Hindawi
Advances in Operations Research
Volume 2024, Article ID 6905897, 22 pages
https://doi.org/10.1155/2024/6905897

*Research Article*

# Malmquist–Luenberger Productivity Index for a Two-Stage Structure in the Presence of Undesirable Outputs and Uncertainty

**Rita Shakouri** [1] **and Maziar Salahi** [2]

[1] *Department of Computer Engineering, Faculty of Mathematical Sciences, Technical and Vocational University (TVU), Tehran, Iran*
[2] *Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Guilan, P.O. Box 1914, Rasht, Iran*

Correspondence should be addressed to Rita Shakouri; rshakori@tvu.ac.ir

Received 12 November 2023; Revised 16 January 2024; Accepted 18 January 2024; Published 6 February 2024

Academic Editor: Yi-Kuei Lin



Network Data Envelopment Analysis (NDEA) models assess the processes of the underlying system at a certain moment and disregard the dynamic effects in the production process. Hence, distorted efficiency evaluation is gained that might give misleading information to decision-making units (DMUs). Malmquist–Luenberger Productivity Index (MPI) assesses efficiency changes over time, which are measured as the product of recovery and frontier-shift terms, both coming from the DEA framework. In this study, a form of MPI involving network structure for evaluating DMUs in the presence of uncertainty and undesirable outputs in two periods of time is presented. To cope with uncertainty, we use the stochastic p-robust approach and the weak disposability of Kuosmanen (American Journal Agricultural Economics 87 (4):1077–1082, 2005) proposed to take care of undesirable outputs. The proposed fractional models for stages and overall system are linearized by applying the Charnes and Cooper transformation. Finally, the proposed models are applied to evaluate the efficiency of 11 petroleum wells to identify the main factors determining their productivity, utilizing the data from the 2020 to 2021 period. The results show that the management of resource consumption, especially equipment and capital, is not appropriate and investment is inadequate. Although the depreciation rate of capital facilities in this industry is high, the purpose of the investment is not to upgrade the level of technology.

## 1. Introduction

Since its beginning (see [1]), data envelopment analysis (DEA) has been applied to evaluate the efficiencies of a collection of decision-making units (DMUs) in which estimating the efficiency frontier does not require the recognition of the production function. Classical DEA models consider each DMU as a black box ignoring the internal relations of processes. However, in real world, DMUs may contain several linked processes. Network DEA (NDEA) models, an extension of classical DEA models, are developed for the efficiency evaluation of DMUs taking into consideration their internal relations via intermediate products in assessing efficiency. However, NDEA models disregard the dynamic effects within the production processes which is the case various real world applications Cook and Zhu [2]. Malmquist–Luenberger Productivity Index (MPI) presented by Malmquist [3] is a quality index for analyzing the consumption of production resources in different time periods. The MPI not only defines patterns of productivity change and renders a new interpretation along with the managerial implication of each Malmquist component but also identifies strategic directions of an organization in the past time periods for proper choice in future periods. Also, it appears as a shape of time series one which includes a particular construction in each period. Table 1 summarizes some of the MPI- DEA developments and applications.

However, in production processes besides desirable outputs, there might be undesirable outputs whose decrease results in improved performance. For example, Pittman et al.

Table 1: Recent advances and applications in MPI-DEA/NDEA.

| Authors | DEA model | Type factor | Applications scope |
|---|---|---|---|
| Yu and Chen [4] | CCR | Output | Airline companies |
| Li et al. [5] | SFA | Output | Forestry company |
| Liang et al. [6] | CCR | Input | Banking sector |
| Wanke et al. [7] | CCR | Output | Banking sector |
| Yang and Zhang [8] | CCR | Output | Regional eco-efficiency |
| Nedaei et al. [9] | CCR | Output | Oil and gas wells |
| Mahmoudi and Emrouznejad [10] | SBM | Output/input | Airline companies |
| Tone et al. [11] | CCR | Output | Insurance companies |

[12] first studied the application of undesirable outputs to do an efficient assessment under the expanded model of Caves et al. [13] so that the efficiency of DMUs could be evaluated in the presence of desirable and undesirable outputs. Then, Tone [14] studied the efficiency of 12 Chinese commercial banks from 2005 to 2013 based on undesirable outputs and investigated the truth that considering undesirable outputs can make research results more reliable by comparing with the result gained without considering undesirable output. Generally, treating undesirable outputs has proven to be a complete challenge for scholars working on DEA (see a critical review of Halkos and Petrou [15]). Table 2 summarizes some recent advances of undesirable output developments in MPI-DEA models.

In all the abovementioned research studies, input and output parameters are considered to be exact and the effect of uncertainty is ignored. Research detected that a small perturbation in the problem data may lead to critical variation in ranking. To treat uncertainty in the DEA models, various approaches such as fuzzy programming, stochastic programming, and robust optimization are used in the literature. Table 3 summarizes recent progress of the DEA models under uncertainty.

Although the present literature has progressed significantly, all of the available DEA models consider either the pure undesirable outputs or uncertainties in problem data. So, in this paper, we present a combined model to measure the performance of DMUs with uncertain perspectives in the presence of undesirable outputs in dynamic settings. In the MPI framework, we apply the stochastic p-robust approach to attain robustness against the existing uncertainty for the CCR-DEA model. The stochastic approach searches to minimize the total expected cost among all scenarios. The optimal solution gained by applying it is probably very good for some scenarios but very poor for others. Also, the weak disposable production technology of Kuosmanen [41] is employed for modelling undesirable outputs which is a widely used technology in dealing with undesirable outputs. The MPI not only reveals patterns of productivity change and presents a new interpretation along with the managerial implication of each Malmquist component but also recognizes the strategy shifts of exclusive companies under isoquant changes. We applied the proposed approach to the dataset of 11 petroleum wells from the 2020 to 2021 period. The contribution of the paper can be summarized as follows:

(i) Assessing the efficiencies of an NDEA system and its internal processes over time by dynamic models, simultaneously in the presence of undesirable outputs and uncertainty

(ii) Applying Kuosmanen's weakly disposable technology that is convex and more flexible with regard to the choice of nonuniform pollution abatement factors and preserving the linear structure

(iii) Describing the uncertainty in two worst-case and the best-case scenarios, defining a robustness level for two-stage DEA-based MPI that reflects the DMU's regress or progress

(iv) Adjusting the conservatism degree in the proposed approach and providing a deterministic solution approach

(v) Computing MPI to evaluate the total efficiency of 11 petroleum wells in two time periods

The remainder of this paper unfolds as follows: in the next section, a summary of a two-stage DEA model is given and a brief review of weakly disposable technology and stochastic $p$-robust approach is as follows. In Section 3, by considering undesirable outputs and p-robust approach, a model is proposed that calculates MPI under the NDEA model. Section 4 presents the efficiency measurement of the overall NDEA and substages. Ultimately, to show the applicability of the proposed approach, it is applied to a real dataset in Section 5, which is followed by conclusions, and some directions for future research are given in the last section.

## 2. Preliminaries

A petroleum well system is commonly considered to be a multistage process with substages each of which includes several initial inputs, intermediate inputs and outputs, feedbacks, and final outputs. Since the complexity index of each petroleum well is different, the operating units dealing with the corresponding refining and purification processes face different requirements across petroleum wells. Here, in order to have a homogenous structure in all petroleum wells, all conversion and purification processes have been considered as a sub-DMU. Figure 1 shows such a structure where the production process of a petroleum well system like a DMU under evaluation has two main stages: the oil production and the wastewater treatment system as the first and second stages, respectively.

Table 2: Recent advances of environmental factors in MPI- DEA/NDEA.

| Authors | DEA model | Environmental factor | Applications scope |
|---|---|---|---|
| Wang and Feng [16] | CCR | Output | Industrial |
| Zhu and Lin [17] | CCR | Output | Iron and steel industry |
| Li et al. [18] | SBM | Output/input | Industrial systems |
| Li et al. [19] | BCC | Output | Forestry |
| Aghayi et al. [20] | CCR | Output | Banking industry |
| Shirazi and Mohammadi [21] | CCR | Output | Airline |
| Toloo and Hanclova [22] | CCR | Output | Countries |
| An et al. [23] | SBM | Output/input | Trunk streams |
| Asanimoghadam et al. [24] | ASBM | Output | Industrial airline |
| Salahi et al. [25] | ASBM | Output | Provinces in China |
| Shakouri and Salahi [26] | CCR | Output | Oil generation |
| Zhang et al. [27] | CCR | Output | Industrial system |
| Chen et al. [28] | SBM | Output/input | Public health center |
| Arabi et al. [29] | SBM | Output/input | Power plants |

Table 3: Recent progress in stochastic, fuzzy, and robust optimization with MPI in the NDEA and DEA.

| Authors | DEA/uncertainty parameters | Robust approach | Applications scope |
|---|---|---|---|
| Peykani et al. [30] | CCR | Fuzzy | Investment firms |
| Dar et al. [31] | CCR/BCC/input | SFA | Banking sector |
| Salahi et al. [32] | CCR/CSW/in-output | Interval | Energy/forest district |
| Salahi et al. [33] | Russell/in-output | Interval | Banking sector |
| Salahi et al. [34] | CCR-CSW/output | Bertsimas | Banking sector |
| Soltanzadeh and Omrani [35] | CCR/in-output | Fuzzy | Airline companies |
| Akbarian [36] | BCC | Interval | Numerical example |
| Shakouri et al. [37] | CCR/input | Stochastic $p$-robust | Banking sector |
| Shakouri et al. [38] | CCR | Stochastic $p$-robust | Banking sector |
| Mehdizadeh et al. [39] | CCR | Stochastic | Commercial banks |
| Tavana et al. [40] | CCR | Fuzzy | Refinary |

