Research Article

Structural Damage Identification Method of Girder Bridges Based on Multilevel Data Fusion Theory

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1. Introduction

When bridge structures are in service [1], due to influence of uncertain factors such as environment and vehicle load, structural stiffness and material properties will be degraded to a varying degree, which would affect normal use of structure and pose a threat to driving safety. Therefore, it is particularly important to monitor health of structure, to determine damage position and its extent in time, and to repair defect or to strengthen structure as soon as possible.

Presently, damage identification methods based on dynamic fingerprint are most commonly used, which include natural frequency [2], curvature mode [3–5], flexibility matrix [6, 7], and modal strain energy [8, 9]. Ji-Ling et al. [10] applied primary frequency change of local dynamic response to identify local damage of truss structure. Chang-Sheng et al. [11] proposed a damage identification method based on modal curvature utility information entropy by utilizing advantage of modal curvature sensitive to damage, to realize damage localization and quantitative analysis of beam structure. Zi-Chuan et al. [12] used the method of modal strain energy change rate to identify damage of L-shape pipeline. However, the methods above all have their own limitations; their recognition accuracy will also vary with influence of factors such as index selection, test error, and environmental noise. Meanwhile, their recognition results are also changeful when different modal data are adopted for identification, especially for damage quantitative analysis, in which the error obtained by only single indicator will be still large; if it is applied to actual project, it would certainly lead to fault choices of subsequent repair measures.

Data fusion is a method where multisource information is comprehensively considered, which has good fault tolerance and reliability [13]. Combining data fusion
technology with structural damage identification can effectively improve the accuracy of damage diagnosis. Through redundant or complementary information obtained from multiple damage features, data fusion is carried out according to the fusion algorithm, and then the best synergistic result is obtained, which improves the accuracy of damage identification [14, 15]. Data fusion methods commonly include weighted average criterion [16], Bayes fusion principle [17–19], D-S evidence matrix [20, 21], BP neural network [22–24], and support vector machine (SVM) [25]. Grande and Imbimbo [26] proposed a damage identification method based on a combination of modal strain energy index and D-S evidence criterion, which significantly improved identification efficiency, but the recognition ability of this method depends on the sensitivity of vibration modes to damage scenarios. Ji-Lin et al. [27] proposed a method to use D-S evidence criterion to fuse damage identification results of multiple finite element models that consider environmental variation, which has high identification accuracy and practical applicability. Jun et al. [28] found that the damage identification method based on Bayes data fusion can more accurately and effectively determine the location of single damage and multiple damage in beam structures. Xue-Yan and Li-Chang [13] used Bayes data fusion theory to fuse multigroup damage vectors measured under various test environment and multiple sensors, which could improve damage identification accuracy. Bin et al. [29] conducted two-stage information fusion method for multiposition damage identification in a truss frame by using D-S evidence matrix and weighted average criterion and proved that it has better noise immunity.

However, the existing data fusion methods still have shortcomings. For example, when using the Bayesian fusion method for damage identification, the prior probability of damage to each element of the structure is not easy to obtain [30]. Due to the uncertainty and subjectivity of prior data, D-S evidence criterion usually faces the problem of evidence conflict [31, 32]. The extraction of feature parameters easily affects the learning time and reasoning performance of neural network fusion system [33].

In this paper, modal strain energy dissipation rate (DR), change rate of cross-model modal strain energy (CR), and difference of modal curvature ratio (RD) are selected as basic indexes, to construct multilevel fusion model for structural damage localization and quantitative analysis based on Bayes theorem, weighted average criterion, and BP neural network.

2. Data Fusion Method

2.1. Fusion Rule

2.1.1. Bayes Theorem. Bayes theorem is one of common theories used in multiobjective information fusion decision. Its principle will constantly update prior probability through last observation data for acquiring posterior probability.

Equation of Bayes is known as

$$P(A_m|B) = \frac{P(B|A_m)P(A_m)}{\sum_{j=1}^{m} P(B|A_j)P(A_j)}$$

(1)

where $P(A_m)$, $P(A_m|B)$ are prior probability of event $A_m$ and its posterior probability when event $B$ occurs.

