# Determination of Constitutive Parameters of Crystalline Limestone Based on Improved RHT Model 

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#### Abstract

This study improves the RHT constitutive model based on some shortcomings, and the main improvements work are as follows: (1) hyperbolic function is used to make up for the deficiency of tensile strain rate enhancement factor of RHT constitutive model at high strain rate; (2) considering the constant change of hydrostatic pressure, the initial damage value of rock is introduced into the constitutive model, and the improved damage accumulation expression is derived based on this value. (3) In the equation of residual failure surface, the influence of the Lode angle factor is considered, the RHT constitutive parameters of crystalline limestone are determined by the SHPB experiment, and the difference in rock failure forms before and after the correction of constitutive parameters is simulated by ANSYS/LS-DYNA software. The results show that the improved constitutive parameters can effectively represent the yield strength surface and residual strength surface of the rock, which verifies the rationality of the improved RHT constitutive model, and also shows that it is an effective and simple method to obtain RHT constitutive parameters by SHPB experiment.


## 1. Introduction

Compared with the HJC [1] and K \& C [2] model, the RHT model [3] fully considers the compression effect, strain rate effect, and damage accumulation on the failure intensity of the rock under explosive impact. The elastic limit surface equation, failure surface equation, and residual strength surface equation related to pressure are embedded in this model, which is used to describe the variation law of yield strength, failure strength, and residual strength of concrete and brittle materials under high strain rate. With the rapid development of numerical simulation technology, the model is widely used in the numerical simulation of high strain rates such as explosion impact. When Tu, Hansson, and Skoglund [4-8] et al. applied the RHT model, they found that the model was incomplete for tensile damage description, so the model was modified for tensile and high strain rate effects. Zhang ruoqi and Pavlovic A [9-14] et al. adjusted and improved the failure surface equation and residual surface
equation in RHT constitutive parameters based on concrete experiments and verified them by AUTODYN element simulation, all of which achieved good results. A total of 34 constitutive parameters of the RHT model need to be determined. Most scholars can only refer to Riedel's [3, 15-17] research results because of its complexity when applying the RHT model. Li [18] used marble as an example to explain in detail the method for determining the constitutive parameters of RHT. Xie [19, 20] and Wang [21-23] et al. studied the constitutive parameters of rock under the action of high ground stress and successfully applied them to the numerical simulation analysis of rock crack propagation, cut hole blasting, and cyclic blasting. For the convenience of parameter determination, Xu and Ye [24], Liu et al. [25], and Li et al. [26-28] used an orthogonal experimental method and optimized Latin hypercube design method to analyze the sensitivity of the parameters. The analysis results show that the strength parameters of failure surface and residual surface have a sensitive influence on the fracture
morphology and damage accumulation of rock. Therefore, some parameters that have little influence on rock failure can be neglected when determining rock constitutive parameters, which makes the RHT model more convenient to use.

The study of the above scholars for the RHT constitutive model mainly includes the following: the modification of the constitutive equation, the sensitivity analysis of the model parameters, and the method of determining the constitutive parameters. In the improvement of the constitutive equation, most scholars use the linear interpolation method for the residual strength of the rock in the damage softening stage, while the initial damage of rock has not been considered. In fact, for rocks in the karst area, due to the development of karst phenomena, the rock has certain initial damage, which will reduce the strength of the rock. In addition, when rock is subjected to dynamic loads such as explosion, vibration, and impact, the hydrostatic pressure on its failure surface often changes with the change of load, so it is necessary to update the expression of damage accumulation, considering that many complex experiments are usually needed in the process of determining RHT constitutive parameters. Based on this, the residual surface equation of the RHT constitutive model is improved, and the
constitutive parameters of crystalline limestone are determined by the Hopkinson bar impact test.

## 2. RHT Constitutive Model

2.1. Failure Surface Equation. In the RHT model, the equivalent stress intensity $\sigma_{\text {eq }}^{*}$ of the failure surface is a function of normalized pressure $p^{*}=p / f_{c}$, Lode angle $\theta$, and strain rate $\dot{\varepsilon}$.

$$
\begin{equation*}
\sigma_{\mathrm{eq}}^{*}(p, \theta, \dot{\varepsilon})=Y_{\mathrm{TXC}}^{*}(p) R_{3}(\theta) F_{\mathrm{rate}}(\dot{\varepsilon}) \tag{1}
\end{equation*}
$$

In formula (1), the * in the upper right corner of the formula indicates the normalized uniaxial compressive strength $f_{c}$ (unit: MPa) of the material, and $Y_{\text {TXC }}^{*}(p)$ is the compression meridian strength.

