Hindawi
Advances in Operations Research
Volume 2022, Article ID 1133882, 14 pages
https://doi.org/10.1155/2022/1133882

Research Article

Stochastic P-Robust Approach to a Centralized Two-Stage DEA System with Resource Waste

Rita Shakouri [1] and Maziar Salahi [2]

[1] *Department of Computer Engineering, Faculty of Mathematical Sciences, Technical and Vocational University (TVU), Tehran, Iran*
[2] *Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Guilan, Rasht, P. O. Box 1914, Iran*

Correspondence should be addressed to Rita Shakouri; rshakori@tvu.ac.ir

Received 11 July 2022; Revised 12 August 2022; Accepted 17 August 2022; Published 19 September 2022

Academic Editor: Zeljko Stevic

Uncertain data and undesirable outputs are two challenging issues in traditional data envelopment analysis (DEA) models while dealing with the environmental efficiency estimation of decision-making units (DMUs). This study considers Stackelberg and the centralized game theory approach in a two-stage DEA model for evaluating DMUs in the presence of uncertainty and undesirable outputs simultaneously. To tackle the uncertainty, we apply the p-robust technique and assume that undesirable outputs are weakly disposable. The proposed fractional models are linearized using the Charnes and Cooper transformation. We utilize the new models for a real dataset drawn from 11 oil generation ports in the Persian Gulf region consisting of two stages: an oil production stage and a wastewater treatment stage. The results revealed that the managers should take different strategies in environmental efficiency evaluation including undesirable impacts and also efficiency improvement in increasing oil generation. Further, the empirical results showed that the stochastic p-robust approach for controlling the conservatism level leads to a more conservative solution, and policymakers could recognize the significant steps that should be followed to improve each oil generation unit's environmental performance. Also, to show the reliability and accuracy of the results and the effect of the decision-maker's preference, a detailed sensitivity analysis is performed.

1. Introduction

Environmental issues, such as resource shortage and environmental pollution have attracted significant attention in recent years[1, 2]. In fact, overuse of natural resources for economic growth and development on the one hand and pollution from the production process on the other hand causes irreparable damage to the environment. To resolve these issues, in the production process in addition to increasing outputs with a certain level of inputs, one has to focus also on the environmental aspect. Data envelopment analysis (DEA) is a widely used methodology for the efficiency estimation of decision-making units (DMUs) in various settings such as environmental analysis [3]. As a nonparametric approach, it has the advantage that it does not require any prior assumptions on the underlying functional relationships between variables of inputs and outputs (e.g., [4–6]. Classical DEA models, with outputs maximization and inputs minimization, evaluate the performance of DMUs without considering environmental factors. Thus, various extension of classical DEA models are proposed in the last decades taking into account environmental factors (*undesirable input/output*) that are summarized in Table 1.

Moreover, original DEA models treat each DMU as a black box and neglect their internal structure. However, in the real production process, the internal structures of DMUs are often multistage. That is outputs from the one stage

Table 1: Some advances of environmental factor applications in DEA.

Authors	DEA model	Environmental factor	Applications scope
Aslam et al. [7]	CCR	Output	Financial systems
Matsumoto et al. [8]	CCR	Output	European countries
Mozaffari et al. [9]	CCR/FUZZY	Output	Petrochemical sector
Li et al. [10]	CCR	Output	Water pollution
Wang and Zhao [11]	ADD	Output	Energy
Murty and Russell [12]	CCR	Output	Emission-generating technology
Zhang et al. [13]	SBM	Output/Input	Manufacturing industry

become the inputs to the other stage (e.g., [14–17]. The multistage structures also have been applied for the efficiency evaluation taking into account environmental factors. Table 2 gives an overview of network DEA (NDEA) research with a focus on environmental factors.

In all the abovementioned research, input and output parameters are presumed to be precise and the effect of uncertainty is ignored. Research indicated that a small noise in the problem data can lead to serious variation in ranking. To treat uncertainty in the DEA models, various approaches such as fuzzy programming, stochastic programming, and robust optimization are used in the literature. Table 3 summarizes some of DEA and NDEA models under uncertainty.

Albeit the extant literature has progressed significantly, but all of the available two-stage NDEA consider either the pure intermediate undesirable outputs or uncertainties in problem data. So, in this paper, we present a centralized additive model to measure the efficiency of the whole system with both undesirable outputs and uncertainty perspectives. We apply a stochastic p-robust approach to attain robustness against the existing uncertainty for centralized game-theoretic DEA models of Liang et al. [14]; also, the weak disposable production technology of Kuosmanen [35] is applied for modelling undesirable outputs. The stochastic approaches search to minimize the total expected cost among all scenarios. The optimal solution gained by applying it probably is very good for some scenarios but very poor for others. However, decision makers are often motivated to search for min-max regret solutions that appear effective no matter which scenario is realized. Indeed, the aim of this study is to introduce a more stable system for two-stage NDEA models in which major decreases in regret are possible with small increases in the expected efficiency of DMUs. The proposed model has the following contributions in comparison with existing models: First, both undesirable outputs and uncertainty of data are considered in the model. Second, an efficient scenario generation approach is adopted to model the uncertain parameters to ensure the correctness and reliability of the final solutions that are close to the optimal real-world solution. Third, a robustness factor is adopted in the proposed approach, which bounds the relative regret in each scenario when minimizing the expected objective function.

The remainder of the paper is unfolded as follows. In the next section, a short summary of the centralized and noncentralized models utilized in two-stage DEA models and a brief review of weakly disposable technology will follow. Section 3 presents the stochastic p-robust approach for centralized and noncentralized models. Then, the new model and related results are presented. We apply the proposed model to a real dataset to demonstrate its efficiency in Section 4. Sensitivity analysis is discussed in Section 5. Concluding remarks and some directions for future research are given in the last section.

2. NDEA Two-Stage Proposed Structure with Resource Waste

Consider the two-stage structure in Figure 1 in which each DMU is composed of two sub-DMUs sequentially, and undesirable outputs from stage 2 are wastages that can be sent back as inputs to stage 1.

Suppose we have n DMUs. In the first stage, each $\text{DMU}_j\,(j = 1, \ldots, n)$ uses m inputs $x_{ij}^{1s}\,(i = 1, \ldots, m)$ and produces H outputs $y_{hj}^{1s}\,(h = 1, \ldots, H)$ and D intermediate outputs $z_{dj}^{s}\,(d = 1, \ldots, D)$ under scenario $s\epsilon S$ that serve as the inputs to the second stage. Also, there are T inputs $x_{tj}^{2s}\,(t = 1, \ldots, T)$ of the second stage under scenario $s\epsilon S$. Outputs from the second stage take three forms; desirable outputs $y_{rj}^{2s}\,(r = 1, \ldots, A)$, undesirable outputs $z_{qj}^{2s}\,(q = 1, \ldots, Q)$ and a feedback variable $f_{gj}^{s}\,(g = 1, \ldots, G)$ under scenario. $s\epsilon S$.

