.

\[ S_g + S_w = 1. \]  

(11)

(8) The coal and rock mass are in the elastic deformation stage and obey the generalized Hooke’s law:

\[ \sigma_{ij} = \lambda\delta_{ij}\epsilon + 2\mu\epsilon_{ij}. \]  

(12)

(9) The effective stress law of the coal body under the action of coalbed methane follows the modified Biot effective stress law [25]:

\[ \sigma_{ij}' = \sigma_{ij} - \alpha_1 P_g \delta_{ij}. \]  

(13)

The effective stress coefficient can be expressed as:

\[ \alpha = \alpha_1 - \alpha_2 \Theta + \alpha_3 p - \alpha_4 \Theta p. \]  

(14)

Considering the influence of adsorbed gas and pore pressure on solid deformation, the law of effective stress is as follows:

\[ \sigma_{ij}' = \sigma_{ij} - \alpha_1 p \delta_{ij} - \omega C\delta_{ij}. \]  

(15)

In the formula, \( p \) is the gas pore pressure, \( \alpha_1 \) is the Biot coefficient, obtained by the experiment. \( \omega \) is the stress variation coefficient of coal and rock solid skeleton caused by adsorbed gas. \( C \) is the adsorbed gas content of coal and rock mass. \( \delta_{ij} \) is the Kronecker symbol.

(10) The density of water is no longer a constant, but a coefficient related to temperature and pressure, which is expressed as [24]:

\[ \frac{1}{\rho_w} = 3.086 - 0.899 \times (4014.15 - T)^{0.1471} - 0.39 \times (658.15 - T)^{-1.6} ( P - 225.5) + \delta_w. \]  

(16)

4.2. The Seepage Equation of Coalbed Methane. Studying the mass conservation of any control volume unit, there are the following equations:

\[ \text{div}(\rho_g \mathbf{q}_g) = \frac{\partial C}{\partial t}. \]  

(17)

Expansion formula:

\[ \frac{\partial (\rho_g \mathbf{q}_g)}{\partial x} + \frac{\partial (\rho_g \mathbf{q}_g)}{\partial y} + \frac{\partial (\rho_g \mathbf{q}_g)}{\partial z} = \frac{\partial C}{\partial t}. \]  

(18)

Darcy’s law and coalbed methane content (5) are substituted into (17).

\[ \text{div}(\rho_g \mathbf{q}_g) = \frac{\partial}{\partial x} \left( \frac{\rho_g \mathbf{q}_g}{\rho_g \mathbf{q}_g} \right) \]

\[ = \sum \frac{\partial}{\partial x} \left( \frac{p \mathbf{q}_g}{RT \mu_g \delta_x} \right) \]  

(19)

\[ \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( \rho_g q_{t1} + \rho_g q_{t2} \right) \]

\[ = \frac{\partial}{\partial t} \left( nS_g P_g M + \rho_g q_{t3} \right) \]  

(20)

Separate calculation:

<table>
<thead>
<tr>
<th>Axial pressure/MPa</th>
<th>Coal sample</th>
<th>Fitting curve</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Guandi1#</td>
<td>( \varepsilon = 0.4643 (e^{0.2538 C} - 1) )</td>
<td>0.9979</td>
</tr>
<tr>
<td></td>
<td>Guandi2#</td>
<td>( \varepsilon = 0.6252 (e^{0.2375 C} - 1) )</td>
<td>0.9906</td>
</tr>
<tr>
<td></td>
<td>Guandi3#</td>
<td>( \varepsilon = 0.4751 (e^{0.2172 C} - 1) )</td>
<td>0.9952</td>
</tr>
<tr>
<td></td>
<td>Tunlan1#</td>
<td>( \varepsilon = 0.0302 (e^{0.36 C} - 1) )</td>
<td>0.9891</td>
</tr>
<tr>
<td></td>
<td>Tunlan2#</td>
<td>( \varepsilon = 0.0146 (e^{0.3691 C} - 1) )</td>
<td>0.9903</td>
</tr>
<tr>
<td></td>
<td>Tunlan3#</td>
<td>( \varepsilon = 0.0404 (e^{0.3365 C} - 1) )</td>
<td>0.9995</td>
</tr>
<tr>
<td></td>
<td>Guandi1#</td>
<td>( \varepsilon = 0.1628 (e^{0.2514 C} - 1) )</td>
<td>0.9961</td>
</tr>
<tr>
<td></td>
<td>Guandi2#</td>
<td>( \varepsilon = 0.1499 (e^{0.2439 C} - 1) )</td>
<td>0.9761</td>
</tr>
<tr>
<td></td>
<td>Guandi3#</td>
<td>( \varepsilon = 0.174 (e^{0.229 C} - 1) )</td>
<td>0.9614</td>
</tr>
<tr>
<td>12</td>
<td>Tunlan1#</td>
<td>( \varepsilon = 0.0264 (e^{0.3052 C} - 1) )</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td>Tunlan3#</td>
<td>( \varepsilon = 0.0231 (e^{0.3101 C} - 1) )</td>
<td>0.9556</td>
</tr>
</tbody>
</table>