When data fusion theory is applied in research area of structural damage identification, it can be express that there are $n$ information sources (or $n$ evaluation indicators) $S_1, S_2, \ldots, S_n$, and $m$ mutual independent recognized targets (or $m$ elements) $A_1, A_2, \ldots, A_m$, so according to equation (1), $P(A_m|B)$ can be expressed as

$$P(A_m|S_1, \ldots, S_n) = \frac{P(S_1|A_m)P(A_m)}{\sum_{j=1}^{m} P(S_1|A_j)P(A_j)}$$

$$= \frac{P(S_1, \ldots, S_n|A_m)P(A_m)}{\sum_{j=1}^{m} P(S_1, \ldots, S_n|A_j)P(A_j)}.$$  

(2)

While prior probability of target needs to be provided in advance, the damage position of structure usually cannot be foreseen. Usually, damage probability at mid-span is largest and will be smaller at support; therefore, we apply displacement influence line for calculating prior probability [34, 35].

As a case study, a simply supported beam is established as shown in Figure 1, and based on virtual work principle, structure displacement can be expressed as the following equation:

$$\Delta = \int \frac{M_p}{EI} ds.$$  

(3)

The bending moment at any cross section $x$ is

$$M_p(x) = \begin{cases} \frac{F_p (l-x)}{l} & (0 \leq x < \infty), \\ \frac{F_p (l-x)}{l} & (\infty \leq x < l). \end{cases}$$

(4)

According to equations (3) and (4), displacement influence line then can be derived as

$$\varphi_n = \begin{cases} \frac{1}{6EI} \frac{l-s}{l} F_p x^3 - \frac{1}{6EI} \frac{F_p x}{l} (l-s)^3 + \frac{1}{6EI} F_p x l (l-s) & (0 \leq x < s), \\ \frac{1}{6EI} \frac{F_p s}{l} (l-x)^3 - \frac{1}{6EI} \frac{l-x}{l} F_p s^3 + \frac{1}{6EI} F_p s l (l-s) & (s \leq x < l). \end{cases}$$

(5)
When \( s = l/2 \), equation (5) can be easily transformed as follows:

\[
\varphi_u = \begin{cases} 
- \frac{1}{12EI} F_p x^3 + \frac{1}{16EI} F_p x^2 (0 \leq x < \frac{l}{2}) \\
- \frac{1}{12EI} F_p (l-x)^3 + \frac{1}{16EI} F_p (l-x)^2 \left( \frac{l}{2} \leq x < l \right)
\end{cases}
\]  

(6)

In equation (6), mid-span displacement influence line consists of two continuous and derivable piecewise functions, and it can also be drawn as shown in Figure 2.

From Figure 2, the damage prior probability of any interval \([a, b]\) is calculated by its area as the following equation:

\[
P(A_{[a,b]}) = \frac{\int_a^b \varphi_u \, dx}{\int_l^0 \varphi_u \, dx} \quad (7)
\]

If interval \([a, b]\) is at mid-span, the area of mid-span element should be integrated into two parts, left side and right side of mid-span.

2.1.2. Weighted Average Criterion. The principle of weighted mean method [16] is to assign each index with different weight factor, calculating their weighted mean value, to reach the target that sum of square of Euclidean distance between weighted mean value and each index recognition value is smallest.

Mean value of each index is defined as \( \overline{S}_j \),

\[
\overline{S}_j = \frac{\sum_{i=1}^n S_{ij}}{n}
\]

(8)

where \( S_{ij} \) is \( i^{th} \) indicator value of \( j^{th} \) element, and \( \overline{S}_j \) is the mean of each index.

Weight factor \( w_i \) of each index is

\[
w_i = \frac{(1/c_i)}{\sum_{i=1}^n (1/c_i)} \quad (c_i \neq 0),
\]

(9)

where \( c_i = \| \overline{S}_j - S_j \| = \sqrt{\sum_{j=1}^m (\overline{S}_j - S_j)^2} \) is Euclidean distance between each index recognition value and their mean value.

So, the weighted mean value can be obtained.

\[
DF = \sum_{i=1}^n w_i S_i
\]

(10)

2.1.3. BP Neural Network. The backpropagation (BP) neural network first obtains an uncertain reasoning method through a specific learning algorithm and then uses this mechanism to carry out information fusion and relearning through continuous update of network weights. The BP neural network is composed of input layer, output layer, and one or more hidden layers as shown in Figure 3, which includes \( m \), \( n \), and \( p \) neurons in input, output, and hidden layer respectively; here, \( w \) is weight value, and \( h \) is hidden layer output.

When BP neural network is applied to structural damage identification [36, 37], the measured structural features, structural features in the known normal state, and labeled damage feature database are input into neural network for matching. Then, measured features are classified by the self-learning of neural network. Finally, the damage classification results are output.