$$
\begin{equation*}
Y_{\mathrm{TXC}}\left(p^{*}\right)=A\left[p^{*}-p_{\text {spall }}^{*} F_{\text {rate }}(\dot{\varepsilon})\right]^{N} \tag{2}
\end{equation*}
$$

In formula (2), $A$ is the failure surface constant; $N$ is the failure surface index; and $p_{\text {spall }}^{*}$ is the normalized layer crack strength.
$F_{\text {rate }}(\dot{\varepsilon})$ is the strain rate dynamic enhancement factor:

$$
F_{\text {rate }}(\dot{\varepsilon})= \begin{cases}\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{0}^{c}}\right)^{\alpha}, & p \geq \frac{f_{c}}{3},  \tag{3}\\ \frac{p+f_{t} / 3}{f_{c} / 3+f_{t} / 3}\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{0}^{c}}\right)^{\alpha}+\frac{p-f_{c} / 3}{-f_{t} / 3-f_{c} / 3}\left(\frac{\dot{\varepsilon}}{\bar{\varepsilon}_{0}^{c}}\right)^{\delta}, & -\frac{f_{t}}{3}<p<\frac{f_{c}}{3}, \\ \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{0}^{t}}\right)^{\delta} . & p \leq-\frac{f_{t}}{3}\end{cases}
$$

In formula (3), $f_{t}$ is uniaxial tensile strength (unit: $\mathrm{MPa}) ; \quad \dot{\varepsilon}_{0}^{c}=30 \times 10^{-6} \mathrm{~s}^{-1} ; \quad \dot{\varepsilon}_{0}^{t}=3 \times 10^{-6} \mathrm{~s}^{-1} ; \quad \dot{\varepsilon}$ is the current strain rate; $\alpha=4 / 20+3 f_{c}$ is a compression strain rate index; and $\delta=2 / 20+f_{c}$ is the tensile strain rate index.
$R_{3}(\theta)$ is the Lode angle factor: it is a function of the Lode angle $\theta$ and the tension-compression meridian ratio $Q$.

$$
\begin{equation*}
R_{3}(\theta)=\frac{2\left(1-Q^{2}\right) \cos \theta+(2 Q-1) \sqrt{4\left(1-Q^{2}\right) \cos ^{2} \theta+5 Q^{2}-4 Q}}{4\left(1-Q^{2}\right) \cos ^{2} \theta+(2 Q-1)^{2}} \tag{4}
\end{equation*}
$$

In formula (4), $\theta=\cos ^{-1}\left(3 \sqrt{3} / 2 J_{3} / J_{2}^{3 / 2}\right) / 3 \quad \theta \in[0$, $\pi / 3] ; J_{2}$ and $J_{3}$ are deviatoric stress invariants; and $Q$ is the meridian ratio of tension and compression, and in order to consider the effect of pressure on strength, it is expressed as follows:

$$
\begin{equation*}
Q=Q\left(p^{*}\right)=Q_{0}+B p^{*} 0.51 \leq Q \leq 1 \tag{5}
\end{equation*}
$$

In formula (5), $Q_{0}$ is the initial meridian ratio of tension and compression, and $B$ is the material constant.
2.2. Elastic Ultimate Surface Equation. In the RHT model, the elastic limit surface equation is derived from the failure surface equation:

$$
\begin{equation*}
\sigma_{\mathrm{el}}^{*}(p, \theta, \dot{\varepsilon})=\sigma_{\text {eq }}^{*} F_{\text {elastic }} F_{\text {cap }}\left(p^{*}\right) \tag{6}
\end{equation*}
$$

In formula (6), $F_{\text {elastic }}$ is the elastic scaling function and $F_{\text {cap }}\left(p^{*}\right)$ is the cap function, which can effectively solve the problem that materials are always in the elastic state without
yielding under high hydrostatic pressure. The expressions for $F_{\text {elastic }}, F_{\text {cap }}\left(p^{*}\right)$ are as follows:

$$
F_{\text {elastic }}= \begin{cases}g_{c}^{*}, & p \geq \frac{f_{c, e l}}{3}  \tag{7}\\ \frac{p+f_{t, e l} / 3}{f_{c, e l} / 3+f_{t, e l} / 3} g_{c}^{*}+\frac{p-f_{c, e l} / 3}{-f_{t, e l} / 3-f_{c, e l} / 3} g_{t}^{*}, & \frac{-f_{t, e l}}{3}<p<\frac{f_{c, e l}}{3}, \\ g_{t}^{*}, & p \leq-\frac{f_{t, e l}}{3}\end{cases}
$$