For each DMUj, the efficiency score of the first stage (leader) is denoted by e_0^{1s} and the efficiency of the second stage (follower) is denoted by e_0^{2s} under the s^{th} scenario. Model (1) displays a generic form of the efficiency evaluation of the follower stage that is computed by replacing the efficiency of leader stage equal to e_0^{1s*} as a constraint which can be obtained by a linear CCR-type model:

$$
\begin{aligned}
& e_0^{2s*} = \max e_0^{2s} \\
& s.t \\
& e_j^{1s} \le 1, \qquad \forall j, \forall s\epsilon S, \\
& e_j^{2s} \le 1, \qquad \forall j, \forall s\epsilon S, \\
& e_0^{1s} = e_0^{1s*} \qquad \forall s\epsilon S.
\end{aligned} \tag{1}
$$

2.1. Undesirable Outputs. A production technology using the weakly disposable axiom of outputs to model undesirable outputs in the DEA framework is propounded in [35]. Under this technology, inputs and desirable outputs are

Table 2: Some advances of environmental factor applications in NDEA.

Authors	DEA model	Environmental factor	Applications scope
Lozano [18]	SBM	Output	Coal-fired power plant
Chen and Zhu [19]	CCR	Output	Industrial systems
Li and Xiao[20]	SBM	Output	Pulp and paper industry
Michali et al. [21]	CCR	Output	Railway transport
Asanimoghadam et al. [22]	ASBM	Output	Airport
Salahi et al. [23]	ASBM	Output	Industry
Wang and Feng[24]	CCR	Output	Industrial eco-efficiency
Mozaffari et al. [9]	CCR/Fuzzy	Output	Petrochemical sector
Wu et al. [25]	BCC/output	Output	Provincial environmental
Vaezi et al. [26]	CCR	Output	Factory

Table 3: Progress in stochastic, fuzzy and robust optimization applications in DEA and NDEA.

Authors	DEA/Uncertainty parameters	Robust approach	Applications scope
Blagojević et al. [27]	SBM/Input	Fuzzy/AHP	Railway undertaking
Rasoulzadeh et al. [28]	CCR/input	Fuzzy	Portfolio of finance
Wu et al. [25]	CCR/output	Max-min	Hybrid poplar clones
Salahi et al. [29]	CCR-CSW/in-output	Interval	Energy/forest district
Salahi et al. [30]	Russell/in-output	Russell measure	Banking sector
Zhou et al. [31]	CCR	Stochastic	Banking sector
Shakouri et al. [32]	CCR/input	Stochastic p-robust	Banking sector
Huang et al. [33]	CCR	Stochastic	Banking sector
Peykani et al. [34]	CCR	Fuzzy	Investment firms

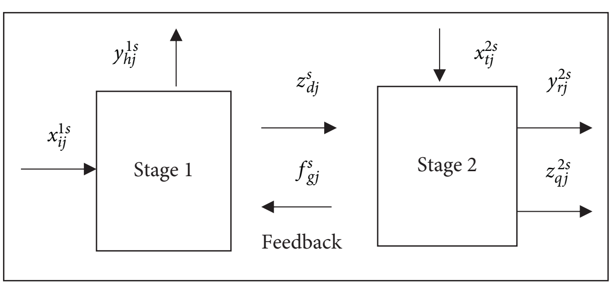

Figure 1: A DMU_j in the two-stage process with an undesirable output.

considered to be freely disposable, undesirable outputs are weakly disposable, and the outputs set is assumed to be convex under a constant return to scale (CRS) assumption. The linear programming model of this technology to evaluate the performance of a DMU is as follows (for further details, see [36]):

$$\begin{aligned}
&\max \sum_{r=1}^{A} u_r y_{rj_o}^{2s} - \sum_{q=1}^{D} \vartheta_q z_{qj_o}^{2s} \\
&s.t \\
&\sum_{r=1}^{A} u_r y_{rj}^{2s} - \sum_{q=1}^{D} \vartheta_q z_{qj}^{2s} + \sum_{i=1}^{m} v_i x_{ij}^{1s} \le 0, \ \ \forall j, \forall s \epsilon S \\
&u_r,\ v_i \ge 0\,,\ \forall r, i,\ \vartheta_q\ free\, \forall q.
\end{aligned} \tag{2}$$

In model (2), the v_i, u_r , and ϑ_q are decision variables of inputs, desirable outputs, and undesirable outputs, respectively. Constraints (2) guarantee that efficiency value is less than or equal to one for each DMU. In the sequel, according to the concepts of the leader-follower or the Stackelberg game theory [14], we will discuss the efficiency of the substages under the noncentralized model in the presence of undesirable outputs.

2.2. Generic Noncentralized Model with Undesirable Outputs. As mentioned above, by accepting that the first and second stages are the leader and the follower, respectively, model (3) computes the maximum achievable value for the efficiency of the first stage under the s^{th} scenario, based on the original CRS-DEA model. Thus we have the following input-oriented DEA model for the first stage in the presence of undesirable outputs according to Kuosmanen [35]'s production technology:

$$\begin{aligned}
&e_0^{1s\,*} = \max \frac{\sum_{h=1}^{H} \eta_h y_{hj_o}^{1s} + \sum_{d=1}^{D} w_d\, z_{dj_o}^{s}}{\sum_{i=1}^{m} v_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{s}} \\
&s.t \\
&\frac{\sum_{h=1}^{H} \eta_h y_{hj}^{1s} + \sum_{d=1}^{D} w_d\, z_{dj}^{s}}{\sum_{i=1}^{m} v_i x_{ij}^{1s} + \sum_{g=1}^{G} \partial_g f_{gj}^{s}} \le 1, \ \ \forall j, \forall s \epsilon S, \\
&\partial_g,\ w_d,\ \eta_h,\ v_i \ge 0,\ \forall\, g, d,\ h,\ i.
\end{aligned} \tag{3}$$

Model (3) is nonlinear but can be linearized using the Charnes and Cooper [37] transformation as follows:

Let $\alpha = 1/\sum_{i=1}^{m} v_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{s}$, then $\sum_{i=1}^{m} \alpha v_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \alpha\partial_g f_{gj_o}^{s} = 1, \alpha v_i = \overline{v}_i$, $\alpha\partial_g = \overline{\partial}_g$, $\alpha\eta_h = \overline{\eta}_h$, $\alpha w_d = \overline{w}_d$. Now, model (3) becomes the following linear model:

$$
\begin{aligned}
e_0^{1s*} &= \max \sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{s} \\
&s.t \\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} \le 0, \quad \forall j, \forall s \epsilon S, \\
&\sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} = 1, \quad \forall s \epsilon S, \\
&\overline{\partial}_g, \overline{w}_d, \overline{\eta}_h, \overline{v}_i \ge 0, \forall\, g, d, h, i.
\end{aligned} \tag{4}
$$