2.2. Index Selection for Fusion

2.2.1. Modal Strain Energy Dissipation Rate (DR). The index based on modal strain energy can well recognize local damage of structure, and so does the modal strain energy dissipation rate (DR) [38]. Modal strain energy before and after damage is known as

\[
MSE_{ij}^u = \Phi_i^{ut} K_j \Phi_i^u,
\]

(11)

\[
MSE_{ij}^d = \Phi_i^{dt} K_j \Phi_i^d,
\]

(12)

where \( K_j \) is stiffness matrix of element \( j \); \( \Phi_i^u \) and \( \Phi_i^d \) represent \( i^{th} \) modal displacement vector of undamaged and damaged elements, respectively.

Considering the damage process of the structure as the dissipation process of modal strain energy of structural element, the DR of \( j^{th} \) element of structure can be expressed as

\[
DR_{ij} = \frac{|MSE_{ij}^d - MSE_{ij}^u|}{|MSE_{ij}^d - MSE_{ij}^u| + MSE_{ij}^u}.
\]

(13)

where \(|MSE_{ij}^d - MSE_{ij}^u|\) is the change of modal strain energy of \( j^{th} \) element of damaged structure.
2.2. Change Rate of Cross-Model Modal Strain Energy (CR).

The cross-model modal strain energy (CMMSE) proposed in paper [9] can effectively improve noise immunity of index. Therefore, its change rate (CR) can also be one of damage fusion indexes. The CMMSE before and after damaged can be calculated by equations (11) and (12), respectively, shown as follows:

\[ CMMSE_{ij}^d = \Phi_i^{dT} K_j \Phi_i^d, \]
\[ CMMSE_{ij}^u = \Phi_i^{uT} K_j \Phi_i^u. \]

So, index CR can be expressed as

\[ CR_{ij} = \frac{CMMSE_{ij}^d - CMMSE_{ij}^u}{CMMSE_{ij}^u}. \]

2.2.3. Difference of Modal Curvature Ratio (RD).

Based on method of central difference, modal curvature of structure can be approximately calculated as follows:

\[ \Phi_{ik} = \frac{\Phi_{i(k+1)} - 2\Phi_{ik} + \Phi_{i(k-1)}}{h^2}, \]

where \( \Phi_{ik} \) represents modal curvature and displacement of \( k \)th node of \( i \)th mode, respectively. \( h \) is distance between adjacent nodes.

The ratio of \( k \)th node modal curvature to all nodes is defined as \( R \), and its values before and after damaged can be acquired by the following equations:

\[ R_{ij}^n = \frac{\Phi_{ik}^{nT}}{\sum_{k=1}^{n} \Phi_{ik}^{nT}}, \]
\[ R_{ij}^d = \frac{\Phi_{ik}^{dT}}{\sum_{k=1}^{n} \Phi_{ik}^{dT}}. \]

Here, we introduce a new index named RD, which is the difference of \( R \) damaged and undamaged as equation (18) shown as follows:

\[ RD_{ik} = R_{ij}^d - R_{ij}^u. \]

2.3. Procedure of Multilevel Fusion

2.3.1. First-Level Fusion Method Based on Bayes Theorem.

The first-level fusion flow chart of simply supported beam is shown in Figure 4; it is a kind of fusion within each index itself, respectively, by index values of the first-three order modes based on Bayes fusion theorem. The detailed steps are as follows:

1. Calculating first-three order modal information of structure by finite element software, and then according to three indices \( DR, CR, \) and \( RD \) mentioned above, their values can be achieved from equations (13), (15), and (18) by MATLAB software.

2. Calculating probability distribution vectors \( [z_1, z_2, z_3] \) of first-three order modes of each index, respectively.

3. Fusing first-three order modal probability assignments \( [z_1, z_2, z_3] \) of each index, and obtaining first-level fusion indexes \( DR_1, CR_1, \) and \( RD_1 \) for damage location judgement. The greater the fusion probability is, the greater the element damage possibility would be.

2.3.2. Second-Level Fusion Method Based on Weighted Average Criterion.

Second-level fusion process is shown in Figure 5, and the concrete steps are as follows:

1. According to the first-level fusion recognition method, calculating values of \( DR_1, CR_1, \) and \( RD_1 \) for every element, respectively, and substituting them into equation (8), so that we can obtain mean S.

2. Substituting \( S \) into equation (9), we can obtain a set of weight factors \( w_1, w_2, \) and \( w_3 \) for indexes \( DR_1, CR_1, \) and \( RD_1 \) respectively.