In equation (7), $f_{t, e l}$ is the uniaxial tensile elastic ultimate stress, $f_{c, e l}$ is the uniaxial compressive elastic ultimate stress, $g_{c}^{*}=f_{c, e l} / f_{c}$ is the compressive yield surface parameter, and $g_{t}^{*}=f_{t, e l} / f_{t}$ is the tensile yield surface parameter.

$$
F_{\text {cap }}\left(p^{*}\right)= \begin{cases}1, & \left(p^{*} \leqslant \frac{1}{3}\right),  \tag{8}\\ \sqrt{1-\left(\frac{p^{*}-1 / 3}{p_{0}^{*}-1 / 3}\right)^{2}}, & \left(\frac{1}{3}<p^{*}<p_{0}^{*}\right), \\ 0, & \left(p \geqslant p_{0}^{*}\right) .\end{cases}
$$

In equation (8), $p_{0}$ is the pressure at which the void material begins to crush.
2.3. Residual Strength Surface Equation. When the equivalent stress strength of the material is greater than the failure stress strength, the damage of the material begins to accumulate and enters the damage softening phase. Damage variable $D$ is the ratio of cumulative equivalent plastic strain increment to final failure equivalent plastic strain. The plastic strain of the material at ultimate failure is as follows:
$\varepsilon_{p}^{\mathrm{f}}= \begin{cases}D_{1}\left[p^{*}-(1-D) p_{\mathrm{t}}^{*}\right]^{D_{2}} & p^{*} \geqslant(1-D) p_{\mathrm{t}}^{*}+\left(\frac{\varepsilon_{\mathrm{p}}^{m}}{D_{1}}\right)^{1 / D_{2}} \\ \varepsilon_{\mathrm{p}}^{m} & p^{*}<(1-D) p_{\mathrm{t}}^{*}+\left(\frac{\varepsilon_{\mathrm{p}}^{m}}{D_{1}}\right)^{1 / D_{2}},\end{cases}$

In formula (9), $\varepsilon_{\mathrm{p}}^{m}$ is the minimum plastic strain at the time of material damage, and $p_{\mathrm{t}}^{*}$ is the destruction of the cutoff pressure. $D_{1}$ and $D_{2}$ are material parameters. In fact, when the pressure of the material exceeds the yield stress, the friction and confining pressure still exist in the broken part of the material, which makes it have the ability to resist shear. Therefore, the residual strength surface equation is introduced into the RHT model.

$$
\begin{equation*}
\sigma_{r}^{*}\left(p^{*}\right)=A_{f}\left(p^{*}\right)^{n_{f}} \tag{10}
\end{equation*}
$$

In formula (10), $A_{f}$ is the residual stress intensity parameter and $n_{f}$ is the residual stress intensity index. The equivalent stresses between the failure strength surface and the residual strength surface are obtained by linear interpolation.

## 3. Improvement of the RHT Model

3.1. Correction of Strain Rate Enhancement Factor. In the RHT model, the dynamic strain rate enhancement factor (DIF) is sensitive to the high strain rate region of materials in tensile, which is different from most impact tests. Qi and Qian [29] research suggests the following: the strength of the material is not always infinitely enhanced with the increase in strain rate, when the strain rate is in the high strain rate region, and if the strain rate continues to increase, the strength of the material will slowly increase. Malvar and Ross [30] proposed a modified CEB model, which is consistent with the results of most experiments. However, the model still cannot show the characteristics of the explosion impact problem under a high strain rate. Based on this, this study uses the hyperbolic function (tanh) proposed by Gebbeken and Greulich [31] to describe the tensile strain rate enhancement factor of rock materials under a high strain rate. This function divides the strength change of rock into low strain rate region, medium strain rate region, and high strain rate region. As shown in Figure 1, the modified formula $D I F_{t}$ is as follows:
$F_{t}(\dot{\varepsilon})=\frac{f_{t d}}{f_{t}}=\left\{\left[\tanh \left(\left(\lg \left(\dot{\varepsilon} / \dot{\varepsilon}_{0}\right)-W_{x}\right) S\right)\right]\left[\frac{F_{m}}{W_{y}}-1\right]+1\right\} W_{y}$.

In formula (11), $\dot{\varepsilon}_{0}$ is the reference strain rate. According to the data of Tedesco and Ross [32] experiments, the fitting formula (11) gives $F_{m}=10, W_{x}=1.6, S=0.8$, and $W_{y}=5.5$
3.2. Model Modification Considering Initial Damage. In the RHT model, the initial damage value of brittle materials such as rock is 0 by default, but in practical engineering, the rock will be affected by different degrees of excavation disturbance, weathering, and groundwater dissolution, which leads to the initial damage value of rock not being zero.


