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Research Article

A Three-Stage Game Analysis of Private Lending Rate Ceiling: Necessity, Impact, and Solution

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The ceiling on private lending interest rates is a powerful financial tool to maintain financial stability and reduce usury. Especially now that peer-to-peer lending has encountered some challenges, the ceiling is an essential way to regulate this market. The purpose of this paper is to analyze the necessity and the impact of such a powerful tool and also to find the optimal solution to determine it. The paper proves the ceiling on private lending interest rates is an inevitable choice by using an evolutionary game. However, it is shown that the lowering of the ceiling on private lending interest rates will increase both the difficulty of financing for SMEs and the default rate. Two optimal solutions to the ceiling are obtained in this study, which also prevent an increase in borrowing costs.

1. Introduction

With the development of online platforms, loans between individuals have become more convenient. Peer-to-peer (P2P) lending, a form of online personal loans, can help connect borrowers and lenders. Since the traditional financial sector is unfriendly to small- and medium-sized businesses (SMEs), private lending of the market can help remedy the situation. Simultaneously, private lending invariably results in difficulties such as usury. It is therefore imperative that regulators employ financial instruments to standardize the market. The interest rate ceiling of private lending is such a useful tool to regulate private lending.

The lending rate ceiling that this paper focuses on is the highest interest rate protected by the law for the private loan, which includes all the loans in the peer-to-peer market. Even if the borrower chooses not to repay, the interest rate below this ceiling can be assured to be repaid by legal procedures. But the necessity of it is controversial. On the one hand, the private lending rate ceiling is a necessary tool to protect borrowers from predatory loans [1], prevent interest rate risks, and avoid usury to a certain extent. Also, it can reduce the financing costs of SMEs to a certain extent and ensure the effective operation of private lending [2]. On the other hand, it may lead to increased noninterest expenses and commissions of loans [3], reduced price transparency of loans, and decreased credit supply and loan approval rates for low-risk and high-risk borrowers [4].

Different countries have different implementation of interest rate ceiling. For instance, some nations set a uniform ceiling on interest rates for all transactions, but this strategy has the drawback of cutting off the credit market for a few select high-risk loan products [5]. Therefore, other countries alleviate this problem by setting multiple ceilings on interest rates. In terms of setting methods, setting an absolute upper bound is the first approach of setting the ceiling. However, the majority of nations prefer to employ an interest rate benchmark, such as Loan Prime Rate (LPR), and the ceiling is usually set to a specific range around the benchmark [6].

1.1. Objective. This paper aims to study this powerful tool and answers the three following questions: 1. Is it necessary to set a ceiling on the interest rate of private loans? 2. How does the ceiling on private lending interest rates affect the loan relationship? 3. Is there a reasonable interest rate ceiling of private lending? This study is expected to promote the effective development of the private lending market.

1.2. Findings. To address the aforementioned problems, the following findings are obtained:

The paper first shows that establishing a lending rate ceiling in the private lending market is a reasonable steady-state social choice by using evolutionary game method.

However, by examining the impact of ceiling modifications on the lending relationship, it is discovered that lowering the ceiling rate increases the difficulty level of obtaining the loan, as well as the nonrepayment rate.

The Nash bargaining game solution and the efficiency solution are determined to be the best options from the perspective of the bargaining game, and the proposed solutions take into account the industry's average return rate, risk-free interest rate, and original interest rate ceiling, which can make them more practical and acceptable.

The remainder of the paper is organized as follows: In Section 2, related literature is shown and compared. The novelty of our paper is also displayed. In Section 3, the variables included in the model are presented, and then, in Section 4, the necessity of a ceiling on private lending interest rates is explored. Section 5 explores the game relationship between borrowers and lenders and elaborates on the impact of the ceiling on lending relationships. From the perspective of bargaining games, Section 6 examines how to establish a reasonable ceiling on private lending interest rates. Section 7 discusses our results and the pros and cons of the existing determination methods of the ceiling. Finally, Section 8 provides the conclusions and outlook.

2. Literature Review

The interest rate ceiling of private lending is still a frequently used tool for bringing down loan rates and expanding access to financing. Existing research recognizes the critical role played by the ceilings and reveals the rationality of setting the ceilings of private loan interest rate as well as the potential risks. Specifically, Miller [5] finds that the ceilings can promote credit access and limit usury from the standpoint of supply and demand. Furthermore, he considers that the adoption of interest rate caps is more effective as a short-term transition than as a long-term economic policy. Maimbo [3] examines the ceilings of private lending rates focusing on the main features of the systems adopted by countries, including the source of power to set interest rates, methods, and standards. Madeira [7] explores how the ceiling makes an impact on the householder's credit. These studies, however, rely on aggregate statistics derived from cross-country data, which may be skewed by other institutional variations.

