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*Research Article*

# On the Odd Perks Exponential Model: An Application to Quality Control Data

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In the present study, the group acceptance plan is examined when the lifetime of an item follows the odd Perks exponential distribution, and a large number of items regarded as a group are evaluated simultaneously. The crucial parameters are derived from the consumer risk and the test termination period. The operating characteristics function values are generated for various quality levels. An optimized group acceptance plan and comparison of group acceptance sampling plan with the ordinary sampling plan are also presented. Additionally, a graphical illustration of operating characteristics for diverse groups and parametric values is provided. The minimum ratios of the actual average life to the stipulated average life are likewise computed at the prescribed producer's risk. Examples are used to illustrate the outcomes via our algorithm under the odd Perks exponential distribution setting. It is explained using a quality control dataset to establish its practical versatility.

## 1. Introduction

The impact of nanotechnology on our daily lives has become a well-established fact. Without the employment of appropriate statistical techniques, advancements in nanotechnology could not have been imaginable. The use of statistical approaches in nanoscale applications was reviewed in depth by [1]. Although, conventional sampling methods such as systematic sampling and simple random sampling and their variants are widely employed to identify a representative sample. However, novel nanotechnology sampling methods have been developed [1]. The sampling approach is intended always to save money and time. In a competitive global business market, quality control has evolved as one of the key instruments for separating apart the various competing businesses. Acceptance sampling and statistical quality control are two crucial strategies for assuring the quality of a product. The basic purpose of the acceptance sampling strategies is to accept or reject submitted lots of a size based on the quality of the products that were examined in a sample obtained from the lot. An acceptance sampling plan (ASP) is a predefined plan to obtain the minimum sample size to be used for testing. The key obstacle in most ASPs for a truncated life test is figuring out how big of a sample to take from the concerned lot.

A group acceptance sampling plan (GASP) can help you achieve the best sample size and trial length possible. A GASP based on truncated life tests is the ultimate result of combining GASP with truncated life testing, and it is based on the assumption that the lifespan of a product fits a specific probability distribution. The ordinary sampling plan (OSP) makes the implicit assumption that each tester will contain just one item. Nevertheless, in practice, testers that can test numerous items at once are used because doing so can reduce testing costs and time. Jun et al. [2] developed the characteristics of GASP for the truncated life test under the

assumption that the lifespan of every item followed Weibull distribution. In such a sampling plan, the number of groups and acceptance numbers are obtained simultaneously for the specified producer and consumer risks.

Several researchers developed a GASP predicted on a truncated life test for instances when the product followed various sorts of probability distributions. For example, [3] investigated GASP based on the log-logistic and inverse Rayleigh distributions, and the same for [4] with the Marshall-Olkin extended Lomax distribution, [5] with the Marshall-Olkin extended Weibull distribution, [6] with the generalized exponential distribution, [7] with the inverse Weibull distribution, [8] with the odd generalized exponential log-logistic distribution, [9] with the Marshall-Olkin Kumaraswamy exponential distribution, and [10] with the exponentiated Bell exponential distribution.

With the potential to induct new parameters, there has been an increase in interest in developing novel distributions based on baseline distributions and compounding techniques. In fact, parameter induction has been shown to be beneficial for examining skewness and tail features as well as improving the convergent validity of the developed model. A detailed review of it is presented by [9]. The most pertinent source for the theoretical framework of the present study is [11], where the authors used the Perks distribution [12] as the baseline distribution in the exponential distribution [13] and investigated many properties of the odd Perks exponential (OPE) distribution. The main approach of this study is to investigate GASPs for the OPE distribution as no GASP has been found in the literature for the Perks distribution. As described in [11], the following are the main implications of taking into account the OPE distribution: (i) it outperforms alternative developed models in terms of fit; (ii) it improves kurtosis adjustability; (iii) it has greater flexibility in terms of probability density function (pdf) and hazard rate function (hrf) shapes; (iv) the Perks distribution has been shown to be more effective in analyzing various types of lifetime data ([14–18]); and (iv) distinctive submodels of the OP-G family have been demonstrated to be especially effective at analyzing lifetime data of diverse types presented by [11]. Through the use of a basic ratio approach and many carefully chosen scaling parameters, the OPE distribution expands the functionality of the exponential distribution along with some of its potent exponentiated variants. We therefore propose the OPE distribution as a perfect potential distribution for GASP.

Plans for acceptance sampling come in a variety of forms which include variable acceptance plans, attributes acceptance plans, accelerated and progressively plans, and group acceptance plan. However, the major objective of these plans is to defend both the producer and the customer while making a judgement on the supplied lot with a minimal sample size. Acceptance sampling plans (ASP) are frequently created in order to provide criteria by which items might be accepted or rejected depending on sample data. The development of ASP is a frequent subject in reliability and quality control for lot acceptability objectives. This subject is fundamentally an optimization technique with constraints. Typically, the analyst must reduce the sample size or a certain cost function while taking into account a number of restrictions put forward by the manufacturer and the customer. The lifespan of a product is a crucial quality attribute in many practical disciplines, particularly in reliability analysis and quality control. In these situations, the optimal life test and reliability strategies offer quick ways to assess the acceptability of a product based on the optimal acceptance design. The optimal designs for developing these sample programmes are subsequently explored by [2, 19–29]. According to the best of our knowledge, there is no work on a sampling plan using the OPE model. In this paper, we will design GASP for the OPE model.

