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Research Article

The Secure Metric Dimension of the Globe Graph and the Flag Graph

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Let $G=(V,E)$ be a connected, basic, and finite graph. A subset $T=\{u_1,u_2,\ldots,u_k\}$ of $V(G)$ is said to be a resolving set if for any $y\in V(G)$, the code of y with regards to T, represented by $C_T(y)$, which is defined as $C_T(y)=d(u_1,y),d(u_2,y),\ldots,d(u_k,y)$, is different for various y. The dimension of G is the smallest cardinality of a resolving set and is denoted by $\dim(G)$. If, for any $t\in V-S$, there exists $r\in S$ such that $(S-\{r\})\cup\{t\}$ is a resolving set, then the resolving set S is secure. The secure metric dimension of ? is the cardinal number of the minimum secure resolving set. Determining the secure metric dimension of any given graph is an NP-complete problem. In addition, there are several uses for the metric dimension in a variety of fields, including image processing, pattern recognition, network discovery and verification, geographic routing protocols, and combinatorial optimization. In this paper, we determine the secure metric dimension of special graphs such as a globe graph Gl_n, flag graph Fl_n, H- graph of path P_n, a bistar graph $B^2_{n,n}$, and tadpole graph $T_{3,m}$. Finally, we derive the explicit formulas for the secure metric dimension of tadpole graph $T_{n,m}$, subdivision of tadpole graph $S(T_{3,m})$, and subdivision of tadpole graph $S(T_{n,m})$.

1. Introduction

Let $G=(V,E)$ be a connected, basic, finite graph. Let $T=\{u_1,u_2,\ldots,u_k\}$ on which the ordering $(u_1,u_2,\ldots,u_k)$ is imposed. The ordered k-tuples $r(w\,|\,T)=(d(u\_(1),w), d(u\_(2),w),\ldots,d(u\_(\mathrm{k}),w))$ are referred to as the metric description of w with regards to T for every $w\in V(G)$. The set T is said to be a resolving set if $r(u|T)$ is not equal to $r(w\,|\,T)$ for any u, w in $V(G)$. A minimum resolving set or basis is a resolving set of G with minimum cardinality, and the cardinality of a minimum resolving set is referred to as the dimension of G, which is indicated by $\dim(G)$ [1].

In the literature, the concept of finding sets in a connected network has already been discussed [2, 3]. Nearly forty years ago, Slater introduced the ideas of locating sets (also known as resolving sets) and a reference set (metric dimension). Later, the aforementioned theory was independently discovered by Harary and Melter [4]. Sebo and Tannier [5], Mohamed et al. [6], Amin et al. [7], and Borchert et al. [8] have all conducted different research on the idea of the metric dimension of graphs. Meanwhile, Imran et al. [9] studied the metric dimension of the generalized Petersen multigraphs $P(2n, n)$, which are the barycentric subdivision of Möbius ladders and established that they have metric dimensions of 3 and when n is even and 4 when n is odd. Jäger et al. [10] showed that the metric dimension of $Z_n \times Z_n \times Z_n$, $n \geq 2$ is $\lfloor 3n/2 \rfloor$. The complement metric dimension and specifics of particular graphs were studied by Susilowati et al. [11]. The dominant metric dimension of the joint product graphs was investigated by Purwati et al. [12]. The metric dimension of corona product graphs was discovered by Iswadi et al. [13]. In certain networks, including the trapezoid network, the $Z-(P_n)$ network, the open ladder network, the tortoise network, and the $P_{2n}\overline{V}P_n$ network, Mohamed et al. [14] studied the exact value of the secure resolving set. The exact values of the local metric basis and local metric dimension of the cyclic split graph were examined by Cynthia et al. [15]. Grigorious et al. [16] identified the metric dimension for all n-values of the circulant graphs $C(n, \pm\{1, 2, 3, 4\})$. The metric dimension of the family of generalized wheels was established by Sooryanarayana et al. [17]. Several different kinds of sets in a graph are related to the concept of security. For instance, a secure set is a dominating set D of G if for all $v \in V\text{-}D$, there exists $u \in D$ such that $(D-\{u\}) \cup \{v\}$ is a dominating set [4, 18]. The secure resolving set for various classes of graphs was identified by Subramanian et al. [19]. Numerous graph operations, such as corona product graphs [13], comb product graphs [20], and joint product graphs [21], have been found to have a metric dimension. Metric dimension has been used in many different applications, including robot navigation in networks [22–25], pharmaceutical chemistry Chartrand et al. [1], pattern recognition Melter et al. [26], and localization of wireless sensor networks [27].

