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Research Article

Lot Size Decisions for Vendor-Buyer System with Quantity Discount, Partial Backorder, and Stochastic Demand

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This paper presents production-inventory model for two-echelon system consisting of single vendor and single buyer. The proposed model contributes to the current inventory literature by incorporating quantity discount scheme into stochastic vendor-buyer model. Almost all vendor-buyer inventory models have discussed this scheme in single-echelon system and deterministic demand situation. Here, we assume that the demand of the buyer is normally distributed and the unmet demand is considered to be partially backordered. In addition, the lead time is variable and consists of production time and nonproductive time. The quantity discount is developed by using all-units quantity discounts. Finally, an iterative procedure is proposed to obtain all decision variables and numerical examples are provided to show the application of the proposed procedure.

1. Introduction

Supply chain is the sequence of business processes and activities from suppliers through customers that provide products, services, and information to achieve customer satisfaction, that is, a chain that can quickly respond to customer's requirement. Integration of different entities in the supply chain is an important way to gain competitive advantage and customer satisfaction [1]. In recent years, research dealing with inventory management in supply chain system has attracted considerable attention from many scholars. Goyal [2] is among the first authors who studies integrated inventory model for single-vendor single-buyer system. He introduces a model for situation in which vendor produces a lot based on an infinite production rate and transfers it to the buyer by a lot-for-lot policy. He shows that making inventory decisions jointly among vendor and buyer can result in a substantial cost reduction compared to individual decisions. The framework proposed by Goyal [2] has encouraged many researchers to present various types of integrated vendor-buyer system. Banerjee [3] relaxes the assumption of lot-for-lot policy and infinite production rate and proposes a model where the vendor produces a batch at finite production rate and then delivers it equally to the buyer. Goyal [4] also relaxes the lot-for-lot assumption and introduces a more general lot sizing model. He argues that producing a batch which is made up of equal shipments generally produced lower total cost, but the whole batch must be completed before the first shipment is made. A number of researchers, including Goyal [5], Hill [6], Hill [7], Goyal and Nebebe [8], Hoque and Goyal [9], Hill and Omar [10], and Zhou and Wang [11] develop a model with unequal-sized shipments, in contrast to the previous models that assume equal-sized shipment policy. For a comprehensive review of vendor-buyer models, the reader can refer to Glock [12].

The vendor-buyer model under stochastic demand has been studied extensively by many researchers. One of the first models dealing with stochastic demand is due to Ben-Daya and Hariga [13], who consider the problem of variable lead time on two-echelon system consisting of a vendor and a buyer. They assume that the lead time of delivering the product from vendor to buyer is formulated by considering production time, nonproductive time, and transportation time. Hsiao [14] then proposes a new formulation of lead time in stochastic vendor-buyer inventory model. The lead time for the first delivery is formulated by considering production time, nonproductive time, and transportation time while the lead time of $2, \ldots, n$ delivery is only the transportation time considered in the model. Mitra [15] investigates returns policy on stochastic environment. He uses simulation model to study proposed stochastic inventory model and shows that the model performs very well with respect to best solution obtained from simulation. Ertogral [16] proposes a service-level-based model for integrated inventory problem where the demand follows a stationary normal distribution. Jauhari et al. [17] develop single-vendor single-buyer model considering stochastic demand and giving flexibility to both parties in determining the order quantity and production batch based on optimal shipment size. Jauhari and Pujawan [18] extend the previous model to include raw material procurement decision and adjusted production rate. Further, Glock [19] studies vendor-buyer model with stochastic demand under different lead time reduction strategies. He assumes that the lead time can be reduced by shortening setup and transportation time or by adjusting production rate.

