Research Article

Factors Influencing Piezoelectric Response of Horizontally and Vertically Embedded PZT Patch in Confined Granular Fills

Nisha Kumari and Ashutosh Trivedi

1Research Scholar, Civil Engineering Department, Delhi Technological University, Delhi-110042, India
2Civil Engineering Department, Delhi Technological University, Delhi-11042, India

Correspondence should be addressed to Nisha Kumari; nishasoni.ce@gmail.com

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Abstract

The energy harvesting from ambient vibrations in the confined granular fill along the expressways and highways is a prospective power source for varied engineering applications. This paper presents the study of charge density, voltage output, and power from the PZT (lead zirconate titanate) patches embedded in dynamically loaded confined granular fill. The effect of the alignment of PZT patches, thickness ratio, material properties of the confined granular fill, and retaining structure on the voltage output is investigated. It provides the scope for the evaluation of voltage output directly from the stress-strain response, position of the PZT patch, and the engineering properties of the confined granular fill. The alignment of PZT patches in the horizontal and vertical directions has been examined analytically to optimize voltage outputs. The results indicate that the modulus ratio of the material, alignment of PZT patches, and gradation of infill material significantly affect the voltage generation. A relationship for voltage and power output has been proposed for a set of engineering applications. The observed voltage output is found appropriate for wide-ranging implements classified as low- (LRA), medium- (MRA), and high- (HRA) resistance applications. It is proposed to be up-scaled using multiple patches embedded throughout the confined granular fill and pavements subjected to continuous dynamic loads.

1. Introduction

The dynamically loaded transportation infrastructures find a unique application for energy harvesting and structural health monitoring due to the profound developments in sensor technology. This energy can be captured using a suitable mechanism for various micro- and macro-engineering operations, including electrical devices and transportation as a renewable energy system [1]. The piezoelectric material provides higher energy density and flexibility in being integrated into a system than the electromagnetic and electrostatic units [2, 3]. Many researchers [4–6] have analysed the effectiveness of embedded PZT (lead zirconate titanate) patches for energy harvesting and health monitoring structures.

Many studies [7–9] considered vibration-based energy harvesting using piezoelectric material for civil infrastructure systems due to dynamic loads of traffic movements. An analytical expression has been proposed for a small PZT patch attached to the surface of a bridge structure. The Hamiltonian principle was considered to account for all the vibration modes of the structure and pointed out that the surface strain fluctuations influence voltage output [10]. The electrical energy generation was investigated in a prestressed tuneable piezoelectric beam harvester from a bridge’s vibrations under ambient loading conditions [11]. They concluded that the maximum power generation from the PZT patch is a function of the excitation frequency of the dynamic loads. Experimental validation for a rail bridge in their operational condition for energy harvesting from the random vibrations of the built infrastructure within the laboratory environment was presented where the theoretical predictions were less than the experimental output [12]. The feasibility of energy harvesting from friction-induced vibration was associated with stronger vibrations [13]. A few researchers [14, 15] have analysed the performance and
structural response of the pavement with an embedded piezoelectric energy harvester. The structural response of the asphalt pavement embedded with a piezoelectric energy harvester (PEH) provided a couple of optimization solutions for design [16]. Table 1 presents a brief review of specific power production capabilities of alternative energy harvesting sources, including PZT patches. It shows the vast potential of PZT patches for various micro- and macro-engineering operations.

Few investigators performed field observation for the piezoelectric cantilever installed above the pavement and embedded underground [17]. They found that a significant amount of electrical energy is generated from the embedded piezoelectric cantilever. The stress on the surface influences the piezoelectric effect of the PZT patches. The voltage polarization of the PZT patch is significantly affected by its embedment depth and orientation. It was found that the vertical compressive stress is decreased when depth is increased effectively [18]. The performance of the PZT patches for energy harvesting from the pavement combines the stiffness and load pattern. Comparing the impact of the dynamic loads, the influence of the pavement thickness on the power generation is negligible [19].

The literature review indicates that the application of piezoelectric material in civil structures is an abundant source of energy. The existing experimental and numerical analysis usually focuses on the methods of piezoelectric energy from various structural elements, namely beams, bridges, roads, and rail tracks. By contrast, there are no significant studies on the stress response of confined granular fill on power generation due to vibrations. The PZT patches embedded in the pavement undergo reversal in the voltage polarity for both horizontal and vertical faces [18]. Therefore, the piezoelectric power output is not necessarily higher for the stronger vibrations. The power output depends on the magnitude of strain generation due to vertical and horizontal vibrations. From previous analytical studies [18, 38], it appears that the resultant vertical stresses and horizontal stresses are affected by the placement of the PZT patch on the retaining face compared to the horizontal pavement structure. Similar observations were made [39] for the increasing depth of embedment of the PZT patch. The analytical study helps us to identify the set of design parameters that can improve the efficiency of the piezoelectric energy output. The efficiency of charge output depends upon the magnitude of the force excitation, the material characteristics, and the electromechanical coupling of the piezoelectric material structure [40, 41]. The piezoelectric response of the PZT patch can be optimized by changing the mechanical properties through variation in the modulus [42].

The objective of this study is a parametric investigation of the factors influencing the piezoelectric voltage generation considering the installation position of the PZT patch, and geometric and mechanical properties of the embedded PZT patch in confined granular fill. The confined granular fill is contained by the pavement and the retaining structure. The confined granular fill experiences vibrations due to dynamic loads. The strain fluctuation is absorbed by the PZT patch, which polarized and induced a voltage. This deformation per unit length is analysed by the stress-strain response of the confined granular fill due to the ambient vibration. The deformation of the PZT patch is used to calculate the charge density and voltage output. The model that consists of confined granular fill with embedded PZT patches is analysed theoretically. The electromechanical process is used to convert mechanical energy into electrical energy. The charge density and voltage output from the PZT patch are discussed with a focus on the influence of the embedded position, piezoelectric properties, geometric properties of the PZT patch, and confined granular fill.

