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*Research Article*

# Analysis of Damage Constitutive Model for Frozen Red Clay under Uniaxial Compression

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At present, it is in the peak period of infrastructure construction in western China, and due to the natural climate, the soil has been frozen for a long time, but there is little research on frozen red clay at present. Therefore, it is necessary to further study the mechanical properties of frozen red clay. In this paper, taking the remolded red clay as the test material, the change characteristics of uniaxial compression and uniaxial creep of frozen soil under different test factors are studied by artificial freezing method, and the damage theory is introduced to analyze the test results. The results of uniaxial compression test of frozen soil show that temperature has a great influence on the uniaxial compressive strength, and the compressive strength increases with the decrease of temperature, and the damage theory is introduced. It is found that the initial damage stress increases with the decrease of temperature, while the initial strain changes little with temperature. A damage constitutive equation which can well reflect the whole damage process of frozen red clay is established. The experimental research results of this paper can provide some theoretical reference for the engineering of soil quality for freezing red clay.

## 1. Introduction

Soil can be seen everywhere in nature, but different areas are affected by climate, geological structure, and other factors. The reason for the complexity of permafrost is that permafrost is a four-item system composed of soil particles, ice, liquid water, and gas. Because of the existence of water and unfrozen water in permafrost, they fluctuate left and right due to the influence of temperature. As a result, permafrost shows strong temperature sensitivity and rheological properties under external loads [1]. As early as the 1930s, experts and scholars began to study the mechanical properties of frozen soil and made a lot of research results, among which Vialov and Tsytovich et al. [2, 3] frozen soil mechanics experts as the representative, these achievements mainly frozen soil uniaxial strength, triaxial strength, and some simple constitutive relations, forming the preliminary stage of the frozen soil mechanics.

In the study of the strength and damage constitutive model of frozen soil, Lai et al. [4] considered the random distribution of internal defects such as cracks and voids in high temperature frozen soil with the help of the continuous damage theory and probability and mathematical statistics. A uniaxial random damage constitutive model is established, and the test data at different temperatures are compared with the model, and the reliability is analyzed. The test results show that the model can well reflect the strength distribution of high temperature permafrost. Li et al. [5] think that the discreteness and randomness of mechanical properties of high temperature frozen soil are due to a large number of initial defects of cracks and voids in it. According to the test data under different confining pressures, a damage statistical constitutive model based on continuous damage theory and the test model is verified. Zhu et al. [6] established a damage constitutive model of frozen soil based on meso-mechanical mechanism and verified the test data of frozen sand under different temperature and ice volume content. The model can well reflect the stress-strain curve of frozen sand. Ning et al. [7] established an elastic constitutive model of the frozen soil with damage based on the meso-mechanical mechanism of composite materials and verified the measured data of frozen sand with different ice volume content and temperature. Other scholars have also made good achievements in the damage constitutive model of geotechnical materials [8–14]. The traditional damage constitutive model is difficult to accurately reflect the stress release and transfer mechanism, which makes it difficult to capture the effect of strain localization. The nonlocal model has solved this problem to a certain extent [15–17].

The nonlocal model introduces a nonlocal integral operator to weighted average the physical quantities to be concerned (such as plastic strain and damage) [18, 19]. Combining the fractional plasticity theory and the concept of characteristic stress, Liang et al. proposed a fractional critical state model in the characteristic stress space, aiming at the fact that the existing fractional constitutive model of the soil cannot describe the critical state of the soil and the different deformation and strength characteristics of triaxial tension and compression. The model extends the elastic-plastic theory of soils [20]. Sumelka considers a special case of the general fractional plastic flow rule. The fractional plastic flow is obtained from the classical flow rule by generalisation of the classical gradient of a plastic potential with a fractional gradient operator. It is important that, contrary to the classical models, nonassociativity of fractional flow appears without introduction of the additional potential [21]. Sun and Sumelka developed the stress fraction viscoplasticity model based on the over stress theory of Perzyna. The viscoplastic flow direction of soil is simulated by the unit tensor obtained by the fractional derivative of the yield surface. With the increase of fractional order, the predicted compressive shear strength changes, and the transition from pure strain hardening to strain hardening and softening is observed [22]. Liang et al. applied the nonorthogonal plastic flow rule to capture the shear expansion behavior of sand. By applying the nonorthogonal plastic flow rule to the elliptic yield function in the eigenstress space, the plastic flow direction under 3D stress can be determined directly. The combination of the nonorthogonal plastic flow rule and the proposed hardening parameters enables the proposed model to reasonably describe the stress-strain relationship of sand with dilatancy [23].

Red clay itself is a kind of special soil, and engineering disasters caused by its bad soil properties frequently appear in engineering construction. At present, there is certain research understanding of red clay in various countries. It is found that red clay has the characteristics of high-water content, high porosity, and high saturation, and its physical property index is poor, but it has high strength and low compressibility, and the mechanical property index is very good. In addition, the bad properties of red clay, such as swelling and shrinkage, porosity, and inhomogeneity of strata, are also gradually understood by the engineering community. The obvious expansion and crack caused by water loss shrinkage and water absorption have brought serious damage to the actual engineering construction [24–26]. In permafrost engineering, only when permafrost has sufficient resistance to failure and deformation under external load can the safety and stability of the project be ensured [27]. The mechanical properties of frozen soil deformation and the simulation of the whole deformation process are important research contents of frozen soil mechanics. The existing constitutive models either do not consider the effect of damage or the initial condition of damage, that is, the problem of damage threshold, it is difficult to reflect the linear elastic stage deformation characteristics of frozen soil at low stress level, which needs to be improved. At present, there are few studies on these properties of frozen red clay.

In this paper, the basic physical index and mineral composition of red clay and the variation characteristics of instantaneous uniaxial strength of frozen red clay with temperature are determined. Determine the damage threshold, select the appropriate damage variables, establish the damage constitutive relation of frozen red clay, and explore the relationship between damage threshold and temperature. The rationality of the model is verified by the means of damage theory and Kachanov [28] damage evolution law. The failure process of frozen soil under uniaxial compression is analyzed from the perspective of damage, and the damage constitutive equation of frozen red clay is constructed, which can provide reference value for practical engineering.