The rest of the article is structured as follows: In Section 2, we lay forth the theoretical foundation for OPE distribution with cdf, pdf, and quantile function (qf). Section 3 encompasses the development of GASP for the lifespan percentiles, followed by a truncated life test along with optimal GASP. An illustrative example of the GASP for the OPE distribution is presented in Section 4. An application and a summary of the suggested approach using actual data are furnished in Section 5. A comparative study of GASP with the OSP is presented in Section 6. The outcomes of the articles are finally summarized in Section 7.

## 2. Odd Perks-G Exponential Distribution

First, let us describe the cdf, pdf, and qf of the odd Perks-G (OP-G) family taken from [11], under which the cdf, pdf, and qf of odd Perks exponential (OPE) distribution are generated. The cdf of the OP-G family is indicated as

$$F(x) = 1 - \frac{1+\beta}{1+\beta e^{\theta(G(x;\delta)/\overline{G}(x;\delta))}}, \quad x \in \mathbb{R}, \tag{1}$$

where $\theta > 0$, $\beta > 0$, and $G(x;\delta)$ is the baseline cdf of an absolutely continuous distribution with a parameter vector $\delta$, and $\overline{G}(x;\delta) = 1 - G(x;\delta)$ is the baseline reliability function. With these notations, the pdf of the OP-G family is expressed as follows:

$$f(x) = \frac{\beta\theta(1+\beta)g(x;\delta)e^{\theta[G(x;\delta)/G(x;\delta)]}}{\overline{G}(x;\delta)^2\left[1+\beta e^{\theta(G(x;\delta)/\overline{G}(x;\delta))}\right]^2}, \tag{2}$$

where $g(x;\delta)$ is the pdf of a baseline distribution. The associated qf is given as

$$Q(u) = G^{-1}\left\{\frac{(1/\theta)\log[\beta + u/\beta(1-u)]}{(1/\theta)\log[\beta + u/\beta(1-u)] + 1}; \delta\right\}, \tag{3}$$

where it is understood that $G^{-1}(x;\delta)$ is the qf of the baseline distribution. Now, taking into account the cdf and pdf of exponential distribution with parameter $\delta = \lambda$ as $G(x;\delta) = 1 - e^{-\lambda x}$ and $g(x;\delta) = \lambda e^{-\lambda x}$, with $\lambda > 0$, in equations (1) and (2), the cdf and pdf of the OPE distribution is taken from [11] and are as follows:

$$F(x) = 1 - \frac{1+\beta}{1+\beta e^{\theta(e^{\lambda x}-1)}}, \quad x > 0, \tag{4}$$

and the pdf of the OPE distribution is given by

$$f(x) = \frac{\beta\theta(1+\beta)\lambda e^{\theta\left(e^{\lambda x}-1\right)}}{e^{-\lambda x}\left[1+\beta e^{\theta\left(e^{\lambda x}-1\right)}\right]^2}, \quad x > 0. \tag{5}$$

It is understood that $f(x) = F(x) = 0$ for $x \le 0$.

Furthermore, the qf of the OPE distribution using equation (4) is given by

$$q(u) = \frac{1}{\lambda}\left\{\log\left[1+\frac{1}{\theta}\log\left(\frac{\beta+u}{\beta(1-u)}\right)\right]\right\}, \quad u \in (0,1). \tag{6}$$

The functions will be used to elaborate our GASP algorithm.

## 3. Description of the GASP under the OPE Model

For the present research, the median is used as the quality index. As [3] claimed, the median outperforms rather than the mean as a criterion for a skewed distribution. Because of the skewed distribution of the OPE distribution, the percentile point must be employed as the quality index. As a result, the key goal of the current research is to provide a GASP presuming that the lifespan of an item followed an OPE model with known parameters $\beta, \theta$, and $\lambda$ having cdf in equation (4). GASP is a valuable quality control instrument that enables a corporation to test randomly picked samples and check the performance of an overall product lot using statistical techniques. The phases for taking the GASP into practice and acquiring the design parameters of the OPE model were taken from [3, 4, 9] and those are as follows:

(1) Selecting the number of groups $g$ and assigning specified $r$ items to each group, yielding a sample size for each lot as $n = g \times r$

(2) Choosing an acceptance number $c$ for each group with a set experiment time $t_o$

(3) Carrying out the experiment for all $g$ groups simultaneously, and making a count of how many times each group fails

(4) Accepting the lot if each of the groups has at most $c$ failures by the end of the trial

(5) Terminating the trial and discarding the entire lot when more than $c$ components fail in any group

Thus, for a given $r$, the hypothesized GASP described by two design parameters $(g, c)$ is defined for the OPE model. The cdf of the OPE model is expressed in equation (4) and is dependent on $\beta, \theta$, and $\lambda$, whereas the median life is expressed in equation (6). The probability of accepting a lot is expressed in the following expression:

$$p_{a(p)} = \left[\sum_{i=0}^{c}\binom{r}{i}p^i(1-p)^{r-i}\right]^g, \tag{7}$$

where $p$ denotes the probability that an item in a group would fail before $t_o$ and is obtained by inserting equations (6) in (4). Based on equation (6), we put

$$m = \frac{1}{\lambda}\left\{\log\left[1+\frac{1}{\theta}\log\left(\frac{\beta+0.5}{0.5\beta}\right)\right]\right\}, \tag{8}$$

and

$$\eta = \left\{\log\left[1+\frac{1}{\theta}\log\left(\frac{\beta+0.5}{0.5\beta}\right)\right]\right\}. \tag{9}$$