The interest rate ceiling of private lending can sometimes have unintended side effects. Ferrari [8] implements case studies of six different types of interest rate ceilings and recognizes that these side effects included higher noninterest fees and commissions, less price transparency, and lower credit accessibility and loan approval ratios for low-risk and high-risk borrowers. Chatterji [9] examines the effect of state-mandated credit card interest rate ceilings on entrepreneurial entry and proposes that credit cards be deregulated. What is more, in Kenya, the interest rate control law has the opposite effect of what was intended. Specifically, it has led to a collapse in credit to small- and medium-sized businesses (SMEs) as well as a shrinkage of the loan book in small banks [4].

Additionally, some quantitative studies verify the relationship between interest rate ceiling and availability of funds. A study conducted by Lukongo [10] shows that the 17% interest rate ceiling stipulated in the Arkansas Constitution affected access to credit by increasing search costs. Ochieng [11] uses Logit model to demonstrate the negative correlation between interest rate ceiling and credit availability and default rate. Most recently, Amvella [12] has established a mathematical model for determining the floor of an institution's lending interest rate interest based on its cost-based approach, which is the minimum rate of interest that an institution should set so that its credit operations are profitable. There is limited research examining how to determine the interest rate ceiling of private lending. Zhang et al. [13] and Jin et al. [14] use different mathematical methods to determine the market interest rate ceiling.

The ceiling of the private lending interest rate is an essential financial adjustment tool and it inevitably affects all the stakeholders in the financing system, that is, lending relationship between the borrower and the lender. Determining the private lending rate ceiling is a process by which regulators weigh the benefits of various aspects. This process needs to first consider its necessity, its impact on both parties in the lending relationship, and how adjustments can better balance the interests of all parties. This is also the focus of the research in this paper.

2.1. Novelty. The majority of research focuses on the bank lending market, and only a few studies investigate the private loan market's interest rate ceiling. Additionally, this study carries out a thorough analysis of the interest rate ceiling, taking into account its necessity, impacts, and finding the optimal solutions. Additionally, this study examines the stability of each market choice from the perspective of an evolutionary game, which can reason about the choices of social evolution in the natural state. Also, not only is the game relationship of the participants in the personal loan market studied, but also all feasible and rational optimal solutions are obtained and compared. The comparison of this study with other literature is displayed in Table 1.

3. Model Preparation

In the following analysis, the following parameters are used:

TABLE 1: A summary of lending rate ceiling's study contribution.

Author (s)	Ceiling type	Ceiling impact	Game process capture	Ceiling setting method	Ceiling stability	Optimal solutions comparison
Miller [5]	Bank	Included	—	—	—	—
Madeira [7]	Bank	Included	—	—	—	—
Ferrari et al. [8]	Bank	Included	—	—	—	—
Chatterji and Seamans [9]	Bank	Included	—	—	—	—
Alper et al. [4]	Bank	Included	—	—	—	—
Lukongo and Miller [10]	Bank	Included	—	—	—	—
Ochieng and Odondo [11]	Bank	Included	—	—	—	—
Maimbo [3]	Private	Included	—	—	—	—
Amvella [12]	Bank	Included	Included	Included	—	—
Zhang et al. [13]	Market	Included	—	Included	—	—
Jin et al. [14]	Market	Included	—	Included	—	—
This study	Private	Included	Included	Included	Included	Included

w: the return rate of the project which the loan will be invested to

s: the loss rate caused if the borrower fails to get the loan

r: the interest rate of the loan

r_f: the risk-free interest rate of this industry

c_a: appeal cost when the loan is not repaid

c_c: loss of credibility caused by not repaying the loan

p: the punishment to the borrower if he fails to repay the loan

r_l: the interest rates ceiling in the private lending market

b: benefits to society that the ceiling of private interest rates can bring

c_r: the cost incurred by the society by setting the private interest rates ceiling

Δw: increased reputation brought by borrowers' long-term compliance

r_h: the return rate on higher-yield investments which is not compliant

f_c: risks when lenders do not invest in compliant loans and seek higher-yield investments

l_r: social losses from lenders seeking noncompliant investments, such as the loss caused by loan shark

4. The Necessity of the Ceiling

The lending interest rate ceiling in private lending market (referred to as ceiling in the following parts) is a double-edged sword, and various countries have different attitudes towards it [8]. Therefore, it is a must to verify whether it is necessary to set up lending interest ceiling in private lending market. Evolutionary game is a sticky tool that fits the realities of life and can reason about the choices of social evolution in the natural state [15]. Here, the society, lenders, and borrowers are taken as the analysis objects, and the theory of evolutionary game is used to verify the necessity of the ceiling in private lending market.

