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Research Article

Research Thermal Fields in the Crystallization Process of Steel Cast Parts

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In foundry production, the duration of metal crystallization of the cast part, as well as the distribution of the thermal field, are described by systems of analytical equations. There are a number of methods for modeling these processes, but their theoretical basis was formed at the end of the twentieth century, and it needs to be updated and clarified. Therefore, the aim of our work is to develop a new analytical method for researching the distribution of thermal fields in cast parts during crystallization. For the first time, on the basis of own analytical and thermophysical developments, it developed the analytical method for researching the thermal field of the casting during crystallization and cooling, which is based on establishing the kinetics of cooling of the casting surface in the foundry mold, the advancement of the crystallization front from the surface to the center of the casting, and the distribution of temperatures in the solid and liquid parts of the casting. The method is expressed in a number of analytical formulas, each of which describes the specific thermal process that occurs in the casting. The developed analytical method was used to research the crystallization of the casting in a one-time sand mold, using the example of researching the thermal field of the cast part made of carbon steel with 0.25% C, hollow cylindrical shape, with the wall thickness of 100 mm. The developed analytical method for researching the thermal fields of cast parts is the analytical basis for the refinement of applied computer programs for modeling crystallization and cooling processes in foundry production.

1. Introduction

The casting process is associated with a number of alloy crystallization conditions that affect the formation of the microstructure of the final product. The size and morphology of grains, the distance between dendrites, segregation and precipitation of dissolved substances, porosity, and other defects significantly depend on the thermal behavior of the metal-mold system during crystallization [1]. In particular, casting defects and the solidification process were analyzed in the work [2]. The results show that the defects can be predicted by the simulation result exactly. This creates a high correlation between the crystallization features, the resulting microstructure, and the final properties of the resulting material. Casting in sand molds, gravity or pressure die casting, continuous casting, and squeeze casting are some of the casting processes where product quality is affected by metal-mold heat-transfer conditions [3]. A modified micro-macro model was presented and used to study the solidification of equiaxed alloys in the work [4]. Nucleation and growth kinetics were coupled to the macroscopic heat transfer equation through the latent heat evolution. The influences of mold temperature and casting speed on the temperature field, microstructures, and mechanical properties were investigated by numerical simulation and experimentation in the work [5]. As a result, cylindrical samples with high surface quality, dimensional accuracy, and uniform microstructure were successfully

obtained. Therefore, the peculiarities of temperature changes over time during alloy crystallization are closely related to the structural integrity of castings, and the issue of clarifying the behavior of thermal fields during the crystallization process of the material in cast parts is very relevant.

Crystallization and solidification are fundamental aspects of casting processes, the control of which makes it possible to control the structure and final mechanical properties of the casting. It is extremely important to take this into account when designing shells (for example, bearing links of shock absorbers) [6], plates (for example, power elements of centralizers) [7], and layered structures [8] that are subjected to extremely high-operating loads throughout the life cycle.

The success of the casting process depends on careful control of all input parameters and proper control of the metal solidification dynamics. A slight change in any of the input parameters affects the outcome of the process and can lead to defective castings. Research methods that use mathematical models are a basic field of research in many metallurgical processes [9]. Therefore, today's foundry engineers make significant efforts to develop new analytical models to achieve precise adjustments to the foundry process [10].

The first step in solving analytical problems related to the distribution of thermal fields was the calculation of soil freezing, performed in 1890 by I. Stefan [11]. It was established that the increase in the thickness of the crystallized layer is proportional to the square root of time, and this regularity was repeatedly confirmed later. This result can be seen both in modern publications [12] and in studies that have become a kind of bestsellers in their countries for engineers in the foundry industry [13] and textbooks for students [14].

In the 20s of the twentieth century, Saito was the first to solve the thermal problem regarding the crystallizing metal and took into account the release of the heat of crystallization. At the beginning of the 30s of the twentieth century, Schwartz was the first to establish approximate formulas for calculating the temperature distribution along the cross-section of the casting and foundry mold. Dependencies are built on the basis of the Gaussian probability integral [15].

In the future, the analytical methods were improved by introducing additional parameters and specifying the coefficients. By the middle of the twentieth century, calculations of the advance of the crystallization front inside the casting were performed, and Sahoo and Sundari proposed an approximate formula for determining the temperature distribution in its crystallized part [16]. However, it is based on empirical coefficients.

The authors of [17] proposed analytical formulas for temperature distribution in the casting. However, at the same time, the temperature of the casting surface is considered constant and independent of time.

Balandinsummarized the information on previously conducted calculations and supplemented it with his own developments, on the basis of which he suggested analytical formulas for determining the duration of crystallization and cooling of castings in one-time (nonmetallic) foundry molds. These formulas are the basis of most modern analytical methods and software for foundry processes [18].