The secure metric dimension for various graphs, including the globe graph Gl_n, the flag graph Fl_n, the H-graph of the path P_n, the bistar graph $B^2_{n,n}$, and tadpole graph $T_{3,m}$ is determined in this paper. The secure metric dimension of the tadpole graphs $T_{n,m}$, the subdivision of the tadpole graph $S(T_{3,m})$, and the subdivision of the tadpole graph $S(T_{n,m})$ are finally derived explicitly.

2. Preliminary Notes

Definition 1 (see [28]). Tadpole graph $T_{n,m}$ is a special type of graph that consists of a bridge connecting a cycle graph with n (at least 3) vertices and a path graph with m vertices.

Definition 2 (see [29]). A globe graph Gl_n is a graph obtained from two isolated vertex connected by n paths of length two.

Definition 3 (see [29, 30]). A bistar graph also known as $B_{n,n}$, is the graph created by connecting the centre (apex) vertices of two copies of $K_{1,n}$ by an edge and it is denoted by $B_{n,n}$. The vertex set of $B_{n,n}$ is given by $V(B_{n,n}) = \{v_1, v_2, \ldots, v_n, v, u, u_1, u_2, \ldots u_n\}$, where v and u are apex vertices and $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots u_n$ are pendent vertices. The edge set of $B_{n,n}$ is $E(B_{n,n}) = \{vv_1, vv_2, \ldots, vv_n, v_u, uu_1, uu_2, \ldots uu_n\}$.

3. Secure Metric Dimension

Definition 4. If a subset T of G is resolving and there is a $y \in T$ such that $(T-\{y\}) \cup \{x\}$ is a resolving set of G for any $x \in V-T$, then the subset T is an *SR* set of G. The secure resolving dimension of G, denoted by $\mathrm{sdim}(G)$, is the least cardinality of an *SR* set of G.

The notion of metric dimension, which was first presented separately by Subramanian [19] and Mohamed [14]. Robot navigation in a network represented by a graph is one of the main motivating factors behind its study (see [31]).

The robot uses "landmarks," or transmitters, set at sites (the graph's vertices) to establish its location inside the network; so, the metric dimension and the secure metric dimension are the bare minimum of transmitters needed for the robot to always be aware of its position inside the network that the graph represents. The metric dimension of a graph is an NP-hard problem, as was mentioned in [32]. Furthermore, business networks, chemical structures, wireless communication networks, and electrical networks [33–44] all make use of secure metric dimension theory.

Remark 5. There is a guarantee that an SR set exists. Because the vertex set $V(G)$ in any graph is both a resolving set and a secure set.

Remark 6. $\dim(G) \leq \mathrm{sdim}(G)$.

3.1. Secure Metric Dimension for Several Certain Graphs [19]

(1) $\mathrm{sdim}\ (K_n) = n-1 = \dim(K_n)$

(2) $\mathrm{sdim}\ (K_{1,n}) = n > \dim(K_{1,n})$

(3) $\mathrm{sdim}\ (K_{m,n}) = m+n-2 = \dim(K_{m,n})\ (m, n \geq 2)$

(4) $\mathrm{sdim}\ (P_n) = 2 > \dim(P_n) = 1\ (n \geq 3)$

(5) $\mathrm{s\,dim}(C_n) = 2\dim(C_n)$

4. Main Results

Here, we demonstrate that the secure metric dimension of the globe graph Gl_n, flag graph Fl_n, H- graph of path P_n, and the bistar graph $B^2_{n,n}$. We also derive the explicit formulas for the secure metric dimension of tadpole graph $T_{3,m}$, tadpole graph $T_{n,m}$, subdivision of tadpole graph $T_{3,m}$, and subdivision of tadpole graph $T_{n,m}$.

