Research Article

Research on the Method of Improving the Precision of Parts in Pearl River Delta Gear Enterprises: Analysis of Gear Transmission Error and Tooth Friction Coefficient Based on the Influence of Tooth Surface Roughness

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Tooth surface roughness is an important index to evaluate the quality of gear processing, which affects the gear transmission error and tooth friction coefficient and changes the gear dynamics. Based on the fractal theory, the gear error correction model considering time-varying roughness is derived, and the new model of time-varying roughness and gear error is introduced into the Xu model to construct the friction coefficient model accounting for time-varying roughness and gear error, which has great reference value for improving the accuracy of gear error. The research results show that the root mean square value of gear error increases by 45.39% and 101.08% under comparison $R_a = 0.8\ \mu m$, $R_a = 1.6\ \mu m$, and $R_a = 3.2\ \mu m$, respectively, and the root mean square value of time-varying friction coefficient increases by 45.74% and 148.18%, respectively, which provides theoretical method support for precision management and quality management of gear processing in machinery enterprises.

1. Introduction

With the development of the national economy, the market demand for gear parts for the machinery industry is increasing, and gear manufacturers are becoming busy due to the increase in orders. However, gears in the test run stage often appear to have various failure problems, these problems seriously limit the amount of construction machinery out of the factory, and the production of products cannot be delivered to the hands of users, so a large backlog of orders seriously affects the sales of products. Technical personnel in the analysis and inspection of these test failures found that most of the reasons for failure are due to the quality of gear parts processing problems caused by; therefore, to improve the rate of construction machinery products, it is necessary to ensure the quality of gear parts processing.

Pearl River Delta gear products mass production enterprises are generally backward in terms of testing technology; there are two main problems: ① there is lack of gear single geometric error measurement instruments such as many factories cannot detect tooth profile error and helix error of the two important error items and thus cannot effectively control the geometric accuracy of the gear and cannot establish the absolute benchmark of the gear and ② due to the characteristics of gears in processing and use, there is a lack of unified geometry and the corresponding accuracy of the benchmark.

In recent years, a large number of scholars at home and abroad have proposed methods for measuring tooth surface roughness [1–4]. Some studies have analyzed the variation law of tooth surface roughness [5–7] and explored the characterization form of tooth surface roughness in-depth...
[8–10], and some scholars have studied the gear error under the influence of tooth surface roughness [11]. The research results show that the gear error under the consideration of tooth surface roughness effect has a significant effect on gear dynamics.

Tooth surface roughness is an important index to evaluate the quality of gear processing. Based on this, this paper adopts the experimental method to introduce the new model of time-varying roughness and gear error into the Xu model, constructs a friction coefficient model that accounts for time-varying roughness and gear error, and proposes a method aimed at optimizing gear process parameters to improve gear product quality management, which provides some theoretical guidance for gear processing enterprises to enhance part accuracy, improve product quality, and establish competitive advantages.

2. Methods and Materials

2.1. Mathematical Characterization of Tooth Surface Roughness Based on Fractal Theory. Gear is an important basic part of electromechanical products. Gear transmission is a major form of transmitting machine power and motion. The gear industry is the most technology-intensive and capital-intensive industry in the machinery industry.

The methods of describing rough surfaces by fractal theory mainly include the W-M function simulation method, time series simulation method, and fractal interpolation simulation method. Among them, the W-M function has a self-affine fractal curve with continuous and nondifferentiable characteristics, which is good for describing the roughness parameters of the actual tooth surface and thus is widely used in modern engineering fields and scientific research.

The construction principle applied to the gear tooth profile height \( Z(x) \) considering roughness can be expressed as follows:

\[
z(x) = G^{(D-1)} \sum_{n=0}^{\infty} y^{(D-2)n} \cos (2\pi^n x),
\]

\[1 < D < 2, \quad \gamma > 1,\]

where \( D \) is the fractal dimension, \( G \) is the characteristic coefficient, \( \gamma \) is a constant to meet the probability statistical distribution, in order to more truly reflect the tooth surface roughness characteristics, so that the tooth surface profile height to meet the requirements of high spectral density and random phase, often taken as 1.5 and is the minimum truncation frequency of the surface profile.