The introduction of the finite element method and the development of engineering process modeling tools based on it have significantly changed the world of materials' design and production. The concept of dividing and discretizing the complex process area into simple subspaces (so-called grids or elements), the behavior of which can be described using approximate solutions, has found many applications among material scientists [19]. Here, in spite of other applications, it should be noted for the emergence of the possibilities of refined prediction of strength [20], ensuring the accuracy of geometric dimensions [21] and obtaining the required operational stiffness [22] of the final product. The simulation results can be used for optimization of casting process parameters, eliminating casting defects, and improving product quality [23]. However, sometimes, the results of modeling the foundry process, which are obtained by using engineering software packages, can be wrong. The most critical input values for such simulation software are the thermophysical properties of the cast metal and mold, as well as interface boundary conditions. But these values are temperature-dependent. Therefore, the values are difficult to acquire for different metal-mold-process combinations. As a result, the outcome of simulation software may deviate from reality [24].

As a result of these features, modern computer programs for modeling foundry processes "MAGMASOFT," "ProCAST," "WinCAST," "LVMFlow," "NovaFlow," and "Polygon" [25, 26] do not provide the required accuracy of thermal calculations. The theoretical study of heat transfer processes with the corresponding formulation of analytical models, which reproduce the essence of the processes as accurately as possible, practically stopped with the advent of computer technology. At the same time, a number of important thermal problems were not finally solved. Analytical solutions for the thermal field distribution in the metal melt and casting during crystallization and subsequent cooling have not been developed.

Programs for modeling foundry processes are built on finite element or finite difference methods [27]. The authors [28], for example, present the results of computer simulation of thermal fields of aluminum casting in foundry molds made of various nonmetallic materials by using the «ANSYS Fluent» software package. In [29], an online version of the program for simulation crystallization processes and the distribution of thermal fields is proposed.

Almost all researchers agree on the importance of the comprehensive assessment of the thermal fields of the "casting-mold" system [30]. This is also relevant from the point of view of the creation and widespread use of new environmentally friendly materials for foundry molds, such as orthophosphates and pyrophosphates of silicon and zirconium [31], meta-phosphates and pyrophosphates of sodium [32], as well as binders synthesized from various aluminum-containing compounds [33], thermophysical properties of which differ from previously known ones. The basis for the analysis of the entire system is a detailed and

accurate determination of the dynamics of the thermal field behavior for the main source of heat generation, that is, the casting.

Available analytical methods are based on programs that require the accurate assignment of boundary conditions, one of which is the casting surface temperature. In addition, it is necessary to establish the law of its change over time, which none of the existing methods makes it possible to do. Therefore, computer programs that describe temperature fields do not give sufficiently accurate results.

Sufficient accuracy is concerned, in foundry production, most calculations are approximate, and this is due to the multifactorial nature of the processes. Therefore, it is not only impossible to ensure the accuracy of the calculation near 1.0 K or even higher but also completely impractical for real conditions. On the other hand, if the process is considered in the temperature range of 1700–1800 K, then even with the fairly accurate calculation with a confidence probability of 0.95, the error can be 85–90 K, and it is often not taken into account.

The analysis of accumulated experience from the standpoint of mathematics and thermal physics showed that none of the previously mentioned analytical methods could be called universal, i.e., one that gives a complete picture of the real distribution of thermal fields inside the casting.

All numerical methods and simulation programs give solutions only if simplifications are introduced. The first common simplification is to take the variable physical quantity (for example, the surface temperature of the casting) as the constant. The second one is the introduction of two or more interdependent quantities into the calculation. For example, the temperature of the casting surface depends on the temperature of the mold surface and vice versa. Both of these shortcomings are common in thermal field calculations. Creation of new analytical methods or improvement of the existing ones should eliminate these shortcomings.

The research objective is to create the analytical method that allows obtaining the distribution of thermal fields in cast parts during crystallization by using the system of analytical formulas.

2. Materials and Methods

The research method of the behavior of the thermal field is implemented in the real example. The calculation object is the cylindrical casting made of steel with 0.25% C, which has an outer diameter of 250 mm and a central hole with a diameter of 50 mm, which is reproduced by the foundry core (Figure 1). This casting belongs to thick-walled castings. Its crystallization occurs from the surfaces (external and internal) to the thermal center.

