The newly developed two-component polymer (TFPU) cut-off walls have great application potential in civil engineering for water-seepage prevention. Investigating the mechanical interactions between TFPU and soil from the theoretical perspective can provide a basis for furtherly evaluating the stability of TFPU cut-off walls, which, however, has not been conducted. In this study, based on the shear testing results, the shear damage model of the TFPU-bentonite contact surface was established. Studies show that the TFPU-bentonite contact surface performs strain-softening behavior under shear and that the strain-softening behavior becomes less and less obvious when the normal stress increases. The theoretical shear stress-strain curves are in good consistent with the experimental ones, and the maximum differences between the theoretical shear stress and shear strain at yield and the experimental ones are about 3.88% and 3.40%, respectively, indicating that the shear damage model can reflect the mechanical properties of the TFPU-bentonite contact surface. The critical damage of the contact surface increases from 0.05 to 0.50 in the power function way when normal stress increases from 75 to 1200 kPa, implying that the shear failure of the contact surfaces changes from brittleness to ductility. This study provides a theoretical base for evaluating the influences of the TFPU-soil contact surface on the stability of the TFPU cut-off wall structures.

1. Introduction

In geotechnical engineering, water seepage has become the main cause of the various engineering accidents [1–5]. The cut-off walls are the most commonly adopted antiseepage measurement [6–8]. The polymer cut-off wall technique is a newly proposed method that uses the two-component foaming polyurethane as an alternative to the traditional concretes to construct cut-off walls [9–11]. As shown in Figure 1, the two-component polymer (TFPU) cut-off wall is built in the soil using the grouting technology [9, 10]. By virtue of the features of light-weight, low-stiffness, fast-construction, easy construction, good durability, strong permeability resistance, and environmental protection [12, 13], TFPU cut-off walls have found their applications in enhancing the permeability resistance of soil dams [9, 14, 15] and underground ancient grave [16]. Moreover, TFPU cut-off walls also hold a great application potential in the landfills and underground storage where seepage prevention is required [9, 15, 17, 18].

In actual engineering, except for playing the role of water barriers, the constructed TFPU cut-off walls sustain the external load [9]. Previous studies on the mechanical behaviors of TFPU cut-off walls used in soil dams have claimed that, under the action of the horizontal hydrostatic pressure, the TFPU cut-off walls will deform along with the deformation of the soil masses [19–21]. Studies have claimed that the deformation of the TFPU cut-off walls is mainly the flexural and stretching deformation [19, 20]. TFPU belongs to the category of high polymer, and its strength is mainly due to the entanglement between adjacent molecular chains [12, 13], but soil is a collection of solid particles, between which the Van der Waals force contributes to the cohesion strength [22]. The mechanical properties of TFPU are often...
different from the soil [13, 22–24]. Considering that TFPU cut-off walls are directly contacting with the soil, the bending and stretching deformation of TFPU cut-off walls will lead to the slip failure of the contact surface [11, 19, 25]. With the development of the TFPU cut-off wall technique, the contact surface between the TFPU matrices and soil becomes more and more in civil engineering. The investigation on the mechanical interactions of the TFPU-soil contact surface is of great engineering significance [9]. In our previous study [26], the shear strength of the TFPU-bentonite contact surface has been experimentally measured. The study was focused on the effects of the moisture content of bentonite and the density of TFPU on the shear strength of the TFPU-bentonite contact surface. Establishing a mechanical model that can reflect the shearing mechanical behavior of the TFPU-bentonite contact surface can provide a base for the further revealing of the mechanism of the coordination deformation between the cut-off wall structures and soil [20, 27–29], which, however, has not been carried out yet.

The damage mechanics theory has been widely used to describe the mechanical laws of heterogeneous materials such as concrete [30, 31], rock [32–34], soil [35], and polymers [13] in the failure process under external load. It has been proved that the theoretical models established based on the damage mechanics theory can also reflect the strength and deformation characteristics of the soil-rock or soil-concrete contact surfaces under shear [28, 29, 36]. Here, the primary experimental procedure and results of our previously work on the shear strength of the TFPU-bentonite contact surface are presented. Then, based on the experimental results, by combining the damage mechanics theory and the Weibull damage statistical equation, the shear damage model of the TFPU-bentonite contact surface was established and verified. The influences of normal stress on the damage evolution and the critical damage of the contact surface during the shearing process were analyzed. The shear damage model proposed here can describe the softening characteristic and stress state of the TFPU-bentonite contact surface during shear. This study provides a theoretical base for evaluating the effects of the TFPU-soil contact surface on the mechanical behavior of the TFPU cut-off wall structures in the soil.