3. Substituting \( DR_1, CR_1, RD_1, \) and weight factors \( w_1, w_2, w_3 \) into equation (10), second-level fusion index \( DF \) will be obtained, which can be applied for damage location identification, and also as input for quantitative analysis in the third-level fusion.
2.3.3. Third-Level Fusion Method Based on BP Neural Network. Third-level fusion process is shown in Figure 6, and the concrete steps are as follows:

1. Establishing various damage cases, and according to equation (10), the second-level fusion results DF of all damage cases are calculated.
2. The second-level fusion results DF of all cases are divided into two parts: training set and prediction set.
3. The training set DF is input into the neural network for training, and the network parameters are continuously adjusted through the learning of training set until network output is close to expected output, and the training is stopped.
4. Input prediction set DF into the trained neural network for damage degree recognition, and finally output the residual stiffness $DD_3$ of damaged unit.

3. Damage Recognition Results of Simply Supported Beam

3.1. Simple Supported Beam Model. A concrete simply supported beam model is established by using ANSYS 17.0 as shown in Figure 7. Beam length is 6.00 m, cross section dimension (width $\times$ height) is 0.20 m $\times$ 0.12 m, elastic modulus $E = 30 \text{ GPa}$, Poisson’s ratio $\nu = 0.17$, and density $\rho = 2500 \text{ kg/m}^3$. Beam189 element model is selected, and beam structure is divided into 25 elements, 26 nodes.

The structural stiffness is declined by reducing element elastic modulus $E$; to simulate structure damage, in total, 20 damage cases are considered here as shown in Table 1.

3.1.1. Modal Analysis. In order to reduce error for damage identification, the first-three modal shapes are used for analysis, so the first-three order frequencies of undamaged and damaged cases are calculated firstly by ANSYS 17.0 software, shown in Table 2. When beam is damaged, the natural frequency of structure will also change, and the modal shape cloud diagram of nondamage case is shown in Figure 8.

3.1.2. Prior Probability. The displacement influence line is generated when unit load moves on beam, and the prior probability $P(A_m)$ of every element can be calculated by equation (7), as shown in Table 3.

3.2. Damage Localization Analysis of Single Index. First, we use three original indicators (DR, CR, and RD) to analyze structural damage separately, and the results of damage cases are shown in Figure 9.

3.2.1. Single Damage Identification. From Figure 9(a), it is obvious that maximum peaks appear at element No.3 in 1st order modal curves of DR, CR, and RD, which correspond to the damage assumed in case 1, so they all can search damage location. However, in 2nd and 3rd order modal curves, not only element No.3, there would appear other peaks at healthy elements, which cannot be ignored. For example, in the 3rd order modal curve of RD (blue line), the values of undamaged elements No. 8 and No. 9 are very large, so it may lead misjudgment.

3.2.2. Two Damage Cases Identification. In Figure 9(b), for case 15, from 1st and 3rd order modal curves of DR and CR, peaks at damage element No. 3 and No. 9 can be searched, while their 2nd order modal curve would be not significant because other peaks appeared at elements No. 12 and No. 14. In RD modal curves, it would only be effective for 1st order mode to lock damage location.

3.2.3. Three Damage Cases Identification. In Figure 9(c), both methods of DR and CR can identify damage position by searching peaks at damaged elements No. 6, No. 12, and No. 18. But in the curves of RD, values of undamaged elements No. 5 and No. 7 will exceed those of damaged elements No. 12 and No. 18, so it is not easy to distinguish which element is damaged or not.

From the above analysis, when multiple damage occurs, the ability of single index and its accuracy are both...
significantly declined for damage recognition, and its probability of misjudgment would be increased.

3.3. Damage Localization Analysis of First-Level Fusion Based on Bayes Theory. Based on Bayes fusion theory, by fusing damage probabilities of the first three order modes of DR, CR, and RD, respectively, three first-level fusion indexes named DR1, CR1, and RD1 will be obtained, which can be applied for damage location identification according to the flow chart in Figure 4.

3.3.1. Two Damage Cases Identification. In Figure 10, the peaks position of two damage cases for cases 15, 16, 17, and 18 can be easily found, which indicates that DR1, CR1, and RD1 can be qualified for damage localization. But from vertical coordinate, the peak values of damage probabilities are irregular and cannot be in accordance with damage set in cases; therefore, the damage degree will not be directly reflected in fusion results.

3.3.2. Multiple Damage Identification. According to the results of case 20 shown in Figure 11, indexes DR1, CR1, and RD1 can locate damage position by peak searching, and the greater the damage degree is, the higher and more obvious the peaks are; for example, damage degree of element No. 6 is biggest (30%), so its peak is highest.