The initial data for the calculations are as follows [12, 13, 34]: carbon steel with 0.25% C; pouring temperature $T_{pour} = 1873$ K; liquidus temperature $T_L = 1770$ K; solidus temperature $T_S = 1720$ K; heat capacity of the alloy in the liquid state $C_L = 922$ J/(kg • K); density of the alloy in the liquid state $\rho_L = 7000$ kg/m^3; coefficient of heat accumulation of the foundry mold $b_M = 1630$ W • s$^{1/2}$/(m^2 • K); coefficient of thermal conductivity of the alloy in the liquid state $a_L = 0.95 \bullet 10^{-5}$ m^2/s; coefficient of heat conductivity of the alloy in the liquid state $\lambda_L = 23.3$ W/(m • K) and in the solid state $\lambda_S = 54.5$ W(m • K). The solution takes into account the fact that the alloy crystallizes in the temperature range (liquidus-solidus).

Analytical activities are performed in the following sequence:

(1) The duration of complete crystallization of the cast part in the sand mold is determined by using previously known analytical formulas.

(2) The duration of removal of the heat of overheating of the melt, i.e., the period of time from pouring to the beginning of crystallization of the casting surface, is established. For this, the previously known analytical formula is used, which was used for the first time as the basis for determining the cooling dynamics of the casting surface.

(3) Based on the making of the differential equation and its solution, the formula for calculating the dynamics of the temperature change of the casting surface is obtained for the first time.

(4) The real position of the thermal center of the casting is determined by the analytical method, taking into account the features of its design.

(5) The dynamics of temperature changes in the thermal center of the casting are determined. For this, the previously known analytical formula is used.

(6) The dynamics of the advancement of the crystallization front from the surface to the thermal center of the casting are determined, taking into account the features of its configuration. At the same time, for the first time, the transition and the corresponding analytical equations from linear to volumetric advancement of the crystallization front are proposed.

(7) For the first time, it is proposed to consider the casting in the crystallization process as the two-layer object, and the balance of heat flows in its parts is made. Based on this, it is determined about the temperature distribution in the solid and liquid parts of the casting, as well as the dynamics of its surface cooling below the solidus temperature.

So, the originality of the proposed analytical approach is as follows: unlike the number of traditional methods, for the first time, the dynamics of temperature changes of the casting from its pouring into the mold to the final crystallization are taken into account; the advance of the crystallization front in the volume of the casting is taken into account in accordance with the features of its configuration; thermal fields in the liquid and solid parts of the casting are also determined based on the balance of heat flows in these parts.

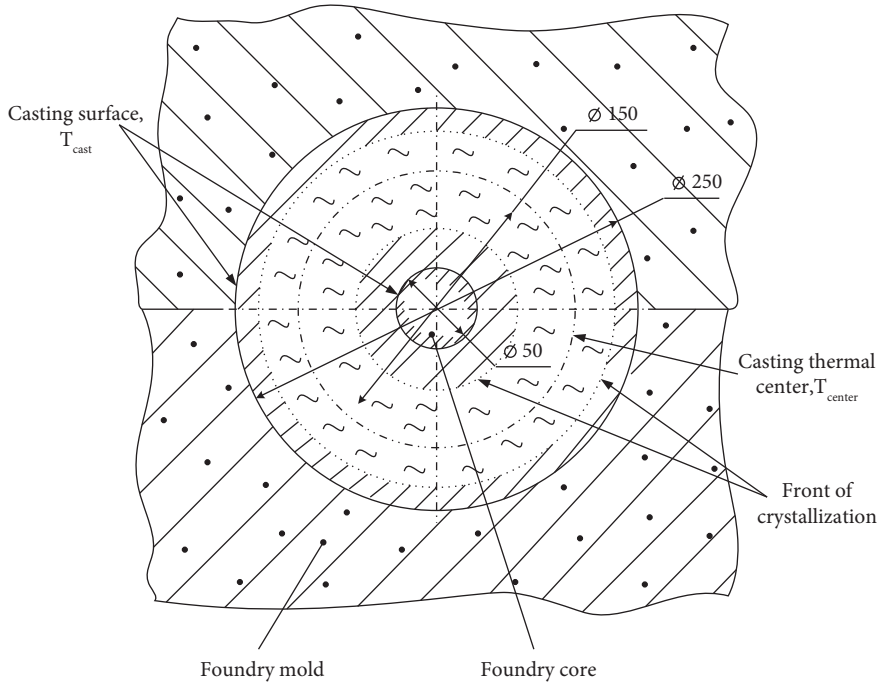

Figure 1: Calculation scheme of the steel casting with the foundry core.

3. Research Results

3.1. Research of the Cooling Dynamics of the Casting Surface. The duration of complete crystallization of the casting is calculated according to known formulas [11, 13]; as a result, 2024 s are obtained.

