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Research Article

Ranking-Theory Methods for Solving Multicriteria Decision-Making Problems

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The Pareto optimality is a widely used concept for the multicriteria decision-making problems. However, this concept has a significant drawback—the set of Pareto optimal alternatives usually is large. Correspondingly, the problem of choosing a specific Pareto optimal alternative for the decision implementation is arising. This study proposes a new approach to select an "appropriate" alternative from the set of Pareto optimal alternatives. The proposed approach is based on ranking-theory methods used for ranking participants in sports tournaments. In the framework of the proposed approach, we build a special score matrix for a given multicriteria problem, which allows the use of the mentioned ranking methods and to choose the corresponding best-ranked alternative from the Pareto set as a solution of the problem. The proposed approach is particularly useful when no decision-making authority is available, or when the relative importance of various criteria has not been evaluated previously. The proposed approach is tested on an example of a materials-selection problem for a sailboat mast.

1. Introduction

This paper considers a novel approach for solving a multicriteria decision-making (MCDM) problem, with a finite number of decision alternatives and criteria. The multicriteria formulation is the typical starting point for theoretical and practical analyses of decision-making problems. Thus, the definition of Pareto optimality and a vast arsenal of different Pareto optimization methods can be used for decision-making purpose.

However, unlike single-objective optimizations, a characteristic feature of Pareto optimality is that the set of Pareto optimal alternatives (i.e., set of efficient alternatives) is usually large. In addition, all these Pareto optimal alternatives must be considered as mathematically equal. Correspondingly, the problem of choosing a specific Pareto optimal alternative for implementation arises, because the final decision usually must be unique. Thus, additional factors must be considered to aid a decision-maker the selection of specific or more-favorable alternatives from the set of Pareto optimal solutions.

The proposed approach is based on ranking-theory methods that used to rank participants in sports tournaments. In the framework of the proposed approach, we build a special score matrix for a given multicriteria problem, which allows us to use the mentioned ranking methods and choose the corresponding best-ranked alternative from the Pareto set as a solution of the problem. Note that the score matrix is built by the quite natural way—it is composed on the simple calculations of how many times one alternative is better than the other for each of the criteria. Hence, there is hope that the proposed approach yields a "notionally objective" ranking method and provides an "accurate ranking" of the alternatives for MCDM. The proposed approach is particularly useful when no decision-making authority is available, or when the relative importance of various criteria has not been evaluated previously.

To demonstrate viability and suitability for applications, the proposed approach illustrated using an example of a materials-selection problem for a sailboat mast. This problem has been addressed by several researchers using various methods and, thus, can be considered as a kind of a benchmark problem. This illustration sheds light on the ranking approach's applicability to the MCDM problems. Particularly, it is shown that the solutions of the illustrative example obtained by the proposed approach are quite competitive.

The rest of this paper is structured as follows. In Section 2, preliminaries regarding MCDM and ranking problems are presented, and the proposed methodology is described; Section 3 considers an illustrative example and Section 4 summarizes the article.

2. Proposed Methods

In what follows, for a natural number n, we denote an n-dimensional vector space by $\mathbb{R}^n$ and $\mathbb{N}(n) = \{1, \ldots, n\}$. If not otherwise mentioned, we identify a finite set A with the set $\mathbb{N}(n) = \{1, \ldots, n\}$, where $n = |A|$ is the capacity of the set A. By necessity, we also identify the matrix $\Pi \in \mathbb{R}^{n\times m}$ with the map $\Pi : \mathbb{N}(n) \times \mathbb{N}(m) \longrightarrow \mathbb{R}$. For a matrix $\Pi \in \mathbb{R}^{n\times m}$, we denote its transpose by $\Pi^T \in \mathbb{R}^{m\times n}$.

2.1. Preliminaries

2.1.1. Background on Multiobjective Decision-Making Problems. The following notation is drawn from a general treatment of multicriteria optimization theory [5, 6]. Let us consider the MCDM problem $\langle A, C\rangle$, where $A = \{a_1, \ldots, a_m\}$ is a set of alternatives and $C = \{c_1, \ldots, c_n\}$ is a set of criteria; i.e., $c_i : A \longrightarrow \mathbb{R}$, $i = 1, \ldots, n$, are given function. Without loss of generality, we may assume that the lower value is preferable for each criterion (i.e., each criterion is nonbeneficial), and the goal of the decision-making procedure is to minimize all criteria simultaneously [7].