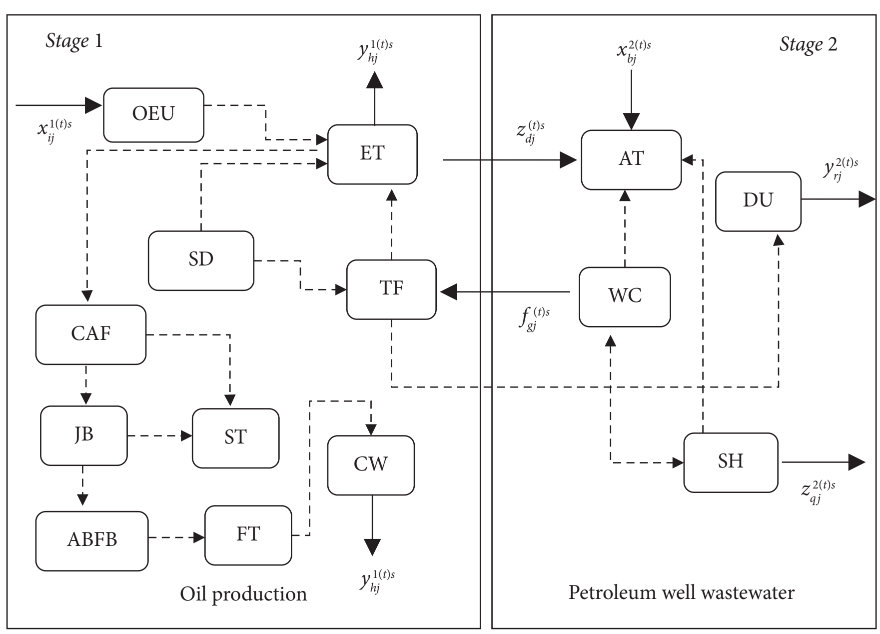


Figure 1: A dynamic petroleum well system for NDEA proposed model. OW: oil extraction unit; ET: equalization tanks; AT: aeration tanks; SD: sludge dewatering; TF: tertiary filters; CAF: coagulation air flotation; JB: JBILAFB; ST: sludge thickener; ABFB: aerobic biological fluidize bed; FT: filter tank; CW: clean water tank; WC: wastewater clarifiers; DU: disinfection units; SH: sludge holding tanks.

Suppose there are $n$ DMUs each of which consists of two sub-DMUs sequentially. In the first stage, each DMU$_j$ ($j = 1, \ldots, n$) uses $m$ inputs $x_{ij}^{1(t)s}$ ($i = 1, \ldots, m$) and produces $H$ final outputs $y_{hj}^{1(t)s}$ ($h = 1, \ldots, H$) and $D$ intermediate outputs $z_{dj}^{(t)s}$ ($d = 1, \ldots, D$) based on the scenario $s \epsilon S$ that assist as the inputs to the second stage. Each DMU consists of two sub-DMUs sequentially, and undesirable outputs from Stage 2 are feedbacks that are sent back as inputs to Stage 1. Also, there are $B$ inputs $x_{bj}^{2(t)s}$ $(b = 1, \ldots, B)$ to the second stage under scenario $s \epsilon S$. Outputs from the second stage take three forms; desirable outputs $y_{rj}^{2(t)s}$ $(r = 1, \ldots, R)$, undesirable outputs $z_{qj}^{2(t)s}$ $(q = 1, \ldots, Q)$, and a feedback variable $f_{gj}^{(t)s}$ $(g = 1, \ldots, G)$ in time $t$ based on the scenario $s \epsilon S$. It is noteworthy that the bold lines in Figure 1 show the used variables in this paper. Further, for each DMUj, the efficiency scores of the first and the second stages are denoted by $e_1^{(t)s}(t)$ and $e_2^{(t)s}(t)$, respectively, under the $s^{\text{th}}$ scenario when all DMUs under evaluation are in time $t$. Also, the efficiency score of the overall process when all DMUs under the assessment for the $s^{\text{th}}$ scenario in period $t$ is shown by $e_o^{(t)*}(t)$.

*2.1. Undesirable Outputs.* A production process may consist of both desirable and undesirable outputs. To take care of undesirable outputs in DEA models, different approaches are developed in the DEA literature (Chavas and Cox [42]; Hailu and Veeman [43]). Weak disposability is an alternative method that models undesirable emissions as outputs, imposing an assumption that these undesirable outputs are weakly disposable. In general, weak disposability means that it is possible to abate emissions by decreasing the level of production activity. Kuosmanen [41] defined a production technology using weakly disposable axiom of outputs to model undesirable outputs in the DEA framework. Based on this technology, inputs and desirable outputs are presented to be freely disposable. Weak disposability hypothesis is used to propose a modern DEA approach for evaluating efficiency of DMUs by taking undesirable outputs into account. This approach is a significant development in computing the efficiency of DMUs with undesirable outputs. The linear programming model of this technology to evaluate the performance of a DMU in time intervals of $t$ is as follows [44]:

$$
\begin{aligned}
&\max \sum_{r=1}^{A} u_r y_{rj_o}^{2(t)s} - \sum_{q=1}^{D} \vartheta_q z_{qj_o}^{2(t)s} - \sum_{i=1}^{m} v_i x_{ij}^{1(t)s},\\
&\text{s.t.} \sum_{r=1}^{A} u_r y_{rj}^{2(t)s} - \sum_{q=1}^{D} \vartheta_q z_{qj}^{2(t)s} - \sum_{i=1}^{m} v_i x_{ij}^{1(t)s} \le 0, \quad \forall j, s \epsilon S,\\
&\sum_{i=1}^{m} v_i x_{ij}^{1(t)s} = 1,\\
&u_r, v_i \ge 0, \quad \forall r, i, \vartheta_q \text{ free } \forall q.
\end{aligned}
\tag{1}
$$

In model (1), $v_i$, $u_r$, and $\vartheta_q$ are decision variables of inputs, desirable outputs, and undesirable outputs, respectively. Constraints (2) guarantee that the efficiency value is less than or equal to one for each DMU.

*2.2. Stochastic p-Robust Concept.* Let $S$ be a collection of scenarios, and $P^{(t)s}$ be a deterministic maximization problem for each scenario $s$ in time $t$ (there is a different problem $P^{(t)s}$ for each scenario $\epsilon S$). For each $s$, let $\mathrm{M}_0^{(t)s*} > 0$ be the optimal efficiency score for $P^{(t)s}$ in time $t$. So, suppose that X is a feasible solution to $P^{(t)s}$ for all $s \epsilon S$, and let $\mathrm{M}_0^{(t)s}(X)$ be the efficiency score of $P^{(t)s}$ under solution X in time $t$. Then, X is called $p$-robust, if for all $s \epsilon S$, the following inequality holds:

$$p \ge \frac{\mathrm{M}_0^{(t)s*} - \mathrm{M}_0^{(t)s}(X)}{\mathrm{M}_0^{(t)s*}}. \tag{2}$$

In (2), the right hand side is the relative regret for scenarios in time $t$, and $p \ge 0$ is a constant that limits the relative regret for each scenario. It is obvious that inequality (2) can be written as follows:

$$(1 - p)\mathrm{M}_0^{(t)s*} \le \mathrm{M}_0^{(t)s}(X). \tag{3}$$

Therefore, for controlling the relative regret relative to all scenarios, the $p$-robust constraints (3) are added to the models.

*Definition 1.* $\text{DMU}_j$ is stochastic $p$-robust efficient in different scenarios if and only if its optimal objective function is one.

## 3. Two-Stage NDEA Model under Undesirable Outputs and Uncertainty

In this section, first, we present a two-stage model in the presence of undesirable outputs, and then it is combined with the p-robust approach to handle the uncertainty. To evaluate the overall efficiency of the whole NDEA model in Figure 1 in time period $t$, we compound the weighted average of the two stages as follows:

$$
\begin{aligned}
&e_o^{(t)*}(t) = \left(\max \xi_1^{(t)s} e_1^{(t)s}(t) + \xi_2^{(t)s} e_2^{(t)s}(t)\right),\\
&\text{s.t. } e_1^{(t)s}(t) = \frac{\sum_{h=1}^{H} \eta_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} w_d z_{dj}^{(t)s}}{\sum_{i=1}^{m} v_i x_{ij}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj}^{(t)s}} \le 1, \quad \forall j, s \epsilon S,\\
&e_2^{(t)s}(t) = \frac{\sum_{r=1}^{s} u_r y_{rj}^{2(t)s} + \sum_{g=1}^{G} \partial_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \vartheta_q z_{qj}^{2(t)s}}{\sum_{d=1}^{D} w_d z_{dj}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj}^{2(t)s}} \le 1, \quad \forall j, s \epsilon S,\\
&u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \ge 0, \quad \forall r, d, g, b, h, i, q,
\end{aligned}
\tag{4}
$$