The present study evaluates the charge density and voltage output over the permittivity, placement of the PZT patch, depth of embedment, and geometric properties of the retaining structure. Figure 1 shows a flow chart of the conversion of mechanical energy into electrical energy from the confined granular fill. The mechanical vibration of dynamic loads for energy harvesting consists of a loading system, which transfers the vibration waves through the mechanical units. The mechanical unit comprises a pavement with a PZT patch embedded horizontally and vertically over the granular fill retained by a retaining wall. Besides the energy generation, the influence of the material properties on the polarization of the PZT patch is a vital engineering parameter influencing charge density.

The results show the geometric properties of the retaining structure and mounted position of the PZT patch on the transformation of mechanical into electrical energy through the change in charge density and voltage output. It has been demonstrated that the piezoelectric response of a PZT patch embedded in the confined granular fill can be tuned through the changes in its thickness and stiffness. A comparison of output voltage among the hard, semihard, and soft PZT patches based on piezoelectric material properties has been made. Based on the theoretical analysis, the influence of different alignments and stiffness of the PZT patch on the output energy is captured.

2. Engineering Properties of PZT Patches

When piezoelectric materials are subjected to mechanical stress, electrical energy generated in proportion to that stress is known as the direct piezoelectric effect. The direct effect occurs through the compression of piezoelectric material, as shown in Figure 2. \( D_3 \) is the electric displacement on the PZT patch, \( E_3 \) is the electric field in direction-3, \( d_{31} \) is the piezoelectric strain coefficient in mode 31, \( \varepsilon^T_{33} \) represents the permittivity of the material in the direction-33 under conditions of constant mechanical stress \( \sigma_{33}^{T_{33}} \) (corresponds to free permittivity), and \( \sigma_{PZT} \) is stress in the PZT layer.

Piezoelectric energy output is commonly based on two coupling modes: 33-mode and 31-mode, as shown in Figures 3–4. The coupling mode defines the capability of piezoelectric material to transform mechanical energy into electrical energy. Figure 3 shows a 33-mode PZT patch where the external stress direction generated an electric field in the same direction. Figure 4 shows 31-mode where the applied
stress is axial and perpendicular to the direction of the electric field.

The parameters influencing the voltage output from the PZT patches are relative permittivity ($\varepsilon_{33}/\varepsilon_0$) (here, $\varepsilon_0$ refers to the absolute permittivity of the vacuum), piezoelectric strain constant ($d$), polarization ratio ($d_{31}/\varepsilon_{33}$), and piezoelectric voltage coefficient ($g$). The charge density of the PZT patch is the most important consideration when choosing a piezoelectric material for energy harvesting. The product of the effective piezoelectric strain constant ($d$) and the effective piezoelectric voltage constant ($g$) determines the magnitude of the charge density [44]. The PZT material considerations are classified as the hard, semihard, and soft PZT patches, as shown in Table 2. The piezoelectric material considerations are:

Table 1: Energy output capacities of varied system configurations.

<table>
<thead>
<tr>
<th>System configuration</th>
<th>Energy output</th>
<th>Estimated cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typically a dynamic loading from the busy expressway$^a$</td>
<td>188 kW</td>
<td>$0.08-0.18/kWh</td>
</tr>
<tr>
<td>PZT system paved in roadway section with very high traffic volume$^b$</td>
<td>4.04 MWh</td>
<td>$1.77/kWh</td>
</tr>
<tr>
<td>Fan-type windmill using induced resonance$^c$</td>
<td>613 $\mu$W</td>
<td>—</td>
</tr>
<tr>
<td>Piezoelectric cantilever embedded in small fan windmill$^d$</td>
<td>363 $\mu$W</td>
<td>—</td>
</tr>
<tr>
<td>Aero-elastic energy harvesting using the piezoelectric transducer from wind tunnel$^e$</td>
<td>27 mw</td>
<td>—</td>
</tr>
<tr>
<td>Dynamic loading from the ocean waves$^f$</td>
<td>60–180 mw</td>
<td>—</td>
</tr>
<tr>
<td>Hydraulic pressure in the hydraulic systems$^g$</td>
<td>1.2 MW</td>
<td>—</td>
</tr>
<tr>
<td>High-rise buildings due to wind and earthquake$^h$</td>
<td>432.21 MW</td>
<td>—</td>
</tr>
<tr>
<td>Building due to pedestrian traffic$^i$</td>
<td>1.1 MWh</td>
<td>$3850/\text{per tile}$</td>
</tr>
<tr>
<td>A vibration-based system installed on bridge cable$^j$</td>
<td>35.67 mW</td>
<td>$80-85/kWh</td>
</tr>
<tr>
<td>Electromagnetic energy harvester from low-frequency mechanical vibrations$^k$</td>
<td>554.7 $\mu$W</td>
<td>—</td>
</tr>
<tr>
<td>Electromagnetic generator utilized the low ambient vibrations$^l$</td>
<td>46 $\mu$W</td>
<td>—</td>
</tr>
<tr>
<td>Oscillation motion of human body and vibration of machines$^m$</td>
<td>95 mW</td>
<td>—</td>
</tr>
<tr>
<td>Power generation utilizing the vibration of moving cars$^n$</td>
<td>3.9 mW</td>
<td>—</td>
</tr>
<tr>
<td>Pavement system supported by solar panels$^o$</td>
<td>1718 MWh</td>
<td>$0.05-0.075/kWh</td>
</tr>
<tr>
<td>PV panels mounted 33-bus distribution network$^p$</td>
<td>14 kW</td>
<td>—</td>
</tr>
<tr>
<td>Households and village power stations$^q$</td>
<td>200–2500 W</td>
<td>$10-$12/W</td>
</tr>
<tr>
<td>Photovoltaic noise barriers with motorways and railway tracks$^r$</td>
<td>800 MWh</td>
<td>—</td>
</tr>
</tbody>
</table>