We say furthermore that A is the set of admissible alternatives and map $\overrightarrow{c} = (c_1, \ldots, c_n) : A \longrightarrow \mathbb{R}^n$ is the criterion map (correspondingly, $\overrightarrow{c}(A) \subset \mathbb{R}^n$ is the set of admissible values of criteria). The following concepts are also associated with the criterion map and the set of alternatives. An alternative $a_* \in A$ is Pareto optimal (i.e., efficient) if there exists no $a \in A$ such that $c_j(a) \le c_j(a_*)$ for all $j \in \mathbb{N}(n)$ and $c_k(a) < c_k(a_*)$ for some $k \in \mathbb{N}(n)$. The set of all efficient alternatives is denoted as A_e and is called the Pareto set. Correspondingly, $f(A_e)$ is called the efficient front.

Pareto optimality is an appropriate concept for the solutions of MCDM problems. In general, however, the set A_e of Pareto optimal alternatives is very large and, moreover, all alternatives from A_e must be considered as "equally good solutions". On the other hand, the final decision usually must be unique. Hence, additional factors must be considered to aid the selection of specific or more-favorable alternatives from the set A_e. The following subsections describe a novel approach that handles this problem objectively.

2.1.2. Ranking Methods. This section gives a brief overview of the basic concepts of ranking theory. References [8, 9] discuss ranking theory in greater detail. For a natural number N, the $N \times N$ matrix $S = [S_{ij}]$, $1 \le i, j \le N$, is a score matrix if $S_{ij} \ge 0$, $S_{ii} = 0$, $1 \le i, j \le N$. To emphasize that this problem was formulated in the context of competitive sports—note also that we can interpret elements of $\mathbb{N}(N)$ as athletes (or teams) who contest matches among themselves—and for each pair of athletes (i, j), $1 \le i, j \le N$, the joint match $\mathrm{M}(i, j)$ includes K games. We interpret entry S_{ij}, $1 \le i, j \le N$, as the number of athlete i's total wins in the match $\mathrm{M}(i, j)$. We also say that the result of the match $\mathrm{M}(i, j)$ is S_{ij} wins of athlete i (losses of athlete j), S_{ji} wins of athlete j (losses of athlete i), and $(K - S_{ij} - S_{ji})$ draws. Hence $G_{ij}(S) = S_{ij} + S_{ji}$, ($G = [G_{ij}(S)]$, $1 \le i, j \le N$; $G = S + S^T$), can be interpreted as the number of decisive games that did not end in a draw in the match $\mathrm{M}(i, j)$, $1 \le i, j \le N$. We also introduce the function $g_i(S) = \sum_{j=1}^{N} G_{ij}(S)$, $1 \le i \le N$, which reflects the number of decisive outcomes in all matches played by athlete i, $1 \le i \le N$.

For natural N and score matrix $S = [S_{ij}]$, $1 \le i, j \le N$, we say that the pair $(\mathbb{N}(N), S)$ is the ranking problem. The weak-order (i.e., transitive and complete) relation $R(N, S) \subset \mathbb{N}(N) \times \mathbb{N}(N)$ represents the ranking method for the ranking problem $(\mathbb{N}(N), S)$. The vector $r \in \mathbb{R}^N$ is a rating vector, where each r_i, $1 \le i \le N$, is the measure of the performance of player $i \in \mathbb{N}(N)$ in the ranking problem $(\mathbb{N}(N), S)$. For the ranking problem $(\mathbb{N}(N), S)$, a ranking method $R(N, S)$ is induced by the rating vector $r \in \mathbb{R}^N$ if

$$\begin{gathered}(i, j) \in R(N, S) \\ (\text{i.e., } R(N, S) \text{ ranks } i \text{ weakly above } j) \text{ if and only if } r_i \ge r_j.\end{gathered} \tag{1}$$

In this article, for illustrative purposes, we consider only a few of the many ranking methods discussed in the literature (note also that the ranking methods considered here, based on the ranking problems involved in chess tournaments, go back to the investigations of H. Neustadtl, E. Zermelo, and B. Buckholdz. For detailed explanations see, e.g., [9] and the literature cited therein). All these methods are induced by their corresponding rating vectors. For a given score matrix $S = [S_{ij}]$, $1 \le i, j \le N$, we consider the following ranking methods.

Score Method. The rating vector for the score method, $r^s = (r^s_1, \ldots, r^s_N) \in \mathbb{R}^N$, is defined as the average score $r^s_i = \sum_{j=1}^{N} S_{ij}/g_i(S)$, $1 \le i \le N$.