2. Materials and Methods

The calcium-based bentonite and TFPU grouting material were used to prepare the tested samples. The bentonite is with a density of 1.61 g/cm³, the montmorillonite content of 75%, and the moisture content of 22%. The bentonite was compacted layer by layer by impacting and each layer was impacted 30 times with a heavy load to a height half the mold size, followed by grouting with TFPU material to fill the remaining spaces [26]. TFPU is the polymerization reaction product of isocyanate and polyols at a mass ratio of 1:1 [15, 26]. The isocyanate and polyols are liquids, and they were firstly atomized into microdrops with the significantly increased specific surface area using the high-pressure nitrogen flow (8.0 MPa), and then the two droplets were simultaneously injected into the closed molds [12, 13, 26]. The polymerization reaction between the raw materials occurs so fast that they cannot be fully mixed with the plain mechanical stirring method [37]. Through the atomization treatment, the well-mixed materials could be obtained [38]. Compared with the compaction load, the expansion force of the TFPU is small [39]; therefore, the grouting process of TFPU brings little influence on the pore structure and moisture content of the bentonite. More details about the fabrication and curing procedures of TFPU-bentonite samples are included in Reference [26].

The prepared samples were cubic with a side length of 100 mm. The moisture content of the bentonite here was the optimum (22%), and the density of the TFPU was 0.488 g/cm³, the commonly designed density in actual engineering [9, 26]. The density of TFPU was controlled by fixing the injection time at 30 s based on the previous experiences of grouting experiment [12]. Studies on the mechanical properties of the two materials show that bentonite was with the cohesion force of about 123 kPa and friction angle of 19°, respectively, and that the shear strength of the TFPU can be about 4 times that of bentonite under the same normal stress [26].

The direct shear test was conducted on the TFPU-bentonite samples to obtain information about the shear strength of the contact surfaces. Before shearing, the designed normal stress (σ_N) was applied to the samples and it was maintained as constant during shearing. Four σ_N (75, 300, 600, and 1200 kPa) were adopted. The shearing of the contact surfaces was achieved by applying the horizontal displacement to the bentonite part at a rate of 1 mm/min for simulating the static shear load [40]. A DH5902 N data acquisition system was used to record real-time horizontal stress and displacement.

3. Results and Discussion

3.1. Experimental Results. Previous studies on the shear strength of the soil-concrete or soil-rock contact surfaces have pointed out that the shear failure mainly happens in the low-strength medium near the contact surfaces, and the thickness of the shear failure zone is usually about 1/100–1/10 of the contact surface length [41, 42]. Here, the thickness of the shear failure zone was determined as 10 mm, about 1/10 of the length of the contact surfaces, which has also been confirmed by the digital photographic test results [26]. According to this assumption and the shear displacement, γ, the shear strain, can be obtained and the shear stress-
shear strain (τ-γ) curves can be drawn (as shown in Figure 2).

From Figure 2, it can be found that τ increases almost linearly with the increasing γ in Stage I and that the increasing rate of τ declines in Stage II and the plastic deformation occurs. The plastic deformation is caused by the gradual accumulation of the damage on the contact surface [30, 32]. When the accumulated damage reaches its critical value, τ reaches the peak value (the yield point), and after that, with the further increasing γ, τ declines gradually (Stage III), implying the strain-softening phenomenon [29]. Finally, the plateaus can be achieved on the τ-γ curves (Stage IV) and τ₀, and the almost unchanged shear stress in Stage IV is known as the residual strength. Figure 2 also presents that in both G₀, the slope of the τ-γ curves in Stage I, τᵣ, and γᵣ (the shear stress and shear strain at the yield point), and τ₀ increases when σN increases and that the strain-softening behavior becomes less significant with the increasing σN. G₀ represents the shear modulus of the TPU-bentonite contact surfaces.