When $\eta$ is substituted in equation (8), $\lambda = \eta/m$ is obtained. Let $m = \eta/\lambda$ and $t_o = a_1 m_o$. The quality level of a product can be expressed as the ratio of its median lifespan to the prescribed lifespan $m/m_o$. Substituting $\lambda = \eta/m$ and $t_o = a_1 m_o$ in equation (4), the probability of failure is expressed as

$$p = 1 - \frac{1+\beta}{1+\beta e^{\theta\left(e^{\lambda t}-1\right)}}, \tag{10}$$

which can also be written as

$$p = 1 - \frac{1+\beta}{1+\beta e^{\theta\left[e^{\eta a_1 (m/m_o)^{-1}}-1\right]}}. \tag{11}$$

Equation (11) can be used to calculate $p$ for specified $\beta$ and $\theta$ when $a_1$ is given and $r_2 = m/m_o$. Now, we have two failure probabilities, $p_1$ and $p_2$, which are used to represent the consumer and producer risks, respectively, where the producer's risk refers to the probability of rejecting a good lot and the consumer's risk refers to the probability of accepting a bad campaign. For given values of $\beta, \theta, a_1$, and $r_2$, we must evaluate the values of $g$ and $c$ that satisfy the following two equations simultaneously:

$$p_{a\left(p_1 | m/m_o = r_1\right)} = \left[\sum_{i=0}^{c}\binom{r}{i}p_1^i(1-p_1)^{r-i}\right]^g \le \gamma, \tag{12}$$

and

$$p_{a\left(p_2 | m/m_o = r_2\right)} = \left[\sum_{i=0}^{c}\binom{r}{i}p_2^i(1-p_2)^{r-i}\right]^g \ge 1-\alpha, \tag{13}$$

where $r_1$ and $r_2$ are the median ratios of consumer risk and producer risk, respectively. The probabilities to be used in equations (12) and (13) are given by

$$p_1 = 1 - \frac{1+\beta}{1+\beta e^{\theta\left[e^{\eta a_1}-1\right]}}, \tag{14}$$

and

$$p_2 = 1 - \frac{1+\beta}{1+\beta e^{\theta\left[e^{\eta a_1 (r_2)^{-1}}-1\right]}}. \tag{15}$$

*3.1. Optimal Group Sampling Plan.* A statistical lifespan test is required to show if the median lifespan of the product under consideration has reached the necessary level. For this purpose, we may specify the corresponding null and alternative hypothesis as

$$H_o: m = r_2 \times m_o;$$
$$H_1: m = r_1 \times m_o, \tag{16}$$

with $r_1 = 1$. The performance of the proposed lifespan test can also be understood by its operating characteristics (OC) curve, that plots the associated probabilities of lot acceptance against predicted lifespan. The purpose of acceptance sampling programmes is often to ensure that the likelihood of producer's risk and the consumer's risk are both relatively low, usually at most $\alpha$ and $\gamma$, respectively ($\alpha$, $\gamma < 0.5$). The specifications of the admissible and rejectable median lifespans, $m_0$ and $m_1$, as well as the choice of the maximum permitted producer and consumer risks $\alpha$ and $\gamma$, are often taken into consideration when developing the appropriate sampling strategy. The least sample size GASP that satisfies the discrepancy constraints defined in equations (12) and (13) would thus be the optimal design. Nonlinear optimization programming may be used to solve the optimization issue to find the least number of failures and $g$ and hence formulated as

$$\begin{cases} \text{Minimize} & g \text{ and } c \\ \text{Subject to} & p_{a\left(p_1 | m/m_o = r_1\right)} \le \gamma, \\ & p_{a\left(p_2 | m/m_o = r_2\right)} \ge 1 - \alpha, \\ & g, r \in Z^+, 0 \le c < r, \end{cases} \tag{17}$$

where $Z^+ = \{1, 2, 3, \ldots\}$ denotes the set of positive integers. For this scenario, the admissible region is determined by

$$\Omega_1 = \left\{(g, r, c):\ p_{a\left(p_1 | m/m_o = r_1\right)} \le \gamma, p_{a\left(p_2 | m/m_o = r_2\right)} \ge 1 - \alpha, g, \quad r \in Z^+,\ 0 \le c < r\right\}. \tag{18}$$

Consequently, we may rewrite the optimization problem defined in equation (17) to make it more concise as min $\{g, c:\ (g, r, c) \in \Omega_1\}$. Thus, for a given $r$, the $g$ and $c$ are the design variables in equation (17) for $0 \le c < r$. During the simulation, for the prefixed values of $r$, it is observed that several combinations of the plan parameters exist that follow the given conditions of equation (18). We selected the optimal values of $g$ and $c$, with maximum tolerated producer's risk of 5%.

## 4. Discussion with Illustrative Examples

Tables 1 and 2 demonstrate the design parameters under GASP for different values of $\theta$ (1.25 and 1.5), $\gamma$ (0.25, 0.1, 0.05, and 0.01), $r_2$ (2, 4, 6, and 8) with $a_1$ (0.5 and 1), and taking $r$ into account (5 and 10). According to the analysis, minimizing $\gamma$ (consumer's risk) tends to increase the number of groups. Furthermore, when $r_2$ increased, the number of groups gradually decreased. However, beyond a certain point with constant $g$ and $c$, $p_{a(p)}$ (probability of accepting a lot) increased. Tables 1 and 2 show the effect of $a_1\,(0.5, 1)$, revealing that when $\gamma = 0.25$, $a_1 = 0.5$, $r_2 = 4$, $\theta = 1.25$, and $r = 5$, 85 $(17 \times 5)$ units should be required to be put on the life test. However, when climbing to 10, 30 $(3 \times 10)$ units are necessary to be put on a life test. In this case, 10 groups would be more appropriate. Under the OPE model, the value of the OC rises and the number of groups reduces as the true median life rises for the considered GASP when median lifetime is used as a quality parameter. When $\gamma = 0.1$, $a_1 = 1$, and $\theta = 1.5$ for $r = 5$, Table 3 represents the true median lifetime with $g$, $c$, and OC values taken from Table 2. It can be observed from Table 3 that as the true median lifetime increased, the values of $g$ and $c$ decreased, but beyond a certain point with constant $g$ and $c$, the $p_{a(p)}$ (OC values) increased. Figure 1 depicts this phenomenon by plotting $r_2$ values with respect to $g$ and OC values from Tables 1 and 2, respectively. It can be seen clearly in Figure 1 that as true median lifetime rises, the $g$ first tends to decrease and then remains constant, but the OC values tend to rise rapidly. Hence, at those points, the investigated lot will be accepted under the OPE model. It would be better to accept the lot for $r = 10$ in Figure 1 as the least number of groups will be tested as compared to $r = 5$ because it would save time and cost.