In summary, compared with single index, after Bayes first-level fusion, damage localization abilities of DR1, CR1, and RD1 are significantly improved, which can reduce the impact of health elements and increase the accuracy of damage identification. Comparing their peak values with each other, the order of their ability is $DR_1 \geq CR_1 \geq RD_1$, so DR1 and CR1 are preferred for structural damage localization in actual engineering application.

3.4. Damage Localization Analysis of Second-Level Fusion Based on Weighted Average Criterion. According to the flow chart in Figure 5, the second-level fusion index will be obtained by applying weighted average criterion. The recognition results are as follows.

In Figure 12, some single damage cases are selected for analysis, and damage elements can be clearly found by searching maximum peaks from the curve. It should be noted that health element values are very small, so we consider that second-level fusion index can weaken “proximity effect” of RD1 in first-level fusion. For multiple damage cases (Figure 13), second-level fusion index also has the same identification ability.

3.5. Damage Quantitative Analysis of Third-Level Fusion Based on BP Neural Network. From the above, we know that whatever the single index is, first-level or second-level fusion, they all cannot easily determine the structural damage degree. Therefore, a third-level fusion model based on BP neural network is established here for further quantitative analysis.

A three-layer BP neural network is adopted for training here, which takes second-level fusion results as input and takes structural residual stiffness as output (DD3). In the training process, maximum iterative steps are 100, and precision is $10^{-5}$. Lastly, the output data can be easily processed to list absolute error (AE), relative percentage error (RPE), mean absolute error (MAE), and mean relative percent error (MRPE), which can be used as output evaluation indices. The detail calculation is as follows:

$$
\begin{align*}
    AE &= |V_k(t) - \bar{V}_k(t)| \\
    RPE &= \frac{|V_k(t) - \bar{V}_k(t)|}{\bar{V}_k(t)} \times 100\% \\
    MAE &= \frac{1}{M} \sum_{i=1}^{M} |V_k(t) - \bar{V}_k(t)| \\
    MRPE &= \frac{1}{M} \sum_{i=1}^{M} \frac{|V_k(t) - \bar{V}_k(t)|}{\bar{V}_k(t)} \times 100\% 
\end{align*}
$$

where $M$ is the number of test samples; $V_k(t)$ is real structural residual stiffness; $\bar{V}_k(t)$ is output residual stiffness of BP neural network.

3.5.1. Taking Cases 4, 8, and 14 as Prediction Set. According to equation (10), the secondary fusion results DF of all damage conditions are calculated, and the DF values of cases 4, 8, and 14 are used as prediction set, and the DF values of other cases are used as the training set. The training set is input into the BP neural network for training. After the accuracy meets the requirements, the prediction set is input into BP neural network for damage degree prediction, and the residual structural stiffness $DD_3$ is output. At the same time, according to equation (2), DR1, CR1, and RD1 of all cases are calculated, respectively, as control group, and they are also divided into training set and prediction set and input into BP neural network, and output $DR_2$, $CR_2$, and $RD_2$. The prediction results of damage degree of prediction set are shown in Table 4.

For further evaluating recognition effect of $DD_3$, a detailed comparison is analyzed for RPE, MAE, and MRPE of...
2. In column of RPE, the maximum value of DD3 is 3.27%, which is 4.93% lower than those of DR2, CR2, and RD2 by 8.80%. For MAE, DD3 result for case 4 can be simply calculated, and its value is 0.0188, less than that of DR2, CR2, and RD2 (0.0216, 0.0415, and 0.0257); MRPEs of cases 4, 8, and 14 for DD3 are 2.35%, 2.28%, and 1.80%, respectively, while those of DR2, CR2, and RD2 are 2.71%, 7.30%, and 6.87%; 5.19%, 4.13%, and 2.99%; and 3.21%, 4.25%, and 6.66%, respectively. It is obvious that mean relative percent error of index DD3 is significantly lower than that of DR2, CR2, and RD2.

3.5.2. Taking Single Case from 1 to 14 as Prediction Set, Respectively. Predicted results of 14 groups will be output following the above steps; specifically, their AE results are drawn as a histogram into Figure 14 for comparison. Maximum AE value of DD3 (black) is 0.014, while the other DR2 (red), CR2 (blue), and RD2 (green) are all beyond 0.04, and also their MAE value is still larger than that of DD3.

Overall, the damage degree actual output of DD3 is closer to ideal output, which means that the third-level fusion method is preferred for structural damage degree identification.

3.6. Noise Immunity Analysis

3.6.1. Noise Simulation Method. In actual working condition, due to external environment, test equipment error, and manual operation irregularities, the noise will be always existing; it would affect measured data so that there is often a certain degree difference between real values. Therefore, in order to better verify effectiveness of proposed methods, certain level noise is added into modal information when simulation, for analyzing their antinoise performance.