where on the basis of the radial CRS-DEA model of Charnes et al. [1], $e_1^{(t)s}$ and $e_2^{(t)s}$ are the efficiency values of the first and the second stages in time $t$ and $\xi_1^{(t)s}$ and $\xi_2^{(t)s}$ show the corresponding weights of stages, respectively, reflecting the importance of the two stages in the overall system ($\xi_1^{(t)s} + \xi_2^{(t)s} = 1$). We let $\xi_1^{(t)s} = (\sum_{i=1}^m v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^G \partial_g f_{gj_o}^{(t)s}) / (\sum_{i=1}^m v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^G \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^D w_d z_{dj_o}^{(t)s} + \sum_{t=1}^T \delta_b x_{bj_o}^{2(t)s})$ and $\xi_2^{(t)s} = (\sum_{d=1}^D w_d z_{dj_o}^{(t)s} + \sum_{t=1}^T \delta_b x_{bj_o}^{2(t)s}) / (\sum_{i=1}^m v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^G \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^D w_d z_{dj_o}^{(t)s} + \sum_{t=1}^T \delta_b x_{bj_o}^{2(t)s})$ in order to linearize the model. Therefore, model (4) becomes

$$
\begin{aligned}
e_o^{(t)s}(t) &= \max \frac{\sum_{h=1}^H \eta_h y_{hj_o}^{1(t)s} + \sum_{d=1}^D w_d z_{dj_o}^{(t)s} + \sum_{r=1}^s u_r y_{rj_o}^{2(t)s} + \sum_{g=1}^G \partial_g f_{gj_o}^{(t)s} - \sum_{q=1}^Q \vartheta_q z_{qj_o}^{2(t)s}}{\sum_{i=1}^m v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^G \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^D w_d z_{dj_o}^{(t)s} + \sum_{t=1}^T \delta_b x_{bj_o}^{2(t)s}},\\
\text{s.t. } e_1^{(t)s}(t) &= \frac{\sum_{h=1}^H \eta_h y_{hj}^{1(t)s} + \sum_{d=1}^D w_d z_{dj}^{(t)s}}{\sum_{i=1}^m v_i x_{ij}^{1(t)s} + \sum_{g=1}^G \partial_g f_{gj}^{(t)s}} \le 1, \quad \forall j, s \in S,\\
e_2^{(t)s}(t) &= \frac{\sum_{r=1}^s u_r y_{rj}^{2(t)s} + \sum_{g=1}^G \partial_g f_{gj}^{(t)s} - \sum_{q=1}^Q \vartheta_q z_{qj}^{2(t)s}}{\sum_{d=1}^D w_d z_{dj}^{(t)s} + \sum_{t=1}^T \delta_b x_{bj}^{2(t)s}} \le 1, \quad \forall j, s \in S,\\
& u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \ge 0, \quad \forall r, d, g, b, h.
\end{aligned}
\tag{5}
$$

Now, let $t_1 = (\sum_{i=1}^m v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^G \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^D w_d z_{dj_o}^{(t)s} + \sum_{t=1}^T \delta_b x_{bj_o}^{2(t)s})^{-1}$, $\overline{v_i} = t_1 v_i$, $\overline{\partial}_g = t_1 \partial_g$, $\overline{\eta}_h = t_1 \eta_h$, $\overline{u}_r = t_1 u_r$, $\overline{\delta}_b = t_1 \delta_b$, $\overline{w}_d = t_1 w_d$, and $\overline{\vartheta}_q = t_1 \vartheta_q$, then model (5) is transformed into the following linear model:

$$
\begin{aligned}
e_o^{(t)s*}(t) &= \max\left(\sum_{h=1}^H \overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^D \overline{w}_d z_{dj}^{(t)s} + \sum_{r=1}^s \overline{u}_r y_{rj_o}^{2(t)s} + \sum_{g=1}^G \overline{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^Q \overline{\vartheta}_q z_{qj_o}^{2(t)s}\right),\\
&\text{s.t. } \sum_{h=1}^H \overline{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^D \overline{w}_d z_{dj}^{(t)s} - \sum_{i=1}^m \overline{v_i} x_{ij}^{1(t)s} - \sum_{g=1}^G \overline{\partial}_g f_{gj}^{(t)s} \le 0, \quad \forall j, s \in S,\\
&\sum_{r=1}^s \overline{u}_r\, y_{rj}^{2(t)s} + \sum_{g=1}^G \overline{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^Q \overline{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^D \overline{w}_d\, z_{dj}^{(t)s} - \sum_{t=1}^T \overline{\delta}_b x_{bj}^{2(t)s} \le 0, \quad \forall j, s \in S,\\
&\sum_{i=1}^m \overline{v_i} x_{ij_o}^{1(t)s} + \sum_{g=1}^G \overline{\partial}_g f_{gj_o}^{(t)s} + \sum_{d=1}^D \overline{w}_d\, z_{dj_o}^{(t)s} + \sum_{t=1}^T \overline{\delta}_b x_{bj_o}^{2(t)s} = 1, \quad s \in S,\\
&u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \ge 0, \quad \forall r, d, g, b, h.
\end{aligned}
\tag{6}
$$

As before, using Charnes and Cooper [45] transformation, both stages are linearized as follows:

$$
\begin{aligned}
e_1^{(t)s}(t) &= \max\left(\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{(t)s}\right),\\
&\text{s.t.} \sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{(t)s} \geq (1-p) e_{10}^{(t)s*}, \quad s \in S,\\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t)s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1(t)s} - \sum\nolimits_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} \leq 0, \quad \forall j, s \in S,\\
&\sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1(t)s} + \sum\nolimits_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} = 1, \quad s \in S,\\
&\overline{\partial}_g, \overline{w}_d, \overline{\eta}_h, \overline{v}_i \geq 0, \quad \forall g, d, h, i,
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
e_2^{(t)s}(t) &= \max\left(\sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2(t)s} + \sum\nolimits_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t)s}\right),\\
&\text{s.t.} \sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2(t)s} + \sum\nolimits_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t)s} \geq (1-p) e_0^{2(t)s*}, \quad s \in S,\\
&\sum_{r=1}^{s} \overline{u}_r\, y_{rj}^{2(t)s} + \sum\nolimits_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^{D} \overline{w}_d\, z_{dj}^{(t)s} - \sum_{b=1}^{B} \overline{\delta}_b x_{bj}^{2(t)s} \leq 0, \quad \forall j, s \in S,\\
&\sum_{d=1}^{D} \overline{w}_d\, z_{dj_o}^{(t)s} + \sum\nolimits_{t=1}^{T} \overline{\delta}_b x_{bj_o}^{2(t)s} = 1, \quad s \in S,\\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1(t)s} - \sum\nolimits_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} \leq 0, \quad \forall j, s \in S,\\
&\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d\, z_{dj_o}^{(t)s} - e_0^{1(t)s*}\left(\sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s}\right) = 0, \quad s \in S,\\
&u_r, \overline{w}_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q > 0, \quad \forall r, d, g, b, h, i, q.
\end{aligned}
\tag{8}
$$

*Definition 2.* The two-stage process is efficient if and only if $e_1^{(t)s}(t) = e_2^{(t)s}(t) = 1$.

Now, to take care of uncertainty, formula (3) can be merged with the expected objective function of the model (6), and in order to control the relative regret related to the scenarios, the $p$-robust restrictions are added to model (6). Thus, the efficiency value for the stochastic $p$-robust version of model (6) is as follows:

$$
\mathrm{M}_0^{(t)s*}(t) = \max \sum_{s=1}^{S} q^s \left[\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t)s} + \sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t)s}\right], \tag{9.a}
$$

$$
\text{s.t.} \sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t)s} + \sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t)s} \geq (1-p) e_0^{(t)s*}, \quad \forall s \in S, \tag{9.b}
$$

$$
\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} \leq 0, \quad \forall j, \forall s \in S, \tag{9.c}
$$

$$
\sum_{r=1}^{s} \overline{u}_r\, y_{rj}^{2(t)s} + \sum\nolimits_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^{D} \overline{w}_d\, z_{dj}^{(t)s} - \sum_{t=1}^{T} \overline{\delta}_b x_{bj}^{2(t)s} \leq 0, \quad \forall j, \forall s \in S, \tag{9.d}
$$

$$\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}+\sum_{b=1}^{B}\overline{\delta}_b x_{bj_o}^{2(t)s}=1,\quad \forall s\in S, \tag{9.e}$$

$$u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \geq 0,\quad \forall\, r, d, g, b, h. \tag{9.f}$$

*Remark 3.* By considering $f_{ko}^{(t)s}=\max\{f_{gj_o}^{(t)s}\mid 1\leq g\leq G\}>0$, $x_{ko}^{(t)s}=\max\{x_{io}^{1(t)s}\mid 1\leq i\leq m\}>0$, $\max\{z_{dj_o}^{(t)s}\mid 1\leq d\leq D\}>0$, and $\max\{x_{bj_o}^{2(t)s}\mid 1\leq b\leq B\}>0$, then setting $(\overline{\eta}_1,\ldots,\overline{w}_1,\ldots,\overline{v}_1,\ldots,\overline{\partial}_1,\ldots,\overline{u}_1,\ldots,\overline{\vartheta}_1,\ldots,\overline{\delta}_1,\ldots)=(0,\ldots,1/z_{dj_o}^{(t)s}, 1/x_{ko}^{1(t)s}, 1/f_{ko}^{(t)s}, 0,\ldots,0,\ldots,1/x_{bj_o}^{2(t)s})$, restrictions $\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}+\sum_{b=1}^{B}\overline{\delta}_b x_{bj_o}^{2(t)s}=1$, and $\sum_{r=1}^{s}\overline{u}_r y_{rj}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj}^{2(t)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj}^{2(t)s}\leq 0$, $\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t)s}\leq 0$, imply $\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}+\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t)s}\leq 1$. Thus, we achieve $e_0^{(t)s*}\leq 1/1-p$. So, for very small $p$s, there may not be $p$-robust solutions for model (9a)–(9f) in periods $t$ and $t+1$ based on scenario $s$; therefore, it may be infeasible.