## 2. Study on Uniaxial Compression Test

All the tests are carried out on the WDT-100 permafrost testing machine developed by Anhui University of Science and Technology (Figure 1). Compared with the mechanical performance testing machine at room temperature, this testing machine has a temperature control system, which has high technical requirements and complex system, but it is easy to operate. It is mainly composed of test host, computer, hydraulic oil pump loading system, low temperature box, electrical control system, and so on. The maximum loading capacity of the testing machine is 10 tons and the accuracy is 1%. The strain or stress control loading mode can be selected in the test. The data in the whole process of the test are automatically collected by the computer, the collection time interval is 0.1 s, and the stress-strain curve is drawn automatically. The main performance, technical parameters, configuration, and performance of the uniaxial testing machine for frozen soil are shown in Table 1.

*2.1. Sample Preparation.* The test material is red clay from a mine in Shanxi Province. Because of the objective conditions of the test, the remolded soil is used to explore the mechanical properties of the red clay. Through laboratory tests, the relative density of soil particles is 2.74, the natural moisture content of soil samples is 21.98%, the liquid limit is 52.20%, and the plastic limit is 23.69%. The calculated dry density is 1.72 g/cm$^3$ and the weight is 20.98 kN/m3. First of all, cut up the undisturbed soil sample, put it in the oven, bake it at a constant temperature of 105°C~110°C for more than 8 hours, remove and cool, crush and sieve the dried soil, distribute water according to the moisture content needed for the test, and after uniform stirring, seal and maintain for more than 24 hours, so that the water is evenly absorbed, the moisture content is about 20%, and then the sample is

Figure 1: WDT-100 frozen soil experiment machine.

Table 1: Main technical parameters single-axle frozen soil experiment machine.

| Main technical parameters | |
|---|---|
| Maximum test force (kN) | 100 kN |
| Load binning | 10%, 20%, 50%, and 100% |
| Axial displacement binning | Could be adjusted freely |
| Axial displacement range | ±50 mm |
| Axial strain loading rate | 1%/min–30%/min |
| Axial stress loading speed | 0.1 MPa/s–0.8 MPa/s |
| Power supply | 380 V |
| Temperature measurement accuracy | ±0.1°C |
| Sampling interval | ≥0.1 s |

prepared in the standard mold (Figure 2). In order to prevent red clay from sticking on the mold. Before sample preparation, apply a layer of vaseline in the mold, layer-by-layer the cured soil into the mold with diameter 50 mm and high 100 mm, adopt the layered compaction method to ensure that the density of each sample is basically the same, pay attention to the brushing treatment of the connecting surface of each layer in the process of layered compaction, ensure to facilitate the combination between layers, seal the prepared soil samples with fresh-keeping bags (to prevent the influence of incubator on the moisture content of the samples), and affix labels. Write a note, put it in a set incubator, maintain it according to the prescribed method, wait for the test, and use the tools as shown in Figure 2.

*2.2. Analysis of the Test Process and Test Results.* In this experiment, red clay with water content of 20% was used to test the uniaxial unconfined compressive strength of frozen soil at −2°C, −5°C, −10°C, and −15°C. The strain rate of 2%/min was used in the test. When the stress-strain curve has peak stress, the stress decreases or the axial strain reaches more than 20%. When the compressive strength has peak stress, the peak stress is taken, and if there is no peak stress, the corresponding stress value is taken when the strain is 20%.

## 3. The Relationship between Uniaxial Strength and Temperature

Through the arrangement of the test data, the uniaxial compressive strength stress-strain curve of frozen red clay is shown in Figure 3.

Through the analysis of the stress-strain curve of frozen red clay in Figure 3, it can be found that the frozen red clay goes through four stages in the process of uniaxial compression: the first stage, the compaction stage, the cracks in the frozen soil are closed, the initial gaps (such as cracks and pores) gradually disappear, and the deformation modulus increases gradually; in the second stage, in the elastic stage, the stress and strain are approximately linear, and the deformation modulus of frozen soil tends to be stable. In the third stage, in the stage of plastic deformation, new cracks appear in the frozen soil, the voidage increases, accompanied by volume expansion, and the tangent slope of the stress-strain curve decreases gradually, that is, the deformation modulus decreases gradually. In the fourth stage, the failure stage, when the stress reaches the strength limit, the frozen soil enters the failure stage. Because there is still a certain friction force in the new fracture surface, the frozen soil still has a certain bearing capacity, and the shear slope of the rock stress-strain curve becomes negative. When the temperature is low, the stress-strain curve has obvious peak stress, and the peak stress softens later, while when the temperature is higher, the curve shows slow strain hardening type. The reason for this phenomenon is that when the temperature is high, the unfrozen water content in the frozen soil is more, the ice content is less, the soil has strong plasticity, along with the decrease of temperature and the increase of ice content, the cementation between solid particles and ice becomes stronger, showing brittle failure. From the structural microstructure analysis, the stress-strain characteristic of frozen soil is the macroscopic expression of dynamic compensation between the loss of interparticle cohesion and the slip resistance caused by the arrangement of new structures. When the slip resistance can compensate for the loss of intergranular cohesion, the stress-strain relationship of frozen red clay is hardened type, otherwise it is softened type, when the two are equal. The stress-strain relationship of frozen red clay is ideal elastoplastic macroscopically.

When the sample is unloaded to the original stress state after loading, part of the strain of the soil is unrecoverable, and this part of the unrecoverable strain is plastic strain. It can be expressed as $\varepsilon = \varepsilon_e + \varepsilon_p$, where $\varepsilon_e$ represents elastic strain; $\varepsilon_p$ represents plastic strain. The limitation of monotone loading is that the strain of soil is regarded as elastic strain, but large plastic strain is not allowed in practical engineering application. In general, the deformation of soil can be approximately considered to be elastic.