Table 1: Fitting results of coal strain and gas content.
\[
\frac{\partial}{\partial t} \left( \frac{nS_g p_M}{RT} \right) = \frac{S_g M p_g}{RT} \frac{\partial n}{\partial t} + \frac{S_g n M \cdot n \cdot p_g}{RT^2} \frac{\partial T}{\partial t} + \frac{n p_g M}{RT} \frac{\partial S_g}{\partial T},
\]
(21)

\[
\frac{\partial}{\partial t} \left( \frac{ab p_g}{1 + bp_g} \frac{p_g M}{RT} \right) = \frac{\rho_0 b_p}{1 + bp_g} \frac{\partial a}{\partial t} + \frac{\rho_0 a p_g}{1 + bp_g} \frac{\partial b}{\partial t} + \frac{\rho_0 ab}{1 + bp_g} \frac{\partial p_g}{\partial T},
\]
(22)

Comprehensive calculation equation (17)–(22).

\[
\sum_{i=1}^{3} \left( \frac{M}{2 RT} \frac{\partial}{\partial x_i} \left( \frac{k_{x_i} \partial^2 p_g}{\mu \partial x_i^2} \right) + \frac{M}{2 RT^2} \frac{k_{x_i} \partial^2 p_g}{\mu \partial x_i \partial x_i^2} \right) \frac{\partial T}{\partial t} = \frac{S_g M p_g}{RT} \frac{\partial n}{\partial t} + \frac{S_g n M \cdot n \cdot p_g}{RT^2} \frac{\partial T}{\partial t} + \frac{E_g M}{RT} \frac{\partial S_g}{\partial T} + \frac{\rho_0 b_p}{1 + bp_g} \frac{\partial a}{\partial t} + \frac{\rho_0 a p_g}{1 + bp_g} \frac{\partial b}{\partial t} + \frac{\rho_0 ab}{1 + bp_g} \frac{\partial p_g}{\partial T},
\]
(23)

The above equation is the seepage equation of coalbed methane transmission considering the factors of solid deformation, temperature effect, and adsorption parameter change.

For the partial derivative of formula (4) with respect to time,

\[
\frac{\partial a}{\partial t} = \frac{a_0 \alpha_T e^{-(\alpha_T - T/T)} - a_0 \alpha_T e^{-\alpha}}{\alpha_T - T/T} \frac{\partial T}{\partial t},
\]
(24)

\[
\frac{\partial b}{\partial t} = \frac{b_0 \beta_T e^{-\beta_T / T}}{\beta_T - (\beta_T - T/T)} \frac{\partial T}{\partial t},
\]

Substitute formula (23) in formula (24) to obtain,

\[
\sum_{i=1}^{3} \left( \frac{M}{2 RT} \frac{\partial}{\partial x_i} \left( \frac{k_{x_i} \partial^2 p_g}{\mu \partial x_i^2} \right) + \frac{M}{2 RT^2} \frac{k_{x_i} \partial^2 p_g}{\mu \partial x_i \partial x_i^2} \right) \frac{\partial T}{\partial t} = \frac{S_g M p_g}{RT} \frac{\partial n}{\partial t} + \frac{S_g n M \cdot n \cdot p_g}{RT^2} \frac{\partial T}{\partial t} + \frac{\rho_0 b_p}{1 + bp_g} \frac{\partial a}{\partial t} + \frac{\rho_0 a p_g}{1 + bp_g} \frac{\partial b}{\partial t} + \frac{\rho_0 ab}{1 + bp_g} \frac{\partial p_g}{\partial T} + \frac{S_g M \cdot n \cdot p_g}{RT^2} \frac{\partial T}{\partial t},
\]
(25)
(25) is the gas seepage equation considering factors such as deformation, saturation, seepage pressure, and temperature.