TABLE 2: Utility matrix if the society chooses to set up the ceiling.

	Compliant lending	Noncompliant lending
Repayment	$[b - c_r, w - r + \Delta w, r]$	$[b - c_r - l_r, 0, r_h - f_c]$
No repayment	$[b - c_r, w - c_c - r_l - p, r_l]$	$[b - c_r - l_r, 0, r_h - f_c]$

TABLE 3: Utility matrix if the society chooses not to set up the ceiling.

	Compliant lending	Noncompliant lending
Repayment	$[0, w - r + \Delta w, r]$	$[-l_r, 0, r_h - f_c]$
No repayment	$[0, w - c_c, 0]$	$[-l_r, 0, r_h - f_c]$

4.1. Assumptions and Model. An important reason why the evolutionary game theory is more realistic is that it assumes bounded rationality and then simulates social choices in the natural state [15]. Here, this paper follows this assumption. This section takes society, lenders, and borrowers as the research objects. The three-party strategy here is as follows: First of all, society has only two choices: the first is set up the ceiling, and the second is not set up the ceiling. The lender can participate in compliant lending so that it is necessary to follow the relevant rules of the ceiling or to invest money in noncompliant loans, such as usury. Therefore, lender's strategy choice is either compliant lending or noncompliant lending. The borrower can choose to comply with the regulations to repay or choose not to repay, so the borrower's strategy is either repayment or no repayment.

According to the above model and the actual situation in reality, the utility matrix for the society is displayed in Tables 2and 3 (society is on the left, the borrower is in the centre, and the lender is on the right).

4.2. Replicated Dynamic Equation. The probability that the society chooses to set up the ceiling is set as α, so the probability of not setting up is $1-\alpha$; the probability of lenders choosing compliant lending is β, and the probability of noncompliant lending is $1-\beta$; the probability of borrowers choosing to repay the loan is γ, and the probability of default is $1-\gamma$. The expected utility of the society choosing to set up the ceiling is U_{11}, and the expected utility of the society choosing not to set up the ceiling is U_{12}; the expected utility of lenders choosing compliant lending is U_{21}, and the expected utility of choosing noncompliant lending is U_{22}; the expected utility when the borrower repays is U_{31}, and the expected utility when the loan is not repaid is U_{32}.

4.2.1. Society Situation. If the society chooses to set up the ceiling, the expected utility can be obtained:

$$U_{11}=[\gamma 1-\gamma]\begin{bmatrix} b-c_r & b-c_r-l_r \\ b-c_r & b-c_r-l_r \end{bmatrix}\begin{bmatrix} \beta \\ 1-\beta \end{bmatrix}=b-c_r-l_r+\beta l_r. \tag{1}$$

If the society chooses not to set up the ceiling, the expected utility can be obtained:

$$U_{12}=[\gamma 1-\gamma]\begin{bmatrix} 0 & -l_r \\ 0 & -l_r \end{bmatrix}\begin{bmatrix} \beta \\ 1-\beta \end{bmatrix}=-(1-\beta)l. \tag{2}$$

From this, the society's replicated dynamic equation is

$$F_1(\alpha,\beta,\gamma)=\frac{d\alpha}{dt}=\alpha(1-\alpha)(U_{11}-U_{12})=\alpha(1-\alpha)(b-c_r). \tag{3}$$

4.2.2. Lenders Situation. If lenders choose to compliant lending, the expected utility can be obtained:

$$U_{21}=[\gamma 1-\gamma]\begin{bmatrix} r & r \\ r_l & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix}=\alpha(1-\gamma)r_l+\gamma r, \tag{4}$$

and if lenders choose not to compliant lending, the expected utility can be obtained:

$$U_{22}=[1-\gamma]\begin{bmatrix} r_h-f_c & r_h-f_c \\ r_h-f_c & r_h-f_c \end{bmatrix}\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix}=r_h-f_c. \tag{5}$$

From this, the lender's replicated dynamic equation is

$$\begin{aligned} F_2(\alpha,\beta,\gamma)&=\frac{d\beta}{dt}=\beta(1-\beta)(U_{21}-U_{22}) \\ &=\beta(1-\beta)[\alpha(1-\gamma)r_l+\gamma r-r_h+f_c]. \end{aligned} \tag{6}$$

4.2.3. Borrowers Situation. If borrowers choose to repay, the expected utility can be obtained:

$$\begin{aligned} U_{31}&=[\beta\ \ 1-\beta]\begin{bmatrix} w-r+\Delta w & w-r+\Delta w \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} \\ &=\beta(w-r+\Delta w). \end{aligned} \tag{7}$$