$^a$Hill et al. [20]; $^b$Guo and Lu [21]; $^c$Yang et al. [22]; $^d$Rezaei-Hosseinabadi et al. [23]; $^e$Sousa et al. [24]; $^f$Murray and Rastegar [25]; $^g$Cunefare et al. [26]; $^h$Xie et al. [27]; $^i$Li and Strezov [28]; $^j$Kim et al. [29]; $^k$Zorlu et al. [30]; $^l$Beeby et al. [31]; $^m$Sasaki et al. [32]; $^n$Glynne-Jones et al. [33]; $^o$Wang et al. [34]; $^p$Jiang et al. [35]; $^q$Zahedi [36]; $^r$Nordmann et al. [37].
for engineering applications include piezoelectric strain constant, piezoelectric voltage constant, and mechanical quality factor ($Q$). The relative permittivity of the selected PZT patches (Table 2) was 1250, 1450, and 2400, while the polarization ratio was 0.112, 0.114, and 0.087, respectively. The polarization ratio is defined as the ratio of piezoelectric strain constant and relative permittivity.

2.1. Stress-Strain and Charge Density. The stress-strain response of the PZT patch embedded in the confined granular fill has been obtained in terms of charge density and voltage using a systematic numerical approach. The position of the PZT patch and its geometric properties influences the magnitude of deformation as described in the following sections.

2.2. Stress-Strain Behaviour. The stress-strain behaviour of the PZT patch has been considered for horizontal and vertical alignments. Initially, a PZT patch is horizontally and vertically embedded in the confined granular fill (Figure 5(a)). It has been assumed that the neutral plane of the PZT patch does not coincide with the geometrical centre of the fill. There is a negligible shear movement or separation among the retaining structure, PZT patch, and granular fill. The retaining structure is considered flexible and elastically constrained at the base. The shape of the useful part of the granular fill is considered a triangular wedge as shown in Figure 5(b). The vibrating system consists of a wall of height $H$, length $L$ (such that $L = 1$), and backfill of thickness $h_R$. The thickness of PZT patch is $h_P$. As the result of the vibration, the stresses are generated in the pavement, granular fill, and PZT patch.

The deformation in the horizontally embedded PZT patch is a function of PZT material properties, bending moment, the width of backfill, modulus ratio, and thickness ratio (Appendix A). The strain in the horizontally embedded PZT patch [45] is expressed as

$$
\varepsilon_{HE} = f(M, b_S E_s, h_R, \mu, n) = \frac{36M}{b_S E_s h_R^2} \left[ \frac{(1 + \mu)(1 + \mu n)}{3\mu^2 n^3 + 6\mu^3 n^2 + 5\mu^4 n + \mu^2 n^2 + \mu^2 + 4\mu n + 2} \right].
$$

The stress in the horizontally embedded patch is a function of strain and modulus of the PZT and is expressed as

$$
\sigma_{HE} = \varepsilon_{HE} \ast E_P = \frac{36M}{b_S h_R^2} \left[ \frac{n(1 + \mu)(1 + \mu n)}{3\mu^2 n^3 + 6\mu^3 n^2 + 5\mu^4 n + \mu^2 n^2 + \mu^2 + 4\mu n + 2} \right],
$$

\[\text{Figure 2: Flow chart for the generation of electric charge due to mechanical vibrations.}\]

\[\text{Figure 3: Mode of operation for the PZT patch (33-mode).}\]

\[\text{Figure 4: Mode of operation for the PZT patch (31-mode).}\]
where \( \varepsilon_{HE} \) and \( \sigma_{HE} \) are the strain and the stress in a horizontally embedded PZT patch. \( E_S \) and \( E_P \) are Young’s modulus of backfill and PZT patch, respectively; \( \mu \) is the ratio of the thickness of the PZT patch to the backfill \( h_P/h_R \); and \( n \) is the ratio of the modulus of the PZT patch to the backfill \( (E_P/E_R) \), and \( M \) is the bending moment PZT patch.

The strain in the vertically embedded PZT patch is the function of bending moment, the height of the retaining structure, modulus ratio, height of the backfill, and thickness ratio [46] is expressed as

\[
\varepsilon_{VE} = \int \left( M, H, h_S, a, b, \mu, n \right) = \frac{36M}{E_s H h_s^3} \left[ \frac{3\mu + 2 - 12a^2b\mu^2n}{24\mu n(a + b) + 1} + 18(3\mu + 2 - 12a^2b\mu^2n) \right].
\]

Similarly, the stress for vertically embedded PZT patch between the granular fill and retaining wall is expressed as

\[
\sigma_{VE} = \varepsilon_{VE} \times E_P = \frac{36nM}{H h_s^2} \left[ \frac{3\mu + 2 - 12a^2b\mu^2n}{24\mu n(a + b) + 1} + 18(3\mu + 2 - 12a^2b\mu^2n) \right].
\]

where \( \varepsilon_{VE} \) and \( \sigma_{VE} \) are the strain and stress in vertically embedded PZT patch, \( a \) is the thickness ratio of retaining wall and PZT patch \( (h_w/h_p) \), and \( b \) is the modulus ratio of retaining wall and PZT patch \( (E_w/E_p) \).

