Research Article

A Novel 2D Metal Flow Model for Hot Rolling of Aluminum Alloy Thick Plate

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Received 25 February 2022; Revised 18 May 2022; Accepted 30 May 2022; Published 13 June 2022

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Considering the serious inhomogeneous distribution of plastic deformation in the rolling process of thick plate, a novel 2D metal flow model is proposed with a quadratic distribution of flow velocity in the thickness direction instead of an equal value used in traditional metal flow models. According to the energy method, this model is solved. Through comparing with the experimental data of rolling force, the accuracy of this model is validated. Then, it is found that with the increase in roll speed, the neutral point moves towards the exit of the deformation zone. However, compared with other models of metal flow in rolling process, the neutral point is distributed much closer to the entrance of the deformation zone. That is because with the quadratic distribution of flow velocity in thickness direction, flow velocity of metal at the surface increases faster than average flow velocity of metal, so that the neutral point appears earlier.

1. Introduction

The metal flow model has serious influence on predicting the rolling force and shape of strips and plates. With high accurate prediction of rolling force, the precise roller gap can be obtained and made sure the thickness and shape of strips and plate are under control. Due to the lack of the measuring method, the law of metal flow is hard to be obtained. Therefore, with the large increase in computing capability, the finite element method becomes a popular method for simulating the metal flow in hot rolling process. Shahani et al. [1] adopted the finite element method to analyze the temperature, stress-strain fields. Liu et al. [2] analyzed the stress field in hot rolling process with a finite element and infinite element coupling method. Li et al. [3] established a finite element model for researching the flatness problem in the production of hot wide strip temper mills with small diameter work rollers. Touloupoulos [4] researched the pure viscoplastic models and their finite element discretization for hot rolling with a continuous finite element method. With proper boundary conditions and constitutive relation, the metal flow and rolling force can be calculated accurately with the finite element method. However, this method takes much time, so it is not suitable for online application. For rapidly obtaining predictions of metal flow and rolling force, more numerical simulation models and analytical models need to be proposed.

Both numerical simulation models and analytical models of metal flow in the rolling process need several assumptions, e.g., the symmetry of the model, form of metal flow in three dimensions and rigid plastic material. In the thin strip rolling process, the distribution of flow velocity in the thickness direction is homogeneous approximately [5], as shown in Figure 1(a). Several analytical models were proposed based on that assumption. Therefore, only the distribution of velocity in the rolling direction needs to be

For solving those metal flow models and obtaining flow velocity fields, there are two methods, energy method and yield criterion linearization method. In the energy method, a permitted velocity field with undetermined coefficients must be offered firstly and then the total deformation power of the rolling process can be calculated. Through optimizing the value of undetermined coefficients in the permitted velocity field to make the total deformation power reach a minimal value, the velocity field can be obtained and considered the most closed to the real situation. Liu et al. [11] analyzed the large cylindrical shell rolling with an improved strip layer method. Ding et al. [12] presented a three-dimensional velocity field with the stream function method for analyzing the chamfer edge rolling of ultraheavy plates. Peng et al. [13] calculated the rolling force of hot tandem rolling with a parabolic velocity field and upper bond method. Li et al. [14] calculated the rolling force of edge rolling based on continuous symmetric parabola curves and energy method. Ren et al. [15] proposed a metal transverse displacement model based on the minimum energy principle. Hamidpour et al. [16] calculated the rolling torque in the wire flat rolling process with the upper bound method. Zhang [17] et al. calculated the plastic deformation of strip rolling process with a new meshless method, which is named the flow function element free Galerkin method. Liu et al. [18] analyzed three-dimensional vertical rolling with the energy method and dual stream function method. Liu et al. [19] proposed a sine function dog-bone model for steady state deformation in vertical rolling with flat rolls, and the model is solved based on the upper bound integration method.

