A Modified Fiber Bridging Model for High Ductility Cementitious Composites Based on Debonding-Slapping Rupture Analysis

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Modified micromechanical bridging model is established with consideration of the fiber rupture effect at debonding and slipping stages. The bridging model includes the debonding and slipping rupture of fibers and establishes the fiber/matrix interfacial parameters (friction stress $\tau_0$, chemical bonding force $G_d$, and slip-hardening coefficient $\beta$). A different interfacial bonding can cause fiber rupture. The influence of the interfacial conditions on the fiber rupture risk was investigated. In the modified bridging model, the effective bridging stress, the debonding rupture stress, and the slipping rupture stress were clearly identified. Finally, single-fiber pullout tests with different embedded lengths were carried out to validate the bridging model. The relationship between the fiber bridging stress and the crack opening predicted by the bridging model was consistent with the experimental results. This modified micromechanical bridging model can be used to quantitatively calculate the actual fiber bridging capacity and to predict the ductility of the high ductility cementitious composites reinforced by different types of fibers.

1. Introduction

High ductility cementitious composites (HDCCs) exhibit an excellent tensile ductility ability accompanied by closely spaced multiple cracks appearing before the final failure [1, 2]. The tensile ductility can reach several hundred times that of traditional cementitious composites, and the average tight crack opening is typically less than 100 $\mu$m [3, 4]. These extraordinary characteristics can be designed by the “bridging law,” which give the fiber bridging stress versus crack opening relation for short randomly distributed fibers [5–7]. With a proper design, the fibers bridging stress can be transferred efficiently back to the HDCC matrix after the first cracking event, which enables the composite to undergo multiple cracking and have a high ductility behavior [8, 9]. The bridging model has been extensively investigated in the past as a key theory for HDCC [10]. Nevertheless, this existing fiber bridging model is usually limited to specific application conditions that do not account for chemical bonding or fiber slipping rupture analysis during crack propagation.

In the fiber bridging model design, handling ways of the fiber/matrix interfacial parameters (friction stress $\tau$, chemical bonding force $G_d$, and slip-hardening coefficient $\beta$) determine the accuracy and the applicable conditions. In simplified bridging model [10], the $G_d$ and $\beta$ are ignored. Besides, the friction $\tau$ was assumed to be constant and equal to an $\tau_0$. Based on those assumptions, only hydrophobic high-strength fibers, such as PE fiber and carbon fiber, are qualified. However, some hydrophilic fibers, such as PVA fiber, can form a high chemical bond and strong slip-hardening effect with the surrounding matrix [11–13], and the simplified model cannot be used in this case because of its restriction. Lin and Li replaced the constant friction by
linearly increasing the interfacial friction and considered the parameters of \( G_d \) and \( \beta \) to improve the bridging model [5]. Therefore, most synthetic fibers can be treated with the model whether they are hydrophilic or hydrophobic.

Another essential element of analysis in the bridging model is the fiber rupture phenomenon during crack propagation. Without considering fiber rupture, the fiber bridging ability will be seriously overestimated [8]. In fact, fibers may rupture at the debonding stage or the slipping stage once the fiber stress exceeds fiber tensile strength. Maalej et al. [14], Kanda et al. [9], and Lin et al. [15] extended the bridging model by including the chemical bonding \( G_d \), the slip-hardening effect \( \beta \), and the fiber debonding rupture analysis. Moreover, the bridging stress versus crack opening relation for a single-fiber pullout with a normal and an inclined angle was derived to explain the fiber strength reduction when an inclined angle is used. Wang [16] and Yang et al. [17] established a two-way fiber bridging model by considering the matrix micro-scrapping effect. Based on Yang’s two-way model, Huang et al. [8] devoted to account for fiber rupture phenomenon including debonding rupture and slipping rupture at the fiber pullout stage. The accuracy of the predicted composite bridging stress versus crack opening relation has greatly been improved compared to previous models. Nevertheless, the slipping rupture analysis is not clearly clarified in Huang’s model. Besides, the existing crack opening \( \delta_0 \) at the debonding stage was ignored in the slipping rupture analysis, which will lead to an inaccurate evaluation for the slipping rupture of fibers. Lu and Leung [18] elaborated on the cracking process and the stress-strain relation by considering the variation of matrix strength along the member, the increase of crack bridging stress in the hardening regime, and the possibility of fiber debonding rupture. As a result, the fiber bridging law was gradually perfected for high ductility cementitious composites. However, the bridging model still needs to be improved to more accurately analyze fiber rupture.