### Table 2: Engineering properties of piezoelectric patches.

<table>
<thead>
<tr>
<th>Engineering properties</th>
<th>Symbol</th>
<th>Hard patch; PZT-141</th>
<th>Semihard; PZT-155</th>
<th>Soft patch; PZT-151</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>( \rho )</td>
<td>7800</td>
<td>7800</td>
<td>7800</td>
</tr>
<tr>
<td>Modulus of PZT patch (GPa)</td>
<td>( E_P )</td>
<td>81</td>
<td>63</td>
<td>60</td>
</tr>
<tr>
<td>Relative permittivity(^a)</td>
<td>( \varepsilon_{ij}/\varepsilon_0 )</td>
<td>1250</td>
<td>1450</td>
<td>2400</td>
</tr>
<tr>
<td>Dielectric loss factor(^a)</td>
<td>( \tan \delta )</td>
<td>5 \times 10(^{-3} )</td>
<td>20 \times 10(^{-3} )</td>
<td>20 \times 10(^{-3} )</td>
</tr>
<tr>
<td>Piezoelectric strain constant(^a) (C/N)</td>
<td>( d_{ij} )</td>
<td>(-140 \times 10(^{-12} )</td>
<td>(-165 \times 10(^{-12} )</td>
<td>(-210 \times 10(^{-12} )</td>
</tr>
<tr>
<td>Piezoelectric voltage coefficient(^a) (Vm/N)</td>
<td>( g_{ij} )</td>
<td>(-13.1 \times 10(^{-3} )</td>
<td>12.9 \times 10(^{-3} )</td>
<td>(-11.5 \times 10(^{-3} )</td>
</tr>
<tr>
<td>Polarization ratio(^b)</td>
<td>( d_{31}/\varepsilon_{33} )</td>
<td>0.112</td>
<td>0.114</td>
<td>0.087</td>
</tr>
</tbody>
</table>

\(^a\)Physik Instrumente GmbH [43]; \(^b\) present work.
2.3. Charge Density for the Horizontally Embedded PZT Patch. The charge density equations for the hard, semihard, and soft PZT patches embedded horizontally have been derived from the stress-strain relationship of the PZT patch subject to vibrations in the confined granular fill. A single layer of PZT patch horizontally embedded between the pavement and the granular fill is shown in Figure 5(a). The deflection induces curvature in the triangular wedge at an arbitrary point on the neutral axis due to bending. The power output and the dynamic response of the piezoelectric system entirely depend on the coupling mode of the PZT patch. The researchers [47, 48] have identified that the natural way of excitation of retaining structure is in d31 mode. Therefore, the d31 mode has been chosen in the present work for analysis. For the so configured system, the tensorial representation of the piezoelectric constitutive equation [49] gives the electric displacement as

\[ D_3 = d_{31} \sigma_{PZT} + \epsilon^T_{33} E_3. \] (5)

The stress generated in the PZT patch due to the dynamic loads on the confined granular fill changes the polarization and the displacement on the upper and lower surface of the PZT patch. As there is no external electric source (\( E_3 = 0 \)), the electric displacement in the PZT patch is expressed as

\[ D_3 = d_{31} \sigma_{PZT}. \] (6)

The charge density function at the top and bottom surfaces of the horizontally embedded PZT patch is expressed as

\[ \rho_{\text{THE}} = D_3 \left( Z, h_P + \frac{h_R}{3} \right), \] (7)

\[ \rho_{\text{BHE}} = D_3 \left( Z, h_P \right), \] (8)

where \( \rho_{\text{THE}} \) and \( \rho_{\text{BHE}} \) are charge density functions at the top and bottom surfaces of the horizontally embedded PZT patch, respectively, and \( D_3 \) is electric displacement according to the piezoelectric coupling constitutive equation, and \( Z \) is the thickness direction coordinate from the neutral plane as shown in Figure 5(b).

From equations (6) and (7), the charge density is expressed as

\[ Q_{\text{THE}} = d_{31} \int_0^{(h_R + (h_P/3))} \sigma_{B} dz, \] (9)

\[ Q_{\text{THE}} = \frac{12d_{31} M}{bh_R} \left[ \frac{n(1 + \mu)(1 + \mu n)(3\mu + 1)}{3\mu^2 n^3 + 6\mu^2 n^2 + 5\mu^2 n + \mu^2 + \mu^2 + 4\mu + 2} \right], \] (10)

\[ Q_{\text{BHE}} = -d_{31} \int_0^{(h_P)} \sigma_{B} dz, \] (11)

\[ Q_{\text{BHE}} = -\frac{12d_{31} M}{bh_R} \left[ \frac{n\mu(1 + \mu)(1 + \mu n)}{3\mu^2 n^3 + 6\mu^2 n^2 + 5\mu^2 n + \mu^2 + \mu^2 + 4\mu + 2} \right]. \] (12)

\[ e = \frac{h_P [6a + 1 - 6a^2]}{18a + 12}, \] (14)

\[ \rho_{\text{BVE}} = D_3 \left( Z, h_P \right), \] (15)

where \( \rho_{\text{THE}} \) and \( \rho_{\text{BVE}} \) are the charge density function at the top and bottom surfaces of the vertically embedded PZT patch in the confined granular fill, \( h_P \) is the thickness of the PZT patch, \( a \) is the thickness ratio of the retaining wall and the PZT patch \( (h_W/h_P) \), \( D_3 \) is electric displacement according to the constitutive equation of the piezoelectric coupling, \( Z \) is the thickness direction coordinate from the neutral plane, and \( e \) is the distance between the neutral plan and bottom surface of the PZT patch as shown in Figure 5(b).

The charge density (equations (13) and (14)) is expressed as

2.4. Charge Density for the Vertically Embedded PZT Patch. The charge density determines the dipole mobility of the charges on the PZT surface and affects the efficiency of power generation. The charge density function at the top and bottom surfaces of the vertically embedded PZT patch is expressed as

\[ \rho_{\text{TEV}} = D_3 \left( Z, h_P + e \right), \] (13)

where \( e \) is the distance between the bottom surface of PZT and the centre of gravity of the granular backfill, and is expressed as

\[ \rho_{\text{BVE}} = D_3 \left( Z, h_P \right). \] (15)
\[ Q_{TVE} = d_{31} \int_{0}^{(h_r+e)} \sigma_d dz, \]

\[ Q_{TVE} = \frac{36d_{31}nM}{hah_z} \left[ \frac{(3\mu + 2 - 12a^2by^2n)(24\mu n(a + b) + 1)(6a + 1 - 6a^2)}{(18a + 12)[(3\mu y^2(1 + ba^2) + 1)(24\mu n(a + b) + 1)^2 + 18(3\mu + 2 - 12a^2by^2n)(2\mu a (1 + ab) + 1)]} \right], \]

\[ Q_{BVE} = -d_{31} \int_{0}^{(e)} \sigma_d dz, \]

\[ Q_{BVE} = -\frac{36d_{31}nM}{hah_z} \left[ \frac{(3\mu + 2 - 12a^2by^2n)(24\mu n(a + b) + 1)(6a^2 + 12a + 11)}{(18a + 12)[(3\mu y^2(1 + ba^2) + 1)(24\mu n(a + b) + 1)^2 + 18(3\mu + 2 - 12a^2by^2n)(2\mu a (1 + ab) + 1)]} \right]. \]