Application of the energy method needs programming and takes time to search the minimal deformation power. For further reducing the calculating time and obtaining an analytical solution, a yield criterion linearization method is proposed. Jiang et al. [20] established the expression of a linear specific plastic power to analyze the energy of the proposed elliptical velocity field. Zhang et al. [21] established a rolling force model with a mean slope yield criterion and obtained the analytical solutions. Wang et al. [22] obtained the theoretical rolling force based on a simple available velocity field and equal perimeter yield criterion.

From the above, it can be known that with the assumption of equal distribution in the thickness direction of flow velocity, a large number of metal flow models of thin strips are presented and the precision can be validated. While in the thick plate rolling process, metal at the center of the plate can hardly be rolled and the distribution of deformation is seriously inhomogeneous in the thickness direction [23], as shown in Figure 1(b). If the same assumptions are adopted in the modelling of thick plate, there will be great error in calculating the rolling force and metal flow. For researching the metal flow of thick plate in rolling process, Zhang et al. [24] established a two-dimensional velocity field with a new parameter called deformation penetration coefficient. This parameter is proposed to describe the inhomogeneous distribution of flow velocity in the thickness direction and can be obtained by experiments. According to the results of the experiments, this parameter is related to the relative reduction and number of rolling passes.

In this paper, considering the inhomogeneous and continuous distribution of strain in the thickness direction and complexity of modelling, the assumption of quadratic distribution in the thickness direction of flow velocity is adopted to establish a novel permitted velocity field. Then, the minimal value of total deformation power is found through optimizing the location of the neutral angle. Finally,
the velocity field that is the most closed to the real situation is obtained.

2. Materials and Methods

2.1. Assumptions. The establishing of the model needs several assumptions. According to literature [5], the assumptions applied in this paper and in other models of rolling processes are listed as follows:

1. The work roller is rigid material
2. The sheet is symmetric on both the material and the width about the symmetry axis
3. The sheet is rigid plastic material
4. The deformation of the sheet is plane strain
5. The friction coefficient of the surface is constant

2.2. Boundary Conditions. According to assumptions (1), (3), and (5), the boundary conditions can be obtained as follows:

\[ v(0, z) = v_0, \]  
\[ v(l, z) = v_1, \]  
\[ v(x_n, h(x_n)) = aR \cos \alpha_n, \]  
\[ u(x, 0) = 0, \]  
\[ h(x) = R + h - \sqrt{R^2 - (l - x)^2}, \]  
\[ l = \sqrt{R^2 - [R - (H - h)]^2}. \]

Considering the deformation of metal in a rolling process is a continuous process, the velocity field must obey the equal flow principle. The equal flow principle can be expressed as follows:

\[ v_0H = v_1h = \int_0^{h(x)} v(x, z)dz. \]  

In rolling theory, the total volume of metal does not change in the rolling process, which calls constant volume principle, so that the sum of the strain in x and z axis is zero according to assumption (4),

\[ \varepsilon_x + \varepsilon_z = 0. \]

2.3. Mathematical Model. In traditional models, the flow velocity in the rolling direction is considered as homogeneously distributed in the thickness direction. While according to several research studies [25–27], the plastic deformation inside the plate and the distribution of velocity are quite inhomogeneous. In this paper, as mentioned in introduction, considering the inhomogeneous and continuous distribution of strain in the thickness direction and complexity of modelling, the distribution of this velocity is considered as a quadratic distribution as follows:

\[ v(x, z) = f(x)z^2 + g(x). \]  

According to (1) and (2), when \( x = 0 \) and \( x = l \), the flow velocity in the rolling direction is not affected by \( z \) coordinate, so it can be obtained as follows:

\[ f(0) = f(l) = 0, \]
\[ g(0) = v_0, \]
\[ g(l) = v_1. \]