The specific values of parameters G₀, τᵣ, γᵣ, and τ₀ obtained from Figure 2 are listed in Table 1. The variations of τᵣ with σN are often linearly fitted, and then the cohesion force (c) and friction angle (φ) of the contact surfaces can be obtained according to the More–Coulomb theory [43]. The calculated c and φ are 174.2 kPa and 40.8°, respectively [26].

Figure 3 shows the variation laws of G₀ and τ₀ with σN, from which one can see that the change of G₀ with σN confirms the form of the power function. In Figure 3(a), the parameter P₀ is introduced to make the expression independent of the choice of the units [44]. The parameter P₀ is commonly seen when describing the variation laws of modulus using the mathematical equations and its value is often determined as 101 kPa (1 atm) following the usual suggestion [44]. This power function can simultaneously reflect the effects of both the material properties of the contact surface and the normal stress. Figure 3(b) shows that τ₀ varies linearly with σN. This is because the residual strength is mainly provided by interfacial friction that is linearly dependent on normal pressure [45, 46]. The two functions are with a high goodness of fit (with the coefficient of determination of higher than 0.95), indicating that they can describe the variation laws of G₀ and τ₀ with σN well.

### 3.2. The Damage Model of the TPU-Bentonite Contact Surface

From the macroscopic perspective, the shear failure is caused by the failure of a thin layer of soil near the contact surface [28, 41, 42]. Figure 4 shows the schematic diagram of the failure process of the shear zone [29], where A’ and A” represent the intact area and the damaged area, respectively. The damage of the shear zone, D, can be calculated as the following:

\[
D = \frac{A''}{A' + A''}. \tag{1}
\]

Due to the existence of normal stress during shearing, the damaged area can bear shear stress through friction as well [45]. Assuming that the shear stress on the intact area and the damaged area is τ’ and τ”, respectively, τ, the total shear stress on the shear fracturing plane can be calculated by

\[
\tau = \tau' \frac{A''}{A' + A''} + \tau'' \frac{A'}{A' + A''} \tag{2}
\]

Substituting (1) into (2) gives the following equation:

\[
\tau = \tau' (1 - D) + \tau'' D. \tag{3}
\]

The variation of the shear stress on the intact area with the shear strain can be considered linearly. Therefore, τ’ can be calculated as the following:

\[
\tau' = G' \cdot \gamma, \tag{4}
\]

where G' represents the initial shear modulus of the materials in the intact area. The τ-γ curves presented in Figure 2 show that in Stage I, the shear modulus of the samples is almost the constant, indicating that there exists little damage [30]. Therefore, G’ can be represented by G₀.

Based on equations (3) and (4), the following equation can be obtained as follows:

\[
\tau = G_0 \cdot (1 - D) \cdot \gamma + \tau'' \cdot D. \tag{5}
\]

According to the statistical strength theory [29, 47–49], the distribution of D of the shear zone can be written as the Weibull formula:

\[
D = 1 - \exp \left[ \left( \frac{\gamma}{\gamma_m} \right)^m \right], \tag{6}
\]

where m and n are the Weibull parameters.
Substituting (6) into (5) gives the shear damage function of the contact surface between TFPU and bentonite:

$$\tau = (G_0 \cdot \gamma - \tau^*) \cdot \exp \left[ \left( \frac{\gamma}{m} \right)^{m} \right] + \tau^{''}. \quad (7)$$

According to (7), the specific form of the damage function of the TFPU-bentonite contact surfaces can be obtained when the specific values of parameters $G_0$, $\tau^*$, $m$, and $n$ are provided. Equation (3) shows that $\tau$ equals $\tau^*$ when $D$ reaches to 1.0, and Figure 2 shows that $\tau$ equals $\tau_0$ when the interfaces are completely failed ($D = 1.0$). Therefore, $\tau^*$ can be represented as the following:

$$\tau^* = \tau_0. \quad (8)$$

The $\tau$-$\gamma$ curves also tell us that the nominal shear stress reaches its maximum at the yield point. Therefore, equation (7) gives the maximum value of $\tau$ when $\gamma$ equals $\gamma_Y$, i.e., the following equation holds:

$$\frac{d\tau}{dy}_{y=\gamma_Y} = 0. \quad (9)$$

By solving (9), the following equation can be obtained as follows:

$$\frac{G_0 \cdot n}{(G_0 \cdot \gamma_Y - \tau_0) \cdot m} = \left( \frac{\gamma_Y}{n} \right)^{m-1}. \quad (10)$$