It is noteworthy that models for the first and the second stages can be linearized similar to the overall efficiency. Models (9a)–(9f) evaluate the relative efficiency of the whole system, and the data for all DMUs are retrieved from period $t$. The objective function of model (9a)–(9f) is to maximize the expected efficiency value of all DMUs. Further, $q^s$ in the objective function is the probability that scenario $s$ happens. The first constraint in all models is called the $p$-robust constraint that may not allow the scenario's efficiency to take value more than $100(1-p)\%$ of the ideal efficiency scores gained by each scenario. The relative regret between all scenarios is controlled by the parameter $p$. The $p$-robust constraints in this model become ineffective if $p=\infty$. It is noteworthy that the $p$ values generally are assumed greater than 0.2 and their upper bound is gained by try and error. Also, these values can be different for any problem and are usually defined by the decision-maker.

*Definition 4.* If $M_0^{(t)s*}(t)=1$ in model (9a)–(9f), then $\text{DMU}_o$ is efficient.

## 4. Malmquist–Luenberger Productivity Index

In this section, first, we compute the efficiency of the overall and the substages NDEA process in periods $t$ and $t+1$, and finally, the MPI is calculated. To measure the efficiency of models (9a)–(9f) when the data for the DMU under evaluation is retaken from period $t+1$ while the data for the other DMUs are retaken from period $t$, the following model applies:

$$M_0^{(t+1)s*}(t)=\max\sum_{s=1}^{S}q^s\left[\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}+\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t+1)s}\right], \tag{10.a}$$

$$\text{s.t.}\ \sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}+\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t+1)s}\geq(1-p)e_0^{(t+1)s*},\quad \forall s\in S, \tag{10.b}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t)s}\leq 0,\quad \forall j,\forall s\in S, \tag{10.c}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}\leq 0,\quad \forall j,\forall s\in S, \tag{10.d}$$

$$\sum_{r=1}^{s}\overline{u}_r y_{rj}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj}^{2(t)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj}^{2(t)s}\leq 0,\quad \forall j,\forall s\in S, \tag{10.e}$$

$$\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t+1)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj_o}^{2(t+1)s}\leq 0,\quad \forall j,\forall s\in S, \tag{10.f}$$

$$\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}+\sum_{t=1}^{T}\overline{\delta}_b x_{bj_o}^{2(t+1)s}=1,\quad \forall s\in S,$$
$$u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \geq 0,\quad \forall r, d, g, b, h. \tag{10.g}$$

In a similar way, to scale the efficiency of the overall network when the data for the DMU under the estimate are recovered from period $t$ while the data for the other DMUs are recovered from period $t+1$, models (11a)–(11g) are provided:

$$\mathrm{M}_0^{(t)s*}(t+1)=\max\sum_{s=1}^{S}q^s\left[\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}+\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t)s}\right], \tag{11.a}$$

$$\text{s.t.}\ \sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}+\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t)s}\geq(1-p)e_0^{(t)s*},\quad \forall s\in S,$$
$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t+1)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s}\leq 0,\quad \forall j, \forall s\in S, \tag{11.b}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}\leq 0,\quad \forall j, \forall s\in S, \tag{11.c}$$

$$\sum_{r=1}^{s}\overline{u}_r\, y_{rj}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj}^{2(t+1)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj}^{2(t+1)s}\leq 0,\quad \forall j, \forall s\in S, \tag{11.d}$$

$$\sum_{r=1}^{s}\overline{u}_r\, y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj_o}^{2(t)s}\leq 0,\quad \forall j, \forall s\in S, \tag{11.e}$$

$$\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}+\sum_{t=1}^{T}\overline{\delta}_b x_{bj_o}^{2(t)s}=1,\quad \forall s\in S, \tag{11.f}$$

$$u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \geq 0,\quad \forall r, d, g, t, h. \tag{11.g}$$

Eventually, models (12a)–(12f) are suggested to compute the relative efficiency of the overall NDEA when the data for all DMUs, including the DMU under evaluation, are retaken from period $t+1$ based on scenario $s$ as below:

$$\mathrm{M}_0^{(t+1)s*}(t+1)=\max\sum_{s=1}^{S}q^s\left[\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}+\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t+1)s}\right], \tag{12.a}$$

$$\text{s.t.}\ \sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}+\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t+1)s}\geq(1-p)e_0^{(t+1)s*},\quad \forall s\in S, \tag{12.b}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t+1)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s}\leq 0,\quad \forall j, \forall s\in S, \tag{12.c}$$

$$\sum_{r=1}^{s}\overline{u}_r y_{rj}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj}^{2(t+1)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj}^{2(t+1)s}\le 0,\quad \forall j,\forall s\epsilon S, \tag{12.d}$$

$$\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}+\sum_{t=1}^{T}\overline{\delta}_b x_{bj_o}^{2(t+1)s}=1,\quad \forall s\epsilon S, \tag{12.e}$$

$$u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \ge 0,\quad \forall r, d, g, b, h. \tag{12.f}$$

*4.1. Efficiency Scaling of the First Stage through Both Periods.* In this subsection, the efficiency of the first stage in the presence of undesirable outputs and uncertainty is calculated. Models (13a)–(13e) compute the maximum achievable value for the efficiency of the first stage under the $s^{\text{th}}$ scenario in periods $t$ and $t+1$:

$$M_{10}^{(t)s*}(t)=\max\sum_{s=1}^{S}q^s\left[\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}\right], \tag{13.a}$$

$$\text{s.t.}\ \sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}\ge(1-p)e_{10}^{(t)s*},\quad \forall s\epsilon S, \tag{13.b}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t)s}\le 0,\quad \forall j,\forall s\epsilon S, \tag{13.c}$$

$$\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}=1,\quad \forall s\epsilon S, \tag{13.d}$$

$$\overline{\partial}_g, \overline{w}_d, \overline{\eta}_h, \overline{v}_i \ge 0,\quad \forall g, d, h, i. \tag{13.e}$$

The objective function of models (13a)–(13e) maximizes the expected efficiency value of $\text{DMU}_0$ when it is under evaluation according to scenarios' data in the first stage. In the objective function, $q^s$ is the probability that scenario $s$ happens (it is unclear which scenario will happen in the future, and in other words, there is no information about the probability of chance of each scenario). In this model, the uncertainty in the parameters is defined by discrete scenarios. The first set of constraints imposes the p-robust measure associated with all scenarios. This set of constraints may not allow the scenario efficiency taking a value of more than $100(1-p)\%$ of the ideal efficiency score obtained by each scenario. The parameter $p$ can flexibly control the relative regret among all scenarios. Note that if $p=\infty$ than the $p$-robust constraints become inactive and model (13a)–(13e) may become infeasible if the $p$ is very small. The rest of the constraints which must be held for all $s\epsilon S$.

Models (14a)–(14f) are presented to scale the efficiency of Stage 1 when the data for the DMU in evaluation are recovered from period $t+1$ while the data for the other DMUs are recovered from period $t$ as below:

$$M_{10}^{(t+1)s*}(t)=\max\sum_{s=1}^{S}q^s\left[\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}\right], \tag{14.a}$$

$$\text{s.t.}\ \sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}\ge(1-p)e_{10}^{(t+1)s*},\quad \forall s\epsilon S, \tag{14.b}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t)s}\le 0,\quad \forall j,\forall s\epsilon S, \tag{14.c}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s} - \sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s} - \sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s} \le 0, \quad \forall j, \forall s \in S, \tag{14.d}$$

$$\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s} + \sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s} = 1, \quad \forall s \in S, \tag{14.e}$$

$$\overline{\partial}_g, \overline{w}_d, \overline{\eta}_h, \overline{v}_i \ge 0, \quad \forall g, d, h, i. \tag{14.f}$$

Also, models (15a)–(15e) are applied to scale the efficiency of Stage 1 when the data for the DMU in evaluation are recovered from period $t$ while the data for the other DMUs are recovered from period $t+1$:

$$\mathrm{M}_{10}^{(t)s*}(t+1) = \max\sum_{s=1}^{S} q^s\left[\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1ts} + \sum_{d=1}^{D}\overline{w}_d\, z_{dj_o}^{ts}\right], \tag{15.a}$$

$$\text{s.t.} \sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1ts} + \sum_{d=1}^{D}\overline{w}_d\, z_{dj_o}^{ts} \ge (1-p)e_{10}^{ts*}, \quad \forall s \in S, \tag{15.b}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t+1)s} + \sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s} - \sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t+1)s} - \sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s} \le 0, \quad \forall j, \forall s \in S, \tag{15.c}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s} - \sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s} - \sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s} \le 0, \quad \forall j, \forall s \in S, \tag{15.d}$$

$$\begin{gathered}\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1ts} + \sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{ts} = 1, \quad \forall s \in S,\\ \overline{\partial}_g, \overline{w}_d, \overline{\eta}_h, \overline{v}_i \ge 0, \quad \forall g, d, h, i.\end{gathered} \tag{15.e}$$