Also, according to model (1), the efficiency score of the second stage with undesirable outputs is computed as follows:

$$
\begin{aligned}
e_0^{2s*} &= \max \frac{\sum_{r=1}^{s} u_r\, y_{rj_o}^{2s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{s} - \sum_{q=1}^{Q} \vartheta_q z_{qj_o}^{2s}}{\sum_{d=1}^{D} w_d\, z_{dj_o}^{s} + \sum_{t=1}^{T} \delta_t x_{tj_o}^{2s}} \\
&s.t \\
&\frac{\sum_{r=1}^{s} u_r\, y_{rj}^{2s} + \sum_{g=1}^{G} \partial_g f_{gj}^{s} - \sum_{q=1}^{Q} \vartheta_q z_{qj}^{2s}}{\sum_{d=1}^{D} w_d\, z_{dj}^{s} + \sum_{t=1}^{T} \delta_t x_{tj}^{2}} \le 1, \quad \forall j, \forall s \epsilon S, \\
&\frac{\sum_{h=1}^{H} \eta_h y_{hj_o}^{1s} + \sum_{d=1}^{D} w_d\, z_{dj_o}^{s}}{\sum_{i=1}^{m} v_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{s}} \le 1, \quad \forall j, \forall s \epsilon S, \\
&\frac{\sum_{h=1}^{H} \eta_h y_{hj_o}^{1s} + \sum_{d=1}^{D} w_d\, z_{dj_o}^{s}}{\sum_{i=1}^{m} v_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{s}} = e_0^{1s*}, \quad \forall s \epsilon S, \\
&u_r, w_d, \partial_g, \delta_t, \eta_h, v_i, \vartheta_q \ge 0, \quad \forall\, r, d, g, t, h, i, q.
\end{aligned} \tag{5}
$$

As before, using Charnes and Cooper [37] transformation, it is linearized as follows:

$$
\begin{aligned}
e_0^{2s*} &= \max \sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2s} \\
&s.t \\
&\sum_{r=1}^{s} \overline{u}_r\, y_{rj}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2s} - \sum_{d=1}^{D} \overline{w}_d\, z_{dj}^{s} - \sum_{t=1}^{T} \overline{\delta}_t x_{tj}^{2} \le 0, \quad \forall j, \forall s \epsilon S, \\
&\sum_{d=1}^{D} \overline{w}_d\, z_{dj_o}^{s} + \sum_{t=1}^{T} \overline{\delta}_t x_{tj_o}^{2s} = 1, \quad \forall s \epsilon S, \\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1s} + \sum_{d=1}^{D} \overline{w}_d\, z_{dj}^{s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} \le 0, \quad \forall j, \forall s \epsilon S, \\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^{D} \overline{w}_d\, z_{dj_o}^{s} - e_0^{1s*}\left(\sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s}\right) = 0, \quad \forall s \epsilon S, \\
&u_r, \overline{w}_d, \partial_g, \delta_t, \eta_h, v_i, \vartheta_q \ge 0, \quad \forall\, r, d, g, t, h, i, q.
\end{aligned} \tag{6}
$$

2.3. Centralized Model with Undesirable Outputs. In this section, we combine the two stages as the weighted sum of efficiency scores of stages 1 and 2 with undesirable outputs under the s^{th} scenario as follows:

$$
\begin{aligned}
e_o^{cs*} &= \max \xi_1^s e_0^{1s} + \xi_2^s e_0^{2s} \\
&s.t \\
e_0^{1s} &= \frac{\sum_{h=1}^{H} \eta_h y_{hj}^{1s} + \sum_{d=1}^{D} w_d\, z_{dj}^{s}}{\sum_{i=1}^{m} v_i x_{ij}^{1s} + \sum_{g=1}^{G} \partial_g f_{gj}^{s}} \le 1, \quad \forall j, \forall s \epsilon S, \\
e_0^{2s} &= \frac{\sum_{r=1}^{s} u_r\, y_{rj}^{2s} + \sum_{g=1}^{G} \partial_g f_{gj}^{s} - \sum_{q=1}^{Q} \vartheta_q z_{qj}^{2s}}{\sum_{d=1}^{D} w_d\, z_{dj}^{s} + \sum_{t=1}^{T} \delta_t x_{tj}^{2}} \le 1, \quad \forall j, \forall s \epsilon S, \\
&u_r, w_d, \partial_g, \delta_t, \eta_h, v_i, \vartheta_q \ge 0, \quad \forall\, r, d, g, t, h.
\end{aligned} \tag{7}
$$

where ξ_1^s and ξ_2^s are the weights of the first and second stages, respectively, reflecting the importance of the two stages in the overall system ($\xi_1^s + \xi_2^s = 1$). We let $\xi_1^s = (\sum_{i=1}^m v_i x_{ij_o}^{1s} + \sum_{g=1}^G \partial_g f_{gj_o}^s)/(\sum_{i=1}^m v_i x_{ij_o}^{1s} + \sum_{g=1}^G \partial_g f_{gj_o}^s + \sum_{d=1}^D w_d z_{dj_o}^s + \sum_{t=1}^T \delta_t x_{tj_o}^2)$ and $\xi_2^s = (\sum_{d=1}^D w_d z_{dj_o}^s + \sum_{t=1}^T \delta_t x_{tj_o}^2)/(\sum_{i=1}^m v_i x_{ij_o}^{1s} + \sum_{g=1}^G \partial_g f_{gj_o}^s + \sum_{d=1}^D w_d z_{dj_o}^s + \sum_{t=1}^T \delta_t x_{tj_o}^2)$ in order to linearize the model. Then, model (7) becomes as follows:

$$
\begin{aligned}
e_o^{cs} &= \max \frac{\sum_{h=1}^H \eta_h y_{hj_o}^{1s} + \sum_{d=1}^D w_d z_{dj_o}^s + \sum_{r=1}^s u_r y_{rj_o}^{2s} + \sum_{g=1}^G \partial_g f_{gj_o}^s - \sum_{q=1}^Q \vartheta_q z_{qj_o}^{2s}}{\sum_{i=1}^m v_i x_{ij_o}^{1s} + \sum_{g=1}^G \partial_g f_{gj_o}^s + \sum_{d=1}^D w_d z_{dj_o}^s + \sum_{t=1}^T \delta_t x_{tj_o}^2} \\
&s.t \\
e_0^{1s} &= \frac{\sum_{h=1}^H \eta_h y_{hj}^{1s} + \sum_{d=1}^D w_d z_{dj}^s}{\sum_{i=1}^m v_i x_{ij}^{1s} + \sum_{g=1}^G \partial_g f_{gj}^s} \le 1, \qquad \forall j, \forall s \epsilon S, \\
e_0^{2s} &= \frac{\sum_{r=1}^s u_r y_{rj}^{2s} + \sum_{g=1}^G \partial_g f_{gj}^s - \sum_{q=1}^Q \vartheta_q z_{qj}^{2s}}{\sum_{d=1}^D w_d z_{dj}^s + \sum_{t=1}^T \delta_t x_{tj}^2} \le 1, \quad \forall j, \forall s \epsilon S, \\
&u_r, w_d, \partial_g, \delta_t, \eta_h, v_i, \vartheta_q \ge 0, \qquad \forall r, d, g, t, h.
\end{aligned}
\tag{8}
$$