Usually, the way to add noise [39] is applied through the following equation:

\[ \Phi_{ij}^n = \Phi_{ij} + \Phi_{max,i} \times \text{randn} \times \delta, \]

where \( \Phi_{ij} \) and \( \Phi_{ij}^n \) represent vibration shape before and after noise addition, respectively; \( \Phi_{max,i} \) is maximum value of \( i \)th order vibration shape; \( \text{randn} \) represents Gaussian white noise, where the mean value and standard deviation are 0 and 1, respectively; \( \delta \) is noise level.

3.6.2. Noise Immunity Analysis of the First-Level Fusion. For noise immunity comparative analysis between single index and first-level fusion index, 5% and 10% level noise are added to damage cases 15 and 20, respectively. Figures 15 and 16 show antinoise analysis results of two and three damage cases, respectively. In (a) and (b), with increase of damage elements or noise level, damage probability and amount of health elements are also increased, which would lead to ambiguity for actual damage judgement. But after first-level data fusion based on Bayes theory, in (c), damage probabilities of health elements are significantly reduced, which makes peaks at damage position more prominent and can be easily and accurately found. So, first-level fusion index can improve recognition accuracy and has strong robustness than single index.

3.6.3. Noise Immunity Analysis of the Second-Level Fusion. Taking damage cases 1, 15, and 20 as examples with 5% and 10% noise, respectively, second-level fusion results will be calculated and drawn into histogram for comparison in Figure 17, in which peaks would be easily searched at damaged elements whatever the single, double, or three damage cases are, which means that the second-level fusion index has strong ability to eliminate noise interference.

4. Damage Recognition Results of a Three-Span Continuous Girder Bridge

The damage identification indexes based on multilevel data fusion has a good effect on simply supported beam, while,

<table>
<thead>
<tr>
<th>Case</th>
<th>1st order mode</th>
<th>2nd order mode</th>
<th>3rd order mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamaged case</td>
<td>5.2425</td>
<td>21.048</td>
<td>47.652</td>
</tr>
<tr>
<td>Case 1</td>
<td>5.2414</td>
<td>21.033</td>
<td>47.586</td>
</tr>
<tr>
<td>Case 2</td>
<td>5.2402</td>
<td>21.015</td>
<td>47.514</td>
</tr>
<tr>
<td>Case 3</td>
<td>5.2300</td>
<td>20.959</td>
<td>47.493</td>
</tr>
</tbody>
</table>

Table 2: Modal frequencies of damage cases.
for actual bridge in complicated loading, practicability of those methods needs to be further verified.

4.1. Finite Element Model. A three-span continuous box girder model is established by ANSYS 17.0 software as shown in Figure 18, which is a single box and single cell structure with three equal spans in total 90 meters (30 m + 30 m + 30 m). Concrete material is C50, elastic modulus is $3.45 \times 10^7$ kN/m$^2$, Poisson’s ratio is 0.2, density is 25 kN/m$^3$, and mass density is 2.549 kN/m$^3$/g. The beam is divided into 90 elements, 91 nodes, and the size of cross section is shown in Figure 19.

4.2. Damage Case and Modal Shape. X$+$he method of stiffness reduction is still applied for simulating damage of continuous girder. Usually, damage near support is not easy to be identified [1], so we particularly set damage elements near support, as shown in Figure 20, and set damage to No. 2, 29, 31, 59, and 61. X$+$he damage cases with different degrees with 5% noise level are shown in Table 5. X$+$he first-three modal shapes of three-span continuous girder under undamaged case can be calculated and shown in Figure 21.

4.3. Damage Localization Analysis of Single Index. Identification curves for DR and CR indices of first-three order are shown in Figure 22. Because RD index is poor for continuous beam damage identification, it will not participate in fusion. From the histogram, peaks would appear not only at damaged elements, but also at health elements, so we

Table 3: Prior probabilities of elements.