References [12, 13] presented experimental equations to obtain the following:

\[D = \frac{1.515}{R_a^{0.088}}\]

\[G = 10^{-5.26/R_a^{0.41}}\]

where \( R_a \) is the roughness.

\[R_a = \frac{1}{n} \sum_{i=1}^{n} |y_i|,\]

where \( n \) is the sampling point and \( y_i \) is the contour offset of the \( i \)-th sampling point.

By measuring the tooth profile along the tooth thickness direction, the measurement results of different positions are counted; thus, the roughness parameters, fractal dimension, and characteristic coefficients in the above empirical formula need to be reasonably corrected.

2.2. Tooth Surface Roughness Measurement Method. In this paper, the ZYGO NewView 9000 optical surface profiler from the United States was selected to measure tooth surfaces, as shown in Figure 1. This profiler provides versatile noncontact optical surface analysis, which can easily and quickly measure a variety of surface types, including smooth, rough, flat, sloped, and stepped surfaces. It is equipped with a 150 × 150 mm test bench and measures samples up to 89 × 203 × 203 mm (H × W × H). Its detection performance shows that the surface topography repeatability is less than 0.08 nm, the optical lateral resolution is 0.34, the spatial sampling resolution is 0.04, the RMS repeatability is 0.008 nm, the field of view is in the range of 0.04–17.49 mm, related to the objective and magnification, can rely on the integrated image stitching function, and can test a larger range. The instrument has the required tooth surface roughness and microprofile inspection requirements.

The ZYGO NewView 9000 uses 3D coherent scanning interferometry combined with SureScan™ antivibration technology to achieve subnanometer accuracy and automatic part focusing and positioning at all magnifications, and its built-in integrated antivibration platform enables the system to achieve high accuracy metrology in vibrating environments, as shown in Figure 2.

In order to make the observation results not lose generality, three gears with different tooth surface roughness of \( R_a = 0.8 \mu m, R_a = 1.6 \mu m, \) and \( R_a = 3.2 \mu m \) were selected for the study, corresponding to IT5, IT6, and IT7 grade accuracy, gear module 4 mm, tooth number 33, and pressure angle 20°, as shown in Figure 3, which were cut and processed as shown in Figure 4, and three sets of gear tooth observation specimens were machined for tooth surface roughness observation experiments, as shown in Figure 5.
**Figure 2:** ZYGO's autofocus analysis function.

**Figure 3:** Machining drawing of gear observation specimen.

**Figure 4:** Drawing of gear cutting process.
The roughness of the tooth surface profile is related to the sampling direction, sampling size, and sampling accuracy. Most of the existing studies have used probabilistic statistics to characterize the complete structure of a tooth surface by taking a generalized profile consisting of lines between sampling points of all surfaces, which does not reflect the complete working tooth surface profile. In order to measure the roughness of the tooth surface more comprehensively, the gear was divided into 1000 intervals along the tooth profile direction, ZYGO’s high-resolution image sensor was made to measure along the tooth width direction, and 10 sample groups were selected in each interval as the sample dataset representing the interval, as shown in Figure 6. Within a single interval, the reference length of the tooth profile roughness curve was set as the sampling length of the tooth profile roughness estimate along the profile direction of the image sensor. The interactive 3D mapping model of the ZYGO tooth profile roughness measurement and the statistical results of the quantified morphological information are shown in Figure 7.

2.3. Maximum Likelihood Estimation Method for Tooth Surface Roughness Evaluation Parameters. To determine whether the surface roughness of a part after machining meets the actual needs of engineering, the mainstream assessment method of roughness assessment parameters is currently based on the contour method of the midline system. The surface profile parameters related to surface roughness specified in the present national and international standards can be roughly divided into three categories: height feature parameters, spacing feature parameters, and shape feature parameters [14].

Among these evaluation parameters, the arithmetic mean deviation of the contour of the height feature parameter can more intuitively and comprehensively reflect a large amount of information on the surface contour of the part, as well as such features as small wave valleys, and the calculation is simple, so it is widely used in practical engineering.

Considering that the evaluation of gear surface roughness is essentially targeted at three-dimensional surface...
topography, most of the current evaluation parameters of surface topography are simplified to two-dimensional parameters, which are processed and evaluated by taking a certain line on the rough surface, and the results obtained only have the local geometric properties of surface topography and do not reflect the overall profile characteristics of the surface. Therefore, the isometric method of dividing the tooth surface into intervals is chosen, and the measurement

Figure 7: Gear surface roughness. (a) Actual measurement chart. (b) Distribution map along the measurement direction.
is performed by drawing lines within the interval several times and characterizing the measurement results parametrically.