Q_{TVE} and Q_{BVE} are the generated charge density at the top and bottom surfaces of the vertically embedded PZT patch, respectively.

### 2.5. Voltage Output for Horizontally and Vertically Embedded PZT Patches

Due to the phase difference, the charge developed at the opposite side of the PZT patch is collected by a separate electrode so that charge cancellation due to the phase difference in the electric displacement is prevented.

Since the charge quantity of the top and bottom surfaces is not the same, the voltage between the two poles is expressed as

\[ V = \int_{e}^{(h_r+e)} E(x, z) dz, \quad (20) \]

\[ E = \frac{d_{31}\sigma_p}{\varepsilon_{33}}. \quad (21) \]

where \( V \) is the voltage between the top and bottom surfaces of the PZT patch due to vibration of the system, \( E \) is the equivalent electric field intensity, \( \sigma_p \) is stress in the PZT patch, and \( \varepsilon_{33}^{T}/\varepsilon_{33} \) is relative permittivity. For a horizontally embedded PZT patch, the voltage output between the top and bottom surfaces of the PZT patch is expressed as

\[ V_{HE} = \frac{h_r^2d_{31}E_p(3\mu + 2)}{12\varepsilon_{33}\rho}, \quad (22) \]

\[ V_{HE} = \frac{3Md_{31}}{b\varepsilon_{33}} \left[ -n\mu(1 + \mu)(1 + \mu)(2 + 3\mu) \right] \left( 3\mu^3 n^2 + 6\mu n^2 + 5\mu^2 n + \mu^2 n^2 + \mu^2 + 4\mu n + 2 \right), \quad (23) \]

where \( V_{HE} \) is the voltage output for a horizontally embedded PZT patch in confined granular fill, \( d_{31} \) is the piezoelectric strain constant for 31-mode, and \( E_p \) is the elastic modulus of the PZT patch. Similarly, for a vertically embedded PZT patch, the voltage output between the top and bottom surfaces of the PZT patch is expressed as

\[ V_{VE} = \frac{h_r^2d_{31}E_p\mu^2(-6\mu^2 + 15\mu + 13)}{2\varepsilon_{33}\rho(18a + 12)}, \quad (24) \]

\[ V_{VE} = \frac{18n\varepsilon_{33}Md_{31}}{H\varepsilon_{33}} \left[ \frac{\mu^2(3\mu + 2 - 12\mu^2by^2n)(24\mu n(a + b) + 1)(-6\mu^2 + 15\mu + 13)}{(18a + 12)[(3\mu y^2(1 + ba^2) + 1)(24\mu n(a + b) + 1)^2 + 18(3\mu + 2 - 12\mu^2by^2n)(2\mu a (1 + ab) + 1)]} \right], \quad (25) \]

where \( V_{VE} \) is the voltage output of vertically embedded PZT patch, \( H \) is the height of the retaining structure, \( a \) and \( b \) denote the thickness ratio of retaining wall with PZT patch and the modulus ratio of retaining wall with PZT patch, respectively.

### 3. Results and Discussion

This section provides the stress-response results on power generation from the ambient vibration in retaining the structure of height \( H \) with horizontally and vertically embedded PZT patches. The charge density and voltage output are obtained, which are significantly affected by the
stress state of the PZT patch. The stress-strain response of horizontally and vertically embedded PZT patches is analysed using equations (1)–(4). The effects of stiffness, polarization per unit permittivity, and geometric properties of the PZT patch have been obtained by equations (5)–(25). It is considered difficult to embed the PZT on the upper surface of the pavement (environmental, abrasion, and iteration effects). Therefore, it is recommended to embed the PZT at depth (d). Accordingly, the stiffness of the base course layer has been considered as 1–4 GPa [39].

The resilient modulus of hardened concrete is reported in the range of 10–30 GPa, while that of aggregate is between 45 and 85 GPa. Hence, the range of values (1–5 GPa), which are well below the hardened concrete, is used in this study.

The strain on the PZT patch is evaluated for varying thickness of the granular backfill. The comparative analysis is shown for varied piezoelectric materials, namely hard (H), semihard (SH), and soft (S) patches. To perform the technical analysis, the most commonly used parameters of the PZT patches, namely the relative permittivity ($\varepsilon_{33}/\varepsilon_0$), piezoelectric strain constant ($d$), piezoelectric voltage coefficient (g), and polarization ratio, are considered as mentioned in Table 2. The modulus of the PZT material varies with the temperature and thickness of the material. The researchers [50, 51] suggested that piezoelectric material has a stiffness of the order of 50–90 GPa. Therefore, to analyse the effect of the modulus ratio, the modulus of the hard, semihard, and soft PZT patch has been taken as 81, 63, and 60 GPa, respectively. The charge density and voltage output have been analysed for hard, semihard, and soft PZT patches.