If borrowers choose not to repay, the expected utility can be obtained:

$$\begin{aligned} U_{32}&=[\beta\ \ 1-\beta]\begin{bmatrix} w-c_c-r_l-p & w-c_c \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} \\ &=\beta[w-c_c-\alpha(r_l+p)]. \end{aligned} \tag{8}$$

From this, the borrower's replicated dynamic equation is

$$\begin{aligned} F_3(\alpha,\beta,\gamma)&=\frac{d\gamma}{dt}=\gamma(1-\gamma)(U_{31}-U_{32}) \\ &=\gamma(1-\gamma)\beta[c_c-r+\Delta w+\alpha(r_l+p)]. \end{aligned} \tag{9}$$

4.2.4. Replicated Dynamic Nonlinear System. Through the above analysis, this paper can get the following equation and form the replicated dynamic nonlinear system:

$$\begin{aligned} x&=\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}, F(x)=\begin{bmatrix} F_1(\alpha,\beta,\gamma) \\ F_2(\alpha,\beta,\gamma) \\ F_3(\alpha,\beta,\gamma) \end{bmatrix} x=F(x) \\ &=\begin{bmatrix} \alpha(1-\alpha)(b-c_r) \\ \beta(1-\beta)[\alpha(1-\gamma)r_l+\gamma r-r_h+f_c] \\ \gamma(1-\gamma)\beta[c_c-r+\Delta w+\alpha(r_l+p)] \end{bmatrix}. \end{aligned} \tag{10}$$

4.3. Model Solving and Stability Analysis. Let the dynamic replication equation be equal to 0; that is, $F_1=0$, $F_2=0$, and $F_3=0$ can solve the replication dynamic equation of society, lenders, and borrowers and obtain the eight following local equilibrium points (α,β,γ): (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), and (1, 1, 1). The equilibrium solution derived by this way is not necessarily the evolutionary stable strategy (ESS) in the game. The stable evolution strategy of a differential equation system can be obtained from the local stability analysis of the Jacobian matrix of the system [16]. From the dynamic replication equation, the Jacobian matrix of the system can be obtained as follows:

$$\begin{bmatrix} (1-2\alpha)(b-c_c) & 0 & 0 \\ \beta(1-\beta)(1-\gamma)r_l & (1-2\beta)[\alpha(1-\gamma)r_l+\gamma r-r_h+f_c] & \beta(1-\beta)(r-\alpha r_l) \\ \gamma(1-\gamma)\beta(r_l+p) & \gamma(1-\gamma)[c_c-r+\Delta w+\alpha(r_l+p)] & (1-2\gamma)\beta[c_c-r+\Delta w+\alpha(r_l+p)] \end{bmatrix}. \tag{11}$$

According to evolutionary game theory, the system's evolutionary stable point (ESS) is reached when all of the Jacobian matrix's eigenvalues are negative. Substituting eight equilibrium points into the Jacobian matrix, three

Table 4: Equilibrium stability analysis.

Equilibrium	Eigenvalues	Eigenvalues	Eigenvalues	Stability
Point (α, β, γ)	λ_1	λ_2	λ_3	
(0, 0, 0)	$b - c_c$	$f_c - r_h$	0	Unstable
(0, 0, 1)	$b - c_c$	$f_c - r_h + r$	0	Unstable
(0, 1, 0)	$b - c_c$	$-(f_c - r_h)$	$c_c - r + \Delta w$	To be determined
(0,1,1)	$b - c_c$	$-(f_c - r_h + r)$	$-(c_c - r + \Delta w)$	To be determined
(1, 0, 0)	$-(b - c_c)$	$f_c - r_h + r_l$	0	Unstable
(1, 0, 1)	$-(b - c_c)$	$r - r_h + f_c$	0	Unstable
(1, 1, 0)	$-(b - c_c)$	$-(r_l - r_h + f_c)$	$c_c - r + \Delta w + r_l + p$	To be determined
(1, 1, 1)	$-(b - c_c)$	$-(r - r_h + f_c)$	$-(c_c - r + \Delta w + r_l + p)$	To be determined

eigenvalues corresponding to each equilibrium point can be obtained. The equilibrium stability analysis is displayed in Table 4.

First of all, according to the actual situation, as long as the overall return of the society setting up the ceiling is higher than the reputation loss caused by borrowers not repaying the money, that is, $b > c_c$, the stability point will tend to set up the interest rate ceiling. In real life, setting up the ceiling to reduce usury and financing costs far exceeds the reputation loss caused by no repayment. As a result, points $(0, 1, 0)$ and $(0, 1, 1)$ cannot be stable.