Quantity discount has been a subject of inventory research for a long time; however, little is known about its influence on vendor-buyer models when demand is stochastic. Quantity discount is a common practice in business and provides economic advantages for supply chain's entities. The vendor can get benefit from sales of larger quantities by reducing the unit order and setup costs, while the buyer can reduce per unit ordering cost and holds more inventory by paying a lower unit price [20]. Monahan [21] probably the first researcher who introduces the concept of quantity discount from the point of view of vendor. Further, Lee and Rosenblatt [22] develop the Monahan [21]'s model and investigate the ordering and price level decisions from the vendor's point of view. A number of researchers including Corbett and De Groote [23], Viswanathan and Wang [24], Lin [25], Yadav et al. [26], and Yang et al. [27] study the effect of incorporating quantity discount on single-echelon inventory system under various assumptions. However, the concept of integration between parties in supply chain is neglected by these one-sided inventory systems.

In recent years, several researchers have studied vendor-buyer inventory model involving quantity discount and deterministic demand. One of the papers dealing with quantity discount in a vendor-buyer model is due to Arcelus et al. [28]. The authors study the ordering and pricing policies on vendor-buyer model with price-dependent demand. In this study, the vendor offers discounted wholesale/regular price during sales subperiods. The objective of the model is to find the maximum profit per unit time which is formulated by considering order quantity, reorder level, and retail price for regular and discount subperiods. Viswanathan [29] investigates discount pricing decision by using a version of Stackelberg game. The game consists of a leader who sells the product to the followers who in turn sell it to the customers. This study shows that perfect coordination among vendor and buyer can be achieved by applying quantity discount scheme. Zhou [30] investigates quantity discount pricing policies of two-echelon system in which two partners have stochastic and asymmetric demand information. Munson and Hu [31] study quantity discounts policies in multiechelon system and investigate their inventory impacts into centralized purchasing decision. They propose procedures for both all-units and incremental quantity discount schedules for four different purchasing scenarios. Giri and Roy [32] propose single-vendor single-buyer model in which the vendor offers discounted price to the buyer and the deliveries are made under unequal-sized shipment policy.

In addition, quantity discount in stochastic vendor-buyer system is rarely discussed in the inventory literature. The above-mentioned papers mostly focus on the single-echelon system or two-echelon system with deterministic demand. Therefore, to close the research gap identified above, we focus on investigating the quantity discount and partial backorder in stochastic vendor-buyer inventory model under variable lead time and partial backorder. We assume that the demand is distributed normally and the lead time consists of production time and nonproductive time. In this study, we develop vendor-buyer model to determine simultaneously safety factor, shipment size, frequency of shipment, and product price. We also propose a procedure to find the solutions of the model.

The remainder of the paper is organized as follows. In Section 2, we describe the notations and problem description used for developing the model. In Section 3, we formulate the model by considering all-unit quantity discount and partial backorder. A solution methodology is given in Section 4. Finally, the numerical example and the conclusions of the paper are presented in Sections 5 and 6.

2. Notations and Problem Description

2.1. Notations. The following notations will be used to develop the model

(i) Input Parameters for Buyer

D: mean of demand per unit time,

σ: standard deviation of demand per unit time,

A: order cost incurred by the buyer for each order of size Q,

F: transportation cost incurred by the buyer for each unit product,

S_j: sum of purchase cost and transportation cost at discount level j,

π: shortage cost per unit short,

π_0: marginal profit per unit for buyer,

α: proportion of shortage that will be backordered,

t: transportation time (in transit time),

L_T: lead time,

T_s: nonproductive time and transportation time,

f_{IT}: in-transit inventory carrying charge incurred by the buyer,

f_{IH}: in-house inventory carrying charge incurred by the buyer,

X_j: quantity breakpoint at discount level j,

OC: ordering cost per unit time,

TC: transportation cost per unit product,

HC_B: total holding cost incurred by the buyer per unit time,

HC_{IT}: in-transit holding cost incurred by the buyer per unit time,

HC_{IH}: in-house holding cost incurred by the buyer per unit time,

SC: shortage cost incurred by the buyer per unit time,

$TC_{B,j}$: expected cost for the buyer per unit time at discount level j.

(ii) Input Parameters for Vendor

P: production rate per unit time,

K: production setup cost,

U: production cost incurred by the vendor for each unit product,

fv: inventory carrying charge incurred by the vendor,

SC: setup cost incurred by the vendor per unit time,

HC_v: holding cost incurred by the vendor per unit time,

TC_v: expected cost for the vendor per unit time.