Finally, models (16a)–(16e) are presented to evaluate the efficiency of Stage 1 when the data for the DMU under evaluation are retaken from period $t$ while the data for the other DMUs are retaken from period $t+1$:

$$\mathrm{M}_{10}^{(t+1)s*}(t+1) = \max\sum_{s=1}^{S} q^s\left[\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}\right], \tag{16.a}$$

$$\text{s.t.} \sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s} \ge (1-p)e_{10}^{(t+1)s*}, \quad \forall s \in S, \tag{16.b}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t+1)s} + \sum_{d=1}^{D}\overline{w}_d\, z_{dj}^{(t+1)s} - \sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t+1)s} - \sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s} \le 0, \quad \forall j, \forall s \in S, \tag{16.c}$$

$$\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s} + \sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s} = 1, \quad \forall s \in S, \tag{16.d}$$

$$\overline{\partial}_g, \overline{w}_d, \overline{\eta}_h, \overline{v}_i \geq 0, \quad \forall\, g, d, h, i. \tag{16.e}$$

*4.2. Efficiency Assessment of the Second Stage through Both Periods.* The efficiency value of the second stage in the presence of undesirable outputs and uncertainty is defined as models (17a)–(17g). It evaluates the efficiency of Stage 2 when the data for the DMU under valuation are regained from period $t+1$ while the data for the other DMUs are regained from period $t$:

$$\mathrm{M}_{20}^{(t)s*}(t) = \max \sum_{s=1}^{S} q^s \left[ \sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t)s} \right], \tag{17.a}$$

$$\text{s.t.} \sum_{r=1}^{s} \overline{u}_r y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t)s} \geq (1-p) e_0^{2(t)s*}, \quad \forall s \in S, \tag{17.b}$$

$$\sum_{r=1}^{s} \overline{u}_r y_{rj}^{2(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t)s} - \sum_{b=1}^{B} \overline{\delta}_b x_{bj}^{2(t)s} \leq 0, \quad \forall j, \forall s \in S, \tag{17.c}$$

$$\sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{(t)s} + \sum\nolimits_{t=1}^{T} \overline{\delta}_b x_{bj_o}^{2(t)s} = 1, \quad \forall s \in S, \tag{17.d}$$

$$\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t)s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} \leq 0, \quad \forall j, \forall s \in S, \tag{17.e}$$

$$\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{(t)s} - e_0^{1(t)s*} \left( \sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t)s} \right) = 0, \quad \forall s \in S, \tag{17.f}$$

$$u_r, \overline{w}_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q > 0, \quad \forall r, d, g, b, h, i, q. \tag{17.g}$$

Models (18a)–(18i) are expanded to measure the efficiency value of Stage 2 as follows, where DMU at period $t+1$ and the frontier at period $t$:

$$\mathrm{M}_{20}^{(t+1)s*}(t) = \max \sum_{s=1}^{S} q^s \left[ \sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t+1)s} \right], \tag{18.a}$$

$$\text{s.t.} \sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t+1)s} \geq (1-p) e_0^{2(t+1)s*}, \quad \forall s \in S, \tag{18.b}$$

$$\sum_{r=1}^{s} \overline{u}_r\, y_{rj}^{2(t)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t)s} - \sum_{t=1}^{T} \overline{\delta}_b x_{bj}^{2(t)s} \leq 0, \quad \forall j, \forall s \in S, \tag{18.c}$$

$$\sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t+1)s} - \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{(t+1)s} - \sum_{t=1}^{T} \overline{\delta}_b x_{bj_o}^{2(t+1)s} \leq 0, \quad \forall j, \forall s \in S, \tag{18.d}$$

$$\sum_{d=1}^{D} \overline{w}_d\, z_{dj_o}^{(t+1)s} + \sum_{t=1}^{T} \overline{\delta}_b x_{bj_o}^{2(t+1)s} = 1, \quad \forall s \in S, \tag{18.e}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t)s}\leq 0,\quad \forall j,\forall s\in S, \tag{18.f}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}\leq 0,\quad \forall j,\forall s\in S, \tag{18.g}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t+1)s}-e_0^{1(t+1)s*}\left(\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t+1)s}\right)=0,\quad \forall s\in S, \tag{18.h}$$

$$u_r,\overline{w}_d,\partial_g,\delta_b,\eta_h,v_i,\vartheta_q>0,\quad \forall r,d,g,b,h,i,q. \tag{18.i}$$

Likewise, models (19a)–(19i) are introduced to compute the efficiency value of Stage 2 when the data for the DMU under evaluation are retaken from period $t$ while the data for the other DMUs are retaken from period $t+1$ as follows:

$$\mathrm{M}_{20}^{(t)s*}(t+1)=\max\sum_{s=1}^{S}q^s\left[\sum_{r=1}^{s}\overline{u}_r y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t)s}\right], \tag{19.a}$$

$$\text{s.t.}\ \sum_{r=1}^{s}\overline{u}_r\, y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t)s}\geq(1-p)e_0^{2(t)s*},\quad \forall s\in S, \tag{19.b}$$

$$\sum_{r=1}^{s}\overline{u}_r\, y_{rj}^{2(t+1)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj}^{2(t+1)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj}^{2(t+1)s}\leq 0,\quad \forall j,\forall s\in S, \tag{19.c}$$

$$\sum_{r=1}^{s}\overline{u}_r\, y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\overline{\vartheta}_q z_{qj_o}^{2(t)s}-\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}-\sum_{t=1}^{T}\overline{\delta}_b x_{bj_o}^{2(t)s}\leq 0,\quad \forall j,\forall s\in S, \tag{19.d}$$

$$\sum_{d=1}^{D}\overline{w}_d\, z_{dj_o}^{(t)s}+\sum_{t=1}^{T}\overline{\delta}_b x_{bj_o}^{2(t)s}=1,\quad \forall s\in S, \tag{19.e}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj}^{1(t+1)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj}^{(t+1)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij}^{1(t+1)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj}^{(t+1)s}\leq 0,\quad \forall j,\forall s\in S, \tag{19.f}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}-\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s}-\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}\leq 0,\quad \forall j,\forall s\in S, \tag{19.g}$$

$$\sum_{h=1}^{H}\overline{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\overline{w}_d z_{dj_o}^{(t)s}-e_0^{1(t)s*}\left(\sum_{i=1}^{m}\overline{v}_i x_{ij_o}^{1(t)s}+\sum_{g=1}^{G}\overline{\partial}_g f_{gj_o}^{(t)s}\right)=0,\quad \forall s\in S, \tag{19.h}$$

$$u_r,\overline{w}_d,\delta_b,\eta_h,v_i,\vartheta_q>0,\quad \forall r,d,g,b,h,i,q. \tag{19.i}$$

Finally, models (20a)–(20g) compute the efficiency value of Stage 2 when the data for all DMUs, containing the DMU under evaluation, are retaken from period $t+1$ as follows:

$$M_{20}^{(t+1)s*}(t+1) = \max \sum_{s=1}^{S} q^{s}\left[\sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t+1)s}\right], \tag{20.a}$$

$$\text{s.t.} \sum_{r=1}^{s} \overline{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj_o}^{2(t+1)s} \geq (1-p)e_0^{2(t+1)s*}, \quad \forall s \epsilon S, \tag{20.b}$$

$$\sum_{r=1}^{s} \overline{u}_r\, y_{rj}^{2(t+1)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t+1)s} - \sum_{q=1}^{Q} \overline{\vartheta}_q z_{qj}^{2(t+1)s} - \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t+1)s} - \sum_{t=1}^{T} \overline{\delta}_b x_{bj}^{2(t+1)s} \leq 0, \quad \forall j, \forall s \epsilon S, \tag{20.c}$$

$$\sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{(t+1)s} + \sum_{t=1}^{T} \overline{\delta}_b x_{bj_o}^{2(t+1)s} = 1, \quad \forall s \epsilon S, \tag{20.d}$$

$$\sum_{h=1}^{H} \overline{\eta}_h y_{hj}^{1(t+1)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj}^{(t+1)s} - \sum_{i=1}^{m} \overline{v}_i x_{ij}^{1(t+1)s} - \sum_{g=1}^{G} \overline{\partial}_g f_{gj}^{(t+1)s} \leq 0, \quad \forall j, \forall s \epsilon S, \tag{20.e}$$

$$\sum_{h=1}^{H} \overline{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \overline{w}_d z_{dj_o}^{(t+1)s} - e_0^{1(t+1)s*}\left(\sum_{i=1}^{m} \overline{v}_i x_{ij_o}^{1(t+1)s} + \sum_{g=1}^{G} \overline{\partial}_g f_{gj_o}^{(t+1)s}\right) = 0, \quad \forall s \epsilon S, \tag{20.f}$$

$$u_r, \overline{w}_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q > 0, \quad \forall r, d, g, b, h, i, q. \tag{20.g}$$

Finally, to assess the progress or regress of a DMU, the MPI is calculated using (21) as technological references to scale the variation in productivity taking place across periods $t$ and $t+1$:

$$\mathrm{MPI}_j(t) = \left[\frac{M_j^{(t)s*}(t+1) M_j^{(t+1)s*}(t+1)}{M_j^{(t)s*}(t) M_j^{(t+1)s*}(t)}\right]^{1/2}, \quad \forall j. \tag{21}$$

In (21), the whole system is considered as a DMU and it explains that productivity decreases if the value of the index is lower than one, stays unchanged if it equals one, and amends if it is larger than one. Based on the value of MPI, the trend of productivity is as follows:

(i) $\mathrm{MPI}_j(t) > 1$ shows increase or progress in the productivity of the $\mathrm{DMU}_0$