Now, let $t_1 = (\sum_{i=1}^m v_i x_{ij_o}^{1s} + \sum_{g=1}^G \partial_g f_{gj_o}^s + \sum_{d=1}^D w_d z_{dj_o}^s + \sum_{t=1}^T \delta_t x_{tj_o}^2)^{-1}$, $\overline{v}_i = t_1 v_i$, $\overline{\partial}_g = t_1 \partial_g$, $\overline{\eta}_h = t_1 \eta_h$, $\overline{u}_r = t_1 u_r$, $\overline{\delta}_t = t_1 \delta_t$, $\overline{w}_d = t_1 w_d$ and $\overline{\vartheta}_q = t_1 \vartheta_q$; then, model (8) is transformed into the following linear model:

$$
\begin{aligned}
e_o^{cs*} &= \max \left(\sum_{h=1}^H \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^D \overline{w}_d z_{dj}^s + \sum_{r=1}^s \overline{u}_r y_{rj_o}^{2s} + \sum_{g=1}^G \overline{\partial}_g f_{gj_o}^s - \sum_{q=1}^Q \overline{\vartheta}_q z_{qj_o}^{2s} \right) \\
&s.t \\
&\sum_{h=1}^H \overline{\eta}_h y_{hj}^{1s} + \sum_{d=1}^D \overline{w}_d z_{dj}^s - \sum_{i=1}^m \overline{v}_i x_{ij}^{1s} - \sum_{g=1}^G \overline{\partial}_g f_{gj}^s \le 0, \qquad \forall j, \forall s \epsilon S, \\
&\sum_{r=1}^s \overline{u}_r y_{rj}^{2s} + \sum_{g=1}^G \overline{\partial}_g f_{gj}^s - \sum_{q=1}^Q \overline{\vartheta}_q z_{qj}^{2s} - \sum_{d=1}^D \overline{w}_d z_{dj}^s - \sum_{t=1}^T \overline{\delta}_t x_{tj}^{2s} \le 0, \quad \forall j, \forall s \epsilon S, \\
&\sum_{i=1}^m \overline{v}_i x_{ij_o}^{1s} + \sum_{g=1}^G \overline{\partial}_g f_{gj_o}^s + \sum_{d=1}^D \overline{w}_d z_{dj_o}^s + \sum_{t=1}^T \overline{\delta}_t x_{tj_o}^{2s} = 1, \qquad \forall s \epsilon S, \\
&u_r, w_d, \partial_g, \delta_t, \eta_h, v_i, \vartheta_q \ge 0, \qquad \forall r, d, g, t, h.
\end{aligned}
\tag{9}
$$

In model (9), e_o^{cs*} displays the average of the efficiency score of the two-stage process.

Definition 1. The two-stage process is efficient if and only if $e_j^{1s} = e_j^{2s} = 1$.

It is noted that, if there is uncertainty in data set, models (4), (6), or (9) might be infeasible at optimal solution of nominal problem. Thus, it is essential to choose alternative models such that small variation in data cannot change the rankings. To cope this case, we apply the stochastic p-robust optimization approach of Snyder and Daskin [38] that will be illustrated in the next section.

3. Proposed NDEA Models with Uncertainty

In this section, first, we depict the stochastic p-robust optimization concept; then, we apply it to the proposed two-stage models of Section 2.

3.1. Stochastic P-Robust Concept. Let S be a collection of scenarios, and P^s be a deterministic maximization problem for each scenario s (there is a different problem P^s for each scenario $s \epsilon S$). For each s,, let $F^{s*} > 0$ be the optimal efficiency score for P^s As well, let X be a feasible solution for P^s for all $s \epsilon S$ and let $F^s(X)$ be the efficiency score of P^s under solution X. Therefore, X is called p robust if for all $s \in S$ the following inequality holds:

$$p \geq \frac{F^{s*} - F^{s}(\mathrm{X})}{F^{s*}}. \tag{10}$$

In equation (10), the right-hand side is the relative regret for scenarios and $p \geq 0$ is a parameter (constant) that limits the relative regret for each scenario. It is obvious that inequality (10) can be written as follows:

$$(1-p)F^{s*} \leq F^{s}(\mathrm{X}). \tag{11}$$

Therefore, in order to control the relative regret pertinent to all scenarios, the p-robust constraints (11) are put in the models.

Definition 2. $\mathrm{DMU_j}$ is stochastic p -robust efficient in different scenarios if and only if its optimal objective function is one.

3.2. Stochastic **p** *-Robust Noncentralized Model.* Here, we present the stochastic p robust Stackelberg game versions of uncertain DEA models (4) and (6). The stochastic p robust model for the first stage of model (4) is as follows:

$$\begin{aligned}
&f_0^{1s*} = \max \sum_{s=1}^{S} q^{s}\left[\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{s}\right] \\
&s.t \\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{s} \geq (1-p)e_0^{1s*}, && \forall s\epsilon S, \\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} \leq 0, && \forall j, \forall s\epsilon S, \\
&\sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} = 1, && \forall s\epsilon S, \\
&\overline{\partial}_g, \overline{w}_d, \overline{\eta}_h, \overline{v}_i \geq 0, && \forall g, d, h, i.
\end{aligned} \tag{12}$$

The objective function of model (12) computes the expected efficiency value of DMUs according to the data from each scenario in the leader stage. Also, q_s in the objective function is the probability that scenario s happens (it is clear that, there is no information about the probability of chance of each scenario). Constraints (12) represent the p robust restrictions. This set of restrictions may not allow the scenario efficiency to take value more than $100(1-p)\%$ of the ideal efficiency scores gained by each scenario. Also, the parameter p controls the relative regret between all scenarios, and if $p = \infty$, then the p robust constraints in model (12) become inactive. The third to the fifth set of constraints are the same constraints as in model (4) which must hold for each $s\epsilon S$.