As to the distribution of flow velocity in rolling direction, according to the equal flow principle, the flow velocity in the rolling direction is increasing and can be calculated in the whole deformation zone where the velocity is equal to the linear velocity of work roller called the neutral point, and the central angle corresponding to the arc between the neutral point and the exit of deformation is called neutral angle. At the two sides of the neutral point, the directions of friction force between the work roller and the plate are opposite. It can be concluded that the flow velocity of metal in rolling direction is increasing continuously, and before neutral point, the increment of flow velocity is increasing, while after neutral point, the increment of flow velocity is decreasing. So, the changing rule of increment of flow velocity is much closed to a quadratic curves and the neutral point is where the quadratic curve reaches its maximum. Because at the two sides of the neutral point, the change of flow velocity is hardly symmetrical about neutral point, so \( f(x) \) must be a piecewise function as follows:

\[ f(x) = \begin{cases} a_1 (x - x_n)^2 + b, & 0 \leq x \leq x_n, \\ a_2 (x - x_n)^2 + b, & x_n < x \leq l, \end{cases} \]

where \( a_1 \) and \( a_2 \) must be negative because the flow velocity at the surface is greater than that at the center of plate.

\[ a_1 x_n^2 + b = 0, \]
\[ a_2 (l - x_n)^2 + b = 0. \]

From (7), (9), and (11), \( g(x) \) can be obtained as follows:

\[ g(x) = \begin{cases} \frac{3v_0H - (a_1 x^2 - 2a_1 x_n x)h(x)^3}{3h(x)}, & 0 \leq x \leq x_n, \\ \frac{3v_0H - (a_2 x^2 - 2a_2 x_n x + 2a_2 x_n l - a_2 l^2)h(x)^3}{3h(x)}, & x_n < x \leq l, \end{cases} \]

when \( x = x_n \) and \( g(x) \) is continuous.

Finally, the flow velocity can be expressed as follows:
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\[
\dot{v}(x, z) = \begin{cases} 
(a_1 x^2 - 2a_1 x_n x) z^2 + \frac{3v_0 H - (a_1 x_n^2 - 2a_1 x_n x) h(x)^3}{3h(x)}, & 0 \leq x \leq x_n, \\
(a_2 x^2 - 2a_2 x_n x + 2a_2 x_n l - a_2 l^2) z^2 + \frac{3v_0 H - (a_2 x^2 - 2a_2 x_n x + 2a_2 x_n l - a_2 l^2) h(x)^3}{3h(x)}, & x_n < x \leq l.
\end{cases}
\]

(15)

From (3) and (12), the undetermined coefficients \(a_1\) and \(a_2\) can be obtained as follows:

\[
a_1 = \frac{3[v_0 H - h(x_n) \omega R \cos \alpha_n]}{2x_n^2 h(x_n)^3},
\]

\[
a_2 = \frac{3[v_0 H - h(x_n) \omega R \cos \alpha_n]}{2(l - x_n)^2 h(x_n)^3},
\]

(16)

Strain in \(x\) axial direction can be obtained as follows:

\[
\dot{\varepsilon}_x = \frac{\partial v}{\partial x} = \begin{cases} 
2a_1 (x - x_n) z^2 - \frac{v_0 H x_n h(x)}{h(x)^2} - \frac{2a_1 (x - x_n) h(x)^2}{3}, & 0 \leq x \leq x_n, \\
2a_2 (x - x_n) z^2 - \frac{v_0 H x_n h(x)}{h(x)^2} - \frac{2a_2 (x - x_n) h(x)^2}{3}, & x_n < x \leq l.
\end{cases}
\]

(17)

According to (8), strain in \(z\)-axial direction can be obtained as follows:

\[
\dot{\varepsilon}_z = -\dot{\varepsilon}_x = \begin{cases} 
-2a_1 (x - x_n) z^2 + \frac{v_0 H x_n h(x)}{h(x)^2} + \frac{2a_1 (x - x_n) h(x)^2}{3}, & 0 \leq x \leq x_n, \\
-2a_2 (x - x_n) z^2 + \frac{v_0 H x_n h(x)}{h(x)^2} + \frac{2a_2 (x - x_n) h(x)^2}{3}, & x_n < x \leq l.
\end{cases}
\]

(18)