Substituting (10) into (7) gives the following equation:

$$\ln \left( \frac{\tau - \tau_0}{G_0 \cdot \gamma - \tau_0} \right) = \frac{G_0 \cdot \gamma_Y}{(G_0 \cdot \gamma_Y - \tau_0) \cdot m} \cdot \frac{1}{m} = n. \quad (11)$$

It can be seen that the specific value of parameter $m$ can be obtained by substituting $G_0$, $\tau_0$, $\gamma_Y$, and $\tau_0$ into (11), and then substituting $m$ and other related parameters into (12) can obtain the specific value of parameter $n$. Table 2 gives the calculated values of parameters $m$ and $n$, and their variation laws with $\gamma$ are shown in Figure 5.

Figure 5 shows that with the increasing $\sigma_N$, the Weibull parameter $m$ declines, whereas the Weibull parameter $n$ increases gradually. The variation laws of the Weibull parameters with $\sigma_N$ conform to the form of the power function. The coefficients of determination $R^2$ of the functions presented in Figure 5 are higher than 0.95, indicating the enough-high goodness of fit. Substituting the functions presented in Figure 3, Figure 5, equation (8) into equation (7) gives the specific form of the shear softening damage function that contains two independent variables: $\sigma_N$ and $\gamma$, i.e., $\tau = f(\sigma_N, \gamma)$.

3.3 Verification of the Damage Model. The damage equation established in Section 3.2 can theoretically describe the variations of $\tau$ with $\gamma$ during shearing. But it is necessary to evaluate its reasonableness. Figure 6 shows the comparison between the experimental and theoretical $\tau$-$\gamma$ curves of TFPU-bentonite contact surfaces under different normal stress, and the probability distribution of differences in the experimental and theoretical data is presented in Figure S1.
Table 2: The calculated values of the Weibull parameters $m$ and $n$.

<table>
<thead>
<tr>
<th>$\sigma_N$ (kPa)</th>
<th>75</th>
<th>300</th>
<th>600</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>31.545</td>
<td>8.238</td>
<td>4.801</td>
<td>4.226</td>
</tr>
<tr>
<td>$n$</td>
<td>0.269</td>
<td>0.356</td>
<td>0.401</td>
<td>0.454</td>
</tr>
</tbody>
</table>

$R^2 = 0.951$ \quad m = 22.16 \left( \frac{\sigma_N}{p_a} \right)^{-0.757}$

Figure 5: Variations of the Weibull parameters ($m$) and ($n$) with $\sigma_N$. ($p_a$ and $R^2$ represent the reference pressure and the coefficient of determination, respectively).

Figure 6: Comparison between the experimental and theoretical $\tau$-$\gamma$ curves of the TFPU-bentonite contact surface under different $\sigma_N$. 

(a) $\sigma_N = 75$ kPa

(b) $\sigma_N = 300$ kPa

(c) $\sigma_N = 600$ kPa

(d) $\sigma_N = 1200$ kPa
The theoretical \( \tau \) and \( \gamma \) at the yield point are calculated and they are listed in Table 3.

Table 3: The theoretical shear stress and shear strain of the contact surfaces at the yield point.