In this study, some noteworthy details including the fiber/matrix interfacial parameters and the fiber rupture growth during crack propagation were investigated. The complex slipping rupture phenomena of the entire rupture process of the fiber in the matrix were accurately analyzed. This work presents the current model for the evolution of the fiber bridging stress with crack opening based on fiber rupture analysis, which included the friction \( \tau_0 \), the chemical bonding force \( G_d \), and the slip-hardening coefficient \( \beta \). Compared to existing bridging models, this model considers the fiber state change during the pullout process and is more realistic. In bridging model, the effective bridging stress, the debonding rupture stress, and the slipping rupture stress were clearly identified. Finally, single-fiber pullout tests with different embedded fiber lengths were carried out to validate the bridging model.

2. Modified Micromechanical Bridging Model

2.1. Single-Fiber Bridging Model

2.1.1. Single-Fiber Pullout Behavior of HDCC. During crack propagation, the uniform randomly distributed fibers in HDCC will have five states, namely, debonding state, slipping state, debonding rupture state, slipping rupture state, and complete pullout state. The specific state depends on the fiber’s spatial location and the interfacial bonding. Fibers in HDCC are assumed to be randomly distributed in all 3 dimensions, and the spatial locations of the fibers are expressed as \( f(z) \) and \( f(\theta) \) [19]. Detailed descriptions are given in section 2.1. The fiber/matrix micromechanical interfacial parameters can be obtained by the single-fiber pullout test. Typical single-fiber pullout curves of PVA-HDCC are shown in Figure 1.

As can be seen in Figure 1, the slip-hardening phenomenon occurs after debonding stage. The slip-hardening effect describes the phenomenon that the pullout load increases continuously after the debonding stage of the fiber. The main reason is that the fiber surface is abraded during pullout process. The pullout channel is blocked by the fiber residue (Figure 2), resulting in an increase in the pullout load. Consequently, an intact pullout curve without fiber rupture can be divided into two major regimes: a debonding stage and a slippage stage. Actually, fibers may rupture in the debonding stage or the slipping stage. Rupture in the debonding stage (RD type), rupture in the slipping stage (RS type), and complete pullout without rupture (CP type) are illustrated with an example in Figure 1. For the PVA fiber, serious abrasion and delamination are observed during the pullout process (Figure 2), which can cause slip-hardening effect and increase the risk of slipping rupture. Furthermore, the fiber tensile strength will also decrease due to the abrasion effect, even if the fibers are embedded vertically. The interfacial parameters \( (G_d, \tau_0, \beta) \) are calculated by equations (1) to (3) [20].

\[
G_d = \frac{2(P_a - P_e)^2}{\pi^2 E_f d_f^3} \quad (1)
\]

\[
\tau_0 = \frac{P_b}{\pi d_f L_e} \quad (2)
\]

\[
\beta = \left( \frac{d_f}{L_e} \right) \left[ \left( \frac{1}{\eta_0 h d_f} \right) \left( \frac{\Delta P_a}{\Delta S} \right) \right]_{S_0}^{S_1} + 1 \quad (3)
\]

where \( G_d \) is the chemical debonding energy value (J/m²), \( \tau_0 \) is the frictional bond strength (Pa), \( \beta \) is the slip-hardening coefficient, \( P_a \) is the peak load of the pullout curve in the debonding stage (N), \( P_e \) is the load after the sudden drop following \( P_a \) (N), \( E_f \) is the fiber modulus of elasticity (Pa), \( d_f \) is the fiber diameter (m), \( L_e \) is the fiber embedment length (m), \( (\Delta P_a/\Delta S)|_{S_0}^{S_1} \) is the slope of the pullout curve after full debonding, and \( S_0 \) is the displacement corresponding to a full debonding (m).

2.1.2. Single-Fiber Bridging Stress. A theoretical single-fiber bridging pullout model was derived by Lin and Kanda [15]. In the single-fiber bridging model, there are several main assumptions: (1) the Poisson effect of fiber is negligible. (2) The elastic deformation of fiber in the slipping stage is negligible compared to the slipping magnitude. (3) The interfacial friction \( \tau_0 \) is considered to be linearly increasing
instead of constant \( \tau_0 \). (4) The longer embedded side of the bridging fiber is always in the debonding stage because it needs a greater applied force to complete the debonding process whereas the shorter embedded side can be sliding.