First, each group of measured rough peak data within each interval on the tooth surface is processed, and the absolute value of the most valuable point within each data group is taken for comparison, as in (5), and the comparison result \( g(n) \) is taken as the numerical characterization of this group of data. The mean square value of the characterized data within each interval is taken as the numerical statistical characteristics of the rough peak measurement sample within this interval.

\[
g(n) = \begin{cases} g_{\text{max}}(n), & |g_{\text{min}}(n)| \leq |g_{\text{max}}(n)|; \\ g_{\text{min}}(n), & |g_{\text{min}}(n)| > |g_{\text{max}}(n)|; \end{cases} \quad n = [1: 1000], s = [1: 10],
\]

(5)

where \( n \) is the number of intervals divided on the tooth surface and \( s \) is the number of groups of rough peaks measured within each interval.

The statistical sample of the rough peak on the tooth surface presents strong randomness, which can be regarded as a random signal. The observations selected in this paper are processed and analyzed and brought into the subsequent experiments, which will not be discussed later. Using the pdf module in Matlab, the sample of the rough peak on the wheel teeth is fitted, and the fitting results are shown in Figure 8. Since the logistics distribution has low requirements for the sample data, it can accept the sample data with nonnormal distribution, and the overall prediction accuracy is high, so the logistics distribution is used. Then, the probability distribution function of the rough peak sample is shown in the following formula:

\[
R_a(x | \mu, \sigma) = \frac{e^{(x-\mu)/\sigma}}{\sigma(1 + e^{(x-\mu)/\sigma})^2} - \infty < x < + \infty,
\]

(6)

where \( \mu \) is the sample mean and \( \sigma \) is the sample variance, which are statistical parameters in a random sample.

The maximum likelihood estimation method is based on the principle of maximum likelihood statistics, which is a widely used statistical method, and the obtained maximum likelihood estimates have good properties of consistency, validity, and invariance. A sample of 10 gear teeth with rough peaks is mathematically characterized using the logistics distribution, and the parameters of the logistics distribution characterization of the selected gear teeth with rough peaks are estimated using the great likelihood estimation method, as in the following formula:

\[
R_a(\mu_2, \sigma_2) = \prod_{i=1}^{10} \frac{e^{(x_i-\mu)/\sigma_2}}{\sigma_2(1 + e^{(x_i-\mu)/\sigma_2})^2} = \left(\frac{\sigma_2}{\sigma_2(1 + e^{(x_i-\mu)/\sigma_2})^2}\right)^{\sum_{i=1}^{10} e^{(x_i-\mu)/\sigma_2}},
\]

(7)
where \( R_a(\mu_z, \sigma_z) \) is the likelihood function of surface roughness peak height; \( \mu_z \) is the sample mean of roughness peak height; and \( \sigma_z \) is the sample variance of roughness peak height.

To simplify the operation, it is transformed into a logarithmic form:

\[
\ln R_a(\mu_z, \sigma_z) = -10 \cdot \left( \ln(\sigma_z) + 2 \ln \left( 1 + e^{x - \mu_z/\sigma_z} \right) \right) + \sum_{i=1}^{10} x_i - \mu_z/\sigma_z. \tag{8}
\]

To estimate the maximum overall parameter \( \mu_z^* \) and \( \sigma_z^* \) value that can be obtained, the partial derivatives of the sums \( \mu_z \) and \( \sigma_z \) in (8) are found separately and then made equal to 0.

\[
\begin{align*}
\frac{\partial \ln (R_a(\mu_z, \sigma_z))}{\partial \mu_z} &= \frac{20 \cdot e^{x - \mu_z/\sigma_z}}{(1 + e^{x - \mu_z/\sigma_z})} - \frac{10}{\sigma} = 0, \\
\frac{\partial \ln (R_a(\mu_z, \sigma_z))}{\partial \sigma_z} &= \frac{20 \cdot \mu \cdot e^{x - \mu_z/\sigma_z}}{(1 + e^{x - \mu_z/\sigma_z})}\sigma^2 - \frac{10}{\sigma} \cdot \frac{\sum_{i=1}^{10} x_i - 10 \mu}{\sigma} = 0. \tag{9}
\end{align*}
\]

The results of tooth surface roughness distribution after great likelihood estimation are shown in Figure 9.