(ii) $\mathrm{MPI}_j(t) = 1$ reflects no changes in the productivity during the two periods of the $\mathrm{DMU}_0$

(iii) $\mathrm{MPI}_j(t) < 1$ reveals regress in the productivity of $\mathrm{DMU}_0$

Given the efficiency scores gained from models (9a)–(9f) to (12a)–(12f), the MPI for the whole process of the $j^{\text{th}}$ DMU can be computed utilizing (21). In a similar way, the efficiency scores deduced from models (13a)–(13e) to (16a)–(16e) can be applied to compute the MPI for the first stage of the $j^{\text{th}}$ DMU utilizing

$$\mathrm{MPI}_{1j}(t) = \left[\frac{M_{1j}^{(t)s*}(t+1) M_{1j}^{(t+1)s*}(t+1)}{M_{1j}^{(t)s*}(t) M_{1j}^{(t+1)s*}(t)}\right]^{1/2}, \quad \forall j. \tag{22}$$

Ultimately, the efficiency scores resumed from models (17a)–(17g) to (20a)–(20g) can be applied to compute the MPI for the second stage of the $j^{\text{th}}$ DMU utilizing

$$\mathrm{MPI}_{2j}(t) = \left[\frac{M_{2j}^{(t)s*}(t+1) M_{2j}^{(t+1)s*}(t+1)}{M_{2j}^{(t)s*}(t) M_{2j}^{(t+1)s*}(t)}\right]^{1/2}, \quad \forall j. \tag{23}$$

## 5. Case Study

More than a century ago, the growth of the Iranian petroleum well industry began. The intricate processes and structures of manufacturing refineries, which include filtration units, catalytic conversion, and the refinement of liquid gas and oil, aim to decrease the energy usage. Also, the intricacy index of each Iranian petroleum well is different as the operating units dealing with the corresponding refining and filtration activities face different necessities than the other units in the other petroleum wells. On the other hand, in the oil generation process, freshwater is required, resulting into a large amount of wastewater as undesirable output in the production process. Thus, in petroleum wells, any decrease in water consumption means a reduction in oil generation. In the situation of restricted resources, it is essential to improve the efficiency of oil generation and wastewater treatment. Moreover, it is required to reuse wastewater to warrant the water reserve in the water-deficient regions [46]. Therefore, in the wastewater treatment stage, some of the undesirable wastewater is treated by using the inputs. It is notable that, due to the existence of some variables, data gained are not precise and they are estimated with a specific error level (e.g., [47]). Here, real and accurate data about key performance criteria of all petroleum wells do not always exist. That is, some of the variable amounts for a petroleum well system are not exactly available. For instance, the number of petroleum wells and price of hydrocarbons are not often reported precisely. Also, it may be beneficial to conceal real information and reveal deceptive input and output data for the petroleum systems. Therefore, it is important to analyze the efficiency of petroleum systems under uncertainty, and managing uncertainty in petroleum well development is an important issue and every decision needs to take into account all the uncertainties in all stages of the field development. Here, we evaluate the offered models under discrete scenarios, provided by the petroleum well system analyzers (i.e., $s_1$ =worst-case, $s_2$ =best-case). According to Snyder and Daskin [48], we assume that all scenarios are equiprobable that is $q^s$ =0.5. This paper separates the whole network system into two stages. The characteristics of the oil production and the wastewater treatment system should be considered together, merging with the characteristics of the expanded two-stage NDEA structure. We consider five inputs in Stage 1, the number of generating wells, cost of oil, cost of water, water increase rate, and reusable water, and desirable outputs and undesirable output are actual oil generation and incremental oil generation, respectively. In Stage 2, undesirable wastewater is refined by applying the inputs as operating costs, consumption cost, construction expenditure, and hydrocarbons; therefore, hydrocarbons removal rates and the quantity of reusable refined wastewater are desirable outputs and unrefined wastewater is undesirable output. All data have been gathered over a period of two years (2020-2021) through consultations with experts that are listed in Tables 4 and 5, respectively. The results gained are displayed in two main sections. In the first section, the whole efficiencies, including those of the first and second stages, are prepared. In the second section, the MPI is computed both for the entire process and each one of the stages.

First, using models (6)–(8), we get the ideal efficiency scores of each DMU based on each scenario in both stages and overall. The results of solving these models with different $p$ values and equal probabilities of 0.5 for each scenario in 2020 and 2021 years are reported in the columns of Table 6. Based on these results, models (9a)–(9f), (13a)–(13e), and (17a)–(17g) give infeasible results for some scenarios when small values are determined for $p$ such as $p = 0.45, 0.46, 0.47, 0.48$ that we did not report those here. As we increase the $p$ values, we observe feasible results. For example, by increasing the $p$ value from 0.49 to 0.52, the efficiency score of DMU9 has improved. This improvement can be seen in other DMUs such as 5, 10, and 11. Models (9a)–(9f), (13a)–(13e), and (17a)–(17g) maximize the weighted efficiency score of each scenario, whereas $p$-robust constraints control the relative difference between their efficiency score generated by the model and ideal efficiency from different scenarios. Accordingly, we can gain each scenario ranking based on the $p$ values in mind.

As can be seen from Table 6, the efficiency scores in models (6) and (7) for most DMUs are equal to one that is 63.3% and 90.9% in two scenarios from 2020 year to 72.7% and 90.9% in two scenarios from 2021 year, respectively. Also, in model (8), 54.54%, 90.9% in two scenarios from 2020 year to 54.54% in two scenarios from 2021 year, respectively. Subsequently, we solved models (9a)–(9f) and (12a)–(12f) to gain the overall efficiency scores for different $p$ values in the years 2020 and 2021. Then, we solved models (13a)–(13e) and (16a)–(16e) for the first stage and models (17a)–(17g) and (20a)–(20g) to get the efficiency scores of the second stages, respectively, in the years 2020 and 2021 which are presented in Tables 7 and 8. It should be noted that the abovementioned models give infeasible results for some DMUs when $p \leq 0.49$ that we do not report those here.

As mentioned before, for small values of $p$, the proposed models give infeasible results in some scenarios. In this study, on the one hand, when $p \leq 0.49$ with respect to the results, our models give infeasible results for some DMUs. As the $p$ value enhances, the efficiency scores get better and the number of infeasible DMUs gradually reduces and we see feasible results. On the other hand, for $p \geq 0.50$, the efficiency scores remain constant. So, we do not carry on and stop it for the other $p$ values. Thus, here, we only consider $p \geq 0.50$ and do not report the results of $p < 0.49$. For example, in Table 7, by increasing the $p$ value from 0.49 to 0.50, the efficiency scores of DMU5 and DMU9 shift. This shift also can be seen in some DMUs of the first and the second stages. It is noted that models (9a)–(9f) to (19a)–(19i) maximize the expected efficiency scores of DMUs in each scenario, while $p$-robust constraints control the respective variation between their efficiency scores produced by the model and ideal efficiency under each scenario. Further, Tables 9–11 show the results of efficiency scores of DMUs for each scenario in the years 2020 and 2021 with models (10a)–(10g), (11a)–(11g), (14a)–(14f), (15a)–(15e), (18a)–(18i), and (19a)–(19i) for $p = 0.50$.

As can be seen in Table 9 and Figure 2, the overall efficiency score of DMU2 is the highest among all DMUs in both years, while DMU4 displays the lowest overall efficiency score in both years. Even though DMU2 is efficient in 2020 and 2021 and its efficiency ratio is constant over the two consecutive periods, however, its performance is not efficient in 2020 compared to 2021. Also, DMU9 is not efficient in 2020 and 2021, but this ratio is constant over the two consecutive periods; however, its performance is not efficient in 2020 in comparison to 2021. Unlike DMU6 and DMU8, the efficiency values of DMU1, DMU3, DMU4, DMU5, DMU7, DMU10, and DMU11 are greater in 2020 proportionate to 2021, compared to 2021 proportionate to 2020. As DMU3 is efficient in 2020 and inefficient in 2021, the ratio of its efficiency values is less than 1 in the two consecutive periods 2020 and 2021, indicating that their annual efficiency is smaller in 2021 proportionate to 2020 than vice versa.