Theorem 7. *Let G is a globe graph Gl_n with k blocks and n vertices, then sdim$(Gl_n) = n$-2.*

Proof. We label the globe graph Gl_n as shown in Figure 1. We choose a subset $S = \{v_1, v_2, \ldots, v_{n-2}\}$, and we must demonstrate that $\mathrm{sdim}(Gl_n) = n$-2 for $n \geq 4$. The following is how we obtained the representations of the vertices in the graph Gl_n with respect to S.

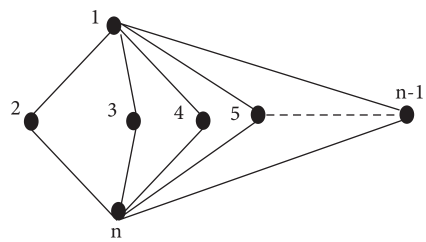

Figure 1: Globe graph Gl_n.

$$\begin{aligned} r\left(v_1 \mid S\right) &= (0,1,1,\ldots,1,1),\\ r\left(v_2 \mid S\right) &= (1,0,2,\ldots,2,2),\\ r\left(v_3 \mid S\right) &= (1,2,0,2,\ldots,2,2),\\ &\vdots\\ r\left(v_{((n/2)+1)} \mid S\right) &= (1,2,0,2,\ldots,2,2). \end{aligned} \tag{1}$$

The vertices of graph Gl_n have distinct representations when viewed from above. Although it implies that S is a secure resolving set, this does not imply that it is the lower bound. $s\dim(Gl_n) \le n-2$ is the upper bound as a result. For the globe graph Gl_n, there is no secure resolving set whose cardinality is 1. $\mathrm{Sdim}(Gl_n) \ge n-2$ is the lower bound and as a result obtained that $\mathrm{sdim}(Gln) \le n-2$ and $\mathrm{sdim}(Gln) \ge n-2$; therefore, $\mathrm{sdim}(Gln) = n-2$. From the aforementioned proof, we deduce that $\mathrm{sdim}(Gl_n) = n-2$. □

Theorem 8. *Let G be a flag graph Fl_n with n vertices, then $sdim(Fl_n) = 2$.*

Proof. We label the flag graph Fl_n as shown in Figure 2. It is obvious that there are n vertices. In order to have two cases in the proof, let $S = \{v_2, v_{n-1}\}$ if n is odd and $S = \{v_1, v_{n-2}\}$ if n is even. □

Case 9. n is odd. Let $S = \{v_2, v_{n-1}\} \subset V(Fl_n)$ be a secure resolving set. The representations of the vertices of Fl_n with regards to S are the following:

$$r\left(v_i, S\right) = \begin{cases} \left(\dfrac{i+1}{2}, \dfrac{n-1}{2}\right), & 1 \le i \le n-2;\ \text{where}\ i = 1,3,\ldots,n-2;\\[2ex] \left(\dfrac{i-2}{2}, \dfrac{n-i-1}{2}\right), & 2 \le i \le n-1;\ \text{where}\ i = 2,4,\ldots,n-1;\\[2ex] \left(2, \dfrac{i+1}{2}\right), & \text{where}\ i = n. \end{cases} \tag{2}$$

Case 10. n is even. Let $S = \{v_1, v_{n-2}\} \subset V(Fl_n)$ be a secure resolving set. The representations of the vertices of Fl_n with regards to S are the following:

$$r\left(v_i, S\right) = \begin{cases} \left(0, \dfrac{n-2}{2}\right), & \text{where}\ i = 1;\\[2ex] \left(\dfrac{i-1}{2}, \dfrac{n-i+1}{2}\right), & 3 \le i \le n-1;\ \text{where}\ i = 3,\ldots,n-1;\\[2ex] \left(\dfrac{i}{2}, \dfrac{n-i-2}{2}\right), & 2 \le i \le n-2;\ \text{where}\ i = 2,4,\ldots,n-2;\\[2ex] \left(1, \dfrac{i}{2}\right), & \text{where}\ i = n. \end{cases} \tag{3}$$

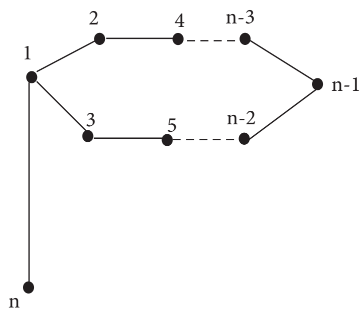

FIGURE 2: Flag graph Fl_n.