The results of Tables 1–3 are illustrated here with an example, and readers are directed to [7] for more information. Let us say the lifespan of a ball bearing is put to the test using an OPE model with a value of $\theta = 1.25$ and the lifespan of a ball bearing is 2000 cycles. When the average lifetime is 3000 and 5000 cycles, the consumer and producer face a 25% and 5% risk, respectively. Now, an investigator wishes to undertake a 2000-cycle experiment with 10 units in each group to test if the median life of the ball bearing exceeds the stated life. For this structure, we have $m_o = 2000$ cycles, $\theta = 1.5$, $a_1 = 0.5$, $\gamma = 0.25$, $r_2 = 4$, $r = 10$, $r_1 = 1$, the producer's risk equal to 0.05, and $r_2 = 4$. Furthermore, from Table 2, we have $g = 16$ and $c = 2$. This involves the establishment of 80 $(16 \times 5)$ units with 5 units being allocated to all of the 16 groups. If no more than two units in all of these groups expire before 2000 cycles, the average life of a ball bearing will statistically be proven to be longer than the prescribed life. If an investigator intends to test the hypothesis that ball bearings have a life span of 5000 cycles, but a true median life of four times, the investigator can test 16 groups of 5 units each. If more than 2 units expire in 2000 cycles, as $a_1 = 0.5$ and the true median life length is in 2000 of cycles, the investigator will conclude with 95% confidence that the life is more than 5000 cycles. As a result, the investigated lot should be accepted.

## 5. Application

The dataset is taken from [30] which consisted of the lifespan of Kevlar 373/epoxy fatigue fractures, which were placed under steady pressure at a 90 percent stress level unless all of

Table 1: GASP under the OPE model, showing minimum $g$ and $c$ when $\beta = 1$ and $\theta = 1.25$.

| $\gamma$ | $r_2$ | $r = 5$ | | | | | | $r = 10$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $a_1 = 0.5$ | | | $a_1 = 1$ | | | $a_1 = 0.5$ | | | $a_1 = 1$ | | |
| | | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ |
| 0.25 | 2 | — | — | — | 44 | 4 | 0.9735 | 112 | 5 | 0.9771 | 3 | 5 | 0.9634 |
| | 4 | 17 | 2 | 0.9791 | 3 | 2 | 0.9698 | 3 | 2 | 0.9636 | 1 | 3 | 0.9842 |
| | 6 | 4 | 1 | 0.9582 | 1 | 1 | 0.9584 | 3 | 2 | 0.9885 | 1 | 2 | 0.9725 |
| | 8 | 4 | 1 | 0.9763 | 1 | 1 | 0.9764 | 2 | 1 | 0.9510 | 1 | 2 | 0.9877 |
| 0.1 | 2 | — | — | — | 73 | 4 | 0.9564 | 186 | 5 | 0.9622 | — | — | — |
| | 4 | 28 | 2 | 0.9658 | 4 | 2 | 0.9599 | 13 | 3 | 0.9855 | 2 | 3 | 0.9686 |
| | 6 | 28 | 2 | 0.9899 | 4 | 2 | 0.9881 | 5 | 2 | 0.9809 | 1 | 2 | 0.9725 |
| | 8 | 6 | 1 | 0.9646 | 2 | 1 | 0.9533 | 2 | 1 | 0.9510 | 1 | 2 | 0.9877 |
| 0.05 | 2 | — | — | — | — | — | — | 242 | 5 | 0.9511 | — | — | — |
| | 4 | 36 | 2 | 0.9563 | 5 | 2 | 0.9501 | 16 | 3 | 0.9822 | 2 | 3 | 0.9686 |
| | 6 | 36 | 2 | 0.9870 | 5 | 2 | 0.9852 | 6 | 2 | 0.9772 | 2 | 3 | 0.9932 |
| | 8 | 8 | 1 | 0.9531 | 2 | 1 | 0.9533 | 6 | 2 | 0.9901 | 2 | 2 | 0.9756 |
| 0.01 | 2 | — | — | — | — | — | — | — | — | — | — | — | — |
| | 4 | 418 | 3 | 0.9863 | 24 | 3 | 0.9867 | 25 | 3 | 0.9723 | 3 | 3 | 0.9533 |
| | 6 | 55 | 2 | 0.9802 | 7 | 2 | 0.9793 | 9 | 2 | 0.9659 | 3 | 3 | 0.9897 |
| | 8 | 55 | 2 | 0.9917 | 7 | 2 | 0.9913 | 9 | 2 | 0.9851 | 2 | 2 | 0.9756 |

Hyphens (—) are presented in the required cells for a large sample size.