The uniaxial compressive strength and elastic modulus obtained from the above stress-strain curves are shown in Table 2, and the elastic modulus is determined as follows:

$$E = \frac{\sigma_S/2}{\varepsilon_{1/2}}. \tag{1}$$

$E$—elastic modulus, MPa; $\sigma_S$—ultimate compressive strength, MPa; $\varepsilon_{1/2}$—the strain value corresponding to half of the ultimate compressive strength.

Table 2 shows that the compressive strength of frozen red clay has a certain relationship with elastic modulus and temperature. On the whole, with the decrease of

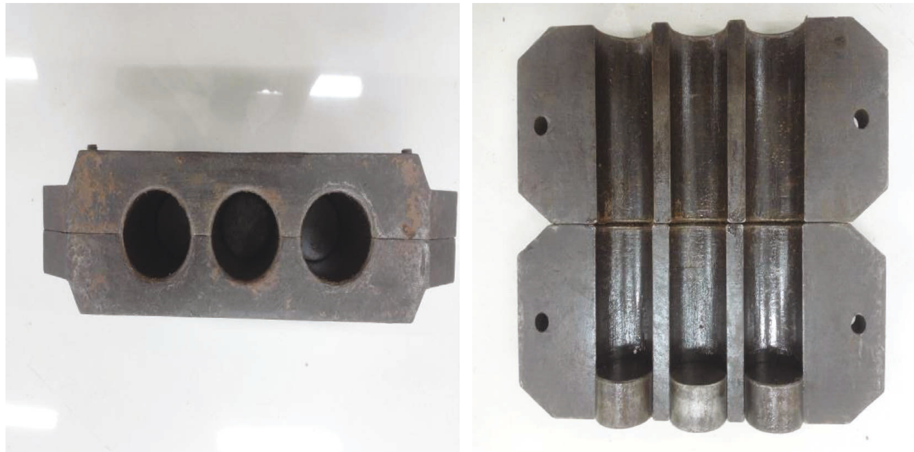


Figure 2: Test sample mold.

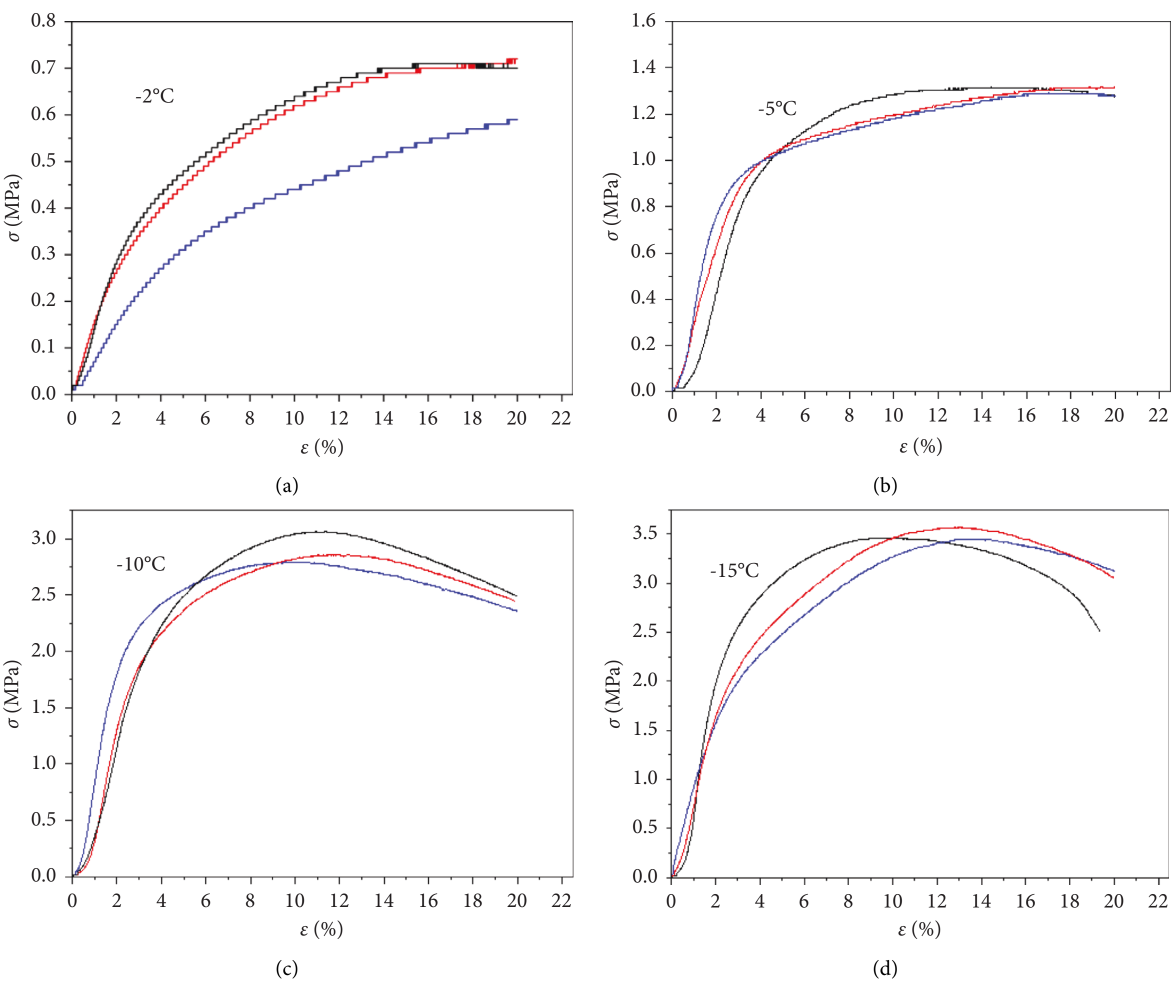



Figure 3: Stress strain curve of frozen red clay at different temperatures.

temperature, the compressive strength and elastic modulus increase, as shown in Figures 4 and 5.

