

Research Article

A Combined Weighting Model Based on Maximizing Deviation for Multiple Attribute Decision-Making

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Multiattribute decision-making is an important part of decision-making theory and modern scientific decision-making. It is widely used in engineering design, economic management, and so on. It is an important part of modern decision science to sort decision objects when considering multiple attributes. Due to time pressure and lack of understanding of decision-making problems, it is difficult for decision makers to accurately express judgment information. Decision-makers' judgment information is more suitable to be expressed by intuitionistic fuzzy sets rather than deterministic numbers or linguistic variables. In the multiattribute decision-making problem, the size of attribute weight reflects the relative importance of each attribute. The research on attribute weight determination method is one of the core problems of multiattribute decision-making. Whether it is the subjective weighting method, the objective weighting method, or the combined weighting method, the research mainly focuses on deterministic multiattribute decision-making, mostly transforming fuzzy information into deterministic information for decision-making, which will lose a lot of information. Due to the differences of objective information data, a combined weighting method in different cases was proposed in this study. The original weight information and the prior information of standardized evaluation can be fully utilized in this model. The results indicate that when decision makers have preferences for different weighting methods, the combined weighting method can be determined according to the preference information of decision makers.

1. Introduction

In the past decades, multiattribute decision analysis has been widely used. The role of multiattribute decision-making in different application fields has been significantly enhanced. Especially, with the development of new methods and the improvement of old methods, multiattribute decision-making is widely used as a model and tool to deal with complex engineering problems [1, 2]. Many problems faced by decision makers and incomplete fuzzy multiobjective decision-making problems because the characteristics of these problems often need the information of this kind of information [3–5]. Many factors will affect the results of engineering decision-making, and there may be contradictions and conflicts among various factors. Various factors

should be weighed against each other to ensure multiattribute [6]. On the one hand is the release of economic factors in the process of earthquake prediction. If the earthquake does not release the prediction, it will cause more serious economic losses and casualties, which should also be considered [7, 8]. These two attributes have contradictory characteristics. Therefore, multiattribute decision-making has the following basic contents and characteristics, as shown in Table 1.

The essence of multiattribute decision-making is to use the existing decision information to sort and select a group (limited) selected schemes in a certain way. Multiattribute decision-making mainly includes two parts: decision information and its determination [9]. Obtaining decision information plays an important role in multiattribute

TABLE 1: The content and characteristics of multicriteria decision-making.

Characteristic	Basic content
The attribute values of quantization, nonquantization, and different dimensions can be processed at the same time	Objective: what needs to be achieved in the decision-making process
Conflicts and contradictions between multiple goals can be handled	Attribute: the factors that affect decision-making and are the criteria for judgment
Problems with different priorities can be dealt with	Decision options

decision-making. Decision information includes attribute weight and attribute value.

- (1) Attribute values and attribute weights: there are three types of attribute values: interval number, real number, and language. Among them, attribute weight plays an important role in determining multiattribute decision-making. The determination method can be divided into objective methods. This method does not contain people's subjective information, but mainly uses the existing information. The subjective method is a weighting method based on experience.
- (2) Decision information needs to be gathered by certain methods in order to make decisions on the results. At present, TOPSIS [10, 11], ELECTRE [12], and LINMAP [13] are widely used.

In real life, there are many engineering multiattribute decision-making problems, such as structural scheme optimization and design scheme evaluation. Such problems generally require decision makers to provide preference information (attribute weight). In recent years, multiattribute decision-making is a very active research field both at home and abroad. Although many meaningful research achievements have been made on the ranking theory of multiattribute decision-making, many decision-making methods have been proposed, such as the simple weighting method, the linear distribution method, and the TOPSIS method; they are still far from perfect. These methods need to determine the weight of attributes in advance [14]. Due to the complexity of objective things and the fuzziness of human thought, in general, it is difficult for people to give clear preference information, which can only provide its possible variation range, or the weight information may be completely unknown. There are few special studies on decision-making in this regard [15–19].