4.3. Water Seepage Control Equation. When water vapor injection enhances coalbed methane mining, the water seepage process also exists in the coal body. The mass of water in any control volume unit is conserved.

\[
\text{div}(\rho_w q_w) = \frac{\partial(n_{w}p_w)}{\partial t} - W_l. \tag{26}
\]

Considering the respective saturations of coalbed methane and water in the pores,

\[
\text{div}(\rho_w q_w) = \frac{\partial(n_{w}p_w)}{\partial t} - W_l \tag{27}
\]

\[
= S_w \rho_w p_w \frac{\partial \rho_w}{\partial t} + n_p w \frac{\partial S_w}{\partial t} + n_{w} \frac{\partial p_w}{\partial t} - W_l.
\]

According to assumption (10), the density of water is a function of temperature and water pressure; thus:

\[
\frac{\partial \rho_w}{\partial t} = \frac{\partial p_w}{\partial T} + \frac{\partial p_w}{\partial T} \frac{\partial T}{\partial T} + \frac{\partial p_w}{\partial T} \frac{\partial T}{\partial T}.
\]

Substituting the hypothesis (5) Darcy’s law of water seepage into (26),

\[
\frac{\partial}{\partial x} \left( k_w \frac{\partial p_w}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_w \frac{\partial p_w}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_w \frac{\partial p_w}{\partial z} \right) = S_w \rho_w \frac{\partial S_w}{\partial t} + n_p w \frac{\partial S_w}{\partial t} + n_{w} \frac{\partial p_w}{\partial t} - W_l.
\]

(29) is the seepage equation of water in the coal body. It can be seen that the seepage pressure is related to many factors, such as the deformation of the coal body skeleton, water saturation, and fluid pressure changes over time.

4.4. The Temperature Field Control Equation considering Convective Heat Transfer and Solid Heat Conduction. In the simulation process of coal seam gas extraction by heat injection, thermal fluid conducts heat transfer by convection, while the skeleton of coal and rock mass conducts heat transfer mainly by heat conduction. The two have different heat transfer characteristics, and the heat conduction coefficient of specific heat of coal and rock mass are different from that of fluid; thus, they need to be re-deduced according to their respective equations.

The thermal conductivity equation of the solid skeleton of coal and rock mass is defined as follows:

\[
(1 - n) \rho_c c_p \frac{\partial T}{\partial t} = (1 - n) \lambda_z \nabla^2 T + Q_l. \tag{30}
\]

The fluid convection equation is defined as follows:

\[
\frac{n}{\partial t} \frac{\partial T}{\partial l} + \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \left( n \lambda_w \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q_l, \tag{31}
\]

where \( \lambda \) is the heat conduction coefficient of the gas–liquid two-phase mixed fluid; \( T \) is temperature; \( c_p \) is the specific heat capacity of the fluid in gas–liquid two-phase turn; \( \rho \) is the density of the gas–liquid two-phase mixed fluid; \( u, v, \text{ and } w \) are the velocity of the mixed fluid, respectively; \( Q_l \) is the source sink term;

The calculation formula for the specific heat, thermal conductivity, and density of the mixed fluid are as follows:

\[
c_p = S_g c_g + S_w c_w,
\]

\[
\lambda_z = S_g \lambda_g + S_w \lambda_w,
\]

\[
\rho = S_g \rho_g + S_w \rho_w.
\]

According to hypothesis (5), Darcy’s law of seepage and (32) are substituted into (31):

\[
n \left( S_g \rho_g + S_w \rho_w \right) \left( S_g c_g + S_w c_w \right) \frac{\partial T}{\partial t} + \left( S_g \rho_g + S_w \rho_w \right) \left( S_g c_g + S_w c_w \right) \lambda_z \nabla p_w \cdot \nabla T = n \left( S_g \lambda_g + S_w \lambda_w \right) \nabla T^2 + Q_l. \tag{33}
\]

4.5. Coal and Rock Mass Deformation Equation. According to the theory of elasticity, the static equilibrium equation of coal and rock mass is shown as follows:

\[
\sigma_{ij} + F_i = 0. \tag{35}
\]

According to assumptions (6) and (7), considering the effect of coalbed methane pore pressure, coalbed methane desorption, and coal seam deformation caused by temperature changes, the stress balance equation in terms of displacement is expressed as follows:
the effect of adsorption expansion. (36)

(36) is the deformation motion equation of coal and rock mass expressed by displacement, considering pore pressure, temperature, and coalbed methane adsorption and desorption.

\[ \begin{align*}
(\lambda + \mu)u_{i,ij} + \mu u_{i,j} + (a\delta_i p)_j + (\omega_i \delta_j C)_j + (\beta \delta_i T)_j + F_i &= 0. \\
\end{align*} \]