This completes the proof.

Since it is obvious that no two vertices have the same labeling, we obtain a secure resolving set S with $|S|$, and as a result, we obtain $\mathrm{sdim}(Fl_n) = 2$.

Theorem 11. *Let G be a H- graph of path P_n with n vertices, then sdim(G) = 2.*

Proof. We label the H- graph of path P_n as shown in Figure 3. Let $S = \{v_1, v_{(n/2)+1}\}$ such that n is the vertices number. We want to show that $d(v_i, S) \neq d(v_j, S)$ for all $i \neq j$ to demonstrate that S is a secure resolving set. Observe the following:

Case 1. For $n = 4$, then $\mathrm{sdim}(G) = 1$.

Case 2. For $n = 6$, then $\mathrm{sdim}(G) = 2$ and $\{v_1, v_3\}$ be a secure metric basis of G.

Case 3. For $n = 8, 10, 12, \ldots, n$ then $\mathrm{sdim}(G) = 2$ and $\{v_1, v_{n/2+1}\}$ be a secure metric basis of G.

Begin
for $(i = 1;\ i \leq (n/4)\ ;\ i{+}{+})$ **do**
$d(v_i, S) = (i - 1, (n/2) - i + 1)$
end
for $(i = (n/4) + 1;\ i \leq (n/2)\ ;\ i{+}{+})$ **do**
$d(v_i, S) = (i - 1, i + 1)$
end
for $(i = (n/2) + 1;\ i \leq (3n/4)\ ;\ i{+}{+})$ **do**
$d(v_i, S) = (n - i + 1, i - (n/2) - 1)$
end
for $(i = (3n/4) + 1;\ i \leq n\ ;\ i{+}{+})$ **do**
$d(v_i, S) = (i - (n/2) - 1, i - (n/2) - 1)$
end
end

This completes the proof.

Since it is obvious that no two vertices have the same labeling, we obtain a secure resolving set S with $|S|$, and as a result, $\mathrm{sdim}\ (G) = 2$. Four for-loops are present in the algorithm used to prove Theorem 11, but they are not inner loops, hence the complexity of the algorithm is $O(n)$, indicating that it is a polynomial time algorithm. □

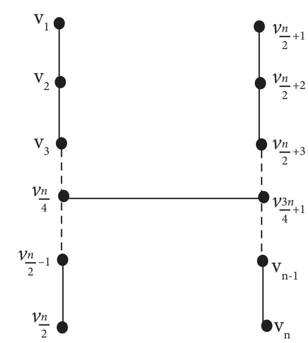

FIGURE 3: H-graph of path P_n.

Theorem 12. *Let G be tadpole graph $T_{3,m}$ with m vertices, and $\{v_1, v_3\}$ be a secure metric basis of G, then sdim $(T_{3,m}) = 2$.*

Proof. We label the tadpole graph $T_{3,m}$ as shown in Figure 4. We choose a subset $S = \{v_1, v_3\}$, and we must show that $\mathrm{sdim}(T_{3,m}) = 2$ for $n \geq 4$. The following is how we obtained the representations of the vertices in the graph $T_{3,m}$ with respect to S.

$$\begin{aligned} r(v_1 \mid S) &= (0, 1),\\ r(v_2 \mid S) &= (1, 1),\\ r(v_3 \mid S) &= (1, 0),\\ &\vdots\\ r(v_n \mid S) &= (i - 2, i - 3). \end{aligned} \tag{4}$$

Obviously, there are no two vertices with the same labeling; we then get a secure resolving set S with $|S|$, so we have sdim $(T_{3,m}) = 2$. □

Corollary 13. *Let G be tadpole graph $T_{n,m}$ with n and m vertices and $\{v_1, v_3\}$ be a secure metric basis of G, then sdim $(T_{n,m}) = 2$.*

Corollary 14. *Let G be subdivision of tadpole graph $S(T_{3,m})$ with m vertices and $\{v_1, v_3\}$ be a secure metric basis of G, then sdim $(S(T_{3,m})) = 2$.*

Corollary 15. *Let* G *be subdivision of tadpole graph* $\mathrm{S}(\mathrm{T}_{\mathrm{n,m}})$ *with n, m vertices and* $\{\mathrm{v}_1, \mathrm{v}_3\}$ *be a secure metric basis of* G, *then sdim* $(\mathrm{S}(\mathrm{T}_{\mathrm{n,m}})) = 2$.