According to aforementioned assumptions, the crack opening on one side should be \( \delta \) for a given general crack opening \( 2\delta \) in the debonding stage, as illustrated in Figure 3(b). Consequently, the stress of a single bridging fiber can be calculated by equation (4) [17]. When full debonding is completed, the crack opening \( \delta_0 \) is calculated by equation (4) for \( \theta = 0 \) and is given by equation (5) for \( \theta = 0 \).

\[
\sigma_b(\delta) = \begin{cases} 
\Delta\sigma_{bd}(\delta) = \sqrt{4(\tau_0\delta + 2Gd)(1 + \eta)\frac{E_f}{d_f}}, & (0 \leq \delta < \delta_0), \\
\Delta\sigma_{bp}(\delta) = \frac{4\tau_0}{d_f^2}(L_\nu - (\delta - 2\delta_0))(d_f + \beta(\delta - 2\delta_0)), & (\delta_0 \leq \delta \leq \frac{L_f}{2}), 
\end{cases}
\]

\[
\delta_0 = \frac{2\tau_0L_\nu^2(1 + \eta)}{E_fd_f} + \sqrt{\frac{8GdL_\nu^2(1 + \eta)}{E_fd_f}},
\]

where \( \sigma_b, \sigma_{bd} \) is the fiber bridging stress in the debonding stage and \( \sigma_{bp} \) is the fiber bridging stress at slipping stage (Pa). \( \delta \) is the single main bridge crack opening, and \( \delta_0 \) is the full debonding crack opening (m). \( \eta = V_f E_f/V_m E_m \), where \( E_m \) is the elastic modulus of the matrix (Pa), and \( V_f \) and \( V_m \) represent the volume fraction of the fibers and the matrix, respectively.

Equation (4) applies to the fibers embedded in a direction perpendicular to the crack surface. However, in the more general case, randomly distributed fibers will intersect the crack plane with different inclined angles \( \theta \). For inclined fibers, the fiber bridging stress is magnified by the snubbing effect and given by equation (6) [21]. Moreover, the apparent fiber tensile strength will be decreased because of the
inclined angle $\theta$. This degradation effect can be represented by equation (7) [19, 22, 23].

$$
s_b(\theta) = \begin{cases} 
\Delta \sigma_{b}(\theta) &= \sigma_{b} e^{f\theta} \\
\Delta \sigma_{p}(\theta) &= \sigma_{p} e^{f\theta} 
\end{cases} \quad \sigma_{f}(\theta) = \sigma_{L} e^{-f\theta},
$$

where $f$ is the snubbing coefficient, $f'$ is the reduction of the apparent fiber strength, $\theta$ is the inclined angle, $\sigma_{f}$ is the apparent fiber tensile strength, and $\sigma_{f}$ is the nominal fiber tensile strength (Pa).

3. Relationship between Bridging Stress and Crack Opening in the Composites

3.1. Without Fiber Rupture Analysis. The composite bridging stress versus crack opening relation is used to link the properties of the matrix, the fiber, and the fiber/matrix interface. The spatial location and interfacial parameters of the randomly distributed fibers are contained in the relationship. Li et al. [15] used a double integral method to add up the contributions of every single fiber in the crack plane:

$$
\sigma_{c}(\delta) = \mathbf{V}_{f} \int_{0}^{\pi/2} \int_{0}^{L_f/2 \cos \theta} \sigma_{b}(\delta, z, \theta) f(z) f(\theta) dz d\theta,
$$

where $\mathbf{V}_{f}$ is the volume fraction of the fibers, and $f(z)$ and $f(\theta)$ are the probability density functions at the inclined angle $\theta$ and centroid distance $z$ of fibers from the crack plane, respectively. For a 3D random distribution, $f(z) = 2/L_f$ and $f(\theta) = \sin \theta(0 \leq \theta \leq \pi/2)$. The geometric relations between $L_c$, $L_f/2$, and $z$ are revealed in Figure 3(a). The embedment length $L_c$ is converted using $L_c = L_f/2 - z/\cos \theta$. Through an integral conversion, an extended expression is given by the following equation [8, 11]:

$$
\sigma_{c}(\delta) = \frac{\mathbf{V}_{f}}{L_f} \int_{0}^{\pi/2} \int_{0}^{L_f/2} \sigma_{b}(\delta, L_c, \theta) e^{f\theta} \sin 2\theta dL_c d\theta.
$$

Ideally, without considering fiber rupture during crack propagation, the general bridging stress $\sigma_{c}(\delta)$ is calculated by equation (10). The fibers with a shorter embedment length go through the debonding stage and then the slipping stage, but the fibers with a longer embedment length are still in the debonding stage until the crack opening expands to $2\delta_{0}(L_c)$. Figure 4 shows the specific state of randomly distributed fibers in the composites for a given crack opening $\delta_{0}$. The full debonding stage is only completed for every single fiber in the case of $\delta_{0} = 2\delta_{0}(L_c = L_f/2)$.