4.6. Fluid–Solid Coupling Mathematical Model considering the Gas Adsorption Expansion Effect. After the above analysis, combined with the related coupling equations, the gas drainage fluid–solid coupling control equation considering the deformation of coal adsorption gas is established, which is expressed as:

\[ \begin{align*}
\frac{\partial}{\partial t} \left( \frac{M}{2 RT} \frac{\partial p_{w}}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{M}{2 RT} \frac{\partial p_{w}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{M}{2 RT} \frac{\partial p_{w}}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{M}{2 RT} \frac{\partial p_{w}}{\partial z} \right) + \rho g_a b p_g \frac{\partial a}{\partial t} + \rho g_a n p_g \frac{\partial b}{\partial t} &= 0, \\
\end{align*} \]

5. Numerical Experiment of Coalbed Methane Drainage considering the Adsorption Expansion Effect

5.1. Simplified Calculation Model. According to the proposed thermal–fluid–solid coupling mathematical model for enhanced coalbed methane drainage by heat injection considering the adsorption expansion effect, taking Tunlan Mine as an example, horizontal holes are drilled at 100m intervals in the coal as heat injection holes and drainage holes, and hydraulic fracturing is used to create fluid migration channels in the coal. Assuming that each section of the coal body along the drainage aperture direction is isotropic, the section perpendicular to the borehole is taken as the calculation model. The model is 100m in length, 0.3 m in width, and 6m in height. The simplified geometric and physical model, initial boundary and coal body mechanical parameters are shown in Figure 4 and Tables 2 and 3.

5.2. Numerical Experiment. This paper takes Tunlan coal mine 2# coal seam as the research object and establishes a numerical model. The left side of the model is provided with a heat injection hole and the right side is provided with a drainage hole. On the basis of considering the gas adsorption expansion effect, the adsorbed gas content and coal deformation in the process of coalbed methane extraction under the cooperation of heat–fluid–solid coupling are simulated and calculated. Taking time as the cyclic variable, the results of adsorbed gas and coal deformation in 3 days, 10 days, 20 days, and 30 days during the implementation of heat injection and extraction project are drawn into Figures 5 and 6.
5.2.1. Variation Law of Adsorbed Gas Content. Figure 5 shows the isoline distribution of adsorbed gas at \( y = 0 \) section. With the progress of heat injection and extraction, the adsorbed gas near the fracturing zone in the coal body is desorbed rapidly, and the content of the adsorbed gas in this area decreases significantly. Under the influence of drainage, the adsorbed gas is characterized by the gradual increase of the fracture zone to the top and bottom plates. On the third day of extraction, as shown in Figure 6(a), the adsorbed gas in the coal body presents an elliptical "funnel" distribution with the fracture zone as the center, the adsorbed gas content in the fracture zone decreases significantly, and the adsorbed gas content in the extraction hole area decreases from 22.7 m³/t to 2 m³/T. At the roof 3M away from the fracture zone, the adsorbed gas content still reaches 22 m³/T, indicating that the gas in the coal far away from the fracture zone has not yet desorbed. This is because the shorter the heat injection and extraction time, the farther away from the heat injection hole, the smaller the temperature rise of the coal body, and the less the gas desorption capacity. Therefore, the adsorbed gas presents a "funnel" distribution. At the same time, due to the existence of the fracture zone, the temperature in the middle of the coal body is significantly higher than that in the top and bottom plates; thus, the gas increases from the fracture zone to the top and bottom plates, respectively.

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**Table 2: Initial and boundary conditions.**

<table>
<thead>
<tr>
<th>Equation category</th>
<th>Initial condition</th>
<th>Boundary condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>The gas seepage equation</td>
<td>Initial temperature of the coal body: ( T(x, y, z, t) = 25 \degree C )</td>
<td>( \partial p_g / \partial n = 0 ) Airtight boundaries</td>
</tr>
<tr>
<td></td>
<td>Initial condition: ( p_g(x, y, z, t)</td>
<td>t = 0) = 13.25 \text{ atm} )</td>
</tr>
<tr>
<td>Temperature field control equation</td>
<td>Injection well temperature: ( T(x_0, y_0, z_0, t) = 250 \degree C )</td>
<td>( -\lambda (\partial T/\partial n)_{w} = 0 ) Both are adiabatic boundaries</td>
</tr>
<tr>
<td>Solid stress equation</td>
<td></td>
<td>Up and down the boundary</td>
</tr>
<tr>
<td>Water seepage equation</td>
<td></td>
<td>Front and rear boundaries</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The displacement boundary: lower bound, ( r_y, w = 0 ); Left boundary, ( u = 0 );</td>
</tr>
<tr>
<td></td>
<td></td>
<td>both are adiabatic boundaries</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water injection well: ( q_w((x_0, y_0, z_0, t) = 0.5 \text{ l/s} )</td>
</tr>
</tbody>
</table>