Figure 1: Law of rock material with tensile strain rate changes.

Therefore, it is necessary to introduce the initial damage value $D_{0}$ to modify the damage accumulation expression of the RHT model, and the improved damage value $D$ is as follows:

$$
\begin{equation*}
0 \leq D=D_{0}+\sum \frac{\Delta \varepsilon_{p}}{\varepsilon_{p}^{\mathrm{f}}}=\int_{0}^{\varepsilon_{p}} \frac{1}{\varepsilon_{p}^{\mathrm{f}}} d \varepsilon_{p} \leq 1 \tag{12}
\end{equation*}
$$

In formula (12), $\Delta \varepsilon_{p}$ is equivalent plastic strain increment, $D_{0}=1-{\overline{V_{c}}}_{c}^{2} / V_{c}^{2}$ is the initial injury of the rock mass, ${\overline{V_{c}}}_{c}^{2}$ is the wave velocity of the rock mass, and $V_{c}^{2}$ is the wave velocity of the rock test piece.
3.3. Correction of Residual Strength Surfaces. The Lode angle factor is considered in the failure surface equation, which can transform the compression meridian into the tension meridian. Drawing on this idea, the rudder angle factor can be taken into account in the calculation formula of residual stress. In the RHT model, the equivalent stress strength between the failure stress surface and the residual stress surface is obtained by linear interpolation, which is inconsistent with the actual stress state in the rock mass, because the hydrostatic pressure in this interval is usually constantly changed. Therefore, the normalized hydrostatic pressure in equation (10) can be corrected to the normalized hydrostatic pressure $p_{p r e}^{*}$ corresponding to the current stress state surface, and finally, the corrected residual surface strength can be obtained as follows:

$$
\begin{equation*}
\sigma_{r-p r e}^{*}\left(p^{*}\right)=A_{f}\left(p_{p r e}^{*}\right)^{n_{f}} R_{3}(\theta) \tag{13}
\end{equation*}
$$

When the strain exceeds the strain corresponding to the yield stress, it can be assumed that there is a linear relationship between the strain increment and the plastic strain increment.

$$
\begin{equation*}
\Delta \varepsilon_{P}=\Delta \varepsilon \cdot\left(\sum \frac{\Delta \varepsilon}{D_{1}\left[p_{p r e}^{*}-(1-D) p_{\mathrm{t}}^{*}\right]^{D_{2}}}\right)^{-1} \tag{14}
\end{equation*}
$$

From formula (14), it can be inferred that

$$
\begin{equation*}
D_{\text {pre }}=\int_{0}^{\varepsilon_{\text {pre }}-\varepsilon_{\text {fail }}} \frac{\mathrm{d} \varepsilon}{D_{1}\left[p_{\text {pre }}^{*}-(1-D) p_{\mathrm{t}}^{*}\right]^{D_{2}}} \times\left(\int_{0}^{\varepsilon_{\max }-\varepsilon_{\text {fail }}} \frac{\mathrm{d} \varepsilon}{D_{1}\left[p_{p r e}^{*}-(1-D) p_{\mathrm{t}}^{*}\right]^{D_{2}}}\right)^{-1} \tag{15}
\end{equation*}
$$

In equation (15), $\varepsilon_{\text {fail }}$ is the strain corresponding to the residual strength and $\varepsilon_{\text {max }}$ is the strain corresponding to the failure strength when a brittle material such as rock loses its load-bearing capacity, and the new equivalent stress intensity between the failure stress surface and the residual stress surface can be obtained by linear interpolation:

$$
\begin{equation*}
\sigma_{\text {damage }}=\left(1-D_{\mathrm{pre}}\right) \sigma_{\mathrm{fail}}+D_{\mathrm{pre}} \sigma_{r-p r e}^{*} \tag{16}
\end{equation*}
$$

## 4. Crystalline Limestone RHT Parameter Determination

### 4.1. Parameter Determination of the Standard RHT Model

### 4.1.1. Determination of Static Load Mechanic Parameters.

 The rock sample used in the test is crystalline limestone in a mining area in Guizhou province. The size of the specimen is $\Phi 50 \times 50 \mathrm{~mm}$ cylinder, the aspect ratio is 1.0 , and core taking, cutting, and polishing of rock samples are carried out by core-taking machine, cutting machine, double-face grinding machine, and sandpaper. The nonparallelism and nonperpendicularity of rock specimens are limited below0.02 mm in order to make the subsequent tests go on smoothly. The rock sample is shown in Figure 2.