To obtain the analytical equation of the temperature change of the casting surface, it is necessary to have reliable information about its value at least at two different points in time. For this, the formula [13] is used to determine the duration of overheating heat removal as follows:

$$\tau_{over} = \left[\frac{C_L \cdot \rho_L \cdot R_C \cdot \left(T_{pour} - T_L\right)}{1.128 \cdot b_M \cdot \left(T_{pour} - T_M\right)}\right]^2, \quad (1)$$

where C_L is the heat capacity of the alloy in the liquid state, J/(kg • K); ρ_L is the density of the alloy in the liquid state, kg/m^3; R_C is the effective size of the casting, m; T_{pour}, T_L, and T_M are the temperatures of alloy pouring, liquidus, mold, K; b_M is the coefficient of thermal accumulation of the foundry mold W • s$^{1/2}$/(m^2 • K).

As a result, it is obtained as $\tau_{over} = 135s$. From the physical point of view, this is the time during which the melt in the mold cools from the initial temperature (1873 K at the time $\tau = 0$) to the beginning of the formation of the solid phase. Since the crystallization of the casting starts from the surface and ends in its center it is obvious that directly on the surface of the casting the temperature is equal to $T_L = 1770$ K at the moment $\tau = \tau$.

Applying formula (1) to determine the duration of cooling of the casting below the liquidus temperature is incorrect because in the casting volume from the time $\tau = \tau_{over}$, in addition to the decrease in temperature, the latent heat of crystallization begins to be released, which the formula does not take into account.

To determine the duration of cooling of the casting surface to the solidus temperature (1720 K), it is necessary to establish the analytical law of cooling of this surface. The formulation of this problem is as follows:

(i) Initial temperature $T_0 = 1873$ K

(ii) Temperature at the time $\tau_{over} = 135s = 2.25$ min: $T_1 = T_L = 1770$ K

(iii) Initial mold temperature $T_M = 293$ K; – solidus temperature $T2 = TS = 1720$ K

The cooling rate v for such process is a variable quantity and is described by the following differential equation: $v = d(T-T_M)/d\tau$. Then, $dT/d\tau = k(T-T_M)$. Hence, $dT/(T-T_M) = kd\tau$; $\ln(T-T_M) = k\tau + \ln C$; $\ln((T-T_M)/C) = k\tau$. Therefore, the analytical law of cooling the casting surface has the following form:

$$T - T_M = C \cdot e^{k\cdot\tau}. \quad (2)$$

To establish the constant C, it is applied for the first boundary condition: $T_{\tau = 0} = T_0 = 1873$ K; $T_M = 293$ K. So,

$$1873 - 293 = C \cdot 1; C = 1580. \quad (3)$$

Then,

$$T - 293 = 1580 \cdot e^{k\cdot\tau}. \quad (4)$$

To set the coefficient k, the second boundary condition is applied: $T_{\tau = 2.25\ \text{min}} = T_1 = 1770$ K. So,

$$1770 - 293 = 1580 \cdot e^{k \cdot 2.25}$$
$$e^{2.25 \cdot k} = 0.935 \quad (5)$$
$$e^{k} = (0,935)^{1/2.25} = 0.935^{0.444}.$$

Thus, the analytical law of cooling the casting surface has the final form as follows:

$$T - 293 = 1580 \cdot 0.935^{0.444 \cdot \tau}. \quad (6)$$

Here, τ is time, min; T is the surface temperature of the casting at the calculated time, K.

According to formula (6), the duration of cooling of the casting surface to the solidus temperature (1720 K) is found:

$$\mathrm{T} = \mathrm{T}_2 = 1720\,\mathrm{K},$$
$$1720 - 293 = 1580 \cdot 0.935^{0.444 \cdot \tau}$$
$$\ln(1720 - 293) = \ln 1580 + 0.444 \cdot \tau \cdot \ln 0.935; 7.26 = 7.37 - 0.03\tau.$$
$$\tau = \frac{7.37 - 7.26}{0.03} = 3.67\ \mathrm{min} = 220s. \quad (7)$$

As a result of solving the task, the following ways is obtained:

(i) At the time $\tau = 0$ at all points (on the surface and in the thermal center, Figure 1), the casting temperature is $\mathrm{T}_{\mathrm{cast}} = \mathrm{T}_{\mathrm{center}} = \mathrm{T}_{\mathrm{pour}} = 1873\,\mathrm{K}$

(ii) At the time $\tau_{\mathrm{over}} = 135s$, temperature on the casting surface is $\mathrm{T}_{\mathrm{cast}} = \mathrm{T}_L = 1770\,\mathrm{K}$

(iii) At the time $\tau_s = 220s$, temperature on the casting surface is $\mathrm{T}_{\mathrm{cast}} = \mathrm{T}_S = 1720\,\mathrm{K}$

(iv) By formula (6), it is possible to determine the surface temperature T_{cast} at any time in the period from 0 to 220 s

3.2. Research of Dynamics of Advancement of the Crystallization Front. The thickness of the crystallized layer increases starting from $\tau_S = 220$ s until the moment of complete crystallization $\tau_T = 2024s$. In the period (2024–220) = 1804 s, the thickness of this layer reaches the thermal center of the casting.