(iii) Decision Variables

n: number of shipments per production batch, which is a positive integer,

Q: the shipment size from the vendor to the buyer,

k: safety factor,

v: purchase cost for each unit product.

2.2. Problem Description. In this model, we present production-inventory model for supply chain system consisting of single vendor and single buyer. The demand is assumed to be normally distributed with mean D and standard deviation of demand σ. The buyer uses continuous review policy to manage his inventory level. Therefore, an order size of Q will be placed when the buyer's inventory drops to reorder point level (ROP). Each time an order is done, the fixed ordering cost A incurs. Moreover, each time the delivery size of Q is made, the buyer incurs transportation cost F. The demand from end customers which is not satisfied is assumed to be partially satisfied. This means that the proportion of shortage may be backordered while the other is lost sale. The vendor produces product with the batch size of nQ. The production rate of the vendor is constant at a rate of P. The vendor will incur setup cost K when the production is started. The vendor adopts lot streaming policy which means that the delivery can be made as soon as possible after the vendor has the quantity of Q.

The vendor offers quantity discount to the buyer to induce the buyer to order a large quantity of product. This policy gives the buyer an opportunity to buy the product with lower purchase price; hence, his inventory cost can be minimized. The discount offered by the vendor is formulated by using all-units quantity discount. This means that purchase cost in the schedule is applied to the product as long as the order quantity is above the breakpoint (X_J). The schedule of quantity discount is presented in Table 1.

Table 1: Quantity discount schedule offered by the vendor.

J	Q (order quantity)	v_j (purchase cost)
0	$0 < Q < X_1$	v_0
1	$X_1 \leq Q < X_2$	v_1
$\vdots$	$\vdots$	$\vdots$
J	$Q \geq X_J$	v_J

3. Model Formulation

In this section, we describe the development of expected total cost for buyer, vendor, and the supply chain system, respectively.

3.1. The Expected Buyer Cost. As mentioned earlier, the demand in buyer side is assumed to be normally distributed. Therefore, to guarantee that the probability of negative demand tend to zero, we use the assumption that $G(-D/\sigma) = 1$, where $G(\cdot)$ is the complementary cumulative distribution function of the standard normal distribution. Further, the optimal policy will be such that there is never more than a single-order outstanding at any time, which means that $Q > \text{ROP}$. It is also assumed that the reorder point (ROP) is positive, which means that there will be no backorders outstanding at the reorder point.

The lead time demand Y has a normal probability density function $f(y)$ with mean DL_T and standard deviation $\sigma\sqrt{L_T}$, where $L_T = Q/P + T_s$. Thus, the reorder point is given by

$$\text{ROP} = D\left(\frac{Q}{P} + T_s\right) + k\sigma\sqrt{\frac{Q}{P} + T_s}. \quad (1)$$

Therefore, the expected shortage at the end of the cycle is given by

$$\int_{\text{ROP}}^{\infty} (y - \text{ROP}) f(y)\, dy = \sigma\sqrt{\frac{Q}{P} + T_s}\psi(k), \quad (2)$$

where

$$\psi(k) = f_s(k) - k\left[1 - F_s(k)\right], \quad (3)$$

$f_s(k)$ is the probability density function and $F_s(k)$ is cumulative distribution function of standard normal distribution. The expected number of backorder per cycle can be determined by considering the expected shortage in (2) and the proportion of shortage that will be backordered (α). Consider

$$\sigma\alpha\sqrt{\frac{Q}{P} + T_s}\psi(k). \quad (4)$$

Further, the expected number of lost sales can be expressed as follows:

$$\sigma(1-\alpha)\sqrt{\frac{Q}{P} + T_s}\psi(k). \quad (5)$$

The expected shortage cost for the buyer per unit time is

$$\mathrm{SC}=\left(\frac{D}{Q}\right)\left(\pi+\pi_0\left(1-\alpha\right)\right)\sigma\sqrt{\frac{Q}{P}+T_s}\psi\left(k\right). \quad (6)$$

Based on Montgomery et al. [33], the expected net inventory level just before receipt of shipment is $\mathrm{ROP}-D(Q/P+T_s)+(1-\alpha)\sigma\sqrt{Q/P+T_s}\psi(k)$ and the expected inventory level immediately after the shipment is $Q+\mathrm{ROP}-D(Q/P+T_s)+(1-\alpha)\sigma\sqrt{Q/P+T_s}\psi(k)$.