<table>
<thead>
<tr>
<th>Element no.</th>
<th>Prior probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.003835904</td>
</tr>
<tr>
<td>2</td>
<td>0.011458567</td>
</tr>
<tr>
<td>3</td>
<td>0.01893376</td>
</tr>
<tr>
<td>4</td>
<td>0.0261632</td>
</tr>
<tr>
<td>5</td>
<td>0.033048576</td>
</tr>
<tr>
<td>6</td>
<td>0.039491584</td>
</tr>
<tr>
<td>7</td>
<td>0.04539392</td>
</tr>
<tr>
<td>8</td>
<td>0.05065728</td>
</tr>
<tr>
<td>9</td>
<td>0.05518336</td>
</tr>
<tr>
<td>10</td>
<td>0.058873856</td>
</tr>
<tr>
<td>11</td>
<td>0.061630507</td>
</tr>
<tr>
<td>12</td>
<td>0.06335488</td>
</tr>
<tr>
<td>13</td>
<td>0.063949312</td>
</tr>
</tbody>
</table>

Note. Prior probabilities of the other 12 elements are the same due to symmetry.
Figure 9: The identification results of single index for single (a), two (b), and three (c) damage cases. (a) Case 1. (b) Case 15. (c) Case 20.

Figure 10: Fusion recognition results of two damage cases for DR₁ (a), CR₁ (b), and RD₁ (c).

Figure 11: Fusion recognition results of multiple damage (case 20).
prove once again that it will cause “false damage” phenomenon and is not easy to identify damage near support.

4.4. Damage Localization Analysis of Multi-Level Fusion. The three-span continuous girder is shown in Figure 23. According to equation (7), damage prior probability of each element can be calculated, and the results are shown in Figure 24.

In order to accurately identify damage near support and reduce interference from undamaged elements, the first-level fusion and second-level fusion methods are introduced for three-span continuous girder damage recognition, and their results for cases 21 and 22 are shown in Figure 25.

Compared with single index results, after Bayes first-level fusion, the value of peaks at damage elements is more higher than that at health elements, which indicates that health element will significantly reduce interference to damaged one, and probability of misjudgment would be declined, so that identification accuracy will be improved.

Moreover, superposing the two results of first-level fusion DR1 and CR1, we can obtain second-level fusion result DF, from the histogram, values at undamaged elements are almost zero, and peaks at damage elements are prominent. Therefore, second-level fusion index will have strong anti-interference performance and present better recognition effect.

5. Test Study

5.1. Test Overview. A test model of simply supported rectangular steel plate beam is constructed here for further verification. Cross section size is 100 mm × 8 mm (width × height), length is 2000 mm, calculated span is 1750 mm, elastic modulus of steel is \( E = 2.0795 \times 10^5 \text{kN/m}^2 \), and density is \( \rho = 7698 \text{kg/m}^3 \), as shown in Figure 26.

Within the calculated span length, 36 white lines are signed on surface of steel girder with equal distance of 5 cm, for accurate positioning acceleration sensors installation. When the test, single-point excitation, and multipoint acceleration collection method are applied, here, six signal sensors are employed to obtain modal information of beam; among them, five sensors are set as a working group with a distance of 5 cm moving on beam, the other one as a base control group, for calculating relative value of working group. The system INV-9812 is used for data collection, and analysis software DASP2003 for signal processing. The sensors layout is shown in Figure 27.

5.2. Damage Pattern and Dynamic Parameter Acquisition. The damage pattern in this experiment is realized by symmetrical cutting method, actual cut form of steel.
### Table 4: Damage quantitative analysis comparison for \( \text{DD}_3, \text{DR}_2, \text{CR}_2, \) and \( \text{RD}_2. \)

<table>
<thead>
<tr>
<th>Index</th>
<th>Case</th>
<th>Ideal output</th>
<th>Actual output</th>
<th>AE</th>
<th>RPE (%)</th>
<th>Ideal output</th>
<th>Actual output</th>
<th>AE</th>
<th>RPE (%)</th>
<th>Ideal output</th>
<th>Actual output</th>
<th>AE</th>
<th>RPE (%)</th>
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<tr>
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<td>0.8261</td>
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<td>3.2675</td>
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<tr>
<td>DD3</td>
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<td>2.6982</td>
<td>0.8773</td>
<td>0.0227</td>
<td>2.5198</td>
<td>0.8759</td>
<td>0.0259</td>
<td>3.0455</td>
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<tr>
<td></td>
<td>3</td>
<td>0.8088</td>
<td>0.0088</td>
<td>1.0938</td>
<td>0.8854</td>
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<td>6.0281</td>
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<td>CR2</td>
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<td></td>
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<td>0.0318</td>
<td>3.9755</td>
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<td>0.0331</td>
<td>3.6825</td>
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<td>DR2</td>
<td>2</td>
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<td>1.7759</td>
<td>0.8671</td>
<td>0.0329</td>
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<td>0.0310</td>
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</table>

**Figure 14:** Comparison of absolute error of damage degree.

**Figure 15:** Antinoise analysis results of case 15. (a) 5% noise level of single index. (b) 10% noise level of single index. (c) 5%, 10% noise level of first-level fusion index.
Beam is shown in Figure 28, cutting width is 10 mm, and cutting depth and position are listed in Table 6.