Proposition 1. *In a private lending market, only two equilibrium points can be stable according to evolutionary game theory and real situation, which are (1,1,0) and (1,1,1).*

According to the proposition, it can be considered that setting up the lending rate ceiling in private lending market is a steady-state choice in society. Under the condition that the society chooses to set up the lending rate ceiling, the stability point of lenders and borrowers needs to be determined according to $r_l - r_h + f_c$, $r - r_h + f_c$, *and* $c_c - r + \Delta w + r_l + p$. *For lenders who want to invest funds in compliant loans, they must ensure that* $r_l + f_c > r_h$ *or* $r + f_c > r_h$, *under the condition that* r_l *and* r *cannot be changed, but* f_c *can be changed. So if it is large enough, the above inequality can be established, that is, the risk of lenders investing in noncompliant loans needs to be increased, which is to increase the related supervision and punishment. If society wants to maintain good loans and make borrowers repay steadily, then* $c_c - r + \Delta w + r_l + p > 0$ *must be guaranteed, and long-term repayment incentives can be established by increasing the penalty of no repayment and the loss of reputation for no repayment to make the above inequality stable.*

In summary, the setting of the interest rate ceiling is a choice of social evolution.

5. The Impact of the Ceiling

5.1. Assumptions and Model. It is assumed that players of the model are one borrower A and one lender B and the loan amount is one unit. The loan period is one year. The utility of borrower and lender is represented by (A, B), and the entire borrowing process is as follows:

(1) First, borrower A decides whether to initiate a loan; that is, the borrower's strategy at this node is either borrow or no borrow. So if borrower A choose no borrow, then the utility is $(-s, r_f)$.

(2) Lender B will choose whether to lend or not when receiving the borrower's borrowing demand; that is, the strategy is either lend or no lend; if it chooses not to lend, then the utility is $(-s, r_f)$.

(3) When the loan expires, borrower A will also choose whether to repay the loan. The strategy is either repay or no repay, and there will be a credit loss if the loan is not repaid. If repay is chosen, then the utility of the two parties is $(w - r, r)$.

(4) If borrower A chooses not to repay, lender B will choose whether to appeal where the strategy is either appeal or no appeal. The appeal will cause an appeal cost, but A will be punished. If the lender chooses no appeal, the utility is $(w - c_c + 1, -1)$.

Here the project assumes that both parties are rational. According to the implementation of the ceiling of lending interest rates in various countries, if the interest rate is below the ceiling, then the society will determine that this interest rate is valid and the borrower needs to compensate the interest rate and principle in full. If the interest rate exceeds the ceiling of interest rate, as long as it does not exceed the invalid interest rate threshold, the excess part will be paid on a voluntary basis [3]. Therefore, if the borrower does not repay the principal, then, in the case of lender's appeal, the utility of the two parties will be affected by the borrowing interest rate at this time. If the borrowing interest rate is below the ceiling, that is, $0 < r < r_l$, then the utility is $(w - c_c - r - p, r - c_a)$. If the loan interest rate is higher than the ceiling, that is, $r_l < r$, then the utility is $(w - c_c - r_l - p, r_l - c_a)$, which can be reformed as follows:

$$\text{Utility}(A, B) = \begin{cases} (w - c_c - r - p, r - c_a) & \text{if } 0 < r < r_l \\ (w - c_c - r_l - p, r_l - c_a) & \text{if } r_l < r \end{cases}. \quad (12)$$

According to the above assumptions, the entire lending process can be represented in Figure 1.

5.2. Equilibrium. After establishing an ordinary lending relationship, how the ceiling affects the strategic choices of the borrower and the lender through the equilibrium is shown (the case of equal conditions is not considered here, since it does not affect the results).