Table 2: GASP under the OPE model, showing minimum $g$ and $c$ when $\beta = 1$ and $\theta = 1.5$.

| $\gamma$ | $r_2$ | $r = 5$ | | | | | | $r = 10$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $a_1 = 0.5$ | | | $a_1 = 1$ | | | $a_1 = 0.5$ | | | $a_1 = 1$ | | |
| | | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ |
| 0.25 | 2 | — | — | — | 44 | 4 | 0.9704 | 100 | 5 | 0.9752 | 3 | 5 | 0.9591 |
| | 4 | 16 | 2 | 0.9779 | 3 | 2 | 0.9667 | 3 | 2 | 0.9595 | 1 | 3 | 0.9822 |
| | 6 | 4 | 1 | 0.9549 | 1 | 1 | 0.9555 | 3 | 2 | 0.9871 | 1 | 2 | 0.9697 |
| | 8 | 4 | 1 | 0.9742 | 1 | 1 | 0.9745 | 3 | 2 | 0.9944 | 1 | 2 | 0.9863 |
| 0.1 | 2 | — | — | — | 73 | 4 | 0.9513 | 166 | 5 | 0.9592 | — | — | — |
| | 4 | 26 | 2 | 0.9643 | 4 | 2 | 0.9559 | 12 | 3 | 0.9844 | 2 | 3 | 0.9647 |
| | 6 | 26 | 2 | 0.9894 | 4 | 2 | 0.9868 | 5 | 2 | 0.9786 | 1 | 2 | 0.9697 |
| | 8 | 6 | 1 | 0.9616 | 4 | 2 | 0.9944 | 5 | 2 | 0.9906 | 1 | 2 | 0.9863 |
| 0.05 | 2 | — | — | — | — | — | — | — | — | — | — | — | — |
| | 4 | 34 | 2 | 0.9536 | 15 | 3 | 0.9900 | 15 | 3 | 0.9806 | 2 | 3 | 0.9647 |
| | 6 | 34 | 2 | 0.9862 | 5 | 2 | 0.9835 | 6 | 2 | 0.9744 | 2 | 3 | 0.9922 |
| | 8 | 34 | 2 | 0.9941 | 5 | 2 | 0.9930 | 6 | 2 | 0.9888 | 2 | 2 | 0.9728 |
| 0.01 | 2 | — | — | — | — | — | — | — | — | — | — | — | — |
| | 4 | 384 | 3 | 0.9853 | 23 | 3 | 0.9848 | 23 | 3 | 0.9704 | 5 | 4 | 0.9875 |
| | 6 | 52 | 2 | 0.9789 | 7 | 2 | 0.9770 | 9 | 2 | 0.9619 | 3 | 3 | 0.9883 |
| | 8 | 52 | 2 | 0.9911 | 7 | 2 | 0.9903 | 9 | 2 | 0.9832 | 2 | 2 | 0.9728 |

Hyphens (—) are presented in the required cells for a large sample size.

them collapsed. As a result, the data with precise failure times were given as

$$\left\{\begin{array}{c} 0.4763, 0.0251, 0.3451, 0.3113, 0.2501, 0.0891, 0.0886, 0.8375, 0.5671, 0.7696, 0.6753, \\ 0.6753, 0.6748, 0.6566, 1.0483, 0.8425, 0.9836, 0.9120, 0.9120, 0.8851, 0.8645, 1.3503, \\ 1.0773, 1.3211, 1.2986, 1.2985, 1.2570, 1.1733, 1.7460, 1.4595, 1.7263, 1.7083, 1.7083, \\ 1.5728, 1.4880, 1.8881, 1.7746, 1.8878, 1.8808, 1.8808, 1.8375, 1.8275, 2.2100, 1.9558, \\ 2.1330, 2.1093, 2.1093, 2.0408, 2.0048, 2.9911, 2.2878, 2.5260, 2.4952, 2.4951, 2.3470, \\ 2.3203, 4.8073, 3.2678, 3.9143, 3.7456, 3.7455, 3.4846, 3.4045, 9.0960, 5.4435, 6.5541, 5.5295 \end{array}\right\}. \quad (19)$$

Table 3: True median lifetime with $c$, $g$, and $p_{a(p)}$.

| $m/m_o = r_2$ | 4 | 6 | 8 |
|---|---|---|---|
| $g$ | 2 | 1 | 1 |
| $c$ | 3 | 2 | 2 |
| $p_{a(p)}$ | 0.9647 | 0.9697 | 0.9863 |