It can be seen from Figures 4 and 5 that the uniaxial compressive strength and elastic modulus of frozen red clay are basically the same with temperature, which can be approximately described linearly, that is, with the decrease of temperature, uniaxial compressive strength and elastic modulus increase. The uniaxial compressive strength of frozen red clay increases with the decrease of temperature. When the temperature field in the soil changes, water migration will occur [29]. The strength growth rate is different at each stage, the strength increases most obviously at

Table 2: Uniaxial compressive strength and elastic modulus at different temperatures.

| Temperatures | Test numbers | Uniaxial strength/MPa | Average strength/MPa | Elastic modulus/MPa | Average elastic modulus/MPa |
|---|---|---|---|---|---|
| | D1-1 | 0.70 | | 13.31 | |
| −2 | D1-2 | 0.59 | 0.67 | 6.75 | 10.40 |
| | D1-3 | 0.72 | | 11.14 | |
| | D2-1 | 1.31 | | 25.01 | |
| −5 | D2-2 | 1.29 | 1.30 | 39.49 | 31.80 |
| | D2-3 | 1.31 | | 30.89 | |
| | D3-1 | 3.07 | | 62.15 | |
| −10 | D3-2 | 2.86 | 2.90 | 66.08 | 74.31 |
| | D3-3 | 2.8 | | 94.72 | |
| | D4-1 | 3.45 | | 75.32 | |
| −15 | D4-2 | 3.57 | 3.49 | 79.62 | 84.86 |
| | D4-3 | 3.46 | | 99.65 | |

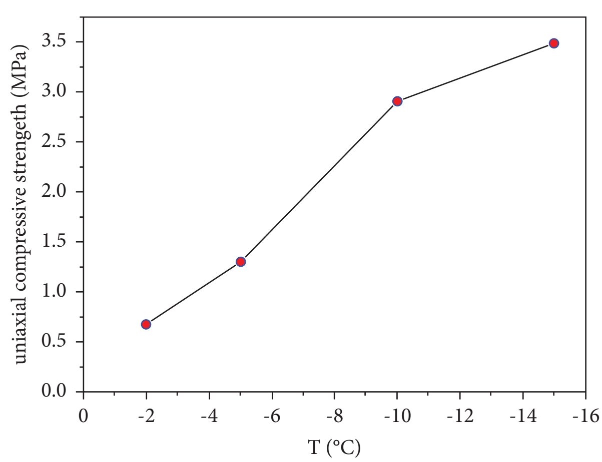



Figure 4: The change of uniaxial compressive strength with temperature.

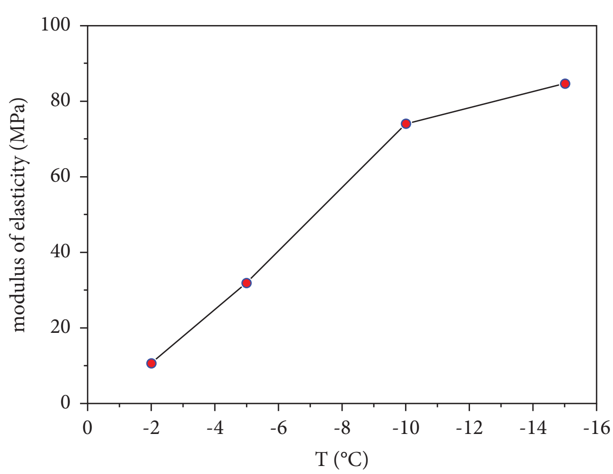



Figure 5: The relationship between the modulus of elasticity and temperature.

−2°C~−10°C, and the effect of temperature on strength decreases between −10°C~−15°C. Therefore, in the actual freezing construction process, we can consider −10°C as a key temperature point of freezing temperature, and the overall strength growth rate is about 0.23 MPa/°C. The change of elastic modulus with the temperature is basically the same as that of strength.

It can be seen from Figure 6 that the failure form of frozen red clay under uniaxial compression is very sensitive to the temperature. When the temperature is low, the specimen will have an oblique crack, and the failure surface will run through the whole specimen. The angle with the direction of axial stress is about 40°, and the failure mechanism belongs to shear failure. When the $\varepsilon < 2.4\%$, the stress-strain curve can be regarded as a linear segment, that is, the elastic stage. After entering the viscoplastic stage, the slope of the stress-strain curve decreases gradually, until the failure, the stress-strain curve will have obvious peak points, such as the stress-strain curve at −10°C and −15°C. When the temperature is high, the frozen red clay shows bulging after failure, and there is no obvious damage trace on the surface. When the $\varepsilon < 2.4\%$, the stress-strain curve can be regarded as a linear segment, that is, the elastic stage. After entering the viscoplastic stage, the slope of the stress-strain curve decreases gradually, and the stress-strain curve shows hardened plastic characteristics, such as the stress-strain curve at −2°C and −5°C. The main reason for the above two failure forms is due to the change of ice content in the frozen soil. When the temperature is low, the proportion of frozen water in the frozen soil increases, and a large number of cementation bodies are formed between ice and soil particles. At this time, the structural cementation force of the frozen soil is much larger than the slip resistance formed when the broken particles are broken, and it shows brittle failure, while when the temperature is higher, the proportion of unfrozen water in the soil is larger. During the failure, the structural cementation force and the slip resistance can compensate each other, and the soil strain increases slowly, which leads to the final failure.