**Table 3: Modeling parameters for the numerical simulation.**

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal friction angle of gas-containing coal rock, ( \phi ) (°)</td>
<td>32</td>
</tr>
<tr>
<td>Young's modulus of gas-containing coal rock, ( E ) (MPa)</td>
<td>2183.1</td>
</tr>
<tr>
<td>Poisson ratio, ( \nu )</td>
<td>0.3274</td>
</tr>
<tr>
<td>Gas power viscosity, ( \mu ) (Pa.s)</td>
<td>( 1.34 \times 10^{-5} )</td>
</tr>
<tr>
<td>Coal body weight, ( Y ) (g/cm³)</td>
<td>1.4</td>
</tr>
<tr>
<td>Single axis strength, (MPa)</td>
<td>8.2</td>
</tr>
<tr>
<td>Porosity, ( n )</td>
<td>6.07%</td>
</tr>
<tr>
<td>Volume modulus of gas-containing coal rock, ( k_v ) (MPa)</td>
<td>170</td>
</tr>
<tr>
<td>Gas density, ( \rho ) (kg/m³)</td>
<td>0.714</td>
</tr>
<tr>
<td>Adsorption constant ( a ), (m³/kg)</td>
<td>24.15</td>
</tr>
<tr>
<td>Adsorption constant ( b ), (MPa⁻¹)</td>
<td>1.32</td>
</tr>
</tbody>
</table>

---

**Figure 4: Mechanical model of the XZ plane.**
When the drainage is carried out to the 10th and 20th days, respectively, the content of the adsorbed gas at the same horizontal position has little difference. With the drainage, the desorption area gradually expands. Near the fracture zone, the maximum value of the adsorbed gas in the coal seam decreased significantly. Figure 5(d) shows that after one month of drainage, the adsorbed gas content of the coal seam in the simulation area has decreased significantly, and the maximum content of adsorbed gas in the coal seam is only 2.6 m$^3$/t. At this time, the adsorbed gas content within 2 m from the fracture zone has decreased to less than 0.1 m$^3$/t.
5.2.2. Coal Deformation Law. Figure 6 is the contour characteristic image of coal deformation up to 3 days, 10 days, 20 days, and 30 days when the heat injection-enhanced gas drainage process is used to inject heat into the coal. Through comparison, it can be found that coal deformation shows three characteristics with the extension of heat injection time: first, when the heat injection fluid does not completely penetrate the heat injection hole and production hole, the coal strain at the same horizontal position shows that the side strain of the heat injection hole is greater than that of the production well. After the fluid migration channel is successfully opened, the internal strain at the same level is almost equal. This is because before the fluid migration channel is connected, there is a temperature difference between the heat injection hole and the extraction hole, and the desorption amount of coaledbed methane is also different. Therefore, the coal strain caused by coaledbed methane desorption is also different.

Secondly, due to the fixed constraint of the lower boundary of the model, when the coal body shrinks due to the desorption of coaledbed methane, the coal body strain shows a decreasing trend from the roof to the floor under the action of self-gravity; after 3 days of heat injection, the maximum strain of the top plate is 0.011. The maximum strain of the roof is 0.015 when the heat injection is 30 days. Thirdly, with the extension of the heat injection time, the coal body strain increases with the progress of extraction, but the strain increase decreases. This is mainly because with the extension of heat injection and extraction time, the content of coaledbed methane desorbed from the adsorbed state to the free state in the coal body continues to decrease, and the shrinkage of the coal body caused by desorption also decreases, resulting in a decrease in the strain change.

6. Conclusion

Based on the in-depth analysis of the existing mathematical model of coaledbed methane transportation, through the method of indoor experiment, using the self-developed fluid–solid coupling triaxial coal adsorption deformation measurement system, this paper obtains the relationship between gas deformation and gas content before and after gas adsorption, and establishes the thermal–fluid–solid coupling mathematical model on the premise of neglecting the adsorption expansion effect. Taking the Tunlan Mine as the research object, the distribution characteristics of adsorbed gas, which shows that the adsorption characteristics of coal and the distribution of the fracture zone has decreased to 0.1m³/t.

(3) On the basis of considering the gas adsorption expansion effect, with the progress of heat injection and drainage, there are differences between the deformation characteristics of coal and the distribution characteristics of adsorbed gas, which shows that the strain decreases from the top plate to the bottom plate. With the increase in the heat injection time, the coal strain will also increase continuously. The maximum strain at the top plate is 0.015 at 30 days of heat injection.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also form part of an ongoing study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Dong ZHAO curated the data; Pengwei LI performed investigation; Jianlin XIE reviewed and edited the original draft.

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