The $f_{c}$ and $f_{t}$ of crystalline limestone can be obtained by conventional uniaxial compressive test, Brazilian splitting test, and rock wave velocity test. The rock density $\rho_{0}$ and initial porosity $\alpha_{0}$ were measured by the balance method and the mass method. The mechanical parameters of crystalline limestone are shown in Table 1:
4.1.2. $p-\alpha$ State Equation Parameter Determination. Brittle materials such as rock and concrete contain a large number of voids in the interior. When the material is under a strong dynamic load, the material will be subjected to the combined action of shear stress and high hydrostatic pressure, which makes the mechanical response of the material more complex. Based on this, Herrman [33] puts forward an equation of state considering the internal voids of brittle materials in 1969, which is called equation $p-\alpha$, and its compression process is shown in Figure 3:

From Figure 3, it can be seen that when the pressure value is $p=0$, the initial density of the brittle material is


Figure 2: Experimental samples of crystalline limestone. (a) Standard sample size. (b) Partial rock specimen.

Table 1: Static load mechanic parameters.

| Type of rock | $f_{c}(\mathrm{MPa})$ | $f_{t}(\mathrm{MPa})$ | $E(\mathrm{GPa})$ | $\mu$ | $\rho_{0}\left(\mathrm{~g} \cdot \mathrm{~cm}^{-3}\right)$ | $\alpha_{0}$ | $v_{p}\left(\mathrm{~m} \cdot \mathrm{~s}^{-1}\right)$ | $v_{s}\left(\mathrm{~m} \cdot \mathrm{~s}^{-1}\right)$ | $G(\mathrm{GPa})$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: | :---: |
| Crystal limestone | 44.2 | 5.34 | 47.5 | 0.36 | 2.68 | 1.025 | 4235 | 2439 | 17.46 |



Figure 3: Pressure versus density diagram.
$\rho_{0}=\rho_{\text {initial }}$, and when the pressure linearly increases to $p_{\text {cursh }}$, then the density of the material is compacted to $\rho_{p}$. When the pressure increases in a nonlinear way until the internal voids of the rock are completely compressed, the corresponding pressure is $p_{\text {lock }}$ and compaction density is $\rho_{\text {lock }}$. Generally, the dense relationship is expressed in MieGreisen [34] form as follows:

$$
\begin{align*}
& A_{1}=\rho_{0} c_{0}^{2}  \tag{17}\\
& A_{2}=A_{1}(2 s-1) .  \tag{18}\\
& A_{3}=A_{1}\left(3 s^{2}-4 s+1\right) .  \tag{19}\\
& B_{0}=B_{1}=2 s-1 . \tag{20}
\end{align*}
$$

$$
\begin{equation*}
T_{1}=A_{1} T_{2}=0 \tag{21}
\end{equation*}
$$

In the above formula, $A_{1}, A_{2}$, and $A_{3}$ are the RankineHugoniot polynomial coefficients. $c_{0}$ is the sound velocity in rock mass when the pressure is zero, and $s$ is an empirical parameter. Calcium is the main component of crystalline limestone. Available through Meyers [35] literature $s=0.95$, $B_{0}, B_{1}, T_{1}$, and $T_{2}$ are the parameters of the equation of state. According to the literature [36], the pressure at the beginning of crushing of the void can be obtained as $p_{\text {crush }}=f_{c} / 3$. Generally, the pressure at the time of crushing of the material is $p_{\text {lock }}=6000 \mathrm{MPa}$ and the compression index is $n=3$
4.1.3. Determination of RHT Constitutive Parameters. Hoek and Brown [37] empirical formula is used to estimate rock strength under different confining pressures, and its expression is as follows:

$$
\begin{equation*}
\sigma_{1}=\sigma_{3}+\sqrt{m_{b} \sigma_{c} \sigma_{3}+s \sigma_{c}^{2}} \tag{22}
\end{equation*}
$$

$\sigma_{1}$ and $\sigma_{3}$ in equation (22) are the maximum and minimum principal stresses at the failure of the rock mass, respectively, $m_{b}$ reflects the soft and hard degrees of rock mass, $m_{b}=24$, and $s$ reflects the degree of fragmentation of the rock mass and is taken as $s=1 . \quad \sigma_{f}=$ $\sqrt{1 / 2\left[\left(\sigma_{1}-\sigma_{3}\right)^{2}+\left(\sigma_{2}-\sigma_{3}\right)^{2}+\left(\sigma_{3}-\sigma_{1}\right)^{2}\right]}$, and $p=$ $-\left(\sigma_{1}+\sigma_{2}+\sigma_{3}\right) / 3$. The mechanical parameters of the rock mass under different loading conditions can be obtained according to equation (22), as shown in Table 2.