## 4. Establishment of Damage Constitutive Model of Frozen Red Clay

*4.1. Analysis of Damage Initiation Conditions of Frozen Soil Structures.* Combined with the whole process of frozen soil deformation, it can be divided into three stages from the point of view of damage. The first stage has no damage or only a small amount of structural damage, which belongs to the elastic deformation stage. The second stage begins to appear as damage stage, the stress exceeds the yield limit,

(a) (b)

Figure 6: Two damage forms of frozen red clay.

enters the stress-strain curve into the nonlinear stage, the soil is constantly damaged, except for the slip between soil particles and a large number of particles damage. The third stage of failure. The slip between particles is the main cause of deformation. For each stage of permafrost deformation, the stress and strain corresponding to the end of the elastic stage can be taken as the starting point of damage of the frozen soil structure, that is, when $\sigma > \sigma_s$ or $\varepsilon > \varepsilon_s$, the frozen soil structure begins to produce damage, but it is difficult to accurately locate how to select this starting point of damage. The WTD-100 tester used in this test has high accuracy and records the values of stress and strain all the time. The damage threshold point can be roughly determined in the stress-strain diagram drawn (Figure 7) in order to explore the influence of frozen soil temperature on frozen soil damage.

According to the results of uniaxial compressive strength test of frozen soil, the stress damage threshold and strain damage threshold at different temperatures are shown in Table 3.

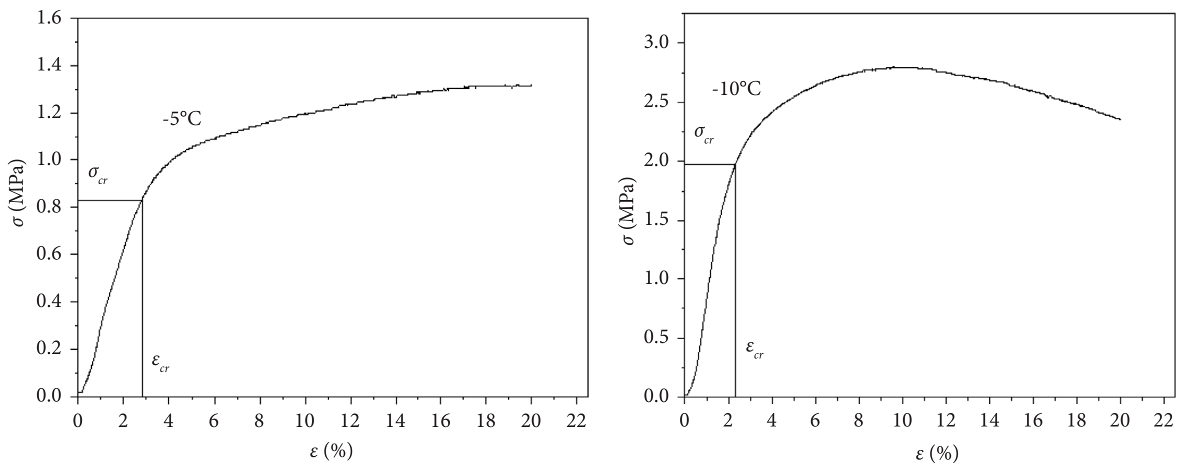
The relationship between strain damage threshold and stress damage threshold and temperature drawn from the data results are shown in Figures 8 and 9.

According to the test data, combined with Figures 8 and 9, we can draw the following conclusions. Temperature has little effect on the strain damage threshold, and the relationship between them is basically linear, ranging from 2.88% to 2.44%, but as the temperature decreases, the strain damage threshold decreases. Temperature has a great influence on the stress damage threshold, and the relationship between them is basically linear, and the range is between 0.29 MPa and 1.92 MPa. As a whole, the stress damage threshold increases with the decrease of temperature.

*4.2. Selection of Damage Variables.* Damage theory is put forward by Kachnov, 1958, when studying fatigue, creep, and plastic deformation of metals, which is mainly used for one-dimensional damage. He proposes to introduce a continuous damage factor to simulate the stress-strain relationship after damage. After this theory is widely used in the damage study of rock and concrete materials, but the structure of soil is different from rock and other materials, so some theories are not fully used. Therefore, it is necessary to introduce an appropriate damage variable when studying the damage mechanical behavior of clay.

The damage variables can be measured directly or indirectly, such as cross section electron microscope, coincidence electron microscope scanning technology, X-ray, CT scanning technology, and so on. This method can directly measure the number, shape, and distribution of pores. This method is more direct and effective, but it is difficult to connect microscopic results with macroscopic mechanical properties. Indirect measurement: some physical parameters of the materials are measured to describe the damage development of materials indirectly. This method is easy to be used in engineering practice, but it is difficult to establish the relationship between internal damage and macroscopic mechanical properties.

In general, the change of damage variable $D$ can be represented by the following three geometric physical quantities: $D = 1 - A_2/A$, $D = 1 - \sigma/\sigma_0$, and $D = 1 - E_0/E$. In the formula, $A$ is the initial effective area, $A_2$ is the undamaged area, $\sigma$ is the external stress, $\sigma_0$ is the effective stress after damage, $E_0$ is the elastic modulus with damage, and $E$ is the elastic modulus without damage. It can be found that the damage variable $D$ has a clear physical meaning, that is, when $D = 0$, it means that the material is not damaged; when $0 < D < 1$, it means that a part of the material is damaged and a part is not damaged; when $D = 1$, it means that the material is completely damaged and the material is in a state of destruction. Because in the actual test process, it is difficult to directly measure the damaged area and undamaged area, so generally do not use the area as the description of the damage variable and use other macromechanical parameters to indirectly obtain the change law of the damage variable.




(a) (b)

Figure 7: Damage starting point of two stress-strain states.