Remark 1. By assuming $f_{ko}^{s} = \max\left\{f_{go}^{s} | 1 \leq g \leq G\right\} > 0$, $x_{ko}^{s} = \max\{x_{io}^{s} | 1 \leq i \leq m\} > 0$ and then setting $(\eta_1, \ldots, w_1, \ldots, , v_1, \ldots, \partial_1, \ldots) = (0, \ldots, 1/x_{ko}^{s}, 0, \ldots, 1/f_{ko}^{s}, 0, \ldots)$, constraints $\sum_{i=1}^{m} v_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{s} = 1$ and $\sum_{h=1}^{H} \eta_h y_{hj}^{1s} + \sum_{d=1}^{D} w_d z_{dj}^{s} - \sum_{i=1}^{m} v_i x_{ij}^{1s} - \sum_{g=1}^{G} \partial_g f_{gj}^{s} \leq 0$ imply $\sum_{h=1}^{H} \eta_h y_{hj_o}^{1s} + \sum_{d=1}^{D} w_d z_{dj_o}^{s} \leq 1$. Thus, we get $e_0^{1s*} \leq 1/1-p$. So for very small p's, there may not be p-robust solutions for model (12); therefore, it may be infeasible.

As retaining the leader's efficiency fixed, the stochastic p robust model for the second stage for all scenarios can be modeled as follows:

$$\begin{aligned}
&f_0^{2s*} = \max \sum_{s=1}^{S} q^{s}\left[\sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2s}\right] \\
&s.t \\
&\sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2s} \geq (1-p)e_0^{2s*}, \ \forall s\epsilon S, \\
&\sum_{r=1}^{s} \overline{u}_r y_{rj}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2s} - \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} - \sum_{t=1}^{T} \overline{\delta}_t x_{tj}^{2} \leq 0, \ \forall j, \ \forall s\epsilon S, \\
&\sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{s} + \sum_{t=1}^{T} \overline{\delta}_t x_{tj_o}^{2s} = 1, \ \forall s\epsilon S, \\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} \leq 0, \ \forall j, \forall s\epsilon S, \\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{s} - e_0^{1s*}\left(\sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s}\right) = 0, \ \forall s\epsilon S, \\
&u_r, \overline{w}_d, \partial_g, \delta_t, \eta_h, v_i, \vartheta_q > 0, \forall r, d, g, t, h, i, q.
\end{aligned} \tag{13}$$

According to data from each scenario, the objective function of model (13) maximize the expected efficiency value of all DMUs. Constraints (13) impose the p robust criterion associated with all expert's probabilities. Further, the relative regret among all scenarios is controlled by the parameter p. Constraints (13) show the efficiency value of the first stage under scenario s. The other set of constraints are the same as in model (6) which must hold for each $s\epsilon S$. In addition, the relation between the overall efficiency and the efficiency score of each stage, i.e., f_0^{s*}, f_0^{1s*} and f_0^{2s*}, respectively, are given below.

Definition 3. DMU_0 for the s^{th} scenario is overall efficient if and only if it is efficient in both stages under the s^{th} scenario.

Definition 4. $\mathrm{DMU}_{0,}$ under the s^{th} scenario and k^{th} stage is efficient if only if $f_j^{ks} = 1$, $j = 1, \ldots, n$ and $k = 1, 2$.

3.3. Stochastic **p***-Robust Centralized Proposed Model.* Similar to the previous subsection, the stochastic p robust centralized model (9) under uncertainty is as follows:

$$\begin{aligned}
& f_o^{cs*} = \max \sum_{s=1}^{S} q^s \left[\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} + \sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2s} \right] \\
& s.t \\
& \sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} + \sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2s} \geq (1-p) e_0^{s*}, \ \forall s\epsilon S, \\
& \sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} \leq 0, \ \forall j, \forall s\epsilon S, \\
& \sum_{r=1}^{s} \overline{u}_r y_{rj}^{2s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2s} - \sum_{d=1}^{D} \overline{w}_d z_{dj}^{s} - \sum_{t=1}^{T} \overline{\delta}_t x_{tj}^{2s} \leq 0, \ \forall j, \ \forall s\epsilon S, \\
& \sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{s} + \sum_{t=1}^{T} \overline{\delta}_t x_{tj_o}^{2s} = 1, \ \forall s\epsilon S, \\
& u_r, w_d, \partial_g, \delta_t, \eta_h, v_i, \vartheta_q \geq 0, \forall\, r, d, g, t, h.
\end{aligned} \tag{14}$$

The objective function of model (14) maximizes the expected efficiency value of all DMUs. Constraints (14) impose the p-robust restrictions that may not permit the scenario efficiency to take value more than $100(1-p)\%$ of the ideal efficiency scores gained by each scenario. The relative regret between all scenarios is controlled by the parameter p. The p robust constraints in model (14) become ineffective if $p = \infty$. The other constraints are the same as in model (9) which must be retained for all $s\epsilon S$. It is noteworthy that the p values generally are assumed greater than 0.2 and their upper bound is gained by try and error. Also, these values can be different for any problem and are usually defined by the decision maker.

Lemma 1. *The expected efficiency values of model (9) for different scenarios are greater than those of the robust model (14).*

Proof. Since the feasible set of model (14) is the subset of the feasible set of model (9); thus, the result follows. □

Theorem 1. *The expected regret values of model (9) in various scenarios are not less than those of model (14).*

Proof. Let the gap between the expected efficiency values of each model with the average ideal efficiency be as follows:

$$\theta = \sum_{s\epsilon S} q^s F^s(\mathrm{X}) - \sum_{s\epsilon S} q^s F^{s*}. \tag{15}$$

Obviously, the smaller θ amount of a model means that the model gives more exact results. According to Lemma 1, we have

$$f_o^{cs*} \leq \overline{\Omega}, \tag{16}$$

in which $\overline{\Omega}$ is the expected optimal objective value of model (9). From inequality (16), we further can obtain

$$f_o^{cs*} - \sum_{s\epsilon S} q^s F^{s*} \leq \overline{\Omega} - \sum_{s\epsilon S} q^s F^{s*}. \tag{17}$$

The left-hand side in (17) is the gap value for model (14) and the right-hand side represents the gap value of the expected value of model (9). Thus, the result follows. □

4. Real Case Study and Discussion

In this section, the advantage of the proposed models for a dataset of Persian Gulf oilfields is discussed. In comprehensive reservoir management, achieving maximum economic advantage is the most important goal in the development of an oilfield. One of the most important variables in achieving this maximum advantage is the number of wells drilled oilfields [39]. When oil production is increased, more water is being used and more oilfield wastewater is being generated, and while any reduction in water use means less oilfield wastewater releases, it also means a reduction in oil production.

Thus, some inputs are utilized to generate both the desirable and undesirable outputs and minimizing total costs and maximizing the desirable outputs are the aims. Moreover, in accordance with the supportable expansion program, it is required to reuse wastewater to warrant the water reserve in the water-deficient regions [40]. Thus, the wastewater here behaves as a feedback variable which makes better the efficiency of the total oilfield system. Further, uncertainty in the number of wells is an important issue in the development of oil fields. Therefore, every decision needs to take into account all the uncertainties in all stages of field development. The flowchart of the methodology is given in Figure 2.