Since the direction was selected along the tooth profile direction when dividing the experimental interval for tooth profile height measurement, a Cartesian coordinate system was established in each sampling interval at this time, as shown in Figure 10. With the starting observation position as the origin, the horizontal coordinate \( x \) is the sampling length, along the tangential direction of the meshing point, and the vertical coordinate \( y \) is the measured tooth surface roughness peak height in the direction of the normal load direction of the meshing point.

Matching the measurement results with the actual gear meshing conditions, the horizontal coordinate is changed to the time domain by the coordinate transformation shown in the following formula:

\[
t = \frac{L}{V_x} = \frac{L}{V_p \cos \alpha} = \frac{L}{\omega_p R_p \cos \alpha}, \tag{11}
\]

where \( \alpha \) is the pressure angle; \( L \) is the sampling length; \( V_x \) is the tangential velocity at the meshing point; \( V_p \) is the instantaneous velocity at the active gear meshing point; and \( \omega_p \) is the angular velocity of the active wheel.

Equation (10) is transformed by (11) to establish the roughness as a function of time as shown in (12), where \( R_a(t) \) is defined as time-varying tooth roughness in this paper.

\[
R_a(t) = \frac{e^{(t-0.0697/1.5103)}}{1.5103 \cdot (1 + e^{(t-0.0697/1.5103)})^2}. \tag{12}
\]

When \( \omega_p = 600r/min \), the roughness after coordinate transformation is shown in Figure 11(a). When \( R_a = 3.2 \mu m \), the roughness at different rotational speeds is shown in Figure 11(b).
Figure 10: Coordinate transformation of tooth surface profile measurement results.

Figure 11: Height of tooth surface roughness peaks in the time domain after a coordinate transformation. (a) Roughness peak of tooth surface under different roughness. (b) Roughness peak of tooth surface at a different speed.
As can be seen from Figure 11(a), the transverse coordinate of the tooth surface roughness peak height still shows strong randomness after transforming to the time domain, and the root mean square values of the tooth surface roughness peak height at $R_s = 1.6 \mu m$ and $R_s = 3.2 \mu m$ increase by 143.62% and 207.22%, respectively, compared to the root mean square values of the tooth surface roughness peak height at $R_s = 0.8 \mu m$. From Figure 11(b), it can be seen that the root mean square values of tooth surface roughness peak height at 400 $r/min$ and 600 $r/min$ increase by 0.31% and 0.53%, respectively, compared to the root mean square values at 200 $r/min$.

2.4. Gear Error considering Time-Varying Tooth Surface Roughness. The mathematical model of time-varying tooth surface roughness with overall probability statistics of tooth surface morphology under great likelihood estimation obtained above is brought into (2) and (3) to obtain the following equations:

$$D(t) = \frac{1.515}{R_s(t)^{0.088}}$$

$$G(t) = 10^{-5.26R_s(t)^{1.012}}.$$ \hspace{1cm} (13)

Substituting (13) and (14) into (1), the time-varying tooth profile height function with the overall probability statistical characteristics of the tooth surface is obtained in the following equation:

$$z(t) = G(t)^{D(t)-1} \sum_{n=1}^{\infty} y^{(D(t)-2)n} \cos \left(2\pi y^n t\right),$$

$$1 < D(t) < 2, \gamma > 1.$$ \hspace{1cm} (15)

Referring to the mathematical model of static gear error established in the literature [11], the mathematical model of gear error under the action of time-varying tooth surface roughness is established in the following formula since only the effect of tooth surface roughness variation on gear error is considered in this paper.

$$e(t) = z_p(t) + z_g(t),$$ \hspace{1cm} (16)

where $z_p(t)$ and $z_g(t)$ are the profile heights of a set of intermeshing gear teeth.

Figure 12 shows the profile heights of a set of meshing master and driven gear teeth at different accuracies, from which it can be seen that the profile heights of the gear teeth increase with the increase in roughness and are highly random. Figure 13 shows the time-varying gear errors calculated according to (16) under different machining accuracies, and the error model reflects the errors during the meshing of a pair of gear pairs at different moments. From Table 1, it can be seen that compared to the root mean square value of the gear error under $R_s = 0.8$, the gear error increases by 45.39% and 101.08% with the increase of roughness, respectively.