Table 2: mechanical parameters of crystalline limestone under different confining pressures.

| $\sigma_{2}=\sigma_{3} / M P a$ | $\sigma_{1}(\mathrm{MPa})$ | $p(\mathrm{MPa})$ | $\sigma_{f}$ | $p^{*}$ | $\sigma_{f}^{*}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 0 | 5.34 | -1.78 | 5.34 | -0.04 | 0.12 |
| 0 | 44.2 | 14.73 | 14.73 | 0.33 | 0.33 |
| 5 | 90.20 | 33.40 | 85.20 | 0.76 | 1.93 |
| 20 | 172.22 | 70.74 | 152.22 | 1.60 | 3.44 |
| 50 | 284.51 | 128.17 | 234.51 | 2.90 | 5.31 |
| 100 | 428.68 | 209.56 | 328.68 | 4.74 | 7.44 |

The Mohr-Coulomb criterion was chosen to calculate the shear strength $f_{s}$ and the cohesion $c$ of the rock mass.

$$
\begin{equation*}
f_{s}=\tau=\frac{\sigma_{1}-\sigma_{3}}{2} \sin (2 \theta) \tag{23}
\end{equation*}
$$

$2 \theta=\pi / 2+\varphi$ in equation (23), and $\varphi$ is the internal friction angle of rock mass, which is determined by MohrCoulomb criterion:

$$
\begin{equation*}
\sigma_{1}=\frac{\sigma_{3}(1+\sin \varphi)}{(1-\sin \varphi)}+\frac{2 \cos \varphi}{(1-\sin \varphi)} \tag{24}
\end{equation*}
$$

The data brought into Table 2 are calculated as $\varphi=36.6^{\circ}$ and $f_{s}=17.69 \mathrm{MPa}$.

When the rock is under quasi-static loading conditions, $F_{r}=1, \dot{\varepsilon}=3.0 E-6$, and the strength of the rock under 5 MPa and 100 MPa confining pressures is selected to determine the failure surface parameters. The equations for the compression meridian are as follows:

$$
\left\{\begin{array}{l}
A\left(0.76-\frac{1}{3}+(A)^{-1 / N}\right)^{N}=1.93  \tag{25}\\
A\left(4.74-\frac{1}{3}+(A)^{-1 / N}\right)^{N}=7.44
\end{array}\right.
$$

According to equation (25), the failure surface parameters $A=2.47$ and $N=0.714$ can be obtained. So far, most of the parameters in the constitutive model have been determined. The determination of parameters $B$ and $Q_{0}$ is complicated, and the sensitivity analysis of Li et al. [26,38] to the parameters of the RHT model shows that it has little influence on the calculation results. Therefore, the value determined by the Riedel experiment can be used. The RHT constitutive parameters of crystalline limestone are shown in Table 3.

### 4.2. Parameter Determination of the Modified RHT Model

4.2.1. SHPB Experiment. Hopkinson pressure bar test was carried out, and the average impact pressures of the projectile in the experiment were $0.2,0.3,0.4,0.5$, and 0.6 MPa , respectively. Based on the theory of one-dimensional elastic waves and the assumption of stress uniformity, the threewave method is used to calculate the stress-strain and the average strain rates of the specimen based on the experimentally data measured, the strain rates corresponding to different shock pressures are $30.37 \mathrm{~s}^{-1}, 49.64 \mathrm{~s}^{-1}, 87.05 \mathrm{~s}^{-1}$,
$124.75 \mathrm{~s}^{-1}$, and $138.18 \mathrm{~s}^{-1}$ respectively, and the resulting stress-strain curve is shown in Figure 4.

In order to determine the elastic ultimate strength, failure strength, and residual strength of rock, it is necessary to smooth the stress-strain curves obtained from experiments. Elastic ultimate strength is the point at which the slope of the stress-strain curve begins to change, failure strength is the point at which the maximum stress value is on the stress-strain curve, and residual strength is the first turning point after the peak value of the stress-strain curve. The stress in the direction of the compression bar is $\sigma_{1}$, the corresponding strain is $\varepsilon_{e f f}$, and the strain rate is $\dot{\varepsilon}_{e f f}$. The strength indexes are shown in Table 4.