$$
\sigma_{c}(\delta) = \begin{cases} 
\frac{\mathbf{V}_{f}}{L_f} \int_{0}^{\pi/2} \int_{L_c}^{L_f} \sigma_{b}(\delta, L_c) e^{f\theta} \sin 2\theta dL_c d\theta + \frac{\mathbf{V}_{f}}{L_f} \int_{0}^{\pi/2} \int_{0}^{L_f/2} \sigma_{b}(\delta, L_c) e^{f\theta} \sin 2\theta dL_c d\theta, & 0 \leq \delta < 2\delta_{0}(L_c = L_f/2) \\
\frac{2\delta_{0}(L_c = L_f/2)}{2} & \frac{L_f}{2} \leq \delta \leq \frac{L_f}{2}
\end{cases}
$$

One case of the relationship between the bridging stress and the crack opening without fiber rupture analysis is shown in Figure 5. The physical properties of the PVA fibers are listed in Table 1 and the volume fraction of the fibers is 2%. The predicted peak bridge stress reaches 51 MPa, which is significantly higher than the actual value. Therefore, the fiber bridging capacity is seriously overestimated. When the crack expands to 0.18 mm, the debonding stage is over. Due
To the slip-hardening effect, the bridging stress still rapidly increases in the slipping stage. However, the fiber stress will be much larger than the fiber tensile strength and the fibers that have been ruptured cannot bridge the crack plane. Consequently, fiber rupture analysis is indispensable to obtain an accurate bridging model, especially for the slipping rupture.

3.2. With Fiber Rupture Analysis. Fiber rupture will occur once the fiber stress at the crack plane reaches the apparent fiber strength. According to section 2.1, debonding rupture and slipping rupture can be used to analyze fiber rupture phenomenon. The boundary between potential debonding rupture and potential slipping rupture can be derived by the potential critical embedment length $L_d(\theta)$ and $L_p(\theta)$.

3.2.1. Fiber Debonding Rupture. The potential critical debonding embedment length $L_d(\theta)$ is calculated for $\sigma_{bd}(\theta)_{\text{max}} = \sigma_{fu}(\theta)$, as shown in equation (11). For selected composites, $L_d(\theta)$ is determined by the angle $\theta$. The larger the angle $\theta$, the smaller $L_d(\theta)$ will be. The maximum value $L_d(0)$ and the minimum value $L_d(\pi/2)$ can be obtained from equation (11). If the embedded length $L_e(\theta)$ exceeds $L_d(\theta)$, the fibers will risk debonding rupture.

$$L_d(\theta) = \frac{\sigma_{fu}^n d_f}{4\tau_0} e^{-(f + f')\theta} - \sqrt{\frac{2G_d E_f d_f (1 + \eta)}{2\tau_0}}$$

(11)

For different fiber/matrix interfacial conditions, the $L_d(\theta)$ curves have three shapes, as shown in Figure 6. If $L_d(\pi/2) > L_f/2$, debonding rupture will never occur no matter the value of $\theta$ (Figure 6(a)). Otherwise, debonding rupture will happen during crack propagation (Figure 6(b), 6(c)). If the $L_d(\theta)$ curve intersects with $\delta$, the angle $\theta_{\delta\delta}$ can be calculated from

$$\theta_{\delta\delta} = \begin{cases} \frac{\pi}{2} & 0 \leq \delta \leq L_d(\frac{\pi}{2}) \\ \Delta - \frac{1}{(f + f')} \ln \frac{4\tau_0}{\sigma_{fu}^n d_f} \left( \frac{\sqrt{2G_d E_f d_f (1 + \eta)}}{2\tau_0} + \delta \right) & L_d(\frac{\pi}{2}) < \delta \leq L_f(\frac{\pi}{2}) \end{cases}$$

(12)

Figure 6 shows the potential debonding rupture zone determined by $L_d(\theta)$. The fibers located in the potential debonding rupture zone are not simultaneously broken, but they are gradually destroyed during crack propagation. The fibers with a longer embedment length and a bigger angle rupture first. Afterward, the fibers with a shorter embedment length and a smaller angle only start to rupture. For a given crack opening $\delta_r$, the corresponding debonding rupture
zone can be depicted by \( l_d(\delta_r) \), which is calculated by 
\[
\sigma_{\theta, d}(\theta) = \sigma_{fu}(\theta)
\]
The debonding rupture length \( l_d(\delta_r) \) is given by
\[
l_d(\delta_r) = \sqrt{\frac{(\tau_0 \delta_r + 2G_d)E_f d_f}{4\tau_0^2(1 + \eta)}} - \sqrt{\frac{G_dE_f d_f}{2\tau_0^2(1 + \eta)}} \quad (13)
\]
Figure 7 shows the developing process of \( l_d(\delta_r) \) when the crack expands from 5 \( \mu \)m to 250 \( \mu \)m. The debonding rupture zone is determined by the intersection of \( l_d(\delta_r) \) and \( L_d(\theta) \). When \( l_d(\delta_r) = L_d(\pi/2) \), the fibers initiate debonding rupture. Here, the crack opening \( \delta_{cd} \) can be calculated from
\[
\delta_{cd} = \frac{d_f \sigma_{fu}^n}{4E_f \tau_0(1 + \eta)} e^{-2(f/f')^0} - \frac{2G_d}{\tau_0} \quad (14)
\]

Note: a is obtained by Yang et al. (2015) and Yang et al. (2008) [8, 17].