It is difficult for decision makers to accurately express judgment information due to time pressure and lack of understanding of decision-making problems. Decision makers' judgment information is more suitable to be expressed by intuitionistic fuzzy sets rather than deterministic numbers or linguistic variables [20]. Therefore, the application of intuitionistic fuzzy set theory in the research of multiattribute group decision-making reflects unprecedented advantages. Szmidt and Kacprzyk proposed a multiattribute group decision-making problem for intuitionistic fuzzy sets and proposed a method to aggregate individual preferences into group preferences [21]. Zeshui Xu proposed using intuitionistic fuzzy arithmetic average operator, arithmetic weighted average operator, intuitionistic fuzzy

mixed average operator, intuitionistic fuzzy geometric operator, intuitionistic fuzzy ordered geometric operator, intuitionistic fuzzy mixed geometric operator, and other methods to solve the aggregation problem of expert information [22]. Then, Xu gives a decision-making method for the multiattribute decision-making problem in which the expert judgment information is intuitionistic fuzzy set and the attribute weight is partially unknown [23]. Li et al. proposed a decision-making method based on fractional programming for the multiattribute group decision-making problem in which the expert judgment information and expert weight are intuitionistic fuzzy sets [24].

In the multiattribute decision-making problem, the size of attribute weight reflects the relative importance of each attribute. The research on the attribute weight determination method is one of the core problems of multiattribute decision-making [25, 26]. According to different data, the determination methods of attribute weight can be divided into three categories: subjective weighting method, objective weighting method, and subjective and objective comprehensive weighting (combined weighting method) [27].

The advantage of the subjective weighting method is that experts can reasonably determine the ranking of each index according to practical problems, that is, although the subjective weighting method cannot accurately determine the weight coefficient of each index, under normal circumstances, the subjective weighting method can effectively determine the order of the weight coefficient given by each index according to its importance to a certain extent. The main disadvantage of this kind of method is that it is subjective and arbitrary. Different experts are selected, and the weight coefficients are also different [27–31].

The objective weighting method is a late weighting method. This method does not have subjective randomness and does not consider the subjective preference of decision makers so that the weight index completely depends on the attribute value. For example, in information theory, entropy is a measure of uncertainty. The greater the amount of information, the smaller the uncertainty and the smaller the entropy. The smaller the amount of information, the greater the uncertainty and the greater the entropy. According to the characteristics of entropy, we can judge the randomness and disorder degree of an event by calculating entropy or judge the dispersion degree of an index by entropy [32]. The greater the dispersion degree of the index, the greater the impact of the index on the comprehensive evaluation. If the difference of attribute values of all schemes under a certain attribute is smaller, it indicates that the attribute plays a smaller role in scheme decision-making. On the contrary, the more important it is, from the perspective of sorting

schemes, attributes with greater deviation of scheme attribute values should be given greater weight. The amount and quality of information people obtain in decision-making is one of the determinants of the accuracy and reliability of decision-making [33].

In the aspect of the multiattribute decision-making ranking method, TOPSIS idea and method is a widely used and very important method. However, this method has shortcomings. It cannot distinguish the points on the vertical line between positive ideal points and negative ideal points, and the research of improved algorithm also has its own defects [34, 35]. For example, VIKOR algorithm cannot solve the sorting problem of points with the same distance from the ideal point, and the included angle measurement method only considers the included angle of the two schemes without considering the length, etc. In order to take into account the preference of decision makers for attributes, we strive to reduce the subjective randomness of attribute weighting, so as to achieve the unity of subjective and objective attribute weighting and to make the decision-making results true and reliable. Therefore, a reasonable weighting method should weight decision indicators based on the internal law between indicator data and expert experience [36, 37].

This study proposes a combined weighting method in different cases, which not only considers the differences of objective information data but also makes full use of the original weight information and the prior information of standardized evaluation.