Therefore, the in-house holding cost for buyer per unit time is given by

$$\begin{aligned}&\mathrm{HC}_{\mathrm{IH}}\\&\quad=f_H S_j\left(\frac{Q}{2}+k\sigma\sqrt{\frac{Q}{P}+T_s}+\left(1-\alpha\right)\sigma\sqrt{\frac{Q}{P}+T_s}\psi\left(k\right)\right).\end{aligned} \quad (7)$$

The formulation of in-transit holding cost per unit is formulated by considering frequency of shipment (D/Q), in-transit inventory carrying charge (f_{IT}), quantity of shipment (Q), and in-transit time (t). Consider

$$\mathrm{HC}_{\mathrm{IT}}=\frac{DQv_j f_{\mathrm{IT}}t}{Q}=Dv_j f_{\mathrm{IT}}t. \quad (8)$$

Consequently, the total holding cost for the buyer per unit time is given by summing up the in-house holding cost in (7) and in-transit holding cost in (8) which is given by

$$\begin{aligned}&\mathrm{HC}_B\\&\quad=Qv_j f_{\mathrm{IT}}t\\&\qquad+f_{\mathrm{IH}}S_j\left(\frac{Q}{2}+k\sigma\sqrt{\frac{Q}{P}+T_s}+\left(1-\alpha\right)\sigma\sqrt{\frac{Q}{P}+T_s}\psi\left(k\right)\right).\end{aligned} \quad (9)$$

The ordering cost can be formulated by multiplying the frequency of order (D/Q) and the order cost (A), while the transportation cost paid by the buyer to the shipper per unit time can be calculated by considering the size of demand (D) and the transportation cost per unit item (F). The buyer's ordering cost per unit time and the buyer's transportation cost per unit time are given by the following equation, respectively,

$$\begin{aligned}\mathrm{OC}&=\frac{DA}{Q},\\\mathrm{TC}&=\frac{D}{Q}FQ=DF.\end{aligned} \quad (10)$$

Thus, considering the above buyer costs, the expected cost for the buyer per unit time (TC_{Bj}) is given by

$$\begin{aligned}&\mathrm{TC}_{Bj}\\&\quad=\frac{DA}{Q}+DF+Dv_j f_{\mathrm{IT}}t\\&\qquad+f_{\mathrm{IH}}S_j\left(\frac{Q}{2}+k\sigma\sqrt{\frac{Q}{P}+T_s}+\left(1-\alpha\right)\sigma\sqrt{\frac{Q}{P}+T_s}\psi\left(k\right)\right).\end{aligned} \quad (11)$$

3.2. The Expected Vendor Cost. The vendor setup cost per unit time can be determined by considering setup cost (K) and the frequency of setup (D/nQ). The formulation of setup cost per unit time is provided as follows:

$$\mathrm{ST}=\frac{DK}{nQ}. \quad (12)$$

The inventory level for vendor can be determined by subtracting the accumulated buyer's consumption from accumulated vendor production. The holding cost for vendor is provided as follows:

$$\mathrm{HC}_v=\frac{Q}{2}f_v U\left(\left(n-1\right)-\left(n-2\right)\frac{D}{P}\right). \quad (13)$$

Finally, the expected cost for the vendor per unit time including setup cost and holding cost is given as follows:

$$\mathrm{TC}_v=\frac{\mathrm{DK}}{nQ}+\frac{Q}{2}f_v U\left(\left(n-1\right)-\left(n-2\right)\frac{D}{P}\right). \quad (14)$$