The modal parameters and corresponding frequency spectrum curve of simply supported beam under different damage forms can be obtained by processing acceleration signal data with analysis system DASP2003. Some frequency spectrum curve diagrams are shown in Figure 29.

According to equation (7), damage prior probability $P(A_m)$ of every element of tested simply supported beam is calculated, and its parabolic diagram is shown in Figure 30.

**Figure 16**: Antinoise analysis results of case 20. (a) 5% noise level of single index. (b) 10% noise level of single index. (c) 5%, 10% noise level of first-level fusion index.

**Figure 17**: Antinoise analysis results of the second-level fusion index.
5.3. Test Results Analysis

5.3.1. Damage Localization Analysis of Single Index. The results of single index recognition are shown in Figures 31 and 32. From the graph, whatever the form is, form 2 or form 4, peaks will not only appear at the actual damaged elements 6, 9, 17, and 26 we set, but also appear at other health elements, so that we cannot distinguish which element will be damaged and which will not.

5.3.2. Damage Localization Analysis of Multilevel Fusion. Because the above result figures are disorganized, and we will be puzzled to find the actual damaged elements, we need to take effective measures for further processing. The first-level data fusion for forms 2 and 4 and their second-level data fusion are calculated, respectively, and drawn in Figures 33 and 34.

From Figure 33, peaks in histogram, whatever the indexes are, DR1, CR1, or RD1, are all obvious and just happen at damaged elements, while values at other position can be almost ignored. Therefore, the first-level data fusion model is a very nice data processing method and will be qualified for damage identification.

In Figure 34, in second-level fusion results, whatever the curve of single, double, or three damage cases is, peaks will still be easily locked, which well match the damage
Figure 21: Vibration modes for 1st (a), 2nd (b), and 3rd (c) order of three-span continuous girder.

Figure 22: Damage identification results for case 21 (a) and case 22 (b) of single index.
Figure 23: Three-span continuous girder model.

Figure 24: Damage prior probability of continuous girder.

Figure 25: Recognition results for continuous girder of multilevel fusion.
Figure 26: Simply supported steel beam test mode.

Figure 27: Simply supported beam sensors layout.

Figure 28: Damage form of steel beam. (a) Steel beam damage case. (b) Steel beam notch form.

Table 6: Damage forms of tested steel beam.

<table>
<thead>
<tr>
<th>Damage form</th>
<th>Damage position and cutting depth</th>
<th>Damage degree (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No damage</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>300 mm cut 25 mm</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>500 mm cut 20 mm</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1250 mm cut 25 mm</td>
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</tr>
<tr>
<td>4</td>
<td>450 mm cut 20 mm</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>875 mm cut 20 mm</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1300 mm cut 20 mm</td>
<td>40</td>
</tr>
</tbody>
</table>
Figure 29: Frequency spectrum curve of simply supported beam. (a) Form 1. (b) Form 2. (c) Form 4.
Figure 30: Damage prior probability of tested simply supported beam.

Figure 31: Damage identification results of form 2 for DR (a), CR (b), and RD (c).

Figure 32: Damage identification results of form 4 for DR (a), CR (b), and RD (c).
forms set before, and the “proximity effect” is also significantly weakened, so the second-level data fusion model is superior method for priority selection.

6. Conclusion

Based on data fusion theories, this paper takes the modal strain energy dissipation rate (DR), change rate of cross-model strain energy (CR), and difference of modal curvature ratio (RD) as basic indicators to construct multilevel data fusion models for structural damage identification; also the noise immunity and experimental verification are analyzed, and the following conclusions are obtained:

(1) In the process of multilevel data fusion method, both first-level and second-level can locate damage positions at wheresoever mid-span or near support, and whatever the simply supported beam or three-span continuous girder is, the second-level proved that its recognition effectiveness is better for multidamage. The third-level method can directly reflect structural damage degree, and it is more superior for the results of second-level as input for BP neural network training than those of first-level.

(2) The indicators based on multilevel data fusion method reinforced each other due to multisource information superposition, so that they can improve reliability of damage localization and accuracy of damage quantification.

(3) By experimental analysis, the multilevel data fusion method can also accurately identify real damage although the modal data are affected by factors such as site test condition and individual error, which indicate that they have strong robustness and fault tolerance. Those methods need further verification by field test.

Data Availability

The data are available upon request to the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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