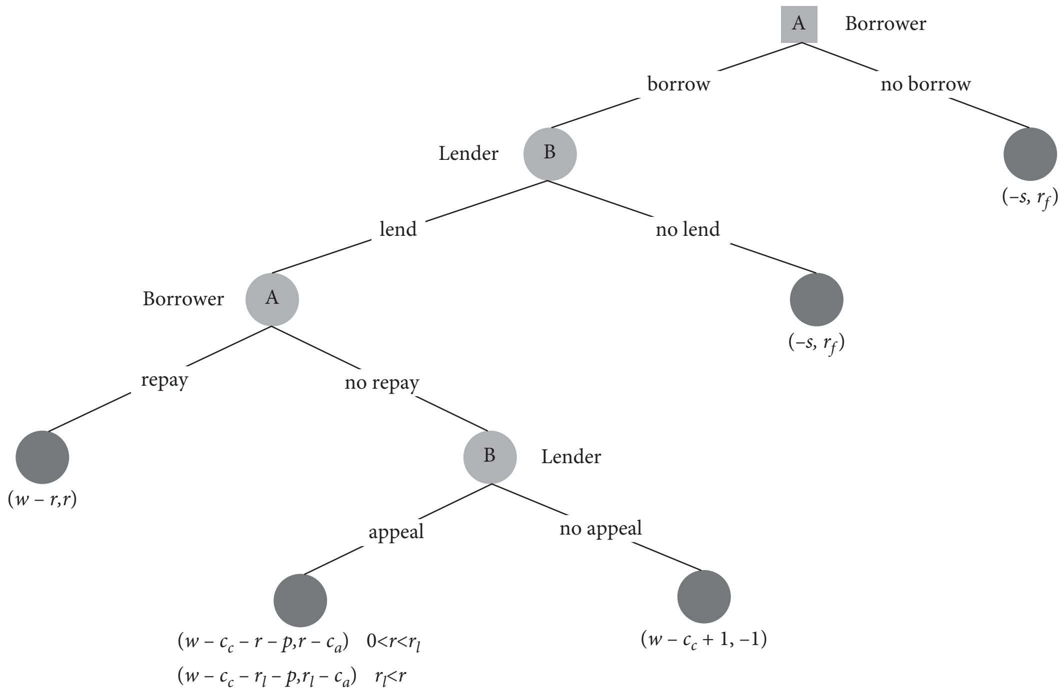

Figure 1: Game relationship analysis.

First, analyze from the final node of the game, that is, whether lender appeals. Compare $r - c_a$, $r_l - c_a$, and -1 here. According to the practical significance, $r - c_a > -1$ and $r_l - c_a > -1$ always hold; that is, in reality, the interest rate minus the appeal fee must be greater than the loss principal. So, at this node, if borrower A chooses not to repay, then lender B's best strategy is appeal.

In the following analysis, the discussion is according to the range of interest rates.

When the loan interest rate is below the ceiling, that is, when $0 < r < r_l$, the following relationship always holds:

$$w - c_c - r - p < w - r. \tag{13}$$

In this case, repayment will be the best choice for borrower, which is repay. Push up again. According to practical significance, the profit generated by borrowing must be greater than the risk-free return in this industrial. The profit generated by borrower minus interest will generally be greater than the loss caused by not borrowing money; that is, the following equation is satisfied:

$$\begin{aligned} r &> r_f \\ w - r &> -s. \end{aligned} \tag{14}$$

So, under this condition, there is only one equilibrium {borrow, lend, repay}.

If the loan interest rate exceeds the ceiling rate, that is, $r_l < r$, the choice of whether to repay by borrower A depends on the comparison of the size of $w - c_c - r_l - p$ and $w - r$, so the following decision conditions exist:

$$\text{strategy}\begin{cases} \text{repay} & c_c + r_l + p > r \\ \text{no repay} & c_c + r_l + p < r \end{cases}. \tag{15}$$

If the condition $c_c + r_l + p > r$ is satisfied, that is, the borrower chooses to repay, then because $r > r_f$ always holds, the final equilibrium is {borrow, lend, repay}.

If the condition $c_c + r_l + p < r$ is met, that is, the borrower chooses not to repay, then $r_l - c_a$ and r_f will be compared. Then there are the following conditional strategies:

$$\text{strategy}\begin{cases} \text{lend} & r_l - c_a > r_f \\ \text{no lend} & r_l - c_a < r_f \end{cases}. \tag{16}$$

If lender B chooses not to lend money, then borrower A chooses borrow and no borrow and the utility is the same. Then the equilibrium of the entire game process is {borrow/no borrow, no lend}.

If lender B chooses to lend money, then borrow will compare $w - c_c - r_l - p$ and $-s$ to get the following conditional strategy:

$$\text{strategy}\begin{cases} \text{borrow} & w - c_c - r_l - p > -s \\ \text{no borrow} & w - c_c - r_l - p < -s \end{cases}. \tag{17}$$

The equilibrium corresponding to the two conditions is {borrow, lend, no repay, appeal} and {no lend}. Because conditions $c_c + r_l + p < r$ and $w - r > -s$ also should be met, here borrow will only choose {borrow}, and only {borrow, lend, no repay, appeal} is the equilibrium.

Proposition 2. *In a private lending market, the equilibrium is as follows:*

If $0 < r < r_l$ holds, the equilibrium is {borrow, lend, repay}.

If $r_l < r$ and $c_c + r_l + p > r$ hold, the equilibrium is {borrow, lend, repay}.

If $r_l < r$, $c_c + r_l + p < r$, and $r_l - c_a > r_f$ hold, the equilibrium is {borrow, lend, no repay, appeal}.

If $r_l < r$, $c_c + r_l + p < r$, and $r_l - c_a < r_f$ hold, the equilibrium is {borrow/no borrow, no lend}.