4. Effect of Stiffness of Confined Granular Fill, Retaining Structure, and PZT Patch

The stiffness is the function of the geometry and the thickness of the material. The stress-strain response has been analysed by considering equations (2)–(4). Figure 6 shows the stress variation of the horizontally embedded PZT patch for varying thickness ratios considered in the range of 0.01 ≤ $\mu$ ≤ 0.20. The PZT patches are commercially available in a range of thicknesses from 0.20 to 20 mm (piezoelectric ceramic product 2011). The thickness ratio range is selected to capture the effect of the thickness of the PZT patch and granular fill. The thickness of the PZT patch was kept 10 mm while that of granular fill was 5–100 cm leading to a thickness ratio of 0.01 to 0.20.

From equation (16), it is observed that charge density is directly proportioned to the stress. Thus, the phenomenon indicates that the thickness of the retaining structure is a vital parameter for voltage output. Similarly, Figure 7 shows the stress response for the vertically embedded PZT patch for thickness ratio in the range of 0.01 ≤ $\mu$ ≤ 0.20. The negative values show the change in the nature of the stress. The change in sign indicates the compression and tension response of the stress. When a thin PZT patch has been embedded vertically, then the stress shows the opposite trend. It shows that the thickness of the surrounded granular
Figure 7: Stress variation with modulus ratio for the vertically embedded PZT patch in the confined granular fill computed for modulus ratio, the width of backfill, the width of retaining structure, and thickness ratio.

Figure 8: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the top surface of the horizontally and vertically embedded hard PZT patch in the confined granular fill.
Figure 9: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the bottom surface of the horizontally and vertically embedded hard PZT patch in the confined granular fill.

Figure 10: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the top surface of the horizontally embedded and from for vertically embedded hard PZT patch in the confined granular fill.
Figure 11: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the bottom surface of the horizontally and vertically embedded hard PZT patch in the confined granular fill.

Figure 12: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the top surface of the horizontally and vertically embedded soft PZT patch in the confined granular fill.
Figure 13: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the bottom surface of the horizontally and vertically embedded soft PZT patch in the confined granular fill.

Figure 14: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the top surface of the horizontally and vertically embedded soft PZT patch in the confined granular fill.
Figure 15: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the bottom surface of the horizontally and vertically embedded soft PZT patch in the confined granular fill.

Figure 16: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the top surface of the horizontally and vertically embedded semihard PZT patch in the confined granular fill.
Figure 17: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the top surface of the horizontally and vertically embedded semihard PZT patch in the confined granular fill.

Figure 18: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the top surface of the horizontally and vertically embedded semihard PZT patch in the confined granular fill.
Figure 19: Charge density computed for retaining structure of unit width, piezoelectric strain, modulus ratio, and thickness ratio on the bottom surface of the horizontally and vertically embedded semihard PZT patch in the confined granular fill.

Figure 20: Voltage output computed for the retaining structure, piezoelectric strain constant, polarization ratio (0.112, 0.087, and 0.114), and thickness ratio of horizontally embedded hard, semihard, and soft PZT patch in the confined granular fill.
fill affects the average stress in the PZT patch. This curve trend owes to the change in the distance between the centroid of the PZT patch and the neutral surface. Suppose the thickness ratio is reduced to 0.01. In that case, the maximum stress in the horizontally embedded PZT patch is more than five times higher than vertically embedded PZT patch. It has been observed that the thickness and material of the wall also affect the strain and stress behaviour of the confined granular fill (equation (4)).

4.1. Charge Density on Hard, Semihard, and Soft PZT Patches. A comparative study is presented for the hard (H), semihard (SH), and soft (S) PZT patches to capture the effect of geometric and mechanical parameters on the charge density on the PZT patch. The dominant parameters for the analysis are the installation position of the PZT patch, the thickness, and the modulus of the backfill material. The engineering properties of the PZT patch are shown in Table 2. As a result of the embedment of the PZT patch between the pavement and the granular material, the charge is induced on the top and on the bottom surface of the PZT patch. The difference between the positive and negative charges is proportional to the energy produced.

Figure 5(a) shows a physical model of the horizontally and vertically embedded PZT patch in the confined granular fill. The vibration in the backfill material deforms the horizontal embedded PZT patch. The piezoelectric phenomenon generates charge density on the surface of the PZT patch. Figures 8–19 show the variation of the charge density for the thickness ratios (μ) in the range of 0.10–0.20. Due to a small polarization ratio (0.114), the lowest charge is generated on the hard PZT patch as shown in Figures 8–11. The maximum charge is generated on the semihard (SH) patch as shown in Figures 16–19. Therefore, a semihard PZT patch is recommended as an effective piezoelectric material. The results also indicate that the energy efficiency of the output voltage is low for the loose granular fills.

For the vertically embedded PZT patch, the maximum charge is generated on the soft PZT patch as shown in Figures 12–15. For the thickness ratios (μ) in the range of 0.10–0.20, the results show that the total charge induced on the horizontally embedded PZT patch is higher than the vertically embedded patch. It indicates that the geometric parameters, modulus of granular fill, and wall significantly affect the charge density on the top and the bottom surface of the PZT patch (equations (17)–(19)).

Table 3 presents the maximum charge density on the top and bottom surfaces of the PZT patches for thickness ratios in the range of 0.01–0.08 and 0.10–0.20 based on the alignment of a patch.

4.2. Comparative Analysis of Voltage Output. In this section, the voltage output from different PZT patches is analysed by considering the variation of the stress-strain response (equations (20)–(25)). The key parameters influencing voltage output are the polarization, alignment of the PZT
Figure 22: (a) The output voltage for different external resistances for high, medium, and low resistance applications. (b) Power output for different external resistances for high-, medium-, and low-resistance applications.
patch, thickness ratio, and material properties. The polarization of the PZT patch depends on the piezoelectric strain and the relative permittivity of the PZT material. The comparison of the voltage output of the hard, semihard, and soft PZT patches is shown in Figures 20 and 21. As the thickness of the retaining wall increases, the voltage output increases up to a critical thickness ratio. A critical thickness ratio is defined as a ratio of the thickness of the granular fill to that of the PZT patch at which the maximum energy is harvested.