Table 4: The 2020 dataset of three scenarios for 11 petroleum wells.

| DMUs | $x_1^1$ | | $x_2^1$ | | $x_3^1$ | | $x_4^1$ | | $y_1^1$ | | $y_2^1$ | | $z_1$ | $x_1^2$ | | $x_2^2$ | | $x_3^2$ | | $f_1$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ |
| 1 | 75000 | 90000 | 0.498 | 0.510 | 92.22 | 121.57 | 67.45 | 60.8 | 510.1 | 638.1 | 102.0 | 68.0 | 433.0 | 477.0 | 117 | 135.0 | 17479.9 | 24279.9 | 92.6 | 239.4 | 489.3 |
| 2 | 48000 | 84000 | 0.442 | 0.453 | 73.36 | 132.05 | 85.5 | 95 | 797.1 | 997.1 | 72.0 | 96.0 | 344.4 | 379.4 | 109 | 126.0 | 12338.8 | 17138.8 | 117.4 | 191.5 | 391.4 |
| 3 | 52000 | 87000 | 0.434 | 0.462 | 113.18 | 117.38 | 55.1 | 64.6 | 542.0 | 678.0 | 87.0 | 116.0 | 531.3 | 585.3 | 113 | 130.5 | 14909.4 | 20709.4 | 47.4 | 478.8 | 978.5 |
| 4 | 4000 | 74000 | 0.533 | 0.546 | 209.60 | 138.34 | 30.4 | 27.55 | 231.2 | 289.2 | 150.0 | 200.0 | 984.0 | 108.4 | 96 | 111.0 | 25705.8 | 35705.8 | 65.5 | 339.9 | 694.7 |
| 5 | 35000 | 63000 | 0.410 | 0.420 | 79.65 | 125.76 | 78.85 | 82.65 | 693.5 | 867.5 | 85.5 | 114.0 | 373.9 | 411.9 | 82 | 94.5 | 14652.3 | 20352.3 | 61.0 | 363.9 | 743.7 |
| 6 | 30000 | 90000 | 0.342 | 0.351 | 127.86 | 117.38 | 48.45 | 40.85 | 342.8 | 428.8 | 87.0 | 116.0 | 600.2 | 661.2 | 117 | 135.0 | 14909.4 | 20709.4 | 103.9 | 215.5 | 440.3 |
| 7 | 18000 | 90000 | 0.513 | 0.525 | 138.34 | 119.47 | 45.6 | 40.85 | 342.8 | 428.8 | 96.0 | 128.0 | 649.4 | 715.4 | 117 | 135.0 | 16451.7 | 22851.7 | 85.8 | 263.3 | 538.2 |
| 8 | 6000 | 95000 | 0.368 | 0.377 | 119.47 | 117.38 | 52.25 | 52.25 | 438.4 | 548.4 | 90.0 | 120.0 | 560.9 | 617.9 | 124 | 142.5 | 15423.5 | 21423.5 | 76.8 | 292.1 | 596.9 |
| 9 | 46000 | 95000 | 0.523 | 0.536 | 85.94 | 123.66 | 72.2 | 64.6 | 542.0 | 678.0 | 52.5 | 70.0 | 403.4 | 444.4 | 124 | 142.5 | 8997.0 | 12497.0 | 173.9 | 129.3 | 264.2 |
| 10 | 23700 | 90000 | 0.564 | 0.577 | 113.18 | 117.38 | 56.05 | 46.55 | 390.6 | 488.6 | 82.5 | 110.0 | 531.3 | 585.3 | 117 | 135.0 | 14138.2 | 19638.2 | 88.1 | 253.8 | 518.6 |
| 11 | 12000 | 84000 | 0.318 | 0.326 | 88.03 | 121.57 | 71.25 | 58.9 | 494.2 | 618.2 | 61.5 | 82.0 | 413.3 | 455.3 | 109 | 126.0 | 10539.4 | 14639.4 | 67.7 | 330.4 | 675.2 |

Table 5: The 2021 dataset of three scenarios for 11 petroleum wells.

| DMUs | $x_1^1$ | | $x_2^1$ | | $x_3^1$ | | $x_4^1$ | | $y_1^1$ | | $y_2^1$ | | $z_1$ | $x_1^2$ | | $x_2^2$ | | $x_3^2$ | | $f_1$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ | $s_1$ | $s_2$ |
| 1 | 75750 | 90900 | 0.503 | 0.515 | 93.1 | 122.8 | 68.1 | 61.4 | 515.2 | 644.5 | 103.0 | 68.7 | 437.3 | 481.8 | 118.2 | 136.4 | 17654.7 | 24522.7 | 93.5 | 241.9 | 486.8 |
| 2 | 48480 | 84840 | 0.446 | 0.458 | 74.1 | 133.4 | 86.4 | 96.0 | 805.1 | 1007.1 | 72.7 | 97.0 | 347.8 | 383.2 | 110.1 | 127.3 | 12462.2 | 17310.2 | 118.6 | 194 | 388.9 |
| 3 | 52520 | 87870 | 0.438 | 0.467 | 114.3 | 118.6 | 55.7 | 65.2 | 547.4 | 684.8 | 87.9 | 117.2 | 536.6 | 591.2 | 114.1 | 131.8 | 15058.5 | 20916.5 | 47.9 | 481.3 | 976 |
| 4 | 4040 | 74740 | 0.538 | 0.551 | 211.7 | 139.7 | 30.7 | 27.8 | 233.5 | 292.1 | 151.5 | 202.0 | 993.8 | 109.5 | 97.0 | 112.1 | 25962.9 | 36062.9 | 66.2 | 342.4 | 692.2 |
| 5 | 35350 | 63630 | 0.414 | 0.424 | 80.4 | 127.0 | 79.6 | 83.5 | 700.4 | 876.2 | 86.4 | 115.1 | 377.6 | 416.0 | 82.8 | 95.4 | 14798.8 | 20555.8 | 61.6 | 366.4 | 741.2 |
| 6 | 30300 | 90900 | 0.345 | 0.355 | 129.1 | 118.6 | 48.9 | 41.3 | 346.2 | 433.1 | 87.9 | 117.2 | 606.2 | 667.8 | 118.2 | 136.4 | 15058.5 | 20916.5 | 104.9 | 218 | 437.8 |
| 7 | 18180 | 90900 | 0.518 | 0.530 | 139.7 | 120.7 | 46.1 | 41.3 | 346.2 | 433.1 | 97.0 | 129.3 | 655.9 | 722.6 | 118.2 | 136.4 | 16616.2 | 23080.2 | 86.7 | 265.8 | 535.7 |
| 8 | 6060 | 95950 | 0.372 | 0.381 | 120.7 | 118.6 | 52.8 | 52.8 | 442.8 | 553.9 | 90.9 | 121.2 | 566.5 | 624.1 | 125.2 | 143.9 | 15577.7 | 21637.7 | 77.6 | 294.6 | 594.4 |
| 9 | 46460 | 95950 | 0.528 | 0.541 | 86.8 | 124.9 | 72.9 | 65.2 | 547.4 | 684.8 | 53.0 | 70.7 | 407.4 | 448.8 | 125.2 | 143.9 | 9087.0 | 12622.0 | 175.6 | 131.8 | 261.7 |
| 10 | 23937 | 90900 | 0.570 | 0.583 | 114.3 | 118.6 | 56.6 | 47.0 | 394.5 | 493.5 | 83.3 | 111.1 | 536.6 | 591.2 | 118.2 | 136.4 | 14279.6 | 19834.6 | 89.0 | 256.3 | 516.1 |
| 11 | 12120 | 84840 | 0.321 | 0.329 | 88.9 | 122.8 | 72.0 | 59.5 | 499.1 | 624.4 | 62.1 | 82.8 | 417.4 | 459.9 | 110.1 | 127.3 | 10644.8 | 14785.8 | 68.4 | 332.9 | 672.7 |

Table 6: Ideal efficiency scores in two scenarios.

| DMUs | Model (6) | | | | Model (6′) | | | | Model (6″) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $s_1$ (2020) | $s_2$ (2020) | $s_1$ (2021) | $s_2$ (2021) | $s_1$ (2020) | $s_2$ (2020) | $s_1$ (2021) | $s_2$ (2021) | $s_1$ (2020) | $s_2$ (2020) | $s_1$ (2021) | $s_2$ (2021) |
| 1 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 2 | 0.770 | 1.000 | 1.000 | 1.000 | 0.257 | 1.000 | 1.000 | 1.000 | 0.128 | 1.000 | 1.000 | 1.000 |
| 3 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.258 |
| 4 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.111 | 1.000 |
| 5 | 0.728 | 0.945 | 1.000 | 1.000 | 0.243 | 0.315 | 1.000 | 1.000 | 0.121 | 0.158 | 1.000 | 1.000 |
| 6 | 1.000 | 1.000 | 0.977 | 0.893 | 1.000 | 1.000 | 0.326 | 0.298 | 1.000 | 1.000 | 0.163 | 0.149 |
| 7 | 1.000 | 1.000 | 0.832 | 1.000 | 1.000 | 1.000 | 0.277 | 1.000 | 1.000 | 1.000 | 0.139 | 1.000 |
| 8 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.119 |
| 9 | 0.955 | 1.000 | 1.000 | 1.000 | 0.318 | 1.000 | 1.000 | 1.000 | 0.159 | 1.000 | 0.128 | 1.000 |
| 10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.261 | 1.000 | 1.000 | 1.000 |
| 11 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.321 |

Table 7: The results of the overall and the stages efficiency scores in 2020.

| Efficiency | Overall | | | | First stage | | | | Second stage | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ value | 0.49 | 0.50 | 0.51 | 0.52 | 0.49 | 0.50 | 0.51 | 0.52 | 0.49 | 0.50 | 0.51 | 0.52 |
| DMUs | | | | | | | | | | | | |
| 1 | 0.718 | 0.887 | 0.888 | 0.889 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 2 | 0.854 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 0.855 | 1.000 | 1.000 | 1.000 |
| 3 | 0.832 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 4 | 0.649 | 0.671 | 0.671 | 0.672 | 0.722 | 0.735 | 0.734 | 0.735 | 0.707 | 0.660 | 0.657 | 0.659 |
| 5 | INF | 0.832 | 0.832 | 0.835 | INF | 0.967 | 0.967 | 0.969 | 0.838 | 0.748 | 0.707 | 0.707 |
| 6 | 0.604 | 0.681 | 0.681 | 0.681 | 0.714 | 0.716 | 0.717 | 0.719 | 0.763 | 0.739 | 0.763 | 0.764 |
| 7 | 0.639 | 0.821 | 0.821 | 0.822 | 0.693 | 0.727 | 0.727 | 0.729 | 0.958 | 0.830 | 0.794 | 0.794 |
| 8 | 0.743 | 0.757 | 0.757 | 0.758 | 0.693 | 0.733 | 0.733 | 0.733 | 0.599 | 0.559 | 0.545 | 0.547 |
| 9 | INF | 0.852 | 0.852 | 0.853 | 0.653 | 0.897 | 0.898 | 0.898 | 0.789 | 0.735 | 0.719 | 0.719 |
| 10 | 0.745 | 0.999 | 0.999 | 0.999 | 0.749 | 1.000 | 1.000 | 1.000 | INF | 0.674 | 0.709 | 0.711 |
| 11 | 0.887 | 0.997 | 0.997 | 0.997 | INF | 0.831 | 0.831 | 0.832 | 1.000 | 1.000 | 1.000 | 1.000 |