Then, the flow velocity in thickness direction can be obtained as follows:

\[
\dot{u}(x, z) = b_1(x), \quad 0 \leq x \leq x_n,
\]

\[
\begin{aligned}
&\frac{2a_1 (x - x_n) z^3}{3} + \left[ \frac{v_0 H x_n h(x)}{h(x)^2} + \frac{2a_1 (x - x_n) h(x)^2}{3} + \frac{2a_1 x_n^2 - 2a_1 x_n x}{3} h(x) h'(x) \right] z + b_1(x), \quad 0 \leq x \leq x_n, \\
&\frac{2a_2 (x - x_n) z^3}{3} + \left[ \frac{v_0 H x_n h(x)}{h(x)^2} + \frac{2a_2 (x - x_n) h(x)^2}{3} + \frac{2a_2 x_n^2 - 2a_2 x_n x + 2a_2 x_n l - a_2 l^2}{3} h(x) h'(x) \right] z + b_2(x), \quad x_n < x \leq l,
\end{aligned}
\]

(19)

Substituting (19) into (4), \(b_1(x)\) and \(b_2(x)\) can be expressed as follows:

\[
b_1(x) = 0, \quad b_2(x) = 0.
\]

(20)
Now, a permitted velocity field that satisfies all boundary conditions and a normal strain field is established. The shear strain field can be obtained based on plastic mechanics as follows:

Friction power can be calculated as follows:

\[ W_1 = \int_S D(\dot{\varepsilon}_{ij}) \text{d}s = \int_0^l h(x) \int_0^{\dot{h}(x)} 0.5942\sigma_s (\dot{\varepsilon}_x - \dot{\varepsilon}_y) \text{d}z \text{d}x. \]  

(22)

\[ W_2 = \int_0^l m k \Delta v \sqrt{1 + h'(x)^2} \text{d}x \]

\[ = mk \int_0^l \Delta v \sqrt{1 + \left( \frac{x - l}{\sqrt{R^2 - l^2}} \right)^2} \text{d}x, \]

(23)

\[ \Delta v = \sqrt{\left( v(x, h(x)) - \omega R \cos \left( \arctan \left( \frac{l - x}{R + h - h(x)} \right) \right) \right)^2 + \left( u(x, h(x)) - \omega R \sin \left( \arctan \left( \frac{l - x}{R + h - h(x)} \right) \right) \right)^2}. \]

(24)

The internal deformation power can be integrated as follows [24]:

\[ W_1 = \int_S D(\dot{\varepsilon}_{ij}) \text{d}s \]

\[ = \int_0^l h(x) \int_0^{\dot{h}(x)} 0.5942\sigma_s (\dot{\varepsilon}_x - \dot{\varepsilon}_y) \text{d}z \text{d}x. \]

(22)

\[ W_2 = \int_0^l m k \Delta v \sqrt{1 + h'(x)^2} \text{d}x \]

\[ = mk \int_0^l \Delta v \sqrt{1 + \left( \frac{x - l}{\sqrt{R^2 - l^2}} \right)^2} \text{d}x, \]

(23)

\[ \Delta v = \sqrt{\left( v(x, h(x)) - \omega R \cos \left( \arctan \left( \frac{l - x}{R + h - h(x)} \right) \right) \right)^2 + \left( u(x, h(x)) - \omega R \sin \left( \arctan \left( \frac{l - x}{R + h - h(x)} \right) \right) \right)^2}. \]

Finally, the total power \( W \) can be calculated as follows:

\[ W = W_1 + W_2 + W_3. \]

(26)

The total power \( W \) is a function of the neutral point location \( x \). Through finding the proper \( x \) to make \( W \) reach the minimum value \( W_{\text{min}} \), the flow velocity field can be figured out.
Then, through recalculating the total power $W$, the torque of motor and rolling force can be calculated as follows [24]:

\[
M_{\text{min}} = \frac{RW_{\text{min}}}{2\nu_R},
\]

\[
F_{\text{min}} = \frac{M_{\text{min}}}{\chi\sqrt{2R\Delta h}},
\]

(27)

where $\chi$ represents the force arm coefficient, which is about 0.5 in the hot rolling process, and $\Delta h$ is the decrease in thickness at exit of deformation zone.