It can be seen from Figure 3 that when the impact pressure is 0.2 MPa , the corresponding yield strength is 61.02 MPa , and with the gradual increase in impact pressure, the failure strength of the rock also gradually increases. When the impact pressure is 0.6 MPa , the failure strength of rock reaches 148.56 MPa , which is 2.43 times higher than the yield strength when the impact pressure is 0.2 MPa and 3.36 times higher than the uniaxial compressive strength. It can be seen that with the increase in impact pressure, the yield strength of the rock is also gradually improved.

Based on the experimental results in Table 4, it is possible to plot the relationship between $\sigma_{r}{ }^{*}$ and $p^{*}$, the curve equation $\sigma_{r-p r e}^{*}=1.94\left(p_{p r e}^{*}\right)^{0.42}\left(0.68+0.05 p_{p r e}^{*}\right)$ is fitted by the modified residual surface equation of equation (13), and decision factor $R^{2}=0.954$. The fitting curve is shown in Figure 4:

From Figure 5 we can obtain $A_{f}=1.94$ and $n_{f}=0.42$, $A=2.95$ and $N=0.76$ are calculated by taking $p^{*}$ corresponding to 0.2 MPa and 0.6 MPa , respectively, and the rest of the parameters are the same as those taken in Table 3.

## 5. Validation of the Modified RHT Model

5.1. Numerical Simulation. In order to verify the correctness of the parameters of the improved RHT model, the numerical model is established by ANSYS/LS-DYNA software according to the experimental process of $1: 1$, in which the incident rod is 2.0 m long, the transmission rod is 1.5 m long, and the diameter of the rod is 0.05 m . The keyword $*$ LOAD_SEGMENT_SET is used to load the sinusoidal stress wave on the end face of the incident rod, which not only ensures the consistency of the waveform but also plays a role in simplifying the model. The model is meshed into 255,400 cells and 274,373 nodes using SOLID 164 cells. The contact between the bar and the specimen is set as surface-to-surface erosion contact, the contact stiffness is the default value, and the dynamic and static friction coefficients are zero. The finite element analysis model is shown in Figure 6. The incident stress wave curve is selected when the impact pressure is 0.2 MPa for loading. When $t=60.373 \mu \mathrm{~s}$, the peak value of incident stress is 134.26 MPa , and the stress wave curve is shown in Figure 7.
5.2. Analysis of Results. In order to verify the rationality of the improved RHT model, it is necessary to compare the

Table 3: Values of RHT constitutive parameters.

| Parameter symbols | Parameter description | Value | Parameter symbols | Parameter description | Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\rho_{0}$ | Density ( $\mathrm{g} \cdot \mathrm{cm}^{-3}$ ) | 2.68 | $N$ | Failure surface index | 0.714 |
| $p_{\text {el }}$ | Pressure in gap compression (GPa) | 0.0147 | G | Shear modulus | 17.46 |
| $p_{\text {comp }}$ | Pressure in gap compression (GPa) | 6 | $Q_{0}$ | Pull pressure meridian ratio | 0.6805 |
| A | Failure surface parameters | 2.47 | $\alpha$ | Compression strain rate index | 0.0262 |
| $A_{1}$ | Rankine-Hugoniot coefficient (GPa) | 48.06 | $\delta$ | Tensile strain rate index | 0.0311 |
| $A_{2}$ | Rankine-Hugoniot coefficient (GPa) | 43.26 | $\dot{\varepsilon}_{0}^{c}$ | Reference compressive strain rate ( $\mathrm{ms}^{-1}$ ) | $3 \times 10^{-8}$ |
| $A_{3}$ | Rankine-Hugoniot coefficient (GPa) | 4.44 | $\dot{\varepsilon}_{0}^{t}$ | Reference tensile strain rate ( $\mathrm{ms}^{-1}$ ) | $3 \times 10^{-8}$ |
| B | Lode angle-related parameters | 0.0105 | $\dot{\varepsilon}^{c}$ | Failure compression strain rate | $3 \times 10^{22}$ |
| $B_{0}$ | Equation of state parameters | 0.9 | $\dot{\varepsilon}^{t}$ | Failure tensile strain rate | $3 \times 10^{22}$ |
| $B_{1}$ | Equation of state parameters | 0.9 | $\varepsilon_{p}^{m}$ | Minimum residual strain of damage | 0.01 |
| $T_{1}$ | Equation of state parameters (GPa) | 48.06 | $D_{1}$ | Initial damage parameters | 0.04 |
| $T_{2}$ | Equation of state parameters (GPa) | 0 | $D_{2}$ | Damage parameters | 1 |
| $n$ | Porosity index | 3 | $\xi$ | Shear modulus reduction factor | 0.5 |
| $\alpha_{0}$ | Initial porosity | 1.1884 | $g_{c}^{*}$ | Compression yield surface parameters | 0.88 |
| $f_{c}$ | Uniaxial compressive strength (GPa) | 0.0442 | $g_{t}^{*}$ | Tensile yield surface parameters | 0.72 |
| $f_{t}^{*}$ | Relative tensile strength | 0.12 | $A_{f}$ | Residual stress strength parameters | 1.62 |
| $f_{s}^{*}$ | Relative shear strength | 0.40 | $n_{f}$ | Residual stress strength index | 0.6 |



Figure 4: Stress-strain curve under different impact pressure.