If the casting is flat, the absolute thickness of the solid layer would increase according to the "square root" law. However, the casting is cylindrical, so it is necessary to take into account its shape.

Since the crystallization process is characterized by the amount of heat released from a unit of volume, the calculation of the advance of the crystallization front should be attributed not to the thickness of the crystallized layer but to its volume.

The nominal thickness of the crystallized layer, which for the flat casting is equal to the real thickness, can be determined by the following formula:

$$\delta_{NT} = K_X \cdot \sqrt{(\tau - \tau_S)}, \quad (8)$$

where δ_{NT} is the nominal thickness of the crystallized layer, m; $\mathrm{K_Ч}$ is the crystallization front speed coefficient, $m/s^{1/2}$; τ is the calculated time, s; τ_S is the duration of cooling of the casting surface to the solidus temperature, s.

To find the coefficient $\mathrm{K_Ч}$, formula (8) is used, assuming $\delta_{NT} = R_C$ (effective size or cooling module of the casting) and $\tau = \tau_{\mathrm{T}}$ (time of complete solidification):

$$K_X = \frac{R_C}{\sqrt{\tau_{\mathrm{T}} - \tau_S}} = \frac{0.05}{\sqrt{2024 - 220}} = 0.001177\,\frac{m}{s^{1/2}}. \quad (9)$$

To take into account the cylindrical shape of the casting, analytical activities are performed in the following sequence:

(i) As a result of dividing the casting into two parts equal in volume, that is, the outer and inner parts, the real position of the casting thermal center is established;

(ii) The calculated volume of the casting is determined. This is its outer part (from the external surface to the thermal center, Figure 1);

(iii) The nominal thickness of the crystallized layer δ_{NT} is determined for the number of consecutive moments of time according to formula (8);

(iv) The relative thickness of the crystallized layer δ_{NT}/R_C is determined for each moment of time;

(v) The relative volume of the crystallized layer is determined for each moment of time;

(vi) The real thickness of the crystallized layer is calculated for each moment of time.

To establish the position of the thermal center, the total nominal volume of the casting is determined and divided into two equal parts. The outer diameter of the casting is 0.25 m, and the inner diameter is 0.05 m. The linear size of the calculated volume is assumed equal to one. Then,

$$V = \frac{\pi}{4} \cdot \left(0.25^2 - 0.05^2\right) \cdot 1 = 0.0471 vol.units. \quad (10)$$

Half of the nominal volume is equal to 0.0235 vol. units. This volume has an outer diameter of 0.25 m, and the inner one passes through the casting thermal center. Then, the diameter of the thermal center is as follows:

$$d_{center} = \sqrt{0.25^2 - \frac{4 \cdot 0.0235}{3.14}} = 0.180\,\mathrm{m}. \quad (11)$$

Thus, the geometric parameters of the calculated volume are established: its inner diameter is 180 mm, and its outer diameter is 250 mm. Its calculated volume is 0.0235 vol. units.

The nominal thickness of the crystallized layer at the time 300 s by formula (8)isas follows:

$$\delta_{NT} = 0.01177 \cdot \sqrt{(300 - 220)} = 0.0105m. \quad (12)$$

The relative thickness of the crystallized layer at the time 300 s is as follows:

$$\frac{\delta_{NT}}{R_C} = \frac{0.0105}{0.05} = 0.210. \quad (13)$$

The relative volume of the crystallized layer:

$$V_{300} = 0.210 \cdot V = 0.210 \cdot 0.0235 = 0.0049 vol.units. \quad (14)$$

The inner diameter of the crystallized layer d_S and its thickness δ_S:

$$\frac{\pi}{4} \cdot \left(0.25^2 - d_S^2\right) \cdot 1 = V_{300},$$

$$d_S = \sqrt{0.25^2 - \frac{4 \cdot V_{300}}{\pi}} = \sqrt{0.25^2 - \frac{4 \cdot 0.0049}{3.14}} = 0.237m, \quad (15)$$

$$\delta_S = \frac{0.25 - d_S}{2} = \frac{0.25 - 0.237}{2} = 0.007m.$$

Similar calculations are made for time points 500, 1000, 1500, 2000, and 2024 s.