2. Preliminaries

The analysis process of attribute decision-making can be summarized as determining the decision-making problem affected by multiple attributes and ranking the decision-making objects. It is an important part of modern decision-making science. Multiple attribute decision-making is widely used in many fields such as engineering, technology, economy, and management. The detailed steps of multiattribute decision-making are as follows:

- (1) Determine the decision object, construct the decision attribute set, and define the decision object set Q :

$$Q = \{q^m | m = 1, 2, 3, \dots, M\}. \quad (1)$$

Each element can be regarded as a point of the decision object, and the decision object is divided into N units, which can be expressed as follows:

$$Q = \{q_{x,y,z}^1, q_{x,y,z}^2, q_{x,y,z}^3, \dots, q_{x,y,z}^n\}, \quad (2)$$

where (x, y, z) represents the position of a point. If $q_{x,y,z}^i = d^i$, then the attribute values under different attributes can be expressed as

$$d^i = (d_1^i, d_2^i, \dots, d_j^i). \quad (3)$$

- (2) Obtain decision information, which includes two aspects. The decision table can be used to represent

the input data of multiattribute decision-making problems, and the decision table can be represented by the decision matrix. According to Malczewski's decision construction elements, the decision matrix B [38] is

$$B = \begin{bmatrix} B_{11} & \cdots & B_{1l} \\ \vdots & \vdots & \vdots \\ B_{lj} & \cdots & B_{lj} \end{bmatrix}, \quad (4)$$

where the decision attribute can be replaced by the column in the matrix, the decision object set can be represented by each row, and b_{ij} represents the score value of the decision object element under the j th decision attribute.

- (3) Determine attribute weights. There are many methods to determine attribute weight. The subjective weighting method is to determine the attribute weight according to the subjective attention of decision makers (experts). The original data of the subjective weighting method is obtained by subjective judgment of experts based on experience. The objective weighting method mainly determines the weight according to the relationship between the original data. Therefore, the weight has strong objectivity and does not increase the burden of decision makers. The method has a strong mathematical theoretical basis. However, this weighting method does not consider the subjective intention of decision makers, so the determined weight may be inconsistent with people's subjective wishes or actual situation, which makes people confused.
- (4) Multiattribute decision aggregation: with the passage of time, the decision system is constantly moving and changing. Based on the real-time comprehensive decision-making, the operation state of the system can be evaluated. The real-time feedback information can be used to update measures and the safe operation of the system can be ensured. The commonly used aggregation methods mainly include the linear weighted synthesis method and the nonlinear weighted synthesis method. The weighted linear combination method is the most commonly used method in multiattribute decision-making [39, 40]. The linear weighted combination method mainly consists of two parts: attribute weight w_n and value function $A(b_{mn})$:

$$A(B_m) = \sum_{m=1}^n w_n a(b_{mn}), \quad (5)$$

where $A(B_m)$ is the comprehensive value of the m th attribute to the decision object. The linear weighted combination method can keep the continuous change of attributes from risk minimum 0 to risk maximum 1, and the attributes can also be compensated. The compensation method is determined by the attribute weight, and the relative

characteristics of attributes can also be expressed by it. Extreme situations and high-risk situations can be avoided.

- (5) Uncertainty analysis: the changeable development of objective things and the limitations of people's understanding of objective things may make the prediction results of objective things deviate from people's expectations, and the available information is often uncertain. The attributes of technology, equipment, engineering scheme, and environmental protection need to be considered, mainly in the decision analysis and evaluation of engineering projects. However, the future of the project may still deviate from the assumption, and the actual results after the implementation of the project scheme may deviate from the predicted results. Therefore, the project construction may face potential dangers. In the process of solving the multiattribute decision-making problem, the change of engineering information attribute should be fully considered. All the data in decision analysis are predicted for a long time in the future according to historical data and experience, and the uncertainty of prediction is well known. Therefore, these data are more or less uncertain. Therefore, it is necessary to analyze the uncertainty of the decision model and results; that is, by analyzing the uncertain factors affecting the decision object and quantitatively calculating the relationship between the change of each uncertain factor and the decision result and its influence degree, we can obtain the most sensitive influencing factors and the critical point of the influence of each influencing factor on the decision result [41]. As an uncertainty analysis method, sensitivity analysis is introduced into the uncertainty analysis of multiattribute decision-making. It involves the influence of the uncertainty of a set of input data on the output of the multiattribute decision-making model. Sensitivity analysis can find out the sensitive factors affecting the decision-making object, analyze the reasons for the change of sensitive factors, and provide basis for further uncertainty analysis (such

as probability analysis). Study the change of uncertain factors, such as the range or limit value of the change of the economic benefit value of the project, and analyze and judge the ability of the project to bear risks. Compare the sensitivity of multiple schemes or decision objects, so as to select insensitive investment schemes when the economic benefit values are similar [42].