3. Results and Discussion

In the actual meshing of a straight cylindrical gear, the two tooth surfaces in contact are in typical elastic flow lubrication. The friction between the meshing tooth surfaces due to relative sliding, as well as the microscopic shape of the gear surface, the operating speed, the lubrication method used in the gear system, and the physicochemical properties of the lubricant have a great influence on the gear dynamics. Xu [15] solved the oil film thickness equation, transient Reynolds equation, and energy balance equation after considering the tooth surface morphology, the lubrication state between the tooth surfaces, and the operating conditions and fitted the regression equation for the tooth surface friction coefficient of the gear meshing process, as shown in the following equation:

$$\mu = e^J \left(\frac{SR, P_h, \omega, b_p, \omega, S}{V_R, P_h, \cos \alpha}\right) \cdot \frac{\cdot |\cdot|}{\cdot |\cdot|} \cdot \cdot,$$

$$f(SR, P_h, S, \omega) = b_1 + b_2 \cdot |SR| \cdot P_h \cdot \log_{10}(\omega),$$

$$+ b_3 \cdot e^{0.3 |SR| P_h \cdot \log_{10}(\omega) + b_4 \cdot e^5,}$$

$$SR = \frac{V_p}{V_e},$$

$$V_s = \left|V_p - V_g\right|,$$

$$V_e = \frac{V_p}{2},$$

$$V_r = V_p + V_g,$$

$$V_p = \omega_p \cdot R_p,$$

$$V_g = \omega_g \cdot R_g,$$

$$P_h = \sqrt{\frac{f_S \cdot R}{\pi \cdot \left(1 - \omega_p^2 / E_p + 1 - \omega_g^2 / E_g\right)^2}},$$

$$f_S = \frac{T_p}{B \cdot R_{bp} \cdot \cos \alpha},$$

$$R = \frac{R_p \cdot R_g}{R_p + R_g},$$

where $SR$ is the slip-roll ratio; $S$ is the root mean square of tooth surface roughness; $V_r$ is the relative sliding speed of the two contacting tooth surfaces; $V_e$ is the entrainment speed (drawing speed); $V_p$ is the relative rolling speed of the two contacting tooth surfaces; $V_p$, $V_g$ is the instantaneous speed of the master-follower gear meshing point; $P_h$ is the maximum Hertzian contact pressure of the tooth surface; $f_S$ is the normal unit load of the tooth surface; $B$ is the tooth width; $T_p$ is the torque of the master wheel; $R_{bp}$ is the radius of the base circle of the master gear; $R_p$, $R_g$ is the radius of curvature of the master-follower wheel meshing; $V_p$, $V_g$ is the...
Figure 12: Tooth profile deviation. (a) Time domain diagram of tooth profile deviation of active wheel teeth. (b) Time domain diagram of tooth profile deviation of driven wheel teeth.

Figure 13: Gear error under tooth surface roughness correction model.
Poisson’s ratio of the master-follower gear; and $E_p$, $E_g$ is the modulus of elasticity of the master-follower gear.

The total efficiency losses of gears during power transmission can be attributed to sliding and rolling friction losses between the gear teeth, wind resistance losses due to the complex interaction of air around the gears, splashing and churning losses of the oil in the gearbox, and losses associated with bearings and seals. Xu considers that the friction loss is mainly related to the sliding speed and load and sets the slip-roll ratio, the roll speed, the equivalent radius of curvature, and the Hertzian contact pressure as the time-varying parameters in the Xu model.

Since the Xu model uses a two-dimensional model, some of the time-varying parameters take an idealized characterization. Time-varying tooth surface roughness and gear errors are affected by time-varying roughness, while the frictional power loss in a single cylindrical spur or helical gear pair is very small, the problem of efficiency loss in a gear system using multiple gear pairs in a gearbox comes to the fore. The transmission efficiency of a gear system formed by tandem gears is the product of the transmission efficiency of each pair of gears, so the implementation of any improvements that can reduce transmission efficiency losses for each pair of gears should not be ignored. After considering the above influencing factors, this paper takes the Xu model as the basis and makes reasonable corrections to the individual parameters affected by gear errors in the Xu model (such as mesh radius of curvature, maximum Hertzian contact pressure, and others) and the expression of roughness.

### 3.1. Gear Geometry Parameters including Time-Varying Tooth Surface Roughness and Gear Errors

Since the tooth friction coefficient is time-varying and strongly nonlinearly distributed, its value depends on many factors such as tooth surface material and surface finish; and too many conditional simplifications often affect the reliability of the analytical conclusions, the equivalent radius of curvature in the Xu model does not reflect well the influence brought by gear errors, and this paper chooses to use the mesh radius of curvature based on the impact friction analysis model [16] with systematic errors and comprehensive deformation of gear sublinear external meshing to replace it.