As the result of the deviation of the advancement dynamics of the crystallization front from the "square root" law, the real thickness of the crystallized layer is significantly different from the nominal one (Figure 2). If the casting is flat, its thermal center would coincide with the geometric center, but due to the cylindrical shape, the thermal center, and accordingly the maximum thickness of the crystallized layer, is not 50 mm but 35 mm from the external surface.

If volumetric units of measurement are taken as the basis of calculation instead of linear units, different results are observed. The growth of the relative volume of the crystallized layer in relation to the entire calculated volume clearly corresponds to the "square root" law and at the end of crystallization reaches unity, which means the complete disappearance of the liquid phase (Figure 3).

Analysis of the results presented in Figures 2 and 3 shows that for the correct interpretation of the progress of the crystallization front inside the casting, it is necessary and sufficient to establish the relative volume fraction of the crystallized and liquid layers.

The dynamics of changes in the thickness of these layers during the crystallization period are shown in Figure 4.

According to the data of Figure 4, in the cylindrical casting, the change in the thickness of the liquid and crystallized layers is approximated to the "square root" law only at the initial stage. During almost the entire period of crystallization of the casting, the dynamics of the advance of the crystallization front is approximated to linear.

3.3. Analytical Calculation of the Thermal Field in the Casting. At this stage, the temperature change of the casting surface is determined only until the solidus temperature is reached. After this moment (220 s), the surface temperature is unknown. The law of its change and specific values must be established.

This requires much more data than is known. To obtain additional information, the temperature change in the thermal center of the casting is calculated.

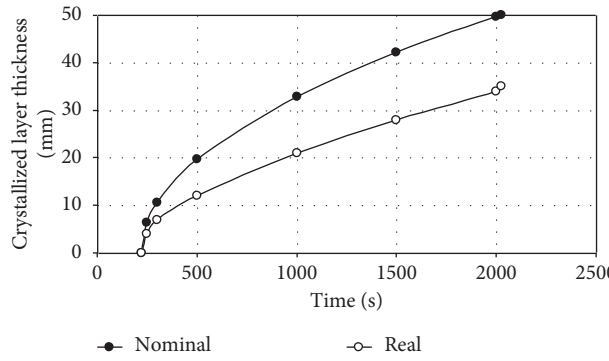

Figure 2: Dynamics of changes in the thickness of the crystallized layer on the casting surface, taking into account its cylindrical shape.

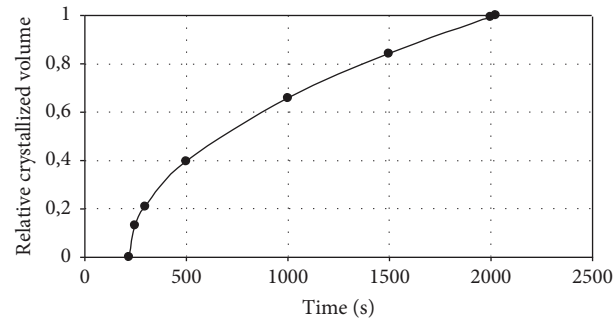

Figure 3: Dynamics of volume growth of the crystallized layer of the cast part in the process of its crystallization.

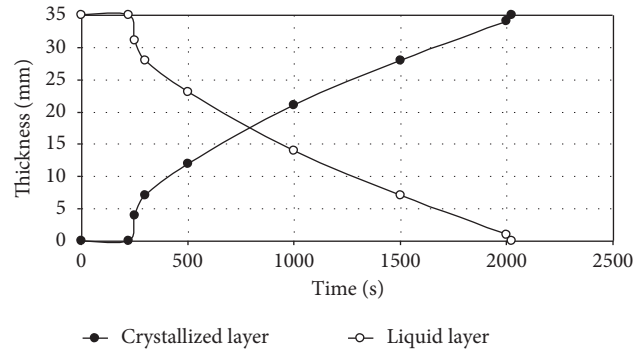

Figure 4: Comparative dynamics of changes in the thickness of the crystallized and liquid layers of the cast part in the crystallization process.

It is known that at the time $\tau = 0$, the temperature in the center is $T_{center} = T_{pour} = 1873$ K, and at the moment of completion of crystallization, the last part of the liquid phase disappears there; that is, the solidus is reached: $T_{center} = T_S = 1720$ K. The temperature change in the center of the casting with sufficient accuracy can be calculated by using the formula of S. Schwartz [11, 13]:

$$T_{center} = T_{pour} - \left(T_{pour} - T_0\right) \cdot \frac{1 - \operatorname{erf}\left(y/2 \cdot \sqrt{a_L \cdot \tau}\right)}{1 - \operatorname{erf}\left(K/2 \cdot \sqrt{a_L}\right)}, \quad (16)$$

where T_{center} is the temperature in the casting center, which is located at the distance y from the surface, at the time τ, K; T_0 is the crystallization temperature of the alloy, K; a_L is the coefficient of thermal conductivity of the alloy in the liquid state, m^2/s; K is the crystallization coefficient, $m/s^{1/2}$.