3. Decision Attribute

In the actual multiattribute decision-making process, the selection of the number of decision-making attributes is very important. Too much and too little have a great impact on the decision-making results. Different attributes play different roles in the decision-making process; some play a larger role and some play a smaller role. In general, in order to grasp the main contradiction of things, we should make decisions with the least key attributes. The principle of the selected attributes can be quantitatively evaluated and measured, so the selection of attributes is the basis of decision-making. Attributes can be divided into two categories: influencing factors and constraints. The influencing factor is the attribute that can improve or reduce the decision-making result, which is a continuously changing quantity in most cases. Each attribute must be comprehensive and measurable. Properties can be explicit or implicit. According to the current statistical literature, the most frequently used attribute types are mainly effective benefit types, which are positively related to the decision-making object. Cost type: the contribution to the decision-making object is negatively correlated. Fixed type: the closer the attribute value is to a fixed value; t_j has a positive correlation with the contribution to the decision object.

Interval type: the closer the decision value is to a fixed interval, especially in the interval $[o_1^j, o_2^j]$, positively correlated is the contribution of attributes to the decision object. Deviating from the interval type, the farther the attribute value is from a fixed value o_j , positively correlated is the contribution of the attribute to the decision object, which is just opposite to the fixed value type. The standardized formulas of benefit type and cost type are

$$y_{ij} = \frac{(x_{ij} - \min_i x_{ij})}{(\min_i x_{ij} - \min_i x_{ij})}, \quad (6)$$

$$y_{ij} = \frac{(\min_i x_{ij} - x_{ij})}{(\min_i x_{ij} - \min_i x_{ij})}, \quad (7)$$

$$y_{ij} = 1 - \frac{(x_{ij} - t_j)}{\max_i |x_{ij} - t_j|}, \quad (8)$$

$$y_{ij} = |x_{ij} - o_j| - \frac{\min_i |x_{ij} - o_j|}{\max_i |x_{ij} - o_j| - \min_i |x_{ij} - o_j|}, \quad (9)$$

$$y_{ij} = \begin{cases} 1 - \frac{\max(o_1^j - x_{ij}, x_{ij} - o_2^j)}{\max[o_1^j - \min(x_{ij}), \max(x_{ij}) - o_2^j]}, & x_{ij} \notin [o_1^j, o_2^j], \\ 1, & x_{ij} \in [o_1^j, o_2^j], \end{cases} \quad (10)$$

where x_{ij} represents the original attribute data and y_{ij} represents the normalized attribute value. Equations (8)–(11) represent the attribute value standardization methods of fixed type, deviation type, interval type, and deviation interval type, respectively:

$$y_{ij} = \begin{cases} \frac{\max(o_1^j - x_{ij}, x_{ij} - o_2^j)}{\max[o_1^j - \min(x_{ij}), \max(x_{ij}) - o_2^j]}, & x_{ij} \notin [o_1^j, o_2^j], \\ 0, & x_{ij} \in [o_1^j, o_2^j]. \end{cases} \quad (11)$$

4. Combined Weighting Model

4.1. Maximizing Deviation. According to the principle of information theory, if a certain attribute makes no significant difference in decision objects at each row level, the ranking of multiattribute decision-making evaluation objects at different levels, independent of standards. If the attribute is a decision object with great difference, this attribute contributes greatly to the decision results and plays an important role [43, 44].