3. Solutions

3.1. Calculating Process. Figure 2 shows the calculation process of the metal flow model.

It is notable that there is a judgement of $a_1$ and $a_2$ because the flow velocity at the surface of the plate has a greater increasing rate than that at the center, so $a_1$ and $a_2$ must be negative. This model is established by the C++ language, through continuously looping, the distributions of flow velocity which can realize minimum energy can be calculated based on the minimum energy method.

4. Results and Discussion

4.1. Rolling Force. Solving of the flow velocity field is to obtain the rolling force, so the rolling force is the most effective indicator to validate the accuracy of this metal flow model. Industry experiments are made to compare the rolling forces obtained by both metal flow model and actual rolling process; the experimental equipment is shown in Figure 3. The forty calculated results of rolling force based on different rolling conditions which are listed in Table 1 are compared with the rolling force obtained from an actual rolling production line with the same rolling conditions, as shown in Figure 4. The rolling force data shown in Figure 3 are arranged in chronological order and are from the former eight passes of five aluminum alloy thick plates. For distinguishing the rolling process of thin strips, the entrance thickness that both adopted in the metal flow model and the actual rolling process is beyond 300 mm [28].

It can be observed that the data points of actual rolling force are distributed around the results obtained by the metal flow model and the maximal error between the metal flow model and actual rolling data is less than 5.6%. This error is considered acceptable in this paper.

In the former eight passes, the rolling force increases firstly and then decreases. This is because firstly, with the time of thick plate stay in air increases, the temperature of the plate decreases through thermal convection and thermal radiation. Then, the lower temperature results in the increasing deformation resistance. Secondly, the reduction increases gradually with the increasing of rolling pass. Under the combined actions of decreasing temperature and increasing reduction, the change law of rolling force is as shown in Figure 4.

4.2. Location of Neutral Point. It is notable that an effective rolling process must be established with a proper matching relation between the roller speed and flow velocity of plate. In rolling theory, the flow velocity of plate at the entrance of the deformation zone is less than the horizontal component of the roller speed [29]. While at the exit of the deformation zone, the flow velocity of the plate is greater than the horizontal component of roller speed. Therefore, the rolling
parameters applied in this paper are listed in Table 2, and the locations of neutral points under all rolling conditions are shown in Figure 5.

It can be observed that under the offered rolling conditions, the neutral points are located in a narrow range of the deformation zone and close to the entrance of the deformation zone. Compared with literatures [11, 13, 20–22], locations of neutral points obtained in this model are more closed to entrance of deformation zone. This is because in their research studies, the flow velocity is equal at different locations in thickness direction. So, under various rolling conditions, neutral points can be located at each location in deformation zone. While in the model in this paper, a quadratic distribution in thickness direction of flow velocity in rolling direction is adopted based on the inhomogeneous deformation in thickness direction, flow velocity at surface of plate increases faster than at center of plate, and due to the thickness of the plate decreasing rapidly near the entrance of the deformation zone, so according to the equal flow principle, the flow velocity increases rapidly, then the flow velocity at surface of plate can be equal to the horizontal component of rotate speed of work roller much earlier, and finally the locations of neutral points obtained in this paper are more close to entrance of deformation zone.

### Table 1: Rolling parameters applied in the comparison of experiments and metal flow model.

<table>
<thead>
<tr>
<th>PASS</th>
<th>Entrance thickness (mm)/reduction (mm)/width (mm)/temperature (°C)</th>
</tr>
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<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>622.85/25.25/1317.8/497.5</td>
</tr>
<tr>
<td>2</td>
<td>597.6/32.4/1319.2/496.2</td>
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<td>565.2/38.5/1321/494.8</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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</tr>
<tr>
<td>8</td>
<td>373.4/20.6/1334.6/487.9</td>
</tr>
</tbody>
</table>

Figure 4: Comparison of rolling force obtained by metal flow model and actual rolling process.

4.3. **Velocity Field.** Figure 6 shows the velocity field and the difference of flow velocity between the surface and the center of the thick plate.