Table 1: Physical properties of PVA fibers.

<table>
<thead>
<tr>
<th>Fiber type</th>
<th>Nominal tensile strength ( \sigma_{fu} ) MPa</th>
<th>Modulus ( E_f ) GPa</th>
<th>Length ( L_f ) mm</th>
<th>Diameter ( d_f ) ( \mu )m</th>
<th>( f )</th>
<th>( f' )</th>
</tr>
</thead>
<tbody>
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<td>PVA fiber</td>
<td>1060(^a)</td>
<td>22(^a)</td>
<td>12(^a)</td>
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Figure 6: \( L_d \) curves with diverse fiber/matrix interface parameters. (a) \( L_d(\pi/2) \geq L_f/2 \). (b) \( L_d(\pi/2) < L_f/2 \) and \( L_d(0) > L_f/2 \). (c) \( L_d(0) \leq L_f/2 \).
growth of the fiber debonding rupture zone is illustrated in Figure 7. When \( l_d(\delta_r) = L_f/2 \), the debonding stage is over and the crack opening \( \delta_{dt} \) is given by

\[
\delta_{dt} = \frac{L_f^2 \tau_0 (1 + \eta)}{E_f d_f} + \sqrt{\frac{8L_f^2 G_d(1 + \eta)}{E_f d_f}}.
\]

When the curve of \( l_d(\delta_r) \) intersects with \( L_d \), the inclined angle \( \theta_{cp} (\delta_r) \) can be calculated from equation

\[
\theta_{cp} = \frac{1}{(f - f')\ln 4E_f (1 + \eta)/d_f \sigma_{fu}^n (\tau_0 \delta + 2G_d)}.
\]

The potential slipping rupture fibers are gradually destroyed during crack propagation. For a given crack opening \( \delta_r \), the current slipping rupture zone can be depicted by \( l_p(\delta_r) \), which is calculated for \( \sigma_{kp}(\theta) = \sigma_{fu}(\theta) \). The current slipping rupture length \( l_p(\delta_r) \) is given by

\[
(l_p(\delta_r) - \delta_r + 2\delta_0)(d_f + \beta(\delta_r - 2\delta_0)) = \frac{d_f^2 \sigma_{fu}^n e^{-(f' + f)\theta}}{4\tau_0}.
\]

Equation (19) can be simplified for \( l_p(\delta_r) \). In Huang's model [8], the crack opening \( 2\delta_0 \) is omitted, which will lead to a larger current slipping rupture zone for a given crack opening \( \delta_r \). In this study, \( 2\delta_0 \) was considered in \( l_p(\delta_r) \).

A case of fiber slipping rupture analysis is shown in Figure 8. It presents a 3D shape of \( l_p(\delta_r) \) with \( \delta_r \) from 50 \( \mu \)m to 300 \( \mu \)m and \( \theta \) from 0 to \( \pi/2 \). \( l_p(\delta_r) \) is not a flat but a coiled surface. The area enclosed by the coiled surface represents the slipping rupture space. The rupture analysis for \( \delta_r \) (100,
150, 200, 250, and 500 μm) is shown in Figure 9. The slipping rupture zone forms a parabolic progression. The slipping rupture zone cannot be neglected compared to the debonding rupture zone.

An interesting case was found for fiber rupture analysis. At the debonding stage, fibers rupture more easily for a longer embedment \( L_e \). However, this may be invalid in the slipping stage. The bridging stress of fibers with a longer embedment may not reach the fiber tensile strength in the slipping stage. In equation (19), \( \sigma_{RD} \) and \( \sigma_{RS} \) are the effective fiber bridging stress and slipping rupture stress, respectively.