Figures 20–21 show a graphical representation of the voltage output at various thickness ratios. The semihard PZT patch shows the highest voltage output due to the highest polarization ratio of 0.114. At the critical thickness ratios of 0.2 and 0.4, the maximum voltage is obtained for horizontally and vertically aligned patches, respectively, as shown in Table 4. The maximum voltage output observed for horizontally embedded patches is 0.44, 0.56, and 0.33 volt. Similarly, for vertically embedded PZT patch voltage output is 0.05, 0.06, and 0.04 volt for hard, semihard, and soft PZT patches, respectively. The variation in output voltage indicates that interaction between the PZT patch and the retaining wall strongly influences the energy output from the confined granular fill.

4.3. Effect of External Resistance on Power Output of the PZT Patch. This section shows the effect of external resistance on the voltage and power output of the PZT patch as per its alignment in the granular fill. Figure 22(a) shows the magnitude of voltage output for the different external resistances. The magnitude of voltage increases logarithmically until a peak voltage of 56 mV for the horizontally embedded PZT patch and 6 μV for the vertically embedded PZT patch. The magnitude of the voltage output from experimental data from Wang et al. [13] and Petroff et al. [42] is appropriate for MRA and HRA, respectively. Eq. (26) shows a generalized relationship for the output voltage based upon a set of design parameters considered by the present study. An empirical relationship of output voltage for high, medium, and low resistance applications for varying design parameters is expressed as

\[ V = \xi R^\eta. \]  

The variables \( \xi \) and \( \eta \) depend upon the design parameters, namely polarization, modulus ratio, position of the PZT patch, strain, geometrical, and engineering properties of the confined granular fill and retaining structure. The response of the PZT patch can be optimized by changing the design parameters and mechanical properties.

### Table 3: Charge density for confined granular fill with PZT patches.

<table>
<thead>
<tr>
<th>Alignment of the PZT patch</th>
<th>Maximum charge density at the top surface (Coulombs/m²)</th>
<th>Maximum charge density at the bottom surface (Coulombs/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>SH</td>
<td>S</td>
</tr>
<tr>
<td>0.10 ≤ µ ≤ 0.20</td>
<td>0.01 ≤ µ ≤ 0.08</td>
<td>0.10 ≤ µ ≤ 0.20</td>
</tr>
<tr>
<td>Horizontal</td>
<td>−26</td>
<td>−256</td>
</tr>
<tr>
<td>Vertical</td>
<td>−3</td>
<td>−30</td>
</tr>
</tbody>
</table>

### Table 4: Voltage output for various PZT patches.

<table>
<thead>
<tr>
<th>Type of the PZT patch</th>
<th>Maximum voltage output for horizontally embedded PZT patch (Volt)</th>
<th>Maximum voltage output for vertically embedded PZT patch (Volt)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard (H)</td>
<td>0.44</td>
<td>0.05</td>
<td>159.18</td>
</tr>
<tr>
<td>Semihard (SH)</td>
<td>0.56</td>
<td>0.06</td>
<td>161.29</td>
</tr>
<tr>
<td>Soft (S)</td>
<td>0.33</td>
<td>0.04</td>
<td>156.75</td>
</tr>
</tbody>
</table>

### Table 5: Classification of PZT applications for output voltage and resistance.

<table>
<thead>
<tr>
<th>Description</th>
<th>Output voltage (Volt)</th>
<th>Resistance (Ω)</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>A flexible piezoelectric energy harvester⁴</td>
<td>10⁻⁴–10⁻¹</td>
<td>10⁸–10⁵</td>
<td>High-resistance applications (HRA)</td>
</tr>
<tr>
<td>A two-degree freedom system with the piezoelectric element⁵</td>
<td>10⁻¹–10⁴</td>
<td>10⁻¹–10⁴</td>
<td>Medium-resistance applications (MRA)</td>
</tr>
<tr>
<td>Horizontally embedded PZT patch⁶</td>
<td>10⁻¹–1</td>
<td>10⁻¹–1</td>
<td>Low-resistance applications (LRA)</td>
</tr>
<tr>
<td>Vertically embedded PZT patch⁷</td>
<td>10⁻³–10⁻¹</td>
<td>10⁻¹–1</td>
<td>Low-resistance applications (LRA)</td>
</tr>
</tbody>
</table>

⁴Petroff et al. [42]; ⁵Wang et al. [13]; ⁶, ⁷present study.
the classification of the application of PZT patches based on the voltage output for external resistance. Based on the range of external resistance, the applications of PZT patches are classified as low resistance, medium resistance, and high resistance applications.

Figure 22(b) illustrates the magnitude of the power of the present work, and experiment results from the literature are plotted with high medium and low resistance applications. The higher power output is observed for the horizontally embedded PZT patch for the same range of the external resistance. The alignment of the PZT patch significantly affects the voltage and power output due to the vertical and lateral vibrations due to dynamic loads.

5. Conclusion

A quantitative estimate of piezoelectric voltage output depends upon the stress-strain response of confined granular fill, retaining structure, and PZT patches. The stress distribution in the embedded PZT patch is affected by the alignment, thickness, and modulus of the material. Three types of PZT patches, namely hard, semihard, and soft PZT patches, have been considered to evaluate charge density and voltage output. The conclusions of this study are summarized as follows:

(i) The charge density is significantly affected by the parameters, namely modulus ratio, the magnitude of strain, placement of PZT patch, relative permittivity, and polarization of the PZT patch.

(ii) Once the PZT patches are embedded horizontally, the maximum charge density is observed to be more than three times compared to the vertically embedded PZT patches for a set of numerical parameters.

(iii) The maximum voltage output was obtained for the thickness ratio of 0.2–0.6. The voltage output for the horizontally and vertically embedded hard, semihard, and soft PZT patches is obtained in the range of 0.01–0.5 volts and 0.001–0.06 volts, respectively.