Neustadt's Method. Neustadt's rating vector, $r^N \in \mathbb{R}^N$, is defined by the equality $r^N = \overline{S} r^s$, where $\overline{S} = [\overline{S}_{ij}]$ and $\overline{S}_{ij} = S_{ij}/g_i(S)$, $1 \le i, j \le N$.

Buchholz's Method. Buchholz's rating vector, $r^B \in \mathbb{R}^N$, is defined by the equality $r^N = [\overline{G}(S) + E_N] r^s$, where $\overline{G}(S) = [\overline{G}_{ij}(S)]$, $\overline{G}_{ij}(S) = G_{ij}(S)/g_i(S)$, $1 \le i, j \le N$.

Fair-Bets Method. The rating vector for the fair-bet method, $r^{fb} \in \mathring{\Delta}_N$, is defined as the unique solution of the following system of linear equations:

$$\sum_{j=1}^{N} S_{ij} r_j^{fb} - \left(\sum_{j=1}^{N} S_{ji} \right) r_i^{fb} = 0, \quad 1 \le i \le N. \tag{2}$$

Maximum-Likelihood Method. The rating vector for the maximum-likelihood method, $r^{ml} = (r_1^{ml}, \ldots, r_N^{ml}) \in \mathbb{R}^N$, is defined by the equality $r_i^{ml} = \ln(\pi_i)$, $1 \le i \le N$, where vector $\pi = (\pi_1, \ldots, \pi_N) \in \mathring{\Delta}_N$ is the unique solution of the following nonlinear system of equations:

$$\pi_i \sum_{\substack{j=1 \\ j \ne i}}^{N} \frac{G_{ij}(S)}{\pi_i + \pi_j} = r_i^s g_i(S), \quad 1 \le i \le N. \tag{3}$$

2.2. Ranking Methods to Solve MCDM Problems. Assume now that $\langle A, C \rangle$ is a MCDM problem with a set of alternatives $A = \{a_1, \ldots, a_m\}$ and a set of nonbeneficial criteria $C = \{c_1, \ldots, c_n\}$ and the decision-making goal is therefore to minimize the criteria simultaneously. Let us consider each element of A as an athlete (e.g., chess player) and assume that, for each pair of athletes $a, a' \in A$, the match $\mathrm{M}(a, a')$ includes m games. The special construction of the score matrix of alternatives, S^A, is defined as follows: for any $a, a' \in A$, we define

$$S^A(a, a') = \sum_{c \in C} s_c^A(a, a'),$$
$$\text{where } s_c^A(a, a') = \begin{cases} 1, & c(a) < c(a'); \\ 0, & c(a) \ge c(a'); \end{cases} \quad \forall c \in C. \tag{4}$$

Thus, the equality $s_c^A(a, a') = 1$ means that $c(a) < c(a')$ for criterion $c \in C$ and the alternative a ("athlete a") receives one point (i.e., the athlete a wins a game $c \in C$ in the match $\mathrm{M}(a, a')$ and, correspondingly, $S^A(a, a')$ indicates the number of total wins of athlete a in the match $\mathrm{M}(a, a')$). Obviously, $m \ge S^A(a, a') \ge 0, S^A(a, a) = 0, \forall a, a' \in A$. We say that an alternative a has defeated an alternative a' if $S^A(a, a') > S^A(a', a)$. We also say that the result of the match $\mathrm{M}(a, a')$ is $S^A(a, a')$ wins of the alternative a (losses of alternative a'), $S^A(a', a)$ wins of the alternative a' (losses of alternative a) and number of draws $(n - S^A(a, a') - S^A(a', a))$. Obviously matrix $S^A = [S^A(a, a')]_{a, a' \in A}$is the score matrix for a set of alternatives in the sense of the definition from the previous subsection.

The following procedure is used for solving MCDM problem $\langle A, C \rangle$:

(i) For the MCDM problem $\langle A, C \rangle$, the score matrix $S^A = [S^A(a, a')]_{a, a' \in A}$ is constructed.

(ii) Using the score matrix S^A, the alternatives from set A are ranked using a method R.

(iii) The alternative from the Pareto set, A_e, ranked best by method R is declared as the R–solution of the considered MCDM problem.

Obviously, it would suffice to rank the Pareto set if Pareto set is known at the beginning of the proposed procedure. Nevertheless, we prefer given above description because it is more convenient in the cases when Pareto set is not known (or partially/approximately known), as it took place usually for the complex MCDM problems.