3.3. The Expected Total Cost. The formulation of expected total cost for supply chain system per unit time can be determined by summing up the expected cost for buyer in (11) and the expected cost for vendor in (14). That is,

$$\begin{aligned}&\mathrm{TC}_j\left(Q,k,v,n\right)\\&\quad=\frac{DA}{Q}+DF+\frac{D\left(Qv_j f_{\mathrm{IT}}t\right)}{Q}\\&\qquad+f_{\mathrm{IH}}S_j\left(\frac{Q}{2}+k\sigma\sqrt{\frac{Q}{P}+T_s}+\left(1-\alpha\right)\sigma\sqrt{\frac{Q}{P}+T_s}\psi\left(k\right)\right)\\&\qquad+\left(\frac{D}{Q}\right)\left(\pi+\pi_0\left(1-\alpha\right)\right)\sigma\sqrt{\frac{Q}{P}+T_s}\psi\left(k\right)\\&\qquad+\frac{DK}{nQ}+\frac{Q}{2}f_v U\left(\left(n-1\right)-\left(n-2\right)\frac{D}{P}\right).\end{aligned} \quad (15)$$

4. Solution Methodology

The optimal shipment quantity can be found by taking the first partial derivative of $\mathrm{TC}_j(Q,k,v,n)$ with respect to Q

then equating it to zero. The formulation of optimal shipment quantity is given by the following equation:

$$Q^* = \left(\left(2D\left\{A + (\pi + \pi_0(1-\alpha))\,\sigma\psi(k)\sqrt{\frac{Q}{P} + T_s} + \frac{K}{n}\right\}\right)\right.$$
$$\times\left(f_v U\left((n-1) - (n-2)\frac{D}{P}\right) + f_{IH}S_j\right.$$
$$\left.\left.\times\left\{1 + \frac{\sigma}{P\sqrt{Q/P + T_s}}\left(k + \frac{\psi(k)}{1 - F_s(k)}\right)\right\}\right)^{-1}\right)^{1/2}. \quad (16)$$

Similarly, the optimal value of safety factor can be determined by taking the first partial derivative of $TC_j(Q, k, v, n)$ with respect to k then equating it to zero. The formulation of optimal safety factor is

$$F_s(k) = 1 - \frac{Q f_{IH} S_j}{D(\pi + \pi_0(1-\alpha)) + Q f_{IH} S_j (1-\alpha)}. \quad (17)$$

When investigating the proposed problem, it can be easily shown that the total cost function is convex in k. However, the total cost function may not be convex in Q. Here, we propose an algorithm to obtain the solution of decision variables. However, the proposed algorithm may result in local optimal solution; hence, global optimality cannot be claimed. For studying this condition, the reader can refer to Goyal [2]. The proposed algorithm of the above problem is provided as follows.

Algorithm 1.

Step 0. Set $n = 1$ and $TC_j(Q^*_{n-1}, k^*_{n-1}, n-1) = \infty$.

Step 1. For each discount level $j = 0, 1, 2, 3, \ldots, J$ do the following steps.

(a) Calculate S_j.

(b) Start with shipment size

$$Q_j = \sqrt{\frac{2D\{A + K/n\}}{f_{IH}S_j + f_v U((n-1) - (n-2)(D/P))}}. \quad (18)$$

(c) Use Q_j resulted from step (b) to calculate k_j by employing (17).

(d) Calculate Q_j using (16) with k_j obtained from step (c).

(e) Repeat step (c)-(d) until no change occurs in the values of Q_j and k_j.

Step 2. Conduct feasibility test for the solutions obtained from Step 1.

(a) For $j = 0, 1, 2, \ldots, J-1$.

Table 2: Quantity discount schedule.

J	Q (units)	v_j ($)
0	$0 < Q < 500$	5
1	$500 \le Q < 750$	4
2	$750 \le Q < 1{,}000$	3
3	$Q \ge 1{,}000$	2

(i) If $Q_j < Q_j^* \le Q_{j+1}$, set Q_j^* and k_j^* as the optimal feasible solutions.

(ii) If $Q_j^* > Q_{j+1}$, the discount level j is not feasible for the solutions obtained and set $TC_j(Q_n^*, k_n^*, n) = \infty$.

(iii) If $Q_j^* \le Q_j$, set Q_j as the optimal feasible solution and compute k_j using (17).