Table 3: The relationship between threshold of strain damage and temperature.

| Temperatures | Threshold of strain damage $\varepsilon$/% | Average strain value $\varepsilon$/% | Threshold of stress damage/MPa | Average stress value/MPa |
|---|---|---|---|---|
| | 2.53 | | 0.33 | |
| −2 | 2.32 | 2.73 | 0.29 | 0.29 |
| | 3.35 | | 0.24 | |
| | 2.31 | | 0.82 | |
| −5 | 2.95 | 2.88 | 0.86 | 0.84 |
| | 3.39 | | 0.85 | |
| | 2.33 | | 1.98 | |
| −10 | 2.86 | 2.67 | 1.81 | 1.84 |
| | 2.82 | | 1.73 | |
| | 2.76 | | 1.91 | |
| −15 | 2.20 | 2.44 | 2 | 1.92 |
| | 2.37 | | 1.9 | |

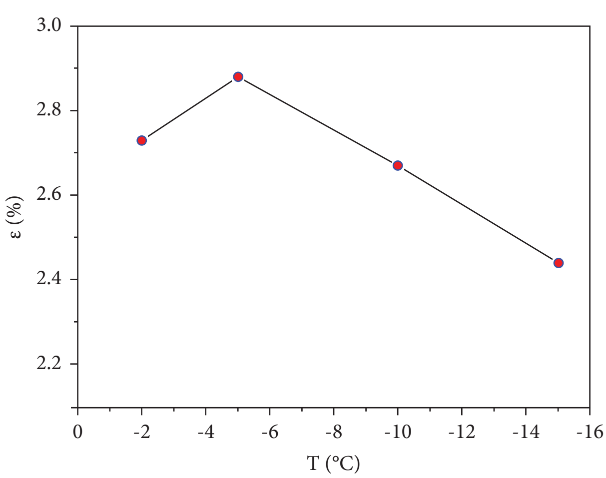



Figure 8: Threshold value of strain damage at different temperatures.

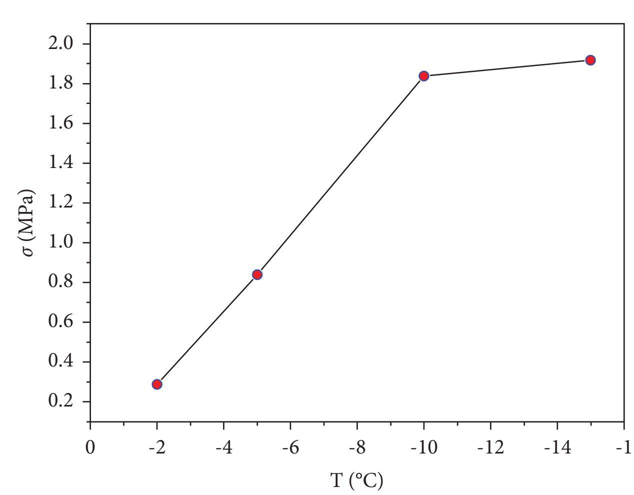



Figure 9: Threshold value of stress damage at different temperatures.

## 5. Establishment of Damage Constitutive Equation and Model Verification

*5.1. Establishment of Damage Constitutive Equation.* According to the Lemaitre equivalent stress principle, in the process of uniaxial compression, the strain caused by stress loading on partially damaged materials is equivalent to that caused by zero equivalent stress acting on lossless materials, so the stress-strain relationship of frozen soil in one-dimensional state is as follows:

$$\varepsilon = \frac{\sigma}{E_0} = \frac{\sigma_0}{E} = \frac{\sigma}{E(1-D)}. \tag{2}$$

In the form, $\sigma_0$—effective stress; $E$—modulus of elasticity without damage; $E_0$—modulus of elasticity with damage; $D$—damage variable.

According to the theory of damage mechanics, it is only necessary to replace the nominal stress in the constitutive relation of lossless materials with the effective stress after damage, that is, the damage constitutive model of frozen soil materials.

$$\sigma = E\varepsilon(1-D). \tag{3}$$

According to the current research results of concrete damage, it is found that in the process of external loading, the material strength obeys the probability and statistical Weibull distribution [30]. Through the analysis of the frozen red clay test data, it is found that the Weibull distribution can well reflect the strength change characteristics of frozen red clay, the damage variable $D$ is expressed by two-parameter Weibull distribution.

$$D = 1 - \exp\left[-\left(\frac{\varepsilon}{a}\right)^m\right]. \tag{4}$$

In the form, $a$—scale parameters of frozen soil; $m$—shape parameters of frozen soil.

Take the formula (4) into the formula (3).

$$\sigma = E\varepsilon * \exp\left[-\left(\frac{\varepsilon}{a}\right)^m\right]. \tag{5}$$

The stress-strain relationship above satisfies the following geometric conditions:

When $\varepsilon = 0$, $\sigma = 0$;

When $\varepsilon = 0$, $d\sigma/d\varepsilon = E$;

When $\varepsilon = \varepsilon_{max}$, $\sigma = \sigma_{max}$;

When $\varepsilon = \varepsilon_{max}$, $d\sigma/d\varepsilon = 0$.

The $\varepsilon_{max}$ and $\sigma_{max}$ in are the strain and peak stress corresponding to the peak stress, respectively.

The strain derivation of equation (5) is as follows:

$$\frac{d\sigma}{d\varepsilon} = E\left[1 - m\left(\frac{\varepsilon}{a}\right)\right]\exp\left[-\left(\frac{\varepsilon}{a}\right)\right]^m. \tag{6}$$

Based on the above stress-strain relationship conditions equations (5) and (6), it can be obtained by solving the following equation:

$$m = \frac{1}{\ln\left(E\varepsilon_{max}/\sigma_{max}\right)}, \tag{7}$$

$$a = \varepsilon_{max}\left(\frac{1}{m}\right)^{-1/m}. \tag{8}$$

When equation (8) is introduced into equation (5), the damage constitutive relation of frozen red clay is as follows:

$$\sigma = E\varepsilon \exp\left[-\frac{1}{m}\left(\frac{\varepsilon}{\varepsilon_{max}}\right)^m\right]. \tag{9}$$

When the two-parameter Weibull distribution is used to describe the damage of frozen red clay, it is considered that the damage of frozen soil is damaged from the very beginning, which is not consistent with the actual situation. Through the analysis of the damage initial condition, a damage initial threshold value is introduced into this formula. In this paper, the strain cr stress is taken as the initial condition, so the damage constitutive equation is as follows:

$$\sigma = E\varepsilon \exp\left[-\frac{1}{m}\left(\frac{\varepsilon - \varepsilon_{cr}}{\varepsilon_{max}}\right)^m\right], \tag{10}$$

where $\langle \varepsilon - \varepsilon_{cr} \rangle$ is the damage switch function, and there are $\langle \varepsilon - \varepsilon_{cr} \rangle = \begin{cases} 0, & \varepsilon \leq \varepsilon_{cr} \\ \varepsilon & \varepsilon > \varepsilon_{cr} \end{cases}.$

From the above formula, we can find that in the process of frozen soil under external load, the stress at any point in the frozen soil is related to the elastic modulus, the strain peak, and the strain max peak of the frozen soil. The value of $m$ is related to the frozen soil material. According to the analysis of the formula, the larger the value of $m$ is, the more brittle the frozen soil tends to be; the smaller the value of $m$, the more the frozen soil tends to plastic failure.