It is clear that, instead of the MCDM problem $\langle A, C \rangle$, we can consider also MCDM problem $\langle C, A \rangle$. Obviously, applying described above procedure to the MCDM problem $\langle C, A \rangle$, we can obtain a ranking of the criteria. However, we omit the corresponding details here.

3. Example

This section discusses the example problem that was solved to demonstrate the practicality of the proposed in Section 2.2 procedure. All the necessary calculations were performed in the MATLAB computing environment. The example considered here is the problem of selecting the material for the mast of a sailing boat. This problem has been addressed by several researchers, using various methods and, thus, can be considered as a kind of benchmark problem.

The component to be optimized, the mast, is modeled as a hollow cylinder that is subjected to axial compression. It has a length of 1,000 mm, an outer diameter $\le$ 100 mm, an inner diameter $\ge$ 84 mm, a mass $\le$ 3 kg, and a total axial compressive force of 153 kN [2]. The following criteria are chosen for the ranking problem at hand: specific strength (SS), specific modulus (SM), corrosion resistance (CR), and cost category (CC) [2]. The choice must be made from 15 alternative materials. The corresponding decision-making data are given in Table 3 of the Appendix, and the normalized decision matrix is given in Table 4 of the Appendix. Note also that, for the problem under consideration, the upper-lower-bound approach was used for normalization of the decision matrix [7]. The Pareto set for the considered problem is $A_e = \{2, 3, 4, 7, 9, 11, 12, 13, 14, 15\}$.

The following methods were used to solve the problem by previous investigators: WPM (weighted-properties method), VIKOR (multicriteria optimization through the concept of a compromise solution), CVIKOR (comprehensive VIKOR), FLA (fuzzy-logic approach), MOORA (multiobjective optimization based on ratio analysis), MULTIMOORA (a multiplicative form of MOORA), RPA (the reference-point approach), and a recently proposed game-theoretic method GTM [1–4, 10, 11]. Note also that the material-selection problem is an important application of MCDM [12, 13]. Table 5 of the Appendix presents the materials ranked by methods other than the one proposed in this paper.

Table 1: Materials ranked by proposed methods.

Material	r^S		r^N		r^B		r^{fb}		r^{ml}	
	Rating	Rank	Rating	Rank	Rating	Rank	Rating	Rank	Rating	Rank
1	0,3529	14	0,1666	14	0,8752	14	0,0335	14	-3,408	14
2	*0,3922*	*12*	*0,1816*	*12*	*0,9136*	*12*	*0,0380*	*12*	*-3,231*	*12*
3	*0,4118*	*10*	*0,1882*	*11*	*0,9274*	*11*	*0,0403*	*11*	*-3,167*	*10*
4	*0,6087*	*5*	*0,2693*	*7*	*1,0945*	*5*	*0,0819*	*7*	*-2,424*	*5*
5	0,2340	15	0,1164	15	0,7342	15	0,0201	15	-4,072	15
6	0,5870	7	0,2716	6	1,0694	7	0,0822	6	-2,532	7
7	*0,6087*	*6*	*0,2843*	*4*	*1,0907*	*6*	*0,0913*	*4*	*-2,438*	*6*
8	0,3673	13	0,1767	13	0,8888	13	0,0371	13	-3,349	13
9	*0,4400*	*9*	*0,2052*	*9*	*0,9563*	*9*	*0,0470*	*9*	*-3,043*	*9*
10	0,4082	11	0,1931	10	0,9288	10	0,0425	10	-3,167	11
11	*0,4600*	*8*	*0,2130*	*8*	*0,9755*	*8*	*0,0502*	*8*	*-2,959*	*8*
12	*0,6481*	*3*	*0,2969*	*3*	*1,1322*	*3*	*0,1022*	*3*	*-2,256*	*3*
13	*0,6800*	*2*	*0,3190*	*1*	*1,1595*	*2*	*0,1232*	*1*	*-2,125*	*2*
14	*0,6875*	*1*	*0,3161*	*2*	*1,1600*	*1*	*0,1222*	*2*	*-2,115*	*1*
15	*0,6250*	*4*	*0,2790*	*5*	*1,1001*	*4*	*0,0884*	*5*	*-2,395*	*4*

Note: italic corresponds to the Pareto optimal (efficient) alternatives.