First, we evaluate ideal efficiency values of the two-stage NDEA models under discrete scenarios, provided by the oilfield system analyzers (i.e., s_1 = pessimistic and s_2 = optimistic). According to the study by Snyder and Daskin [38]; we assume that all scenarios have equiprobable that is $q^s = 0.5$. Further, we consider five inputs in stage1, number of generating wells, cost of oil, cost of water, water increasing rate, and reusable water; desirable outputs and undesirable output are actual oil generation and incremental oil generation, respectively. In stage 2, undesirable wastewater is refined by applying the inputs as operating costs, consumption cost, construction expenditure, hydrocarbons (ammonia); therefore, hydrocarbons removal rates and the quantity of reusable refined wastewater are desirable outputs and unrefined wastewater is undesirable output. Table 4 shows inputs, intermediates, and outputs data of 11 ports of the Persian Gulf oilfields.

First, using models (4), (6), and (9), we obtain the ideal efficiency score of each DMU based on each scenario. Then, we apply models (12)–(14) for the efficiency analysis of DMUs. In Table 5, the relevant results are reported. As well, the ideal efficiency score matching the amounts specified in each scenario is shown in the columns of Table 5.

With respect to Table 5, the efficiency scores in model (4) for most DMUs are equal to one in two scenarios that is 72% of the total oilfield regions in the first scenario and 90% in the second scenario. In model (6), most DMUs gained an efficiency score of one that is 0.81% of total oilfield regions and finally, in model (9) all DMUs are efficient in two scenarios.

Subsequently, we solved models (12)–(14) to gain the efficiency scores for different p values in each scenario that are presented inTables 6–8. As can be seen, models (12)–(14) give infeasible results for some DMUs when $p \leq 0.47$ that we do not report those here.

As mentioned in Section 3.2, for small values of p, the offered models give infeasible results in some scenarios. In this perusal, on the one hand, when $p < 0.47$ with respect to the results, our models give infeasible results for most DMUs. As the p value enhances, the efficiency scores set better and the number of infeasible DMUs gradually reduces and we see feasible results. On the other hand, for $p \geq 0.52$, the efficiency scores remain fixed. So, we do not carry on and stop it for the other p-values. Thus, here, we only consider $p \geq 0.47$ and do not report the results of $p \leq 0.47$. For example, with increasing the p value from 0.47 to 0.50, the efficiency score of DMU #7 shifts. This shift also can be seen in other DMUs. Models (12)–(14) maximize the expected efficiency scores of DMUs in each scenario, while p robust constraints control the respective variation between their efficiency scores produced by the model and ideal efficiency under each scenario. We should note that the overall efficiency of DMU_0 for models (12) and (13) can be specified as $f_0^{s*} = f_0^{1s*} + f_0^{2s*}$ as reported in Table 9.

In order to compare the overall efficiency scores of model (14), we let $p = 0.49$ for each DMU. The related results are given in Table 10, and as well, are shown in Figure 3.

As seen in Table 10, the robust centralized model (i.e., f_o^{cS*}) gives better results for the overall efficiency scores (i.e., f_0^s) compared to the robust noncentralized model. Thus, one can conclude that the centralized approach is preferred over the noncentralized approach.

It is obvious that, for all DMUs, the efficiency scores in model (14)are more premier than the other one. So, model (14) as a superior model is chosen, and to gain a profound understanding of the stochastic p robust NDEA models, we compare model (14) with model (9).

After solving each model and gaining $F^s(\mathrm{X})$, we present two criterions as $\sum_{s\varepsilon S} q^s F^s(\mathrm{X})$ and $\sum_{s\varepsilon S} q^s (F^{s*} - F^s(\mathrm{X}))/F^{s*}$, where the first criterion measures the expected efficiency score of all DMUs by taking into account the occurrence probabilities of each scenario, and the second criterion measures the expected relative regret for each DMU, respectively. Finally, the computed efficiency scores by the robust model (14) are balanced with these two criteria under two scenarios. The results of this comparison are reported in Table 11 and illustrated in Figure 4. It should be noted that, in this trial, we put $p = 0.49$. As seen in Figure 4, both efficiency scores and regret values of model (14) are less than the other one. As mentioned before, the relative regret value demonstrates the relative difference between the ideal efficiency gain in each scenario and the efficiency of a model that is displayed according to Lemma 1 and Theorem 1.

5. Sensitivity Analysis

In this section, we run some experiments for understanding the sensitivity of the proposed model (model (14)) to the experts' probabilities and compare the results to the sensitivity of model (9) . The first row of Table 12 and 13 show the value of probabilities as $q^s = (q^1, q^2)$, and columns 2–4 present the difference between the expected efficiency values of each model and the average ideal efficiency score. In fact, each model is solved using these probabilities and the difference between the expected efficiency of each DMU with the average ideal efficiency (i.e., the gap value) is obtained using equation (15).

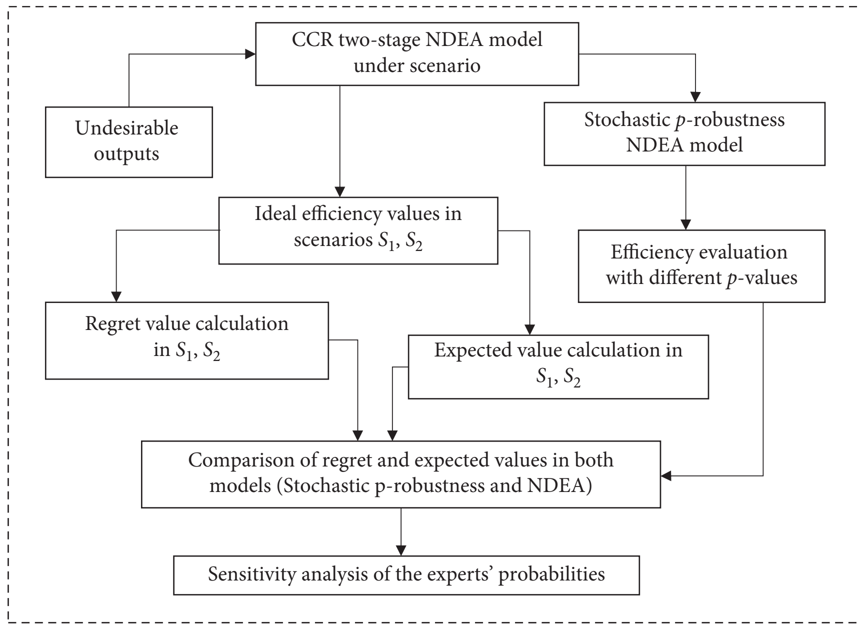

Figure 2: Flowchart of two-stage stochastic p robust.

Table 4: The data set of three scenarios for 11 oilfields.