Based on the analysis of the data and the formula, it is found that this formula can well describe the stress-strain curve of the softening type of frozen soil, while the hardening type is not suitable, so the softening curve at −10°C and −15°C is analyzed with the data. The relationship between the data processing parameter $m$ and temperature is shown in Table 4.

Observing the above data, there is a certain regularity between the temperature and the value of $m$. After sorting out the above data, the relationship between the temperature and the parameter $m$ is shown in Figure 10.

It can be seen from Table 4 that there is a certain relationship between the value of $m$ and temperature. When the temperature is lower, the value of $m$ is larger, and when the temperature is higher, the value of $m$ is smaller, and when the temperature is lower, the stress-strain curve is softened, that is, brittle failure. When the temperature is high, the stress-strain is hardened, which belongs to plastic failure, which is consistent with the relationship between the theoretical value of $m$ and the failure shape of the specimen. The value of $m$ does not show a particularly obvious law with the temperature, which may be due to the small number of test groups, so the law is not obvious, but as a whole, the value of $m$ increases with the decrease of temperature.

TABLE 4: The relationship between temperature and parameter *m*.

| Temperatures | Test numbers | Values of *m* | Average values of *m* |
|---|---|---|---|
| −2 | D1-1 | 0.76 | 0.89 |
| | D1-2 | 1.11 | |
| | D1-3 | 0.8 | |
| −5 | D2-1 | 1.09 | 0.80 |
| | D2-2 | 0.58 | |
| | D2-3 | 0.74 | |
| −10 | D3-1 | 1.27 | 1.07 |
| | D3-2 | 0.99 | |
| | D3-3 | 0.96 | |
| −15 | D4-1 | 1.011 | 1.01 |
| | D4-2 | 0.952 | |
| | D4-3 | 1.064 | |

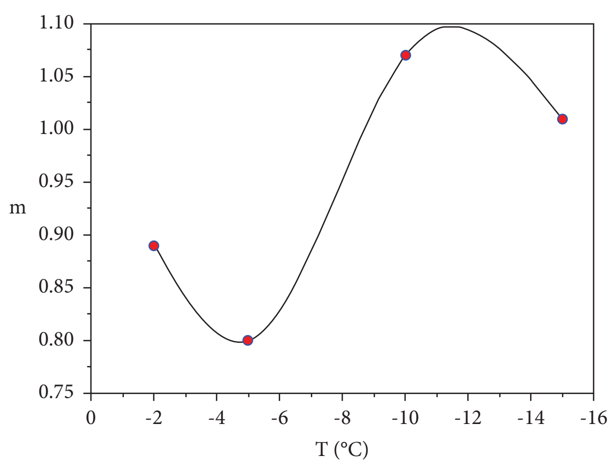



FIGURE 10: The relationship between temperature and parameter *m*.

*5.2. Verification of the Model.* After the analysis and processing of the data, when the damage constitutive relation of frozen red clay is expressed by two-parameter Weibull distribution, the change law of the stress-strain curve is shown in Figures 11 and 12.

It can be seen from Figures 11 and 12 that when the two-parameter Weibull distribution is used to represent the damage constitutive relation of frozen red clay, the change law of the model value is basically consistent with the test value, but when the strain is small, the model stress value is generally smaller than the test value, this is because when the two-parameter Weibull distribution is used to represent the damage of frozen red clay, it is considered that the load or deformation of frozen soil begins to produce negative damage. This is not consistent with the linear elastic deformation of the frozen soil materials at low stress levels, and the model value is smaller than the experimental value.

When the two-parameter Weibull distribution of damage initiation condition is introduced to express the damage constitutive relation of frozen red clay, the calculated stress-strain curve and the experimental stress-strain curve are shown in Figures 13 and 14.

As can be seen from Figures 13 and 14, when the damage constitutive relation of the frozen soil material is expressed by using two-parameter Weibull distribution

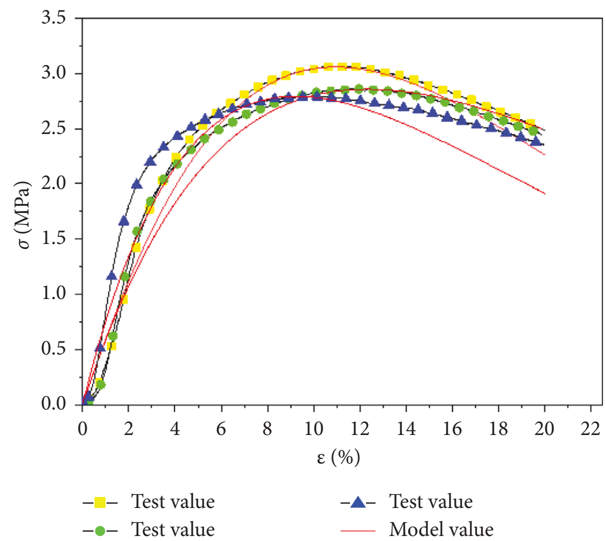



FIGURE 11: Comparison of model values and test values at −10°C.