First, according to the calculation model proposed by Zhou et al. [16], which constructs the equivalent error-integrated deformation of gear system in the direction of gear meshing action line, a gear time-varying mesh radius of the curvature calculation model is established, and the time-varying mesh radius of curvature is obtained by combining the error $e(t)$, which considers the effect of time-varying roughness of tooth surface in the previous paper, as shown in (18). The actual meshing state on the gear meshing line is shown in Figure 14, where the distance from the node to the instantaneous meshing point is $s(t)$.

$$
\begin{align*}
R_p(t) &= r_1 \sin \alpha_p + s(t), \\
R_g(t) &= r_2 \sin \alpha_g + s(t), \\
s(t) &= s_g + e(t),
\end{align*}
$$

where $s_g$ is the mean value of the distance from the instantaneous meshing point to the gear pitch circle on the tooth profile using the geometric relationship, and its calculation model is shown in Figure 15.

Based on the meshing principle and geometric relationship and the geometric model established in the previous work [16], $s$ is found by equation (19), and the mean value of $s_g$ is calculated to improve it ($s_g = 7.3473$ at $R_a = 3.2$ for the gears selected in this paper).

$$
\begin{align*}
l_{O_1D} &= \left[ r_{a2}^2 + (r_1 + r_2)^2 - 2r_{a2}(r_1 + r_2)\cos \eta \right]^{1/2}, \\
s &= l_{O_1D} - r_1,
\end{align*}
$$

where $r_1$ is the radius of the active wheel pitch circle; $r_2$ is the radius of the driven wheel pitch circle; and $r_{a2}$ is the radius of the passive gear tooth top circle.

### 3.2. Time-Varying Friction Factor Model considering the Effects of Time-Varying Roughness and Gear Errors

Based on the meshing principle, the randomness of the peak of tooth surface roughness, and the time-varying nature of the contact state of the surface contact under the influence of gear geometric parameters during the rolling and relative sliding of gear pairs, the gear parameters such as time-varying tooth surface roughness, time-varying gear error, and time-varying gear mesh radius of curvature are improved on the basis of the Xu model, as shown in (20), and the model derives the time-varying friction coefficient as in Figure 16.

In order to more accurately reflect the effect of time-varying tooth surface roughness on the time-varying gear error during gear meshing, as well as to better illustrate the mapping principle between time-varying tooth surface roughness, time-varying gear error, and time-varying friction coefficient, the three mapping relationships are shown in Figure 17.

---

**Table 1: Statistical characteristics of gear errors.**

<table>
<thead>
<tr>
<th>Parameter ($\mu m$)</th>
<th>Root mean square (RMS) value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a = 0.8$</td>
<td>0.0954</td>
</tr>
<tr>
<td>$R_a = 1.6$</td>
<td>0.1387</td>
</tr>
<tr>
<td>$R_a = 3.2$</td>
<td>0.2789</td>
</tr>
</tbody>
</table>
Figure 15: The calculation model of geometric parameters of instantaneous engagement point.

Figure 16: The friction coefficient of tooth surface under time-varying roughness.

Figure 17: Mapping relationships of key parameters of the time-varying friction coefficient model.
Figure 18: Time-varying parameters in the time-varying friction coefficient model. (a) Tooth surface hertz contact pressure. (b) The equivalent radius of curvature.

\[
\mu(t) = e^{f(SR(t), P_h, v_0, Ra(t))} \cdot P_h^b \cdot |SR(t)|^b
\]

\[
\cdot V_{th} \cdot v_0 \cdot R(t)^{b_1}
\]

\[
f(SR, P_h, v_0, Ra(t)) = b_1 + b_4 \cdot |SR(t)| \cdot P_h(t)
\]

\[
\cdot \log_{10}(v_0) + b_5 \cdot e^{-|SR(t)| \cdot P_h(t) \cdot \log_{10}(v_0)} + b_6 \cdot e^{Ra(t)}
\]

\[
SR(t) = \frac{V_s(t)}{V_e(t)}
\]

\[
V_s(t) = |V_p(t) - V_g(t)|
\]

\[
V_e(t) = \frac{V_s(t)}{2}
\]

\[
V_p(t) = V_p(t) + V_g(t)
\]

\[
V_g(t) = \omega_p \cdot R_p(t)
\]

\[
V_p(t) = \omega_p \cdot R_p(t)
\]

\[
P_h(t) = \sqrt{\frac{f_c \cdot R(t)}{\pi \cdot \left(1 - \frac{v_p^2}{E_p} + \frac{v_g^2}{E_g}\right)}}
\]

\[
f_c = \frac{T_p}{(B \cdot R_{bp} \cdot \cos \alpha)}
\]

\[
R(t) = \frac{R_p(t) \cdot R_g(t)}{R_p(t) + R_g(t)}
\]