Theorem 16. *Let G be the bistar graph $B^2_{n,n}$ with n vertices and $\{v_1, v_2, \cdots, v_{n-3}, v_n\}$ be a secure metric basis of G, then sdim $(B^2_{n,n}) = n$-2.*

Proof. We label the bistar graph $T_{3,m}$ as shown in Figure 5. We choose a subset $S = \{v_1, v_2, \ldots, v_{n-3}, v_n\}$, and we must show that sdim $(B^2_{n,n}) = n$-2 for $n \geq 6$. The following is how we obtained the representations of the vertices in the graph $B^2_{n,n}$ with respect to S.

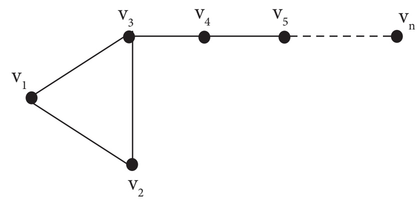

Figure 4: Tadpole graph $T_{3,m}$.

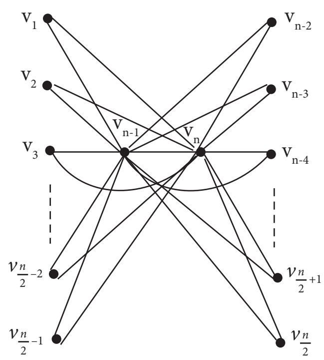

Figure 5: Bistar graph $B^2_{n,n}$.

$$\begin{aligned} r\left(v_1 \mid S\right) &= (0, 2, 2, \cdots, 2, 1), \\ r\left(v_2 \mid S\right) &= (2, 0, 2, \cdots, 2, 1), \\ r\left(v_3 \mid S\right) &= (2, 2, 0, 2, \cdots, 2, 1), \\ r\left(v_{n-3} \mid S\right) &= (2, 2, \ldots, 2, 0, 1), \\ r\left(v_{n-2} \mid S\right) &= (2, 2, \ldots, 2, 1), \\ r\left(v_{n-1} \mid S\right) &= (1, 1, \ldots, 1), \\ r\left(v_n \mid S\right) &= (1, 1, \ldots, 1, 0). \end{aligned} \tag{5}$$

From above, the representations of vertices in graph $B^2_{n,n}$ are distinct. This implies that S is a secure resolving set, but it is not necessarily the lower bound. Thus, the upper bound is $\mathrm{sdim}(B^2_{n,n}) \le n-2$. For $B^2_{n,n}$ there is no secue resolving set that the cardinality is one. Thus, the lower bound is $\mathrm{sdim}(B^2_{n,n}) \ge n-2$ and we obtained that $\mathrm{sdim}(B^2_{n,n}) \le n-2$ and $\mathrm{sdim}(B^2_{n,n}) \ge n-2$; therefore, $\mathrm{sdim}(B^2_{n,n}) = n-2$. From the above proving, we conclude that $\mathrm{sdim}(B^2_{n,n}) = n-2$. □

5. Conclusion

The secure metric dimensions of the globe graph and the bistar graph have the same secure metric dimensions $n-2$. The flag graph H- graph of path P_n, bistar graph, tadpole graph $T_{3,m}$, $T_{n,m}$, and subdivision of tadpole graph $T_{3,m}$, $T_{n,m}$ have the same constant secure metric dimension 2.

6. Future Work

In the future, we plan to determine the secure metric dimension of many graphs, such as subdivisions of crown graphs, cocktail party graphs, triangular pyramid graph P_3, and square pyramid graph P_4.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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