(iv) The magnitude of voltage increases logarithmically until a peak voltage of 56 mV for the horizontally embedded PZT patch and 6 μV for the vertically embedded PZT patch.

(v) A relationship for voltage and power output has been proposed for wide-ranging engineering implementations classified as low (LRA), medium (MRA), and high (HRA) resistance applications.

This study considers a comprehensive set of design parameters for selecting PZT patches for the voltage output. It provides a framework for evaluating power outputs from confined granular fill and improving the conversion efficiency from the strain fluctuations, polarization per unit permittivity, and geometric properties, which play a significant role in the structural health monitoring of the pavement and retaining structures. The observed voltage output is found appropriate for wide-ranging resistance applications. It is proposed to be up-scaled using multiple patches embedded throughout the confined granular fill subjected to continuous dynamic loads.

Abbreviations

PZT: Lead zirconate titanate
ε<sub>HE</sub>: Strain in horizontally embedded PZT patch
σ<sub>PZT</sub>: The stress in the PZT patch
ε<sub>HE</sub>: Stress in horizontally embedded PZT patch
ρ<sub>Top</sub>, ρ<sub>Bottom</sub>: Charge density at the top and bottom surfaces of the PZT patch, respectively
ρ<sub>THE</sub>, ρ<sub>BHE</sub>: Charge density at the top and bottom surfaces of the horizontally embedded PZT patch, respectively
Q<sub>Top</sub>, Q<sub>Bottom</sub>: Charge quantity at the top and bottom surfaces of the PZT patch, respectively
Q<sub>THE</sub>, Q<sub>BHE</sub>: Charge quantity at the top and bottom surfaces of the horizontally embedded PZT patch, respectively
ρ<sub>THE</sub>, ρ<sub>BHE</sub>: Charge density function at the top and bottom surfaces of the vertically embedded PZT patch, respectively
Q<sub>THE</sub>, Q<sub>BHE</sub>: Charge quantity at the top and bottom surfaces of the vertically embedded PZT patch, respectively
h<sub>w</sub>: Thickness of the wall
µ: Thickness ratio of PZT patch and backfill (h<sub>p</sub>/h<sub>w</sub>)
a: The thickness ratio of the retaining wall and PZT patch (h<sub>w</sub>/h<sub>p</sub>)
b: The modulus ratio of retaining wall and PZT patch (E<sub>W</sub>/E<sub>p</sub>)
D<sub>33</sub>: Electric displacement
ε<sub>T</sub>/ε<sub>o</sub>: Relative permittivity
ε<sub>T</sub>: The absolute permittivity of the vacuum
ε<sub>o</sub>: The permittivity of the material in direction-33 under constant stress
ε<sub>T</sub>: The free permittivity of the material in direction-33 under no stress
d<sub>31</sub>: Piezoelectric strain constant in the mode 31
d<sub>33</sub>: Piezoelectric strain constant in the mode 33
d: Depth of embedment of PZT patch
z: The thickness direction coordinate from the neutral plane
e: Distance between the neutral axis and lower surface of the PZT patch
E: Equivalent electric field intensity
K: Constant of proportionality
VE: Subscript for vertically embedded patch
HE: Subscript for horizontally embedded patch
EP: Modulus of the PZT patch
EW: Modulus of the retaining wall
H: Height of the retaining structure
L: Length of the retaining structure
hP: Thickness of the PZT patch
hR: Thickness of the granular backfill
n: Modulus ratio of PZT patch and backfill (E<sub>p</sub>/E<sub>W</sub>)
where $\mu$ is the ratio of the thickness of PZT to the backfill ($h_p/h_R$), $n$ is the ratio of the modulus of the PZT patch and the backfill ($E_p/E_R$), $a$ is the thickness ratio of retaining wall and PZT patch ($h_w/h_p$), and $b$ is the modulus ratio of retaining wall and PZT patch ($E_w/E_p$).

From equations (A.1) and (A.6), the total stress on the PZT patch is expressed as

$$\sigma_{VE} = \frac{ME_p}{E_i H} \left[ \frac{(\bar{z} - z_p)^2}{\left\{nh_p^3/12(1 + ba^2) + h_p^3/36\right\}} + (\bar{z} - z_p)^2\left\{nh_p(1 + ab) + h_p/2\right\} \right]$$

(A.7)

Similarly, the average strain can be calculated for the horizontal embedded PZT patch.

**Charge Density and Voltage.** The electromechanical piezoelectric coupling constitutive equation [49] is expressed as

$$D_3 = d_{31}\sigma_{PZT} + \varepsilon_{35}^T E_3.$$  

(A.8)

According to the stress relationship equation (A.1), the piezoelectric coupling constitutive equation, when the external applied electrical energy is zero ($E_s = 0$), the lateral electric displacement of the top and bottom surfaces of the PZT patch [53], the charge density is expressed as

$$Q_{Top} = \int_0^L bD_3(x, e + h_p)dx.$$  

(A.10)
Similarly, the charge density (equation (A.4)) and total charge quantity on the bottom surface (equation (A.5)) are expressed as
\[ \rho_{\text{Bottom}} = -D_3(x, e) = d_{33} \sigma_{\text{PZT}}. \]  
(A.11)
\[ Q_{\text{Bottom}} = \frac{1}{2} b D_3(x, e) dx. \]  
(A.12)

Thus, by calculating the total charge quantity due to stress on the surface of the PZT patch, the open-circuit voltage between the top and bottom surfaces is expressed as
\[ V = \frac{1/2 b L (|Q_{\text{Top}}| + |Q_{\text{Bottom}}|)}{b L (\varepsilon_{33}/\varepsilon_0)}. \]  
(A.13)
where \( \varepsilon_{33}/\varepsilon_0 \) is the relative permittivity of the piezoelectric material.

Data Availability
All data, models, and code generated or used during the study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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