Table 4: State quantities such as elastic limit, failure strength, and residual strength in the SHPB test curve (unit: MPa).

| The impact pressure | Mean strain rate $\left(\mathrm{s}^{-1}\right)$ | $f_{c, e l}$ | $\sigma_{f}$ | $\sigma_{r}$ | $p$ | $p^{*}$ | $\varepsilon_{\text {eff }}\left(\mathrm{s}^{-1}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.2 | 30.37 | 46.2 | 61.02 | 39.02 | 20.34 | 0.46 | $3.32 \times 10^{-3}$ |
| 0.3 | 49.64 | 65.36 | 82.42 | 49.19 | 27.46 | 0.62 | $2.94 \times 10^{-3}$ |
| 0.4 | 87.0 | 89.57 | 109.18 | 57.24 | 36.39 | 0.82 | $2.82 \times 10^{-3}$ |
| 0.5 | 124.75 | 111.12 | 121.81 | 60.2 | 40.36 | 0.91 | $4.29 \times 10^{-3}$ |
| 0.6 | 138.18 | 133.7 | 148.56 | 68.11 | 49.52 | 1.12 | $4.02 \times 10^{-3}$ |

difference between the rock failure patterns before and after the modification of constitutive parameters in numerical simulation and the experimental results, and the
keyword*MAT_ADD-EROSION in LS-DYNA is used to control the failure of the element in order to simulate the crack propagation in rock. The results of numerical


- Experiment
-     - Fitted curve

Figure 5: Fitted curve of improved normalized pressure versus residual strength.


Figure 6: Hopkinson compression bar model (a) and specimen meshing model (b).


Figure 7: Incident stress wave curve.


Figure 8: Numerical simulation results before modification of constitutive parameters of RHT.


Figure 9: A comparison chart of numerical simulation effect after modification of constitutive parameters of RHT.
simulation before correction and the experimental impact effect are shown in Figure 8.

As can be seen from Figure 8, the stress-strain curves obtained by numerical simulation show that the elastic ultimate strength is 6.98 MPa higher than the experimental value, and the yield strength is 1.45 MPA higher than the experimental value. At the same time, it is not difficult to find that the strain value corresponding to the elastic ultimate strength in the numerical simulation and test results is $5 \times 10^{-4}$ more than the test value. When the strength exceeds the yield value, the residual strength
surface described by the numerical simulation results is different from the experimental results. Although the surface crack propagation trend of numerical simulation is similar to the experimental results, only a few elements fail on the side of rock samples. This is because the unmodified constitutive parameters are conservative in describing the yield strength surface and residual strength surface, which leads to the failure mode of rock not fully displayed. In order to make the simulation results more consistent with the failure mode of rock, the improved constitutive parameters are used for calculation, and the results are shown in Figure 9.

From Figure 9, it can be found that the description of elastic ultimate strength surface, yield strength surface, and residual failure surface by the improved RHT model is basically consistent with the test results. This is because the modified RHT model considers the constant change of hydrostatic pressure in the failure surface equation and the influence of the Lode angle factor in the residual strength surface equation, which makes the constitutive parameters of the RHT model more reasonable to describe the failure strength surface and the residual strength surface, which also verifies the correctness of the modified RHT model.

## 6. Conclusions

In this study, the shortcomings of the RHT constitutive model are studied, and the conclusions are as follows:
(1) It is practical to use the hyperbolic function to describe the tensile strain rate enhancement factor of rock at high strain rates.
(2) The introduction of the initial damage variable $D_{0}$ makes the expression of damage variable $D$ in the RHT constitutive model more complete.
(3) Considering the influence of the Lode angle factor and hydrostatic pressure, the updated damage variable $D_{\text {pre }}$ can better reflect the actual failure mode of the rock mass.
(4) It is a simple and effective method to determine the constitutive parameters of the RHT model by SHPB experiment.

## Data Availability

All data included in this study are available upon request by contact with the corresponding author.

## Conflicts of Interest

The authors declare no conflicts of interest. Tian Hao-fan and Bao Tai equally contributed to this manuscript.

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