\[
\sigma_{RD}(\delta) = \begin{cases} 
\frac{V_f}{L_f} \int_{\theta_{(n)}}^{\pi/2} \int_{L_d(\theta)}^{L_c(\theta)} \sigma_{bd}(\delta, L_c) e^{f\theta} \sin2\theta \ d\theta \ dL_c \ + \ \frac{V_f}{L_f} \int_{\theta_{(n)}}^{\pi/2} \int_{L_d(\theta)}^{L_c(\theta)} \sigma_{bp}(\delta, L_c) e^{f\theta} \sin2\theta \ d\theta \ dL_c, & 0 \leq \delta < 2\delta_0, \\
\frac{V_f}{L_f} \int_{\theta_{(n)}}^{\pi/2} \int_{L_d(\theta)}^{L_c(\theta)} \sigma_{bd}(\delta, L_c) e^{f\theta} \sin2\theta \ d\theta \ dL_c, & 2\delta_0 \leq \delta \leq L_d(\frac{\pi}{2}), \\
\frac{V_f}{L_f} \int_{\theta_{(n)}}^{\pi/2} \int_{L_d(\theta)}^{L_c(\theta)} \sigma_{bd}(\delta, L_c) e^{f\theta} \sin2\theta \ d\theta \ dL_c + \ \frac{V_f}{L_f} \int_{\theta_{(n)}}^{\pi/2} \int_{L_d(\theta)}^{L_c(\theta)} \sigma_{bp}(\delta, L_c) e^{f\theta} \sin2\theta \ d\theta \ dL_c, & L_d(\frac{\pi}{2}) < \delta \leq \frac{L_f}{2}.
\end{cases}
\]

Due to the geometrical irregularity of the current slipping rupture zone, it is very complicated to calculate \( \sigma_{RS}(\delta) \). Figure 12 shows the growth process of the slipping rupture zone \( S1 \). The current slipping rupture zone \( S1 \) was represented by a parabolic enclosure subtract zone \( S2 \). \( \sigma_{RS}(\delta) \) is calculated by integrating the current slipping rupture zone.

3.4. \( \sigma(\delta) \) Relationship When considering Fiber Rupture. Through a comprehensive consideration of debonding rupture and slipping rupture, the effective fiber bridging stress \( \sigma_{\text{effective}}(\delta) \) is calculated from equation (20), where \( \sigma_{RD}(\delta) \) and \( \sigma_{RS}(\delta) \) represent the fiber debonding rupture stress and slipping rupture stress, respectively.

\[
\sigma_{\text{effective}}(\delta) = \sigma_c(\delta) - \sigma_{RD}(\delta) - \sigma_{RS}(\delta). \tag{20}
\]

During crack propagation, \( \sigma_{RD}(\delta) \) is calculated by integrating the zone for which the fiber debonding rupture occurs.
Figure 9: Growth of the slipping rupture space during crack propagation.

Figure 10: Interfacial parameter impact analysis to fiber rupture: (a) the effect of $\tau_0$, (b) the effect of $G_d$, (c) the effect of $\beta$. 
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Figure 11: Fiber rupture analysis with different interfacial parameters when \( \delta = 100 \mu m \).

When \( \delta_r = L_r/16 \), \( L_{r}(\delta_r) \) is in close proximity of \( L_p(\theta) \), \( L_{r}(\delta_r) \) is considered to be equal to \( L_p(\theta) \) and \( \sigma_{RS}(\delta) \) can be calculated by (5).

\[
\sigma_{RS}(\delta) = \frac{V_f}{L_f} \int_0^{\theta_s} \sigma_{bp}(\delta, L_c) e^{f \theta} \sin 2\theta \, d\theta \, dL_c + \frac{V_f}{L_f} \int_{\theta_s}^{\theta} \sigma_{bp}(\delta, L_c) e^{f \theta} \sin 2\theta \, d\theta \, dL_c, \\
\frac{V_f}{L_f} \int_{\theta_s}^{\theta} \sigma_{bp}(\delta, L_c) e^{f \theta} \sin 2\theta \, d\theta \, dL_c, \\
\frac{V_f}{L_f} \int_{\theta_s}^{\theta} \sigma_{bp}(\delta, L_c) e^{f \theta} \sin 2\theta \, d\theta \, dL_c, \\
\frac{V_f}{L_f} \int_{\theta_s}^{\theta} \sigma_{bp}(\delta, L_c) e^{f \theta} \sin 2\theta \, d\theta \, dL_c,
\]

(22)
where $\theta_v$ is the inclined angle when $l_{p1}(\delta_v)$ intersects with $l_{p2}(\delta_v)$ and $\theta_s$ is the inclined angle when $l_p(\delta_r)$ intersects with $L_d(\theta)$.

The process to determine $\sigma_{\text{effective}}(\delta)$ when considering fiber rupture is illustrated by the flowchart in Figure 13. Finally, the current fiber bridging model can be built. Figure 14 shows the calculation result based on the whole flowchart. The influence of fiber debonding rupture and slipping rupture on the bridging stress is clearly shown in Figure 14. The current fiber bridging stress, debonding rupture stress, and slipping rupture stress are perfectly determined during crack propagation. The peak bridging stress is 5.0 MPa corresponding to a peak crack opening of 104 $\mu$m. Compared to the result without considering fiber rupture, the predicted value is more reliable.