<table>
<thead>
<tr>
<th>( \sigma_N ) (kPa)</th>
<th>75</th>
<th>300</th>
<th>600</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_y ) (kPa)</td>
<td>240.31</td>
<td>431.11</td>
<td>754.26</td>
<td>1348.56</td>
</tr>
<tr>
<td>( \gamma_y )</td>
<td>0.241</td>
<td>0.301</td>
<td>0.341</td>
<td>0.423</td>
</tr>
</tbody>
</table>

Figure 6 tells us that even though these exist some differences, the theoretical \( \tau - \gamma \) curves can reflect the primary variation characteristics of \( \tau \) with \( \gamma \). Both the linear growth state, yield stage, strain-softening behavior, and the residual strength can be seen on the theoretical \( \tau - \gamma \) curves. It can also be found from Figure 6 and Figure S1 that with the increasing \( \sigma_N \), the coincidence degree of the theoretical and experimental curves becomes higher and higher and that the two curves almost overlap at \( \sigma_N \) of 1200 kPa. Figure S1 shows that the relative difference between over 80% experimental and theoretical \( \tau \) is lower than 30%. Moreover, compared with the experimental data (Table 1), the maximum relative errors of \( \tau_y \) and \( \gamma_y \) are around 3.88% and 3.40% (at \( \sigma_N \) of 600 kPa), respectively. These differences are acceptable from the actual engineering perspective [50]. Therefore, the shear damage function proposed here can be used to describe the variation of shear stress on the contact surfaces between the TFPU matrix and bentonite.

Figure 7 presents the evolution process of damage during shearing and the variations of the critical damage (\( D_l \)) with \( \sigma_N \). \( D_l \) is derived from equation (7), and it represents the damage when the shear stress of the contact surface reaches the yield strength. The variation curves of \( D \) are drawn based on equation (6), and \( D_l \) was calculated by (13). From Figure 7, one can see that \( D \) remains zero until \( \gamma \) reaches a certain value, after which, it increases with the increasing \( \gamma \) until up to 1. This observation corresponds well with the characteristics of the experimentally obtained \( \tau - \gamma \) curves (Figure 2). When \( \sigma_N \) increases from 75 to 1200 kPa, the rate at which \( D \) increases with the increasing \( \gamma \), whereas \( D_l \) of the TFPU-bentonite contact surface changes from brittleness to ductility. Here, bentonite was used because it is commonly seen in the TFPU cut-off wall engineering [26]. The mechanical properties between bentonite and other types of soil, such as the sand soil, are different [51]; therefore, the shear damage model established here may not be applicable to the contact surfaces between TFPU and other types of soil, but this study highlights the idea of proposing theoretical damage model for the commonly seen contact surfaces.

\[
D_l = \frac{G_0 \cdot \gamma - \tau_y}{G_0 \cdot \gamma - \tau_0}. \quad (13)
\]

4. Conclusions

In this study, the shear strength of the TFPU-bentonite contact surface under different normal stress was experimentally measured and the corresponding shear damage model was established and verified. The following conclusions are drawn:

(1) Under shear, the stress-strain curves of the TFPU-bentonite contact surface contain the linear increasing segment, the yield segment, the strain-softening segment, and the residual strength segment. With the increasing normal stress, the shear...
modulus and residual strength of the contact surface increase in the power function and linear function way, respectively, whereas the strain-softening phenomenon is less and less obvious.

(2) The theoretical shear stress-strain curves have high goodness of fit with those obtained through the direct shear test, and the maximum differences in the theoretical and experimental yield shear stress and yield shear strain are separately about 3.88% and 3.40%. The established shear softening damage model can reflect the strain-softening characteristic and the stress state of the TFPU-bentonite contact surface.

(3) The theoretical variation laws of $D$ versus $\gamma$ correspond well with the characteristics of the experiment $r$-$\gamma$ curves. With the increased normal stress on the TFPU-bentonite contact surface, the rate at which $D$ increases with the increasing $\gamma$ declines, whereas the critical damage increases in the power function way (within $0.05 \sim 0.50$) and the shear failure of the TFPU-bentonite contact surfaces changes from brittleness to ductility.

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors are grateful for the financial support from the National Natural Science Foundation of China (Grant no. 51908515).

Supplementary Materials

Figure S1 shows the probability distribution of differences in the experimental and theoretical $t$-$\gamma$ curves of TFPU-bentonite contact surfaces under different normal stress. In Figure S1, $R_a$ represents the relative error between the experimental and theoretical data of $r$ at different $\gamma$, and $P_d$ and $C_p$ represent the probability density and the cumulative probability density of $R_a$ (Supplementary Materials).

References