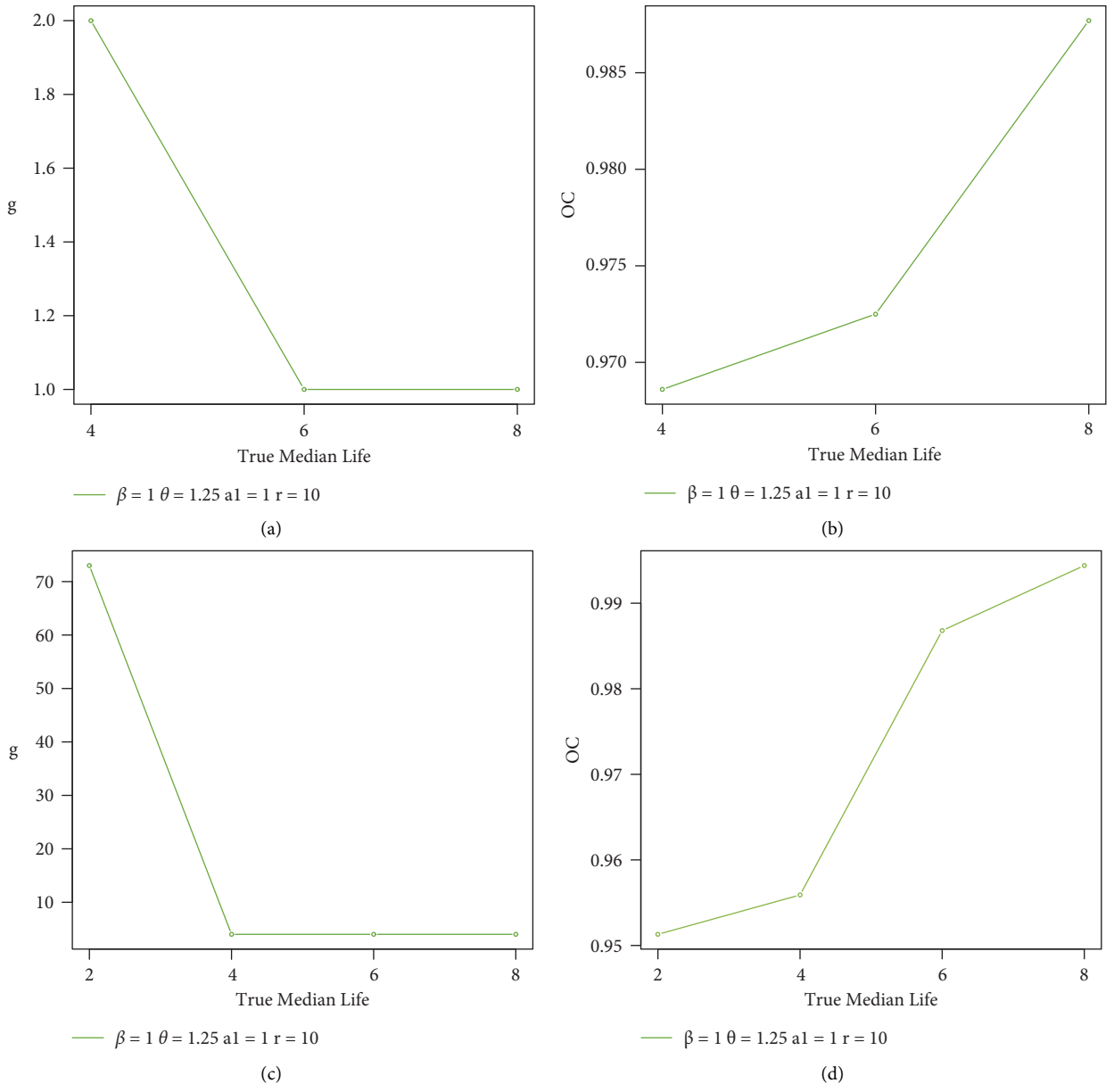


Figure 1: Graphical illustration of $g$ and OC for some parametric values taken from Tables 1 and 2, respectively.

The maximum likelihood (ML) estimates with standard errors (in parenthesis) of the three parameters of OPE for the data are $\widehat{\beta} = 0.525\,(0.332)$, $\widehat{\theta} = 42.258\,(4.545)$, and $\widehat{\lambda} = 0.019\,(0.018)$. According to the Kolmogorov–Smirnov (K–S) test, the maximum distance between the fitted OPE distribution and the data is 0.103 with a $p$ value of 0.4694. Figure 2 is furnished to represent the better fit of the data under the OPE model by incorporating the empirical cdf with the estimated cdf, the quantile-quantile (Q-Q) plot, the probability-probability (P-P) plot, and the histogram of the estimated pdf. Figure 3 presents the total time on test (TTT) plots and the estimated hrf. Both the TTT plot and the