Direct calculations show that the score matrix S^A in the considered case is

$$S^A = \begin{bmatrix} 0&0&0&1&3&1&1&2&2&2&2&1&1&1&1\\ 1&0&1&1&3&1&1&2&2&2&2&1&1&1&1\\ 2&1&0&1&3&1&1&2&2&2&2&1&1&1&1\\ 3&3&2&0&2&1&1&3&3&3&3&1&1&1&1\\ 1&1&1&0&0&0&0&1&1&1&1&1&1&1&1\\ 3&3&3&1&2&0&0&2&2&2&2&2&2&1&2\\ 3&3&3&1&1&1&0&2&2&2&2&2&2&1&2\\ 2&2&2&1&3&2&2&0&0&0&0&1&1&1&1\\ 2&2&2&1&3&2&2&2&0&1&1&1&1&1&1\\ 2&2&2&1&3&2&2&1&1&0&0&1&1&1&1\\ 2&2&2&1&3&2&2&2&1&2&0&1&1&1&1\\ 3&3&3&3&3&2&2&3&3&3&3&0&1&1&2\\ 3&3&3&2&2&1&1&3&3&3&3&3&0&2&2\\ 3&3&3&2&2&2&2&3&3&3&3&2&1&0&1\\ 3&3&3&2&2&1&1&3&3&3&3&1&1&1&0 \end{bmatrix} \quad (5)$$

Using the score matrix S^A, we rank the materials with each of the five methods described in Section 2.1.2. The ranking results are presented in Table 1. These results show that material 14 (*Epoxy–63% carbon fabric*) is ranked best by ranking methods $\boldsymbol{r}^S$, $\boldsymbol{r}^B$, and $\boldsymbol{r}^{ml}$ and material 13 *(Epoxy–70% glass fabric)* is ranked best by ranking methods $\boldsymbol{r}^N$ and $\boldsymbol{r}^{fb}$.

Table 1 also shows that sometimes the case when the alternative which does not belong to the Pareto set is ranked better than some set of the efficient alternatives can be observed (e.g., the efficient alternatives 11,3 and the unefficient alternative 6). However, we should not consider this as contradiction because the Pareto set and the ranking methods are independent objects and only the restriction of the ranking method on the Pareto set is essential.

For comparison, Table 2 presents the correlation coefficients of the alternative ranks as calculated by different methods. As we can see, the results of the proposed ranking methods correlate well with the rankings obtained by FLA, CVIKOR, and VIKOR; they are somewhat correlated with the rankings returned by MOORA, MULTIMOORA, RPA, and WPM and are poorly correlated with the ranking obtained by GTM. Meanwhile, the methods $\boldsymbol{r}^S$, $\boldsymbol{r}^N$, $\boldsymbol{r}^B$, $\boldsymbol{r}^{fb}$, and $\boldsymbol{r}^{ml}$ are very strongly correlated between themselves.

4. Conclusions

In this study, we have proposed a new approach for solving MCDM problems. The proposed approach is based on ranking-theory methods which are used in the competitive sports tournaments. In the framework of the proposed approach, we build a special score matrix for a given multicriteria problem, which allows us to use an appropriate ranking method and choose the corresponding best-ranked alternative from the Pareto set as a solution of the MCDM problem. The proposed approach is particularly useful when no decision-making authority is available, or when the relative importance of various criteria has not been evaluated previously.

To demonstrate the viability and suitability for applications, the proposed approach illustrated using an example of a materials-selection problem. It is shown that the solutions of the illustrative example obtained by the proposed approach are quite competitive. Note also that the proposed approach seems numerically efficient. Namely, our preliminary numerical experiments (unpublished) show that that

Table 2: Correlation between methods.

	r^S	r^N	r^B	r^{fb}	r^{ml}
MOORA∗	0,564286	0,603571	0,578571	0,603571	0,564286
MULTIMOORA∗	0,496429	0,503571	0,521429	0,503571	0,496429
RPA ∗	0,467857	0,492857	0,485714	0,492857	0,467857
FLA∗	0,764286	0,717857	0,792857	0,717857	0,764286
Wpm ∗∗	0,403571	0,410714	0,442857	0,410714	0,403571
CVIKOR ∗ ∗ ∗	0,742857	0,646429	0,739286	0,646429	0,742857
VIKOR ∗ ∗ ∗	0,892857	0,871429	0,907143	0,871429	0,892857
GTM ∗ ∗ ∗∗	-0,12143	-0,07857	-0,09286	-0,07857	-0,12143

Sources:∗[1]; ∗∗[2]; ∗ ∗ ∗[3]; ∗∗∗∗[4].