Many countries seek to keep the ceiling on private lending rates at a low level [17]. This is conducive to reducing the occurrence of usury and reducing the cost of small- and medium-sized financing, and it will also affect the lending relationship to a certain extent. According to the result, for customers whose loan is always below the ceiling rate, the decrease of the ceiling has no effect on them. For customers whose loan is originally below the ceiling rate but left the scope of the ceiling because of the decrease in the ceiling, it can be known that lender's decision mainly relies on comparing $r_l - c_a$ and r_f. If the ceiling is reduced and the condition $r_l - c_a < r_f$ is easier to meet, then lender is more inclined to not lend. Even if the lender decides to lend, the borrower's decision mainly depends on whether the condition $c_c + r_l + p > r$ is met. If the ceiling decreases, obviously this condition is more difficult to meet, so the borrower is more inclined to not repay the money. Therefore, a reduction of the ceiling rate will increase the probability of not being able to borrow money and increase the probability of no repayment.

6. The Solution of the Ceiling

The lowering of the interest rate ceiling is essentially a redistribution of profits; the cooperation and profit distribution between lenders and small- and medium-sized borrowers can be solved by the bargaining model in the cooperative game.

6.1. Assumptions and Model. The game relationship is established from the perspective of profit distribution, so it does not need to consider the impact of repayment factors. The project takes a complete cycle of borrowing and lending as the research object. The time required for the project is recorded as n annually; the return on investment of the project is W. The project uses R_e to represent excess return. The original interest expenditure of the whole process is R, r_{lo} is set as the original ceiling, and $r_{\ln}$ indicates the ceiling that should be set, which is our optimal solution. The return brought by the risk-free interest rate is R_f, because the project here considers the problem of interest rate boundary, so the following formulas exist:

$$\begin{aligned} R &= (1 + r_{lo})^n - 1, \\ R_f &= (1 + r_f)^n - 1, \\ W &= (1 + w)^n - 1, \\ R_e &= R - R_f. \end{aligned} \tag{18}$$

The model uses $S = (s_1, s_2)$ to represent the distribution of benefits in the game, where the distributed benefits are the excess returns generated by lending. So the following formula holds:

$$S = \{(s_1, s_2) \mid 0 \le s_i \le R_e, s_1 + s_2 \le R_e\}. \tag{19}$$

The entire project is equivalent to borrowers and lenders discussing the issue of excess return distribution under the ceiling framework. According to the bargaining game theory, the disagreement point of borrower and lender is $d\colon = (d_1, d_2) = (W - R, R_f)$.

The utility function of both parties can be expressed as follows:

$$\begin{aligned} u_1 &= W - (R - s_1) \\ u_2 &= R_f + s_2 \end{aligned}. \tag{20}$$

So the entire game process can be expressed as $B(S, d; u_1, u_2)$.

6.2. Equilibrium. There are different solutions according to different focuses.

6.2.1. Efficiency Solution. Set R_n as the adjusted total interest expense so that $R_n = (1 + r_{\ln})^n - 1$. Here, first use the efficiency solution of the model, which is the solution when the efficiencies of both parties are equal; the equation can be changed into the following form:

$$\begin{aligned} W - R_n &= R_n \\ R_n &= \frac{W}{2}. \end{aligned} \tag{21}$$

After unfolding the project, we can get

$$r_{\ln} = \sqrt[n]{\frac{(1 + w)^n + 1}{2}} - 1. \tag{22}$$

6.22. Maximum Utility Solution This equilibrium solution essentially is to distribute profits to maximize the overall utility. The total utility can be obtained as follows:

$$\text{Overall profit} = u_1 + u_2 = W - (R - s_1) + R_f + s_2. \tag{23}$$

As $s_1 + s_2 = R_e = R - R_f$, $Overall\ profit = W$ always holds. The result shows that the overall profit is not influenced by the bargaining. So there is no maximum utility solution.

6.2.2. Nash Bargaining Game Solution. The Nash bargaining game solution is based on the two principles of fairness and efficiency. The efficiency principle and fairness principle of Nash bargaining game solution are embodied in two axioms of efficiency and symmetry, respectively. The Nash bargaining solution uses the principle of Pareto efficiency.

We first perform a linear transformation as following:

$$\begin{aligned} u_1^*&\colon = R - W + u_1 \\ u_2^*&\colon = u_2 - R_f \end{aligned}. \tag{24}$$

After transformation, we can get $(u_1^*, u_2^*) = (s_1, s_2)$, satisfying $0 \le s_i \le R_e$ and $s_1 + s_2 \le R_e$. According to the symmetry axiom, the following can be obtained:

$$\begin{aligned} (s_1, s_2) &= \left(\frac{R_e}{2}, \frac{R_e}{2}\right) \\ (u_1, u_2) &= \left(W - \frac{R + R_f}{2}, \frac{R + R_f}{2}\right) \end{aligned} . \tag{25}$$