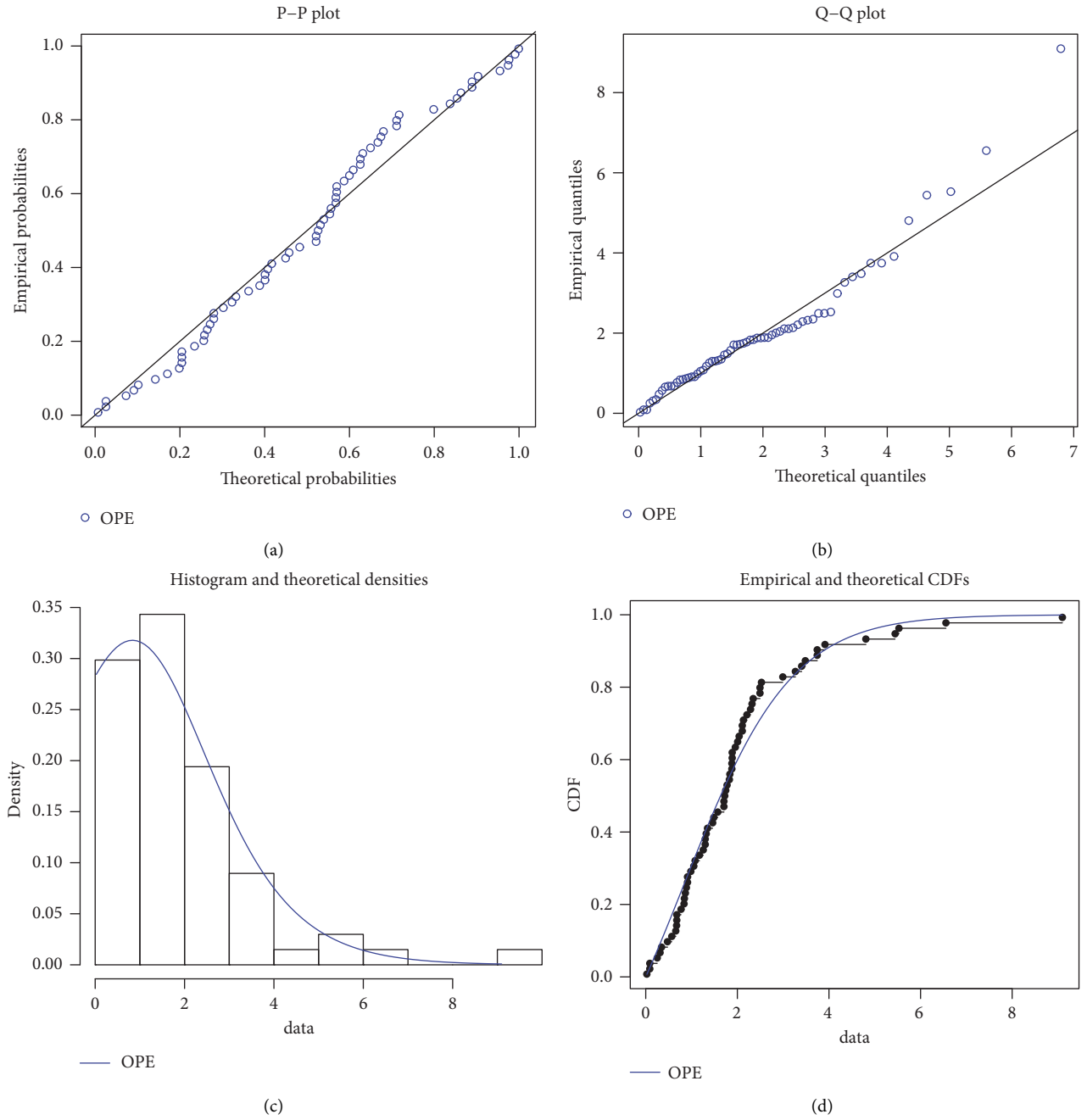


Figure 2: (a) P-P plot, (b) Q-Q plot, (c) histogram fitted by the estimated pdf, and (d) empirical cdf fitted by estimated cdf.

increased estimated hrf showed that the dataset can better be fitted under the OPE model. Similarly, Table 4 demonstrates the GASP for the OPE distribution considering the ML estimates: $\widehat{\beta} = 0.5$ and $\widehat{\theta} = 42$, by taking $r$ into account (5 and 10). It can be observed that the performance of the planned parameters in Table 4 is consistent with the values in Tables 1 and 2.

## 6. Comparison of GASP with OSP

While using sample plans, a process known as lot sentencing determines whether entering or exiting batches should be accepted or rejected depending on a prespecified quality. The two most important elements for professional engineers to think about are the sample size and the length of the trial,

Table 4: GASP under the OPE model, showing minimum $g$ and $c$ when the ML estimates are considered: $\widehat{\beta} = 0.5$ and $\widehat{\theta} = 42$.

| $\gamma$ | $r_2$ | $r = 5$ | | | | | | $r = 10$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $a_1 = 0.5$ | | | $a_1 = 1$ | | | $a_1 = 0.5$ | | | $a_1 = 1$ | | |
| | | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ | $g$ | $c$ | $p_{a(p)}$ |
| 0.25 | 2 | — | — | — | 44 | 4 | 0.9583 | 71 | 5 | 0.9701 | — | — | — |
| | 4 | 13 | 2 | 0.9756 | 3 | 2 | 0.9566 | 6 | 3 | 0.9885 | 1 | 3 | 0.9753 |
| | 6 | 13 | 2 | 0.9926 | 3 | 2 | 0.9867 | 3 | 2 | 0.9825 | 1 | 2 | 0.9604 |
| | 8 | 4 | 1 | 0.9682 | 1 | 1 | 0.9689 | 1 | 1 | 0.9672 | 1 | 2 | 0.9818 |
| 0.1 | 2 | — | — | — | — | — | — | 117 | 5 | 0.9511 | — | — | — |
| | 4 | 22 | 2 | 0.9591 | 12 | 3 | 0.9884 | 10 | 3 | 0.9809 | 2 | 3 | 0.9512 |
| | 6 | 22 | 2 | 0.9876 | 4 | 2 | 0.9823 | 4 | 2 | 0.9767 | 1 | 2 | 0.9604 |
| | 8 | 6 | 1 | 0.9527 | 4 | 2 | 0.9924 | 4 | 2 | 0.9898 | 1 | 2 | 0.9818 |
| 0.05 | 2 | — | — | — | — | — | — | — | — | — | — | — | — |
| | 4 | 192 | 3 | 0.9888 | 15 | 3 | 0.9855 | 12 | 3 | 0.9771 | 2 | 3 | 0.9512 |
| | 6 | 28 | 2 | 0.9842 | 5 | 2 | 0.9779 | 5 | 2 | 0.9710 | 2 | 3 | 0.9886 |
| | 8 | 22 | 2 | 0.9948 | 5 | 2 | 0.9906 | 5 | 2 | 0.9873 | 2 | 2 | 0.9640 |
| 0.01 | 2 | — | — | — | — | — | — | — | — | — | — | — | — |
| | 4 | 295 | 3 | 0.9829 | 23 | 3 | 0.9779 | 19 | 3 | 0.9639 | 5 | 4 | 0.9808 |
| | 6 | 43 | 2 | 0.9759 | 7 | 2 | 0.9692 | 8 | 2 | 0.9540 | 3 | 3 | 0.9830 |
| | 8 | 28 | 2 | 0.9934 | 7 | 2 | 0.9868 | 8 | 2 | 0.9797 | 2 | 2 | 0.9640 |

Hyphens (—) are presented in the required cells for a large sample size.