(20)

Table 2: Statistical characteristics of tooth surface friction coefficient under time-varying roughness.

<table>
<thead>
<tr>
<th>Parameter (μm)</th>
<th>Root mean square value (RMS) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra = 0.8</td>
<td>0.0038</td>
</tr>
<tr>
<td>Ra = 1.6</td>
<td>0.0098</td>
</tr>
<tr>
<td>Ra = 3.2</td>
<td>0.0174</td>
</tr>
</tbody>
</table>

Under the working condition of 600 r/min speed and 100N-m torque, some time-varying parameters in the equation are shown in Figure 18.

Considering the efficiency loss generated by friction during gearing, which may be related to the randomness of gear geometry, the time-varying tooth surface roughness obtained in the previous paper and the gear error affected by it are thus investigated as two time-varying factors that need to be adjusted in the Xu model.

3.2.1. Model of Tooth Surface Friction Coefficient considering Time-Varying Roughness. In the Xu model, tooth surface roughness is in the form of root mean square value, this paper expresses the tooth surface roughness in the form of the time-varying logistic distribution function estimated by great likelihood and brings it into the Xu model as the time-varying parameter for optimization, by bringing the tooth surface roughness into the Xu model in the form of time-varying parameter. The effect of gear error is not considered here; the corrected tooth surface friction coefficient obtained is shown in Figure 16. It can be seen from the figure that as the tooth surface roughness decreases, the friction coefficient curve fluctuation amplitude decreases and its overall change trend becomes more gentle. Combined with Table 2, compared with the root mean square value of the time-varying friction coefficient under Ra = 0.8μm, the root mean square value of the time-varying friction coefficient increases by 157.89% and 77.55%, respectively, with the increase of tooth roughness, and the phenomenon is in line with the engineering reality [17].

The tooth surface time-varying roughness characterized by (12) was brought into the original Xu model to obtain the tooth surface time-varying friction coefficient, which was compared with the time-varying friction coefficient obtained...
using the original Xu model, as shown in Figure 19. The statistical characteristics are shown in Table 3, which shows that the mean square value of the friction coefficient curve increased by 70.59% compared with the mean square value of the friction coefficient curve obtained from the Xu model, and the friction coefficient curve fluctuated sharply compared with that obtained from the Xu model. Theoretically, the roughness in the tooth thickness direction is collected and its statistical characteristics are retained, which can truly reflect the influence of the geometric characteristics of tooth surface morphology on the friction coefficient of the tooth surface.

### Table 3: The comparison of statistical characteristics of tooth friction coefficients with and without consideration of time-varying roughness.

<table>
<thead>
<tr>
<th></th>
<th>The root mean square value obtained by the Xu model</th>
<th>Root mean square values under the time-varying roughness model</th>
<th>Growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0102 mm</td>
<td>0.0174 mm</td>
<td></td>
<td>70.59%</td>
</tr>
</tbody>
</table>

![Figure 19: The time-varying friction factor under the roughness correction model.](image)

![Figure 20: The time-varying friction coefficient obtained after modifying the Xu model.](image)

3.2.2. Tooth Surface Friction Coefficient Model considering the Effect of Time-Varying Roughness and Gear Error. The model of time-varying tooth friction factor under consideration of gear error obtained by combining the gear error model based on the modified tooth surface roughness model using fractal theory with the geometric parameters of the gear and bringing it into the Xu model is shown in Figure 20. Analyzing Table 4, it can be found that compared with the root mean square value of the time-varying friction factor under \(R_a = 0.8\mu m\), as the tooth surface roughness increases, the root mean square value of the time-varying friction factor increases. The phenomenon is in accordance with the objective rule.
A cylindrical spur gear tooth with $Ra = 3.2$ was selected for the study, and a series of geometric parameters affected by gear error were obtained according to the geometric model established above. These geometric parameters and gear error and time-varying tooth surface roughness were brought into equation (20), and the statistical characteristics of the modified time-varying tooth surface friction coefficient are obtained as shown in Table 5. Compare it with the time-varying friction coefficient of tooth surface without considering the error, as shown in Figure 21. The analysis reveals the curve of friction coefficient considering that the influence of the gear error fluctuates more sharply than the curve of friction coefficient without considering the influence of the gear error.