Therefore, under this equilibrium, the interest expense becomes of the following size:

$$\begin{aligned} R_n &= \frac{R + R_f}{2} \\ r_{\ln} &= \sqrt[n]{\frac{(1 + r_{lo})^n + (1 + r_f)^n}{2}} - 1 \end{aligned} . \tag{26}$$

Proposition 3. *The efficiency solution and the Nash bargaining game solution are optimal solutions, which are*

$$\begin{aligned} r_{\ln} &= \sqrt[n]{\frac{(1 + w)^n + 1}{2}} - 1 \\ \text{or } &\sqrt[n]{\frac{(1 + \mathrm{r}_{\mathrm{lo}})^{\mathrm{n}} + (1 + \mathrm{r}_{\mathrm{f}})^{\mathrm{n}}}{2}} - 1, \end{aligned} \tag{27}$$

and the efficiency solution is more inclined to use the return rate of the project to determine the ceiling, while the Nash bargaining game solution tends to focus on the combination of the existing ceiling and the market risk-free interest rate to determine the new ceiling.

Therefore, a reasonable lending rate ceiling needs to consider the industry's average return rate, the current risk-free rate of return, and the existing lending rate ceiling. At the same time, because the risk-free interest rate r_f *and the industry annualized return rate* w *are both floating, the ceiling should also fluctuate with the economic situation.*

7. Discussion

Interest rate marketization has been advocated by many scholars [18, 19], who argue that interest rates should be more determined by supply and demand and the profitability of participants, and our study adds that, in the individual lending market, interest rate ceiling should be established to ensure market stability, which is an evolutionary choice.

Under the world's trend of guiding interest rates downward, the interest rate of the basic interest rate level is in a downward channel in the long run, which is conducive to enterprises to reduce financing costs [8, 20], while our study advocates that the decrease of interest rate ceiling may instead make financing more difficult for enterprises, especially SMEs, and increase the probability of no repayment, which will harm lenders' interests. Our findings coincide with Gonzalez's [21] analysis of agricultural lenders, which argues that interest rate ceiling may be detrimental to lenders. Our findings are also supported by the studies of Schicks [22] and Assefa [23], whose works show that a decrease in interest rates due to excessive competition weakens the relationship between borrowers and lenders and increases the rate of overlending and default.

At present, the determination of the ceiling falls into two ways. The first is a fixed interest rate system, and the second is floating by a basic interest rate level [24]. The use of a fixed ceiling for the lending interest rates is conducive to uniform judgment standards. However, the shortcomings are apparent. For example, they cannot be differentiated according to market changes and different industries. The interest rate ceiling established by this method is easy to cause the phenomenon that the ceiling is not suitable to the actual situation.

Many of the existing interest rates are floating based on the basic interest rate level, such as most mortgages. The basic interest rate level itself also fluctuates according to the market. The ceiling determined by this way is an improvement to the fixed interest rate system, but it cannot treat various industries differently, nor does it take into account the existing interest rate level. In our model, if the efficiency solution is accepted, it can fluctuate according to the industry's average return rate. If the Nash bargaining game solution is adopted, not only the risk-free interest rate but also the current interest rate ceiling is considered, which can make the new ceiling easier accepted.

8. Conclusion and Outlook

This paper systematically studies the interest rate ceiling of private lending. First, it has verified the necessity of the ceiling and revealed that it is an unavoidable and evolutionary result. Additionally, the paper discussed the impact of the adjustment of the ceiling on the lending relationship and concluded that the reduction of the ceiling would increase the financing cost of SMEs and increase the probability of no repayment in terms of lending relationships. Furthermore, two optimal ceilings are obtained through the efficiency solution and the Nash game solution.

The model's ability to distinguish between the various behaviors of borrowers with varying credit quality and the varied behaviors of lenders with diverse risk preferences is constrained. It only takes into account one type of borrowers and lenders. Also, the research in this paper focuses mainly on theoretical analysis and is not yet able to use existing data or experiments to strengthen the theory.

For the future research, verifying the applicability of the various ceiling setting types using market data from various regions or periods will be the first work. For example, using data from several historical eras, Koch [25] has investigated the effect of bank deposit interest rate ceilings on credit size under various setting approaches. It is also an essential future work to design a dynamic adjustment mechanism of lending interest rate ceiling, which is efficient and balances the interests of all parties. For example, Sarkar [26] has designed a two-stage supply chain model for nullifying waste, which can reasonably balance environmental protection and economic efficiency. In the dynamic adjustment model, how to balance the interests of borrowers and lenders is a difficult issue. It is possible to refer to Kugele's [27] method, which uses geometric programming to find the optimal solution. Also, we can refer to Choi's paper [28] and calculate the marginal value of both sides based on the actual data to determine the most suitable balance point.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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