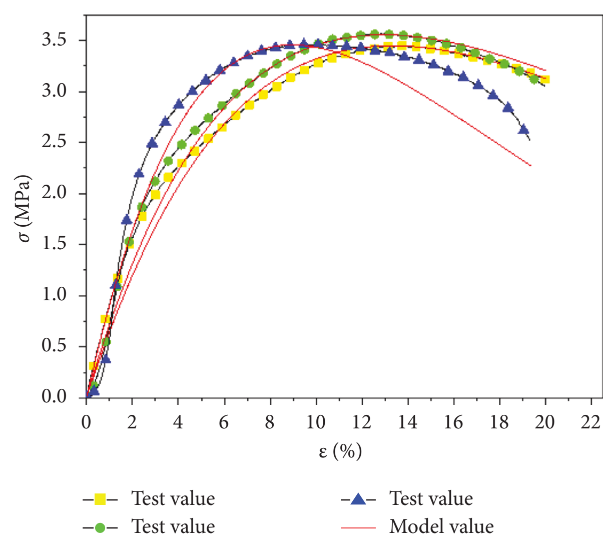



FIGURE 12: Comparison of model values and test values at −15°C.

with damage initial condition, it can well reflect the change process of the stress-strain curve in the whole process of frozen soil damage, eliminate the malpractice that negative damage occurs when the stress-strain curve of two-parameter Weibull distribution reflects the whole process of frozen soil damage, and can reflect the change of frozen soil in the initial elastic stage. It is more consistent with the actual process of the test, in addition, it can be found that there is a large error between the initial elastic stage model value and the test value in some diagrams, because there are cracks in the frozen soil when the internal water is cemented into ice after the soil sample is frozen, so during the initial compaction, the void is compacted and the deformation modulus is small, so the fitting effect of the initial order is not good, because the main body of the constitutive equation adopts Weibull distribution. Therefore, when the elastic section enters the plastic section,

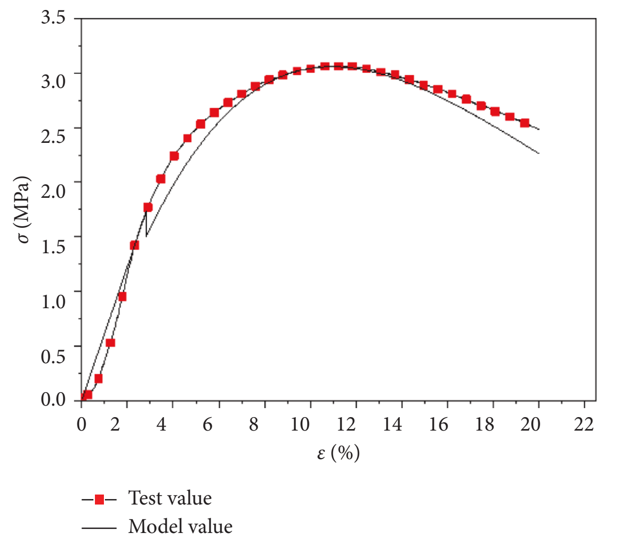



FIGURE 13: Comparison of model values and test values at −10°C.

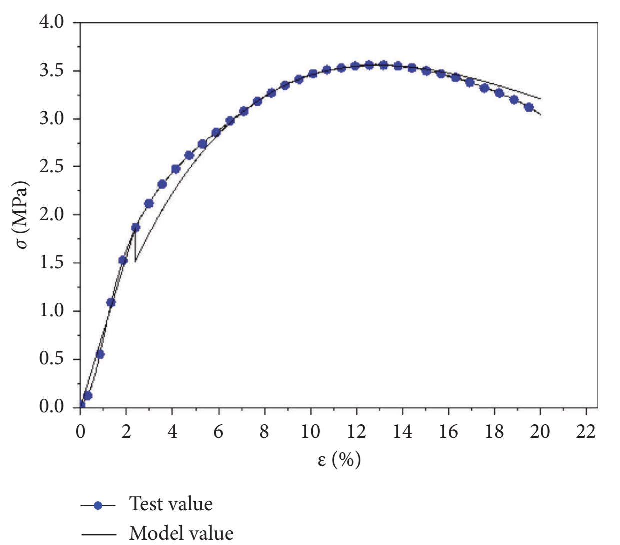



FIGURE 14: Comparison of model values and test values at −15°C.

some damage has occurred, so that the model value of the connection point is less than the test data value, but once the damage occurs, the damage factor $D$ will increase rapidly in a very small strain. The model curve can well reflect the change of stress-strain curve in the damage process of the frozen soil, so the establishment of constitutive equation is reasonable.

## 6. Conclusions

In this paper, through the study of physical properties and uniaxial compression of frozen red clay, analysis of test data, combined with the theory, the main conclusions are as follows:

(1) The shape of the stress-strain curve of frozen red clay has a certain relationship with temperature. The stress-strain curve has obvious peak stress when the temperature is low, and softens after the peak stress, while when the temperature is high, the curve shows slow strain hardening.

(2) The uniaxial compressive strength of artificially frozen red clay is highly sensitive to temperature. With the decrease of temperature, the uniaxial compressive strength increases gradually, but the strength growth rate is different at different stages of temperature. The strength increase is the most obvious at −2°C~−10°C, and the strength growth rate is slightly smaller between −10°C~−15°C, so in the actual freezing construction process we can consider 10°C as a key temperature point of freezing temperature, the overall strength growth rate is about 0.23 MPa/°C, and the change of elastic modulus with temperature is basically the same as that of uniaxial strength.

(3) Due to the development and evolution of internal microstructure defects, there is a certain functional relationship between damage initial strain, initial stress, and freezing temperature. The damage constitutive equation model established by considering the damage threshold value can well reflect the stress-strain relationship under frozen soil damage, in which the material parameter $m$ increases with the decrease of temperature, and when $m$ changes from small to large, the failure mode of frozen soil changes from plasticity to brittleness.

## Data Availability

The data presented in this study are available on request from the corresponding author.

## Conflicts of Interest

The authors declare that there are no conflicts of interest.

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