The predicted critical embedment length $L_e$ and $\sigma(\delta)$ relation will be compared with experimental result in the following section to verify further the accuracy of the current bridging model.

### 4. Model Verification

#### 4.1. Verification of Fiber Rupture Analysis

HDCC matrix composition consists of Type II Portland cement, fly ash, fine sand, and water in a proportion of 0.4 : 0.6 : 0.3 : 0.3. Type II Portland cement conform to the Chinese standard GB175-2007 and Class F fly ash conform to the ASTM C618 standard were used. The river sand used in the experiments had a maximum size and fineness modulus of 0.60 mm and 1.40, respectively. Water reducer and hydroxypropyl methyl cellulose (HPMC) were used to adjust plastic viscosity of the paste. The mixing steps of HDCC paste are as follows: all cementitious materials, fine sand, water reducer, and HPMC were weighed accurately and mixed for 1 min at a speed of 140 rpm. Then, water was added and mixed for 5 min at a speed of 280 rpm.

PVA fibers modified with a mass fraction of 1.2% oil agent were used in this study, and its mechanical and geometrical properties are given in Table 2. The measured average tensile strength of the PVA fibers reached 1260 MPa, as shown in Figure 15(a). To obtain the fiber/matrix interfacial parameters, single-fiber pullout tests were carried out. The fibers were vertically embedded in the matrix, and the embedment length $L_e$ was between 1 mm and 5 mm to determine the critical embedment lengths $L_{d}(\theta)$ and $L_{p}(\theta)$. Single-fiber bridging stress versus pullout displacement curves are shown in Figure 15(b)–15(f). The single-fiber embedment length $L_e$ was set to 1 mm, 2 mm, 3 mm, 4 mm, and 5 mm. Here, the
Interfacial bonding performance was recorded as $L_1, L_2, L_3, L_4$, and $L_5$, respectively.

The fiber rupture phenomenon can be seen in Figure 15 from all pullout curves with $L_e$ from 1 to 5 mm. The rupture strength of PVA fibers only reached 580 to 760 MPa, which was far less than the measured tensile strength. The apparent strength of the PVA fibers dropped drastically even if the fibers were embedded vertically ($\theta = 0$). Figure 16 shows an SEM image of the fiber rupture zone. The fiber pullout zone was seriously abraded. The abrasion effect causes a loss in fiber tensile strength and in the effective diameter. To take into account this effect, the apparent fiber strength was recalculated from equation (22), where $k$ represents the strength reduction coefficient due to abrasion. In this study, $k$ was set to 3.0. Finally, the fiber apparent strength in the matrix was recalculated at 670 MPa.

For fibers with an $L_e$ of 5 mm, the interface parameters could not be exactly obtained because of the fiber ruptured.

**Table 2: Physical properties of the PVA fibers used.**

<table>
<thead>
<tr>
<th>Fiber type</th>
<th>Nominal tensile strength $\sigma_{fu}$ MPa</th>
<th>Modulus $E_f$ GPa</th>
<th>Length $L_f$ mm</th>
<th>Diameter $d_f$ μm</th>
<th>$f$</th>
<th>$f'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVA fiber</td>
<td>1260</td>
<td>30</td>
<td>12</td>
<td>39</td>
<td>0.2</td>
<td>0.33</td>
</tr>
</tbody>
</table>

**Figure 13:** A flowchart for the current bridging model.

**Figure 14:** Fiber bridging stress versus crack opening relation.
before complete debonding. The interface parameters ($G_d$, $\tau_0$, $\beta$) of PVA fiber/matrix were calculated using equations (1)–(4), as shown in Figure 17. The values of $\tau_0$ and $\beta$ slowly decreased when $L_e$ increased from 1 mm to 4 mm. The average values of $G_d$, $\tau_0$, and $\beta$ are given in Table 3.

Figure 18 reveals the critical embedment length $L_d(\theta)$ and $L_p(\theta)$ for different embedment lengths. Due to data fluctuation with the interfacial parameters, $L_d(\theta)$ changed from 3.2 to 5.8 mm when $\theta$ was 0, which meant that the fibers could rupture by debonding rupture when the embedment length is in this range. The experimental results mostly followed the debonding rupture analysis. The most of fibers with an embedment length of 5 mm underwent debonding rupture (Figure 15(f)), and a few fibers underwent debonding rupture when the embedment length was between 3 and 4 mm (Figures 15(d) and 15(e)). In addition, $L_p(\theta)$ changed from 1.2 to 1.9 mm when $\theta$ was 0, which was slightly higher than the experimental result. Figure 15(b) shows that fibers with an embedment length of 1 mm already underwent slipping rupture. The fibers actually undergo

![Figure 17: SEM images of fiber rupture zone. (a) Direct tensile rupture manner. (b) Tensile rupture from matrix manner.](image-url)
slipping rupture more easily, which also revealed how essential it is to include slipping rupture in the analysis to translate the real situation.