To determine the coefficient K, which will provide accurate results in the future, it is taken into account that the condition for the complete crystallization of the casting is to reach the solidus temperature (1720 K) in the center. Therefore, $T_{center} = T_S$ and $T_0 = T_S$ are used in formula (6). In addition, the distance between the casting surface and its thermal center is $y = 0.035$ m. In this case, the following equation is obtained:

$$1 - \operatorname{erf}\left(\frac{0.035}{2 \cdot \sqrt{a_L \cdot \tau}}\right) = 1 - \operatorname{erf}\left(\frac{K}{2 \cdot \sqrt{a_L}}\right), \quad (17)$$

and further,

$$\frac{0.035}{2 \cdot \sqrt{a_L \cdot \tau}} = \frac{K}{2 \cdot \sqrt{a_L}}. \quad (18)$$

By substituting $\tau = \tau_T = 2024$ s, the crystallization coefficient K is determined:

$$K = \frac{0.035}{\sqrt{2024}} = 0.00078 \frac{m}{s^{1/2}}. \quad (19)$$

According to formula (16), the value of the temperature in the casting center is calculated for time points 50, 100, 150, 200, 250, 300, 500, 1000, 1500, and 2000 s, as well as for $\tau_{over} = 135$ s, $\tau_s = 220$ s, and $\tau_T = 2024$ s.

Now, at fixed moments of time, the values of the temperature in the casting center and the position of the crystallization front are known. The casting is conditionally divided into liquid and crystallized parts and considered the two-layer wall (Figure 5); it is correct to assume that the heat flows in these parts are equal [11, 12] $q_S = q_L$, and therefore,

$$\frac{\lambda_L}{\delta_L} \cdot (T_{center} - T_S) = \frac{\lambda_S}{\delta_S} \cdot (T_S - T_{cast}), \quad (20)$$

where λ_L and λ_S are coefficients of thermal conductivity of the alloy in the liquid and solid states, W/(m • K); δ_L and δ_S are thicknesses of the liquid and crystallized layers, m; T_{center} is the temperature in the casting center, K; T_S is the solidus temperature, K; T_{cast} is the surface temperature of the casting, K.

From this equation, the casting surface temperature is

$$T_{cast} = T_S - \frac{\lambda_L \cdot \delta_S}{\delta_L \cdot \lambda_S} \cdot (T_{center} - T_S). \quad (21)$$

To take into account the cylindricity of the casting, instead of the ratio of the thickness of the crystallized and liquid layers, it is necessary to take the ratio of their volumes:

$$T_{cast} = T_S - \frac{\lambda_L \cdot V_S}{V_L \cdot \lambda_S} \cdot (T_{center} - T_S). \quad (22)$$

Based on the calculated values of the volumes of crystallized and liquid layers at different moments of time, the value of the surface temperature is calculated. For example, at time $\tau = 300$ s, we obtain

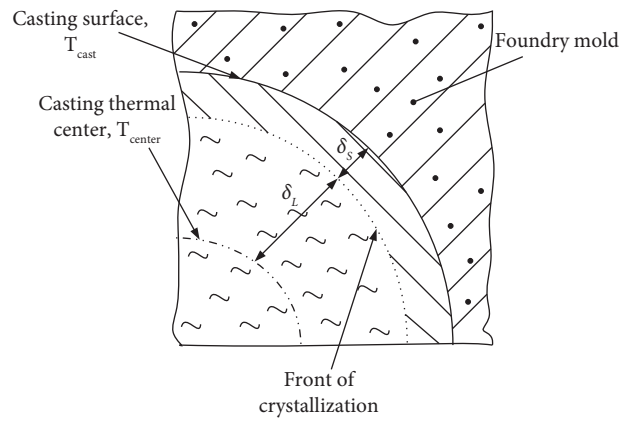

Figure 5: Scheme to determine the temperature distribution in the liquid and crystallized parts of the casting.

$$T_{cast} = 1720 - \frac{23.3 \cdot 0.0049}{54.5 \cdot (0.0235 - 0.0049)} \cdot (1820 - 1720) = 1709\text{K}. \quad (23)$$

The calculated temperature field of the casting is shown in Figure 6.

The temperature difference along the cross-section of the casting during the crystallization process decreases. Since it is minimal at the end of crystallization, it can be neglected for further cooling of the mold and the temperature on the cross-section of the casting can be assumed to be almost the same. Note, in the crystallization process, the temperature difference is large and has a significant impact on the heat transfer processes in the "casting–mold" system and inside the casting itself.