The variance of an attribute can be used to express the deviation influence of an attribute on the decision object. For attribute f_q , $\text{MAX}_{pq}(e)$ represents the deviation between different evaluation objects:

$$\text{MAX}_{pq}(e) = \sum_{k=1}^n |f_{pq}e_q - \overline{f_{kq}}w_q|. \quad (12)$$

Define total deviation $\text{TMAX}_q(e)$:

$$\text{TMAX}_q(e) = \text{MAX}_{pq}(e) = \sum_{p=1}^m \sum_{k=1}^n |f_{ipq} - \overline{f_{kq}}|e_q, \quad (13)$$

where e is the weight coefficient, and $e = (e_1, e_2, \dots, e_n)^T > 0$, which satisfies the unitization constraint:

$$\sum_{q=1}^m e_q^2 = 1. \quad (14)$$

Then, the objective function can be defined as

$$\max \text{TMAX}(e) = \sum_{q=1}^m \text{TMAX}_q(e) = \sum_{q=1}^m \sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}|e_q. \quad (15)$$

Combining equations (14) and (15), the problem of solving e_q is the problem of solving the most optimal solution of the equations. Solve this joint model:

$$\begin{cases} \max & \text{TMAX}(e) = \sum_{q=1}^m \sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}|e_q, \\ \text{s.t.} & e_q \geq 0, \quad q = 1, 2, \dots, m, \quad \sum_{q=1}^m e_q^2 = 1. \end{cases} \quad (16)$$

The maximum point of the objective function can be obtained, w'_j :

$$e'_q = \frac{\sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}|}{\sum_{q=1}^m \left[\sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}| \right]}, \quad q = 1, 2, \dots, m. \quad (17)$$

The traditional weight coefficient can be defined as

$$e_q = \frac{e'_q}{\sum_{q=1}^m e'_q}, \quad q = 1, 2, \dots, m. \quad (18)$$

After normalization, the following results can be obtained:

$$e_q = \frac{\sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}|}{\sum_{q=1}^m \left[\sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}| \right]}, \quad q = 1, 2, \dots, m. \quad (19)$$

This method needs to normalize the obtained weight information, which is applicable to the case where the weight is completely unknown. When some weight information is known, it cannot be considered, which has a certain impact on the accuracy of decision results. Therefore, the variance objective function can be constructed to calculate the weight by calculating the square of the deviation of decision objectives:

$$\begin{aligned} t\sigma_q(e) &= \sigma_{pq}(e) \\ &= \sum_{p=1}^n \sum_{k=1}^n |e_q f_{pq} - e_q \overline{f_{kq}}|^2. \end{aligned} \quad (20)$$

The following optimization problem is constructed:

$$\begin{cases} \max & \sigma(e) = \sum_{q=1}^m \sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}|^2 e_q^2, \\ \text{s.t.} & w_q \geq 0, \quad q = 1, 2, \dots, m, \quad \sum_{q=1}^m e = 1. \end{cases} \quad (21)$$

We introduce Lagrange function to solve this model:

$$\sigma(e, g) = \sum_{q=1}^m \sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}|^2 e_q^2 + 2g \left(\sum_{q=1}^m e_q - 1 \right), \quad (22)$$

where g is the Lagrange operator. Find its partial derivative and let

$$\begin{cases} \frac{\partial \sigma}{\partial g} = 2 \sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}| + 2g = 0, & q = 1, 2, \dots, m, \\ \frac{\partial \sigma}{\partial g} = \sum_{q=1}^m e_q - 1 = 0. \end{cases} \quad (23)$$

Finally, the optimal weight equation can be obtained:

$$e_q = \frac{1}{\sum_{q=1}^m 1 / \sum_{p=1}^n \sum_{k=1}^n |f_{pq} - \overline{f_{kq}}|^2}, \quad q = 1, 2, \dots, m. \quad (24)$$

If the influence degree of an attribute can be determined, for example, the influence degree of an attribute is very small, but the influence degree is calculated to be large when taking the actual value, then the attribute can be limited to $0 \leq e_{1q} \leq e_q \leq e_{2q}$ according to experience, where e_{1q} and e_{2q} are the lower and upper limits of e_q , respectively, and the weight vector can be solved by solving the following linear programming model:

$$\begin{cases} \max & td(e) = \sum_{q=1}^m \sum_{p=1}^n \sum_{k=1}^n (f_{pq} - \overline{f_{kq}})^2 e_q \\ \text{s.t.} & 0 \leq e_{1q} \leq e_q \leq e_{2q}, \sum_{q=1}^m e_q = 1. \end{cases} \quad (25)$$