DMUs	x_1^1		x_2^1		x_3^1		x_4^1		y_1^1		y_2^1		z_1
	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1
1	75000	90000	0.498	0.510	92.22	121.57	67.45	60.8	510.1	638.1	102.0	68.0	433.0
2	48000	84000	0.442	0.453	73.36	132.05	85.5	95	797.1	997.1	72.0	96.0	344.4
3	52000	87000	0.434	0.462	113.18	117.38	55.1	64.6	542.0	678.0	87.0	116.0	531.3
4	4000	74000	0.533	0.546	209.60	138.34	30.4	27.55	231.2	289.2	150.0	200.0	984.0
5	35000	63000	0.410	0.420	79.65	125.76	78.85	82.65	693.5	867.5	85.5	114.0	373.9
6	30000	90000	0.342	0.351	127.86	117.38	48.45	40.85	342.8	428.8	87.0	116.0	600.2
7	18000	90000	0.513	0.525	138.34	119.47	45.6	40.85	342.8	428.8	96.0	128.0	649.4
8	6000	95000	0.368	0.377	119.47	117.38	52.25	52.25	438.4	548.4	90.0	120.0	560.9
9	46000	95000	0.523	0.536	85.94	123.66	72.2	64.6	542.0	678.0	52.5	70.0	403.4
10	23700	90000	0.564	0.577	113.18	117.38	56.05	46.55	390.6	488.6	82.5	110.0	531.3
11	12000	84000	0.318	0.326	88.03	121.57	71.25	58.9	494.2	618.2	61.5	82.0	413.3

DMUs	$\mathbf{x}_1^2$		$\mathbf{x}_2^2$		$\mathbf{x}_3^2$	
	s_1	s_2	s_1	s_2	s_1	s_2
1	477.0	117	135.0	17479.9	24279.9	92.6
2	379.4	109	126.0	12338.8	17138.8	117.4
3	585.3	113	130.5	14909.4	20709.4	47.4
4	108.4	96	111.0	25705.8	35705.8	65.5
5	411.9	82	94.5	14652.3	20352.3	61.0
6	661.2	117	135.0	14909.4	20709.4	103.9
7	715.4	117	135.0	16451.7	22851.7	85.8
8	617.9	124	142.5	15423.5	21423.5	76.8
9	444.4	124	142.5	8997.0	12497.0	173.9
10	585.3	117	135.0	14138.2	19638.2	88.1
11	455.3	109	126.0	10539.4	14639.4	67.7

DMUs	$\mathbf{f}_1$		$\mathbf{z}_1^2$		$\mathbf{y}_1^2$	
	s_1	s_2	s_1	s_2	s_1	s_2
1	239.4	489.3	10.4	30.4	0.41	0.71
2	191.5	391.4	8.3	24.3	0.52	0.9
3	478.8	978.5	20.8	60.8	0.21	0.58
4	339.9	694.7	14.8	43.2	0.29	0.32
5	363.9	743.7	15.8	46.2	0.27	0.83
6	215.5	440.3	9.4	27.4	0.46	0.51
7	263.3	538.2	11.4	33.4	0.38	0.48
8	292.1	596.9	12.7	37.1	0.34	0.55
9	129.3	264.2	5.6	16.4	0.77	0.76
10	253.8	518.6	11.0	32.2	0.39	0.59
11	330.4	675.2	14.4	42.0	0.3	0.75

Table 5: Ideal efficiency scores under scenarios for models (4), (6) and (9).

DMUs	*Model (4)*		*Model (6)*		*Model (9)*	
	$\mathbf{s}_1$	$\mathbf{s}_2$	$\mathbf{s}_1$	$\mathbf{s}_2$	$\mathbf{s}_1$	$\mathbf{s}_2$
1	1	1	1	1	1	1
2	0.7697	1	1	1	1	1
3	1	1	1	1	1	1
4	1	1	1	1	1	1
5	0.7281	0.9451	1	1	1	1
6	1	1	0.9765	0.8936	1	1
7	1	1	0.8320	1	1	1
8	1	1	1	1	1	1
9	0.9549	1	1	1	1	1
10	1	1	1	1	1	1
11	1	1	1	1	1	1

Table 6: The results of solving model (12).

p value DMUs	0.47	0.48	0.49	0.50	0.51	0.52
1	0.058	0.088	0.034	0.041	0.039	0.039
2	0.045	0.045	0.019	0.150	0.150	0.150
3	0.052	0.102	0.064	0.067	0.065	0.065
4	0.054	0.094	0.076	0.069	0.069	0.069
5	INF	0.063	0.038	0.063	0.065	0.065
6	0.043	0.043	0.040	0.061	0.067	0.067
7	0.049	0.098	0.013	0.098	0.096	0.096
8	0.034	0.066	0.025	0.061	0.063	0.063
9	0.046	0.096	0.002	0.013	0.013	0.013
10	0.051	0.015	0.047	0.049	0.049	0.049
11	0.062	0.062	0.041	0.045	0.042	0.042

Table 7: The results of solving model (13).

p value DMUs	0.47	0.48	0.49	0.50	0.51	0.52
1	0.010	0.104	0.058	0.058	0.058	0.058
2	0.123	0.113	0.097	0.101	0.101	0.101
3	0.026	0.024	0.028	0.030	0.028	0.028
4	0.144	0.132	0.037	0.039	0.039	0.039
5	0.091	0.081	0.065	0.070	0.070	0.070
6	INF	INF	0.057	0.060	0.060	0.060
7	0.101	0.101	0.101	0.105	0.105	0.105
8	0.119	0.021	0.035	0.049	0.049	0.049
9	INF	0.090	0.005	0.001	0.002	0.002
10	INF	INF	0.019	0.018	0.017	0.017
11	0.122	0.127	0.019	0.021	0.023	0.023

Table 8: The results of solving model (14).

p value DMUs	0.47	0.48	0.49	0.50	0.51	0.52
1	0.048	0.047	0.224	0.275	0.275	0.275
2	0.051	0.152	0.216	0.260	0.260	0.260
3	0.049	0.247	0.228	0.381	0.381	0.381
4	0.060	0.231	0.295	0.234	0.234	0.234
5	0.044	0.144	0.160	0.257	0.257	0.257
6	INF	0.124	0.207	0.438	0.439	0.439
7	0.042	0.162	o. 184	0.229	0.229	0.229
8	0.048	0.168	0.283	0.402	0.402	0.402
9	0.052	0.182	0.220	0.263	0.265	0.265
10	INF	INF	0.389	0.396	0.398	0.398
11	0.050	0.160	0.184	0.378	0.378	0.378

Table 9: The sum of the efficiency scores of models (12) and (13).

p value DMUs	0.47	0.48	0.49	0.50	0.51	0.52
1	0.068	0.192	0.092	0.099	0.097	0.136
2	0.168	0.158	0.116	0.251	0.251	0.401
3	0.078	0.126	0.092	0.097	0.093	0.093
4	0.198	0.226	0.113	0.108	0.108	0.177
5	INF	0.144	0.103	0.133	0.135	0.200
6	INF	INF	0.097	0.121	0.127	0.194
7	0.150	0.199	0.114	0.203	0.201	0.297
8	0.153	0.087	0.060	0.110	0.112	0.175
9	INF	0.186	0.007	0.014	0.015	0.015
10	INF	INF	0.066	0.067	0.066	0.066
11	0.184	0.189	0.060	0.066	0.065	0.107

Table 10: The results of the overall efficiency of model (14) with $p = 0.49$.