(b) For $j = J$.

(i) If $Q_j^* > Q_j$, set Q_j^* and k_j^* as the optimal feasible solutions.

(ii) If $Q_j^* \le Q_j$, set Q_j as the optimal feasible solution and compute k_j using (17).

Step 3. Compute the feasible $TC_j(Q_n^*, k_n^*, v, n)$ using (15) corresponding to the solutions obtained from Step 2.

Step 4. Set $TC_j(Q_n^*, k_n^*, v, n) = \text{Min } j = 0, 1, 2, \ldots, J\, TC_j(Q_n^*, k_n^*, v, n)$.

Step 5. Set the discount level that generate minimum expected total cost as the optimal discount level (v_n^*) also Q_n^* and k_n^* on that discount level as the optimal solution for n.

Step 6. If $TC_j(Q_n^*, k_n^*, v, n) \le TC_j(Q^*_{n-1}, k^*_{n-1}, v, n-1)$, then repeat Steps 1–5 with $n = n + 1$. Otherwise, go to Step 7.

Step 7. Set $TC_j(Q_n^*, k_n^*, v, n) = TC_j(Q^*_{n-1}, k^*_{n-1}, v, n-1)$ and $(Q_n^*, k_n^*, n^*, \text{dan} v_m^*)$ as the final solution.

5. Numerical Example

To illustrate the application of the above algorithm, we consider an integrated vendor-buyer model with following numerical examples:

$D = 1{,}000$ units/year,

$\sigma = 5$ units/year,

$A = \$100$/order,

$P = 3{,}200$ units/year,

$K = \$400$/setup,

$F = \$0.2$/unit,

$\pi = \$15$/unit,

$\pi_0 = \$45$/unit,

$U = \$1.5$/unit,

Table 3: The optimal solution for numerical example.

n	v_j ($)	Q (units)	k	Vendor cost ($)	Buyer cost ($)	Total cost ($)
1	2	1,207.54	2.121	345.40	692.96	1,038.36
2	2	1,000	2.196	237.50	641.40	878.90
3	2	1,000	2.196	196.61	641.40	838.01
4	2	1,000	2.196	189.06	641.40	830.46
5	2	1,000	2.196	194.84	641.40	836.24

$\alpha = 0.3$,

$t = 0.02$ years,

$T_s = 0.1$ years,

$f_{\mathrm{IT}} = \$0.15/\$/\text{year}$,

$f_{\mathrm{IH}} = \$0.3/\$/\text{year}$,

$f_v = \$0.05/\$/\text{year}$.

The quantity discount schedule is shown in Table 2.

Utilizing a procedure as proposed in Algorithm 1, the summarized optimal values are presented in Table 3. From this table, we can observe that the expected total cost for supply chain has minimum value when $n = 4$. The cost incurred by the vendor is $189.06 while the cost incurred by the buyer is $641.4. The optimal order quantity is 1,000, the optimal production batch is 4,000 units and the optimal safety factor is 2.196.

6. Conclusions

The primary purpose of this paper is to present the single-vendor single-buyer integrated production-inventory model with quantity discount, stochastic demand, and variable lead time. We consider a situation in which the demand in buyer side is assumed to be normally distributed and the shortages are assumed to be partially backordered. To entice the buyer to order in larger quantity, the vendor offers quantity discount. The quantity discount scheme offered by vendor is formulated under all-units quantity discount. In addition, the lead time of transferring the product from vendor to buyer is modeled by considering production time and nonproductive time. We propose an easy algorithm for determining the optimal solutions and provide numerical examples to illustrate the application of the proposed algorithm.

This study can be enhanced in a number of ways. First, one could investigate the impact of quantity and freight discounts on the stochastic vendor-buyer model. In practical situation, it is usually found that the shipper offers freight discount based on the weight of the shipment. Second, another situation would be to study the effect of temporary price discount offered by vendor. Previously, most of the papers discussed temporary price discount under deterministic demand. Therefore, it would be interesting to extend the model by assuming stochastic demand. Third, it would be also interesting to investigate three-stage supply model in which the supplier also provide a discount to raw material procurement.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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