4.2. Information Entropy. Different from the ranking method, the pairwise comparison method, and the variance maximization method, the entropy-based weight solution method is an objective solution method, which does not need to be based on the experience of decision-making experts. This method is mainly calculated based on information entropy [45, 46]. Entropy can be used to measure information, so the weight of attributes can be considered as continuous information. We can calculate the weight of attributes f_{qj} by calculating the content of information in each attribute. The amount of information is measured by entropy, which is defined as E_j

$$\begin{cases} E_j = -\frac{\sum_{q=1}^m P_{qj} \ln(P_{qj})}{\ln(m)}, \\ P_{qj} = \frac{f_{qj}}{\sum_{q=1}^m f_{qj}}. \end{cases} \quad (26)$$

The degree of diversity of information contained in a set of attributes can be defined as b_j :

$$b_j = 1 - E_j. \quad (27)$$

The attribute weight based on entropy can be defined as

$$e_{E_j} = \frac{b_j}{\sum_{j=1}^n b_j}. \quad (28)$$

The weight based on entropy can be combined with the weight e_j obtained by other methods to obtain a new definition of weight:

$$e'_{E_j} = \frac{e_{E_j} e_j}{\sum_{j=1}^n e_{E_j} e_j}. \quad (29)$$

Obviously, the range of e_{E_j} and e'_{E_j} is between 0 and 1. The more kinds of information the attribute contains, the higher its value is. The smaller the entropy value of an attribute, the greater its diversity, which means that this attribute provides more information. If an attribute is uniform, that is, it is a fixed value and its attribute weight is 0, this attribute cannot be used as a decision attribute because it does not transmit information about the decision. The weight determination method based on entropy is an effective multiscale analysis method.

4.3. Combined Weighting Model. Theoretically, in multi-attribute decision-making, the most important attribute does not necessarily make the attribute values of all decision-making schemes have the greatest difference, but the least important attribute may make the attribute values of all decision-making schemes have great differences. In this way, when determining the weight according to the objective weighting method, the least important attribute may have the largest weight, but the most important attribute may not have the largest weight. Moreover, this weighting method depends on the actual problem domain, so the universality and participation of decision makers are poor, the subjective intention of decision-makers is not considered, and most calculation methods are complex. This study adopts the combined weighting method:

$$w_{ci} = sw_s + (1-s)w_o, \quad (30)$$

where w_{ci} represents the combined weight of the i^{th} index and w_s and w_o are the objective weight and subjective weight of each attribute, respectively. The combination of the former is essentially the normalization of multiplication synthesis. This method is used in the case of large number of indicators and uniform weight distribution. The latter is essentially linear weighting, which is called the linear weighted combination weighting method. When the decision maker has a preference for different weighting methods, it can be determined according to the decision-maker's preference information.

5. Conclusion

Aiming at the multiattribute decision-making problem in which the attribute weight is completely unknown or the weight information is partially determined, a combined weighting method based on variance maximization and information entropy is proposed in this study. These two methods not only avoid the difficulty of obtaining preference information but also make full use of the prior information of standardized evaluation. When the decision maker has a preference for different weighting methods, it can be determined according to the decision-maker's preference information. The evaluation results are objective and reliable, and easy to implement on computer.

The advantages and disadvantages of subjective and objective weighting methods are analyzed, and a combined weighting method based on deviation maximization and entropy is proposed. This method overcomes the shortcomings of the single use of the subjective or objective weighting method and avoids the phenomenon that using the maximum sum of deviation squares to allocate weights will amplify the weight difference.

In the future work, the sensitivity of index weight ranking will be studied. And the effects of the evaluation value on the objective weight and the linear representation coefficient of combined weight will be proposed. By deeply excavating the mechanism, the application value of the method will be further improved.

Data Availability

The datasets generated during the current study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare they have no conflicts of interest.

Authors' Contributions

The manuscript is approved by all authors for publication.

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