This chapter investigates the effect of the randomness of tooth roughness topography, the experimental method of roughness measurement, the mathematical characterization of the measured roughness data by probability statistics and parameter estimation, the derivation of the relationship between sampling length and angular velocity, and the establishment of the roughness as a function of time, which is defined as the time-varying roughness. The time-varying roughness is introduced into the gear error model that is established based on the fractal theory, and some parameters in the original error model are modified.

### 4. Conclusion

#### 4.1. Theoretical and Application Values.

In this paper, based on the existing Xu model, the mathematical model of roughness with the overall probabilistic statistical characteristics of tooth surface morphology and the gear error model under the roughness model established by previous authors are introduced. The parameters such as roughness, gear error, and gear mesh radius of curvature in the Xu model are modified. A model of tooth surface friction coefficient considering the effects of roughness and error is established to provide theoretical support for the establishment and analysis of the subsequent gear dynamics model.

By comparing and analyzing the tooth surface friction coefficient model considering the peak roughness variation at different roughness levels, the root mean square value of tooth surface friction coefficient is increased from 157.89% to 77.55% with the increase in roughness, and the root mean square value of tooth surface friction coefficient increased by 70.59% compared with the root mean square value of friction coefficient obtained from the Xu model. By comparing the effect of the error on the tooth surface friction coefficient with and without considering the error, it was found that the root mean square value of the friction coefficient obtained from the tooth surface friction coefficient model considering the peak roughness variation and the gear error increased by 95.40%.

<table>
<thead>
<tr>
<th>Parameters ($\mu m$)</th>
<th>Root mean square (RMS) values (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ra = 0.8$</td>
<td>0.0094</td>
</tr>
<tr>
<td>$Ra = 1.6$</td>
<td>0.0137</td>
</tr>
<tr>
<td>$Ra = 3.2$</td>
<td>0.034</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Root mean square value without considering gear errors</th>
<th>Root mean square value under consideration of gearing error</th>
<th>Growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0174 mm</td>
<td>0.034 mm</td>
<td>95.40%</td>
</tr>
</tbody>
</table>

Figure 21: Time-varying friction factor curve under the modified model considering gear errors.
compared with that obtained from the tooth surface friction coefficient model considering only the peak roughness variation but not the gear error. By comparing the root mean square values of the model at different roughnesses, it was found that the root mean square values of the tooth surface friction coefficient increased from 45.74% to 148.18% with the increase in roughness, respectively.

From the above theoretical analysis and experimental verification, it can be seen that constructing a friction coefficient model that accounts for the time-varying roughness and gear error, comparing and analyzing the impact of the correction before and after and different tooth surface roughness on the friction coefficient, describes the tooth surface roughness more precisely and comprehensively, and makes the tooth pitch error accuracy of gears improve to a certain extent. The roughness model is accurate to better control the machining accuracy of gears, which provides a theoretical basis for the improvement of tooth surface machining accuracy. This method can be achieved without further investment in equipment costs, while improving the accuracy of the equipment spindle, which helps the scale application of the method and provides the basic method support for the in-depth study of gear processing quality management in mechanical engineering enterprises.

4.2. Prospects for Future Work

4.2.1. Improve the Accuracy of Tooth Blank and Tooth Machining. In the process of gear blank machining, it is the most critical to control the dimensional accuracy of the positioning bore of the gear part. In the process of tooth processing, the main factors affecting the tooth error are gear processing machine precision; gear processing tool; tooth blank positioning fixture; and other aspects. These aspects of the study are also very important.

4.2.2. To Ensure the Quality of Gear Materials. Manufacturing gear material is the basis for ensuring the quality of parts processing, so the control of gear material quality is particularly important.

4.2.3. Strengthen the Quality Control and Management of Gear Enterprises. Based on the application of the method proposed in this paper to the scale of gear processing enterprises, the study of production quality control and management methods is also very important.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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