It can be observed from Figure 6(a) that the flow velocity in the rolling direction is increasing continuously from entrance to exit of deformation zone. This is because from entrance to exit of deformation zone, thickness of plate decreases continuously. Because upper geometry boundary is a part of circle, the rate of thickness reduction decreases from entrance to exit of deformation zone. The flow velocity is higher than that at the center of the plate due to the inhomogeneous plastic deformation in the thickness direction, as shown in Figure 6(b). The similar results are shown also in literatures [17, 30, 31]. The distribution of flow velocity in rolling direction shown in literature [17] is so similar to Figure 6(b). Through simulations [30] of rolling process applied for thick plate, it can be found that at the upper and lower surfaces, there are more plastic deformation appearing than that at the center of plate so that the flow velocity at the surface is larger than that at the center of plate. Then, through experimental data [31], it can be found that the elongation at the upper and lower surfaces is larger than that at the center of plate, and the distribution of elongation in thickness direction is closed to a quadratic curve.

However, the velocity difference between those two locations is tiny. According to the rolling theory, even flow velocity in each section inside the deformation zone increases gradually with the thickness of each section decreasing. That is because when the reduction ration is 8%, the flow velocity increases at an even rate less than 8% in
Table 2: Rolling parameters applied in the metal flow model.

<table>
<thead>
<tr>
<th>Initial thickness (m)</th>
<th>Exit thickness (m)</th>
<th>Radius of roll (m)</th>
<th>Entrance velocity (m·s⁻¹)</th>
<th>Exit velocity (m·s⁻¹)</th>
<th>Min linear velocity of roll (m·s⁻¹)</th>
<th>Max linear velocity of roll (m·s⁻¹)</th>
</tr>
</thead>
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<tr>
<td>0.5</td>
<td>0.48</td>
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<td>1.716</td>
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</tr>
</tbody>
</table>

Figure 5: Locations of neutral point under different rolling conditions.

Figure 6: Distribution of flow velocity in the rolling direction with $H = 0.5$ m, $h = 0.46$ m, $R = 0.3$ m, and $v_0 = 1.6$ m/s: (a) 3D contour and (b) comparison of flow velocity between the surface and center of plate.
In this paper, a novel model for predicting the rolling force and calculating the flow velocity field in the rolling process of aluminum alloy thick plate is proposed by considering the quadratic distribution in the thickness direction of flow velocity. This model is solved by the energy method. The conclusions are as follows.

5. Conclusion

In this paper, a novel model for predicting the rolling force and calculating the flow velocity field in the rolling process of aluminum alloy thick plate is proposed by considering the quadratic distribution in the thickness direction of flow velocity. This model is solved by the energy method. The conclusions are as follows.

(1) Through comparing the rolling force calculated by the novel model and obtained by the actual rolling process, the errors are all less than 5.6% so that the accuracy of this novel model is validated.

(2) In the rolling process of thick plate, the neutral points are mostly located near the entrance of the deformation zone. With the increase in roller speed, the location of the neutral point moves towards the exit of the deformation zone.

(3) The field of flow velocity both in rolling and thickness directions obtained in this model is consistent with the experiments of rolling process of thick plate. So, the assumption of quadratic distribution of flow velocity is a useful attempt for establishing more closed to real rolling process of thick plate.

List of Symbols

\( \alpha \): Neutral angle

\( H \): Initial thickness

\( h \): Exit thickness

\( h(x) \): Thickness at the coordinate of \( x \)

\( v_0 \): Initial flow velocity

\( v_1 \): Exit flow velocity

\( v(X,Z) \): Flow velocity in the rolling direction

\( u(x,z) \): Flow velocity in the thickness direction

\( R \): Radius of work roller

\( l \): Length of deformation zone in rolling direction

\( x_0 \): Coordinate in \( x \) axial of neutral point

\( m \): Friction coefficient.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported in part by National Natural Science Foundation of China under Grants 52004029 and 12002236, Natural Science Foundation of Tianjin under Grants 18JCQNJC75000 and 18JCYBJC95200, and Tianjin Science and Technology Plan Projects under Grant 18ZXZNGX00360.

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