Efficiency DMUs	f_0^{1s*}	f_0^{2s*}	$f_0^s = f_0^{1s*} + f_0^{2s*}$	f_o^{cS*}
1	0.034	0.058	0.092	0.224
2	0.019	0.097	0.116	0.216
3	0.064	0.028	0.092	0.228
4	0.076	0.037	0.113	0.295
5	0.038	0.065	0.103	0.160
6	0.040	0.057	0.097	0.207
7	0.013	0.101	0.114	0.184
8	0.025	0.035	0.060	0.283
9	0.002	0.005	0.007	0.220
10	0.047	0.019	0.066	0.389
11	0.041	0.019	0.060	0.184

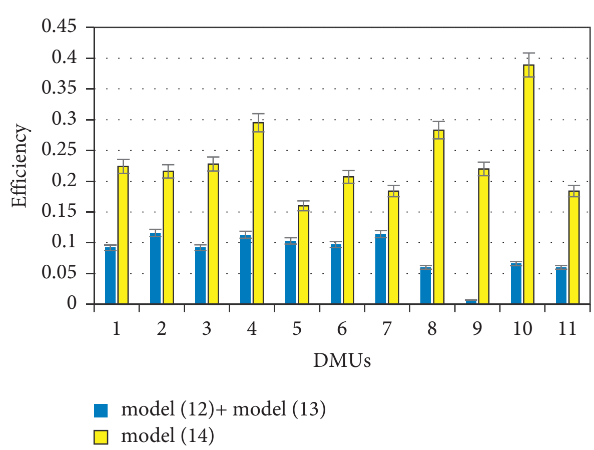

Figure 3: Comparison of the overall efficiency with model (14).

Table 11: The results of expected and regret values of models (9) and (14)in two scenarios.

Efficiency DMUs	*Expected value* Model (9)	*Regret value* Model (9)	**p** *Robust* Model (14)	*Regret value* Model (14)
1	0.671	0.158	0.131	0.063
2	0.587	0.090	0.158	0.043
3	0.782	0.143	0.230	0.049
4	0.528	0.095	0.069	0.034
5	0.792	0.132	0.165	0.046
6	0.556	0.099	0.167	0.051
7	0.727	0.141	0.196	0.062
8	0.586	0.152	0.163	0.056
9	0.501	0.137	0.113	0.052
10	0.858	0.079	0.207	0.044
11	0.762	0.107	0.142	0.066

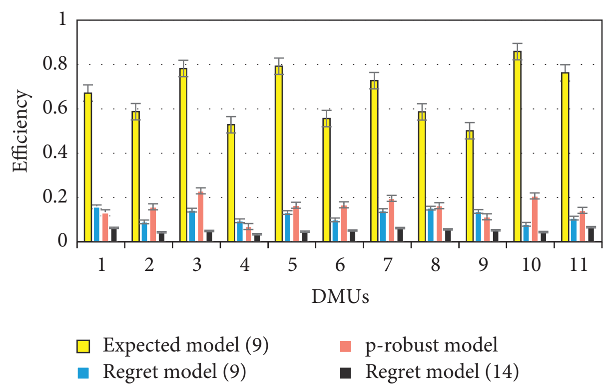

Figure 4: Comparison of regret and expected efficiency of models (9) and (14).

Table 12: The gap values of model (9).

$\mathbf{q}^s$ DMUs	(0.3, 0.7)	(0.6, 0.4)	(0.2, 0.8)
1	0.635	0.659	0.647
2	0.551	0.575	0.563
3	0.746	0.770	0.758
4	0.492	0.516	0.504
5	0.756	0.780	0.768
6	0.520	0.544	0.532
7	0.691	0.715	0.703
8	0.550	0.574	0.562
9	0.465	0.489	0.477
10	0.822	0.846	0.834
11	0.726	0.750	0.738

Table 13: The gap values of model (14).

$\mathbf{q}^s$ DMUs	(0.3, 0.7)	(0.6, 0.4)	(0.2, 0.8)
1	0.635	0.659	0.647
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10	0.822	0.846	0.834
11	0.726	0.750	0.738

The smaller value of the gap for a model approves that the model generates more accurate results, since the expected efficiency of that model is closer to the ideal expected efficiency. It can be viewed from Tables 12 and 13 that model (14) gives better results than the expected model (9). This approves the superiority of our proposed stochastic p-robust model (model (14)) compared to model (9).

6. Conclusions and Future Research

Uncertainty is an essential part of real performance evaluation that imposes serious challenges in this context. In this research, scenario-based robust centralized and noncentralized models are designed for appraising the efficiency of a two-stage NDEA model in the presence of undesirable outputs according to Snyder and Daskin's approach (2006). The objective function of this model sustains the robustness of the solution and achieves the optimal solution from the expected efficiency in each scenario. This model is substantially sensitive to the variation of the parameters pertinent to the robustness level between the values under different scenarios and allows controlling the conservatism level. Further, managers can make a balance between the regret amount and the expected amount of DMUs in feasible scenarios, and as well as a balance between the robustness of the model and the robustness of the solution. Moreover, it combines the benefits of original stochastic and robust optimization models, and the robustness of results is increased significantly without substantial decrease in the expected efficiency. It has relatively low conservatism than the other existing uncertainty models and the solution is immunized against scenarios that are rarely to occur. To appraise the validity and reliability of the proposed models, we applied them to a real dataset drawn from the oilfield system with undesirable outputs in the Persian Gulf region where oil generation is the first stage and wastewater treatment is the second stage. The results indicated, for maximizing the oil generation, the election of the centralized approach for all DMUs gives preferable efficiency in comparison with the noncentralized approach. When oil generation is enhanced, more water is applied and in Stage 2, more oilfield wastewater is obtained, while any decrease in water usage means not so much oilfield wastewater issuances, and it causes a decrease in the input to Stage 1. Moreover, sensitivities analysis on different probability vectors confirms that the proposed p-robust stochastic DEA model produces better results than the differences gained by our model are smaller than those obtained by another model. Extension of the proposed model into the Malmquist indicator Bansal and Mehra [41] under dynamic conditions can be considered as future research direction. In addition, the idea of uncontrollable inputs Zarbakhshnia and Jaghdani [42], and nondiscretionary factors [43] Shakouri and Salahi [44] have also extensive applications, so including them in the proposed models would be an absorbing future research direction.

Data Availability

The data are obtained from a dataset of Persian Gulf oilfields and is available upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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