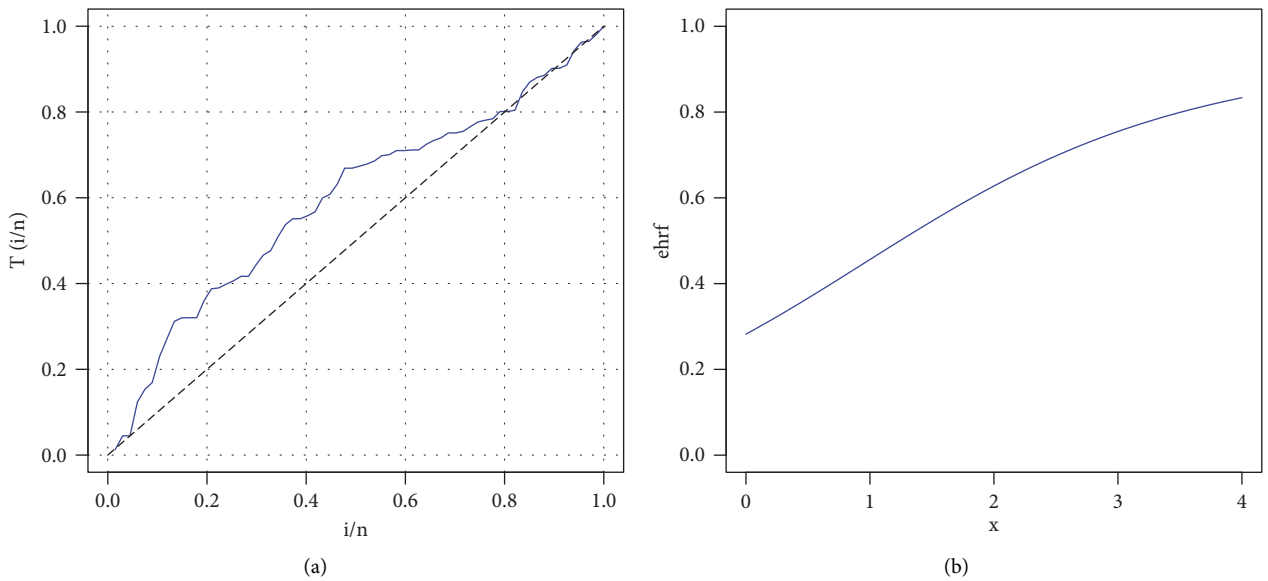


Figure 3: (a) TTT plot and (b) estimated hrf.

and both ought to be maximized. Although OSPs can aid in achieving this optimization, yet it is assumed that only one item will be tested at a time in this scenario. Comparatively, a GASP can also achieve the optimized cost and time when more than one item can be tested in a tester by making groups of the items. The major advantages of GASP as compared to OSPs are as follows:

(i) This approach can be used in sectors with a high level of mass production and industries that adhere to a predetermined manufacturing process

(ii) This approach is practical and simple to comprehend

(iii) This approach required relatively lesser computational labour

(iv) This approach diminishes tiredness and dullness

The suggested GASP is an expansion of the OSP for $g = 1$ when $n = r$. In this section, we compared the suggested GASP with the OSP by taking $g = 1$. For this purpose, we presented a comparison of the suggested GASP under $r = 5$

Table 5: Sample sizes of GASP and OSP when $\widehat{\beta} = 0.5$ and $\widehat{\theta} = 42$.

| $\gamma$ | $r_2$ | $r = 5$ | | $r = 10$ | |
|---|---|---|---|---|---|
| | | GASP | OSP | GASP | OSP |
| 0.25 | 4 | 15 | 17 | 10 | 13 |
| | 6 | 15 | 17 | 10 | 13 |
| | 8 | 5 | 8 | 10 | 13 |
| 0.1 | 4 | 60 | 62 | 20 | 23 |
| | 6 | 20 | 23 | 10 | 10 |
| | 8 | 20 | 23 | 10 | 10 |
| 0.05 | 4 | 75 | 121 | 20 | 23 |
| | 6 | 25 | 28 | 20 | 23 |
| | 8 | 25 | 28 | 20 | 23 |
| 0.01 | 4 | 115 | 118 | 50 | 52 |
| | 6 | 35 | 37 | 30 | 32 |
| | 8 | 35 | 37 | 20 | 22 |

and $r = 10$ with the OSP under $g = 1$ for the OPE distribution with $\widehat{\beta} = 0.525$ and $\widehat{\theta} = 42.258$, for a given $\gamma = (0.25, 0.1, 0.05 \text{ and } 0.01)$ and $a_1 = 1$. Comparison of sample sizes of GASP and OSP from Table 5 revealed that GASP is the best strategy as compared to the OSP as using OSP determined a large sample size, rather than the GASP providing reliable sample size with optimized cost and time by making groups of items and testing multiple items at once.

## 7. Conclusion

In this research, we proposed a GASP using the median as the quality index. The true median life time along with the number of groups, acceptance number, and OC values were obtained and presented graphically. It can be observed that as the true median life time tends to rise, the number of groups and acceptance numbers tend to fall rather than the OC values tend to rise gradually. More precisely, the GASP can be utilized when embedded objects are recruited for a trial run at once, and it will be advantageous in optimizing the test time and cost as several objects can be evaluated at once.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest.

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