4.2. Verification of the $\sigma$-$\delta$ Relationship. Figure 19 shows the relationship between the bridging stress and crack opening for the experimental results and the predicted

![Figure 17: $\tau_0$ and $\beta$ for different embedment lengths.][1]

![Figure 18: Critical embedment length analysis.][2]

<table>
<thead>
<tr>
<th>Series</th>
<th>$\tau_0$ (MPa)</th>
<th>$G_d$ (J/m$^2$)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>2.1 ± 0.12</td>
<td>0.8 ± 0.32</td>
<td>0.21 ± 0.01</td>
</tr>
<tr>
<td>L2</td>
<td>1.2 ± 0.04</td>
<td>0.5 ± 0.10</td>
<td>0.17 ± 0.01</td>
</tr>
<tr>
<td>L3</td>
<td>1.2 ± 0.04</td>
<td>0.2 ± 0.08</td>
<td>0.15 ± 0.02</td>
</tr>
<tr>
<td>L4</td>
<td>1.3 ± 0.08</td>
<td>0.3 ± 0.11</td>
<td>0.17 ± 0.01</td>
</tr>
</tbody>
</table>

Table 3: Fiber/matrix interface parameters for PVA fiber.

![Interface bonding strength $\tau_0$ and Slip-hardening coefficient $\beta$.][3]
Finally, single-fiber pullout tests with different embedded lengths were carried out to validate the modified bridging model. The relationship between the bridging stress and crack opening predicted by the bridging model was consistent with the experimental results.

### Nomenclature

- $P_c$: Peak load of a single-fiber pullout curve (N),
- $\delta_{di}$: Crack opening when the debonding stage is over (m),
- $P_b$: Load after a sudden drop after $P_c$ (N),
- $z$: Centroid distance of fibers from the crack plane (m),
- $E_f$: Fiber modulus of elasticity (Pa),
- $\theta$: Inclined angle,
- $E_m$: Elastic modulus of the matrix (Pa),
- $\theta_{d8}$: Inclined angle when $L_d (\theta)$ intersects with $\delta$,
- $d_f$: Fiber diameter (m),
- $\theta_{p8}$: Inclined angle when $L_p (\theta)$ intersects with $\delta$, $L_p (\theta)$: Fiber length (m),
- $\delta (\delta_r)$: Inclined angle when $l_d (\delta_r)$ intersects with $L_d (\theta)$,
- $V_f$: Fiber volume fraction,
- $\theta_{c1}$: Inclined angle when $l_p (\delta_r)$ intersects with $l_{p2} (\delta_r)$,
- $\theta_c$: Inclined angle when $l_p (\delta_r)$ intersects with $L_d (\theta)$,
- $\sigma_{fa}$: Fiber nominal tensile strength (Pa),
- $\sigma_{b_d}$: Single-fiber bridging stress at debonding stage (Pa),
- $\sigma_{fa}$: Fiber apparent tensile strength (Pa),
- $\sigma_{bp}$: Single-fiber bridging stress at slipping stage (Pa),
- $L_c$: Fiber embedment length (m),
- $\sigma_c$: Fiber bridging stress without consideration of rupture (Pa),
- $\tau_0$: Interfacial friction (Pa),
- $\sigma_R$: Fiber debonding ruptured bridging stress (Pa),
- $G_d$: Chemical bonding force (J/m²),
- $\sigma_{BS}$: Fiber slipping ruptured bridging stress (Pa),
- $\beta$: Fiber slip-hardening coefficient,
- $\sigma_{effective} (\delta)$: Effective fiber bridging stress (Pa),
- $f$: Snubbing coefficient,
- $L_d (\theta)$: Potential critical debonding embedment length (m),
- $f'$: Reduction of the apparent fiber strength,
- $L_p (\theta)$: Potential critical slipping embedment length (m),
- $k$: Fiber tensile strength reduction coefficient due to abrasion,
- $l_d (\delta_r)$: Current debonding ruptured length (m),
- $\delta_r$: Crack opening (m),
- $l_p (\delta_r)$: Current slipping ruptured length (m),
- $\delta_r$: Full debonding crack opening (m),
- $l_{p1} (\delta_r)$: The larger $l_p (\delta_r)$ (m),
- $\delta_{cr}$: Crack opening when fibers initiate debonding rupture (m),
- $l_{p2} (\delta_r)$: The smaller $l_p (\delta_r)$ (m).
Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest
There are no conflicts of interest.

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References