4. Discussion

The studies presented in this article are based on the fundamental laws of mathematics and physics and are devoted to applied fields of knowledge, i.e., thermophysics, the theory of metal crystallization, and the theory of heat transfer. They also have an object-oriented character in the plane of determining the thermal field of a real cast part made of carbon steel in the process of its crystallization.

Using accumulated theoretical knowledge and practical experience in the study of heat transfer in cast parts, the article presents new visions of individual processes that expand their understanding of them. In particular, for the first time, analytical problems about the dynamics of cooling the surface of the cast part from the pouring temperature to the solidus temperature, the dynamics of the advance of the crystallization front inside the part, as well as the problem of thermal fields in the liquid and solid parts of this part were solved.

Solving these problems provided complete knowledge of the distribution of thermal fields in the cast part during the time of crystallization, which is now amenable to the analytical description. The results of the calculations are as close as possible to the results of numerous practical experiments that can be found in the scientific literature. This applies

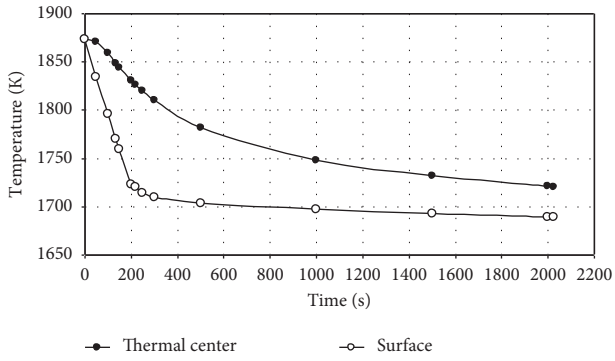

Figure 6: Temperature distribution in the steel casting (Figure 1) during the period from pouring to full crystallization.

primarily to the so-called cooling curves, taken in different areas of the part. The surface cooling curve presented in the article, built on the basis of calculated temperature data, is as close as possible to typical experimental curves. It is observed relatively fast cooling at the stage of heat removal of overheating, slow cooling in the crystallization interval, and practically isothermal aging at the level of the solidus line.

Since the processes of cooling and crystallization of the casting are divided into a number of separate thermal problems, they can easily be the basis of computer programs dedicated to the modeling of crystallization processes. In this case, there will be ample opportunities to analyze the crystallization of complex structures, predict their properties, and eliminate casting defects.

This article provides only one example of the calculation of thermal fields; it is for the steel cylindrical part with a wall thickness of 100 mm. However, the developed mathematical and thermophysical laws are suitable for researching the thermal fields of parts that have different configurations, from different alloys, for casting into molds from different materials. Everything depends on the correct setting of initial conditions, such as initial temperatures, dimensions, configuration, and thermophysical properties of the materials used.

In addition, the method developed in the article can be the basis for calculating the thermal fields of foundry molds and finding theoretical answers to the questions: why does a casting rod with a heat resistance of 400°C not knock out of the cast part after pouring steel with a temperature of 1600°C or how to choose the right composition of the material for the foundry molds and cores to ensure high casting quality.

5. Conclusions

(1) The system of analytical methods has been created to research the thermal fields of castings during their crystallization. The system is based on the combination of previously known analytical formulas and own developments and does not contain empirical coefficients and simplifications.

(2) For the first time, the method is proposed and analytical formulas are presented for determining the dynamics of the temperature change of the casting surface in the interval from pouring to solidus, which consists in solving the differential equation of cooling of this surface in contact with the mold.

(3) The analytical method of researching the dynamics of the advancement of the crystallization front along the cross-section of the casting, taking into account its configuration, has been developed, which consists in determining the amount of heat released during the cooling and crystallization of the metal, per the unit volume of the casting, and not to its linear coordinate.

(4) The method for researching the distribution of thermal fields in the crystallized and liquid parts of the casting has been created, which consists in nominal dividing of the casting into liquid and crystallized parts and determining the thermal fields in these parts taking into account their different thermophysical properties.

(5) According to the developed complex analytical equations, the thermal field of the cylindrical steel casting with a wall thickness of 100 mm during the period of its crystallization in the sand mold is determined.

(6) The developed system of analytical methods can later become the basis for calculations of thermal fields in cast parts, verification and control of the results of direct experiments, and clarification of numerical values of thermophysical properties of metals, alloys, and materials of foundry molds. Also, the proposed formulas can be considered the basis for specifying and improving existing programs for computer modeling of foundry processes.

Data Availability

Data are available upon request.

Disclosure

It was performed as part of the employment of scientific and pedagogical staff in both the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute" and Ivano-Frankivsk National Technical University of Oil and Gas.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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