Table 3: Decision matrix for selecting material for a sailing boat mast.

#	Material	Criteria Specific strength (MPa) SS 1	Specific modulus (GPa) SM 2	Corrosion resistance CR 3	Cost Category CC 4
1	AISI 1020	35.9	26.9	1	5
2	AISI 1040	51.3	26.9	1	5
3	ASTM A242 type 1	42.3	27.2	1	5
4	AISI 4130	194.9	27.2	4	3
5	AISI 316	25.6	25.1	4	3
6	AISI 416 heat treated	57.1	28.1	4	3
7	AISI 431 heat treated	71.4	28.1	4	3
8	AA 6061 T6	101.9	25.8	3	4
9	AA 2024 T6	141.9	26.1	3	4
10	AA 2014 T6	148.2	25.8	3	4
11	AA 7075 T6	180.4	25.9	3	4
12	Ti–6Al–4V	208.7	27.6	5	1
13	Epoxy–70% glass fabric	604.8	28.0	4	2
14	Epoxy–63% carbon fabric	416.2	66.5	4	1
15	Epoxy–62% aramid fabric	637.7	27.5	4	1

Source: [1]. Notes: CR scale: 1 = poor; 2 = fair; 3 = good; 4 = very good; 5 = excellent.
CC scale: 1 = very high; 2 = high; 3 = moderate; 4 = low; 5 = very low.

MCDM problems with the number of alternatives of the order of 1.5 hundred and with the number of criteria of the order of ten can be solved by the proposed method in a few minutes (~5 min, the calculations were conducted on a laptop with 2.59GHz, 8GB RAM, 64-bit operation system, MATLAB environment, and not making any effort to optimize the code).

Due to the simplicity and flexibility of the implementation, the proposed approach can be also used in a few interesting directions. For example, if we consider the "transposed" MCDM problem (i.e., the problem, for which the criteria of the original problem are alternatives and the alternatives of the original problem are criteria), the proposed approach also allows ranking the criteria and identified a "leading criterion". On the other hand, an "objective" ranking of the criteria may stimulate the development of other instruments for the Pareto optimization. It also seems possible that the proposed approach will find applications in the (e.g., evolutionary) Pareto optimization algorithms. However, we will limit ourselves here only to mention these directions for further investigations.

Appendix

See Tables 3, 4, and 5.

Data Availability

Previously reported data were used to support this study. These prior studies are cited at relevant places within the text as references.

Conflicts of Interest

The author declares that he has no conflicts of interest.

Table 4: Normalized decision matrix for the material selection problem.

		Criteria			
		1	2	3	4
Materials	1	0.9832	0.9565	1.0000	0.0000
	2	0.9580	0.9565	1.0000	0.0000
	3	0.9727	0.9493	1.0000	0.0000
	4	0.7234	0.9493	0.2500	0.5000
	5	1.0000	1.0000	0.2500	0.5000
	6	0.9485	0.9275	0.2500	0.5000
	7	0.9252	0.9275	0.2500	0.5000
	8	0.8753	0.9831	0.5000	0.2500
	9	0.8100	0.9758	0.5000	0.2500
	10	0.7997	0.9831	0.5000	0.2500
	11	0.7471	0.9807	0.5000	0.2500
	12	0.7009	0.9396	0.0000	1.0000
	13	0.0537	0.9300	0.2500	0.7500
	14	0.3619	0.0000	0.2500	1.0000
	15	0.0000	0.9420	0.2500	1.0000

Note: italic denotes Pareto optimal (efficient) alternatives.

Table 5: Materials ranked by comparable methods.

Material	MOORA*	MULTIMOORA*	RPA *	FLA*	Wpm **	CVIKOR ***	VIKOR ***	GTM****
1	14	14	14	14	14	12	14	14
2	15	15	13	13	13	6	11	10
3	13	13	12	15	15	9	13	11
4	12	12	15	4	11	4	4	2
5	4	4	4	11	10	15	15	9
6	7	11	11	9	9	14	10	8
7	6	10	10	10	8	11	5	7
8	11	9	9	8	7	13	12	5
9	10	7	8	12	2	8	7	4
10	9	6	7	7	4	10	9	3
11	5	8	6	6	6	5	6	1
12	8	5	2	5	3	7	8	12
13	2	2	3	3	12	2	2	6
14	3	3	1	2	1	1	1	15
15	1	1	5	1	5	3	3	13

Sources: ∗[1]; ∗∗[2]; ∗ ∗ ∗[3]; **** [4].

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