Table 8: The results of the overall and the stages efficiency scores in 2021.

| | Overall efficiency | | | | First stage | | | | Second stage | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ value | 0.49 | 0.50 | 0.51 | 0.52 | 0.49 | 0.50 | 0.51 | 0.52 | 0.49 | 0.50 | 0.51 | 0.52 |
| DMUs | | | | | | | | | | | | |
| 1 | 0.594 | 0.619 | 0.619 | 0.619 | 0.732 | 0.724 | 1.000 | 1.000 | 0.238 | 0.238 | 0.238 | 0.239 |
| 2 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 3 | 0.748 | 0.858 | 0.858 | 0.859 | 0.515 | 0.594 | 0.831 | 0.831 | INF | 0.845 | 0.846 | 0.845 |
| 4 | 0.468 | 0.518 | 0.519 | 0.519 | 0.587 | 0.662 | 0.663 | 0.664 | 0.300 | 0.301 | 0.302 | 0.302 |
| 5 | INF | 0.617 | 0.617 | 0.617 | INF | 0.774 | 0.721 | 0.722 | 0.448 | 0.466 | 0.466 | 0.466 |
| 6 | 0.758 | 0.857 | 0.859 | 0.859 | 0.577 | 0.627 | 0.531 | 0.532 | 1.000 | 1.000 | 1.000 | 1.000 |
| 7 | 0.600 | 0.789 | 0.790 | 0.791 | 0.577 | 0.588 | 0.793 | 0.793 | 0.830 | 0.851 | 0.851 | 0.851 |
| 8 | 0.736 | 0.769 | 0.769 | 0.769 | 0.532 | 0.512 | 0.535 | 0.536 | 0.746 | 0.804 | 0.805 | 0.805 |
| 9 | INF | 0.852 | 0.853 | 0.854 | 0.942 | 0.929 | 0.723 | 0.723 | 0.733 | 0.896 | 0.897 | 0.897 |
| 10 | 0.687 | 0.714 | 0.716 | 0.716 | 0.725 | 0.738 | 0.713 | 0.717 | 0.670 | 0.528 | 0.529 | 0.529 |
| 11 | 0.604 | 0.851 | 0.853 | 0.853 | 0.535 | 0.529 | 0.682 | 0.683 | 0.763 | 0.871 | 0.872 | 0.871 |

In the sequel, MPIs are computed according to Equations (21)–(23) all together with the relative models accounting for the efficiency of the overall process and the first and the second stages. The MPI includes two major elements, the variation in technology and technical efficiency change taking place during the two periods of 2020 and 2021. Table 12 shows the MPI together with the efficiency and technology changes for the overall procedure and its first and second stages. Also, the efficiency values of the first set of columns (for two scenarios) display that DMU2, DMU3, DMU4, DMU6, DMU8, DMU9, and DMU10 have improved the MPI of their total processes from 2020 to 2021, while all the other DMUs have experimented with a worsening. It should be noted that the mean MPI for the total process presents progress from 2020 to 2021.

Table 9: Overall efficiency scores in both time periods for $p = 0.50$.

| DMUs | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $M_0^{(t)s*}(t)$ | 0.887 | 1.000 | 1.000 | 0.671 | 0.832 | 0.681 | 0.821 | 0.757 | 0.852 | 0.999 | 0.997 |
| $M_0^{(t+1)s*}(t+1)$ | 0.619 | 1.000 | 0.858 | 0.518 | 0.617 | 0.857 | 0.789 | 0.769 | 0.852 | 0.714 | 0.851 |
| $M_0^{(t)s*}(t+1)$ | 0.351 | 0.249 | 0.489 | 0.490 | 0.322 | 0.491 | 0.179 | 0.395 | 0.467 | 0.500 | 0.242 |
| $M_0^{(t+1)s*}(t)$ | 0.463 | 0.182 | 0.306 | 0.143 | 0.219 | 0.084 | 0.169 | 0.225 | 0.233 | 0.047 | 0.231 |

Table 10: The efficiency scores of the first stage in both time periods for $p = 0.50$.

| DMUs | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $M_{10}^{(t)s*}(t)$ | 1.000 | 1.000 | 1.000 | 0.735 | 0.967 | 0.716 | 0.727 | 0.733 | 0.897 | 1.000 | 0.831 |
| $M_{10}^{(t+1)s*}(t+1)$ | 0.724 | 1.000 | 0.594 | 0.662 | 0.617 | 0.627 | 0.588 | 0.512 | 0.929 | 0.738 | 0.529 |
| $M_{10}^{(t)s*}(t+1)$ | 1.000 | 1.000 | 0.426 | 1.000 | 1.000 | 0.089 | 0.233 | 1.000 | 1.000 | 0.276 | 1.000 |
| $M_{10}^{(t+1)s*}(t)$ | 0.080 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.323 | 1.000 | 0.182 |

Table 11: The efficiency scores of the second stage in both time periods for $p = 0.50$.

| DMUs | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $M_{20}^{(t)s*}(t)$ | 1.000 | 1.000 | 1.000 | 0.660 | 0.748 | 0.739 | 0.830 | 0.559 | 0.735 | 0.674 | 1.000 |
| $M_{20}^{(t+1)s*}(t+1)$ | 0.238 | 1.000 | 0.845 | 0.301 | 0.466 | 1.000 | 0.851 | 0.804 | 0.896 | 0.528 | 0.871 |
| $M_{20}^{(t)s*}(t+1)$ | 1.489 | 0.536 | 0.064 | 0.503 | 0.495 | 1.000 | 1.000 | 0.038 | 1.000 | 1.000 | 0.244 |
| $M_{20}^{(t+1)s*}(t)$ | 0.003 | 0.135 | 0.061 | 0.012 | 0.367 | 1.000 | 1.000 | 1.000 | 0.048 | 0.167 | 1.000 |

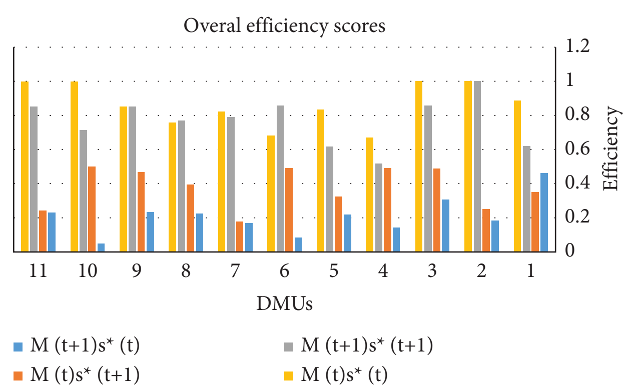


Figure 2: Overall efficiency changes throughout 2020 to 2021.

Table 12: MPI for the overall process and the first and the second stages.

| DMUs | Overall process 2020-2021 | | | First stage 2020-2021 | | | Second stage 2020-2021 | | |
|---|---|---|---|---|---|---|---|---|---|
| | EC | TC | MPI | EC | TC | MPI | EC | TC | MPI |
| 1 | 0.619 | 1.042 | 0.645 | 0.724 | 4.155 | 3.008 | 0.238 | 45.667 | 10.869 |
| 2 | 1.000 | 1.170 | 1.170 | 1.000 | 1.000 | 1.000 | 1.000 | 1.993 | 1.993 |
| 3 | 0.858 | 1.365 | 1.171 | 0.594 | 0.847 | 0.503 | 0.845 | 1.114 | 0.941 |
| 4 | 0.518 | 2.107 | 1.091 | 0.901 | 1.054 | 0.950 | 0.456 | 9.587 | 4.372 |
| 5 | 0.617 | 1.408 | 0.869 | 0.638 | 1.252 | 0.799 | 0.623 | 1.471 | 0.916 |
| 6 | 0.857 | 2.155 | 1.847 | 0.876 | 0.319 | 0.279 | 1.353 | 0.860 | 1.164 |
| 7 | 0.789 | 0.492 | 0.388 | 0.809 | 0.537 | 0.434 | 1.025 | 0.988 | 1.013 |
| 8 | 0.769 | 1.315 | 1.011 | 0.698 | 1.197 | 0.836 | 1.438 | 0.163 | 0.234 |
| 9 | 0.852 | 1.416 | 1.206 | 1.036 | 1.729 | 1.791 | 1.219 | 4.134 | 5.039 |
| 10 | 0.714 | 3.858 | 2.755 | 0.738 | 0.612 | 0.452 | 0.783 | 2.765 | 2.165 |
| 11 | 0.851 | 1.108 | 0.943 | 0.637 | 2.938 | 1.872 | 0.871 | 0.529 | 0.461 |
| Average | 0.768 | 1.585 | 1.191 | 0.786 | 1.422 | 1.084 | 0.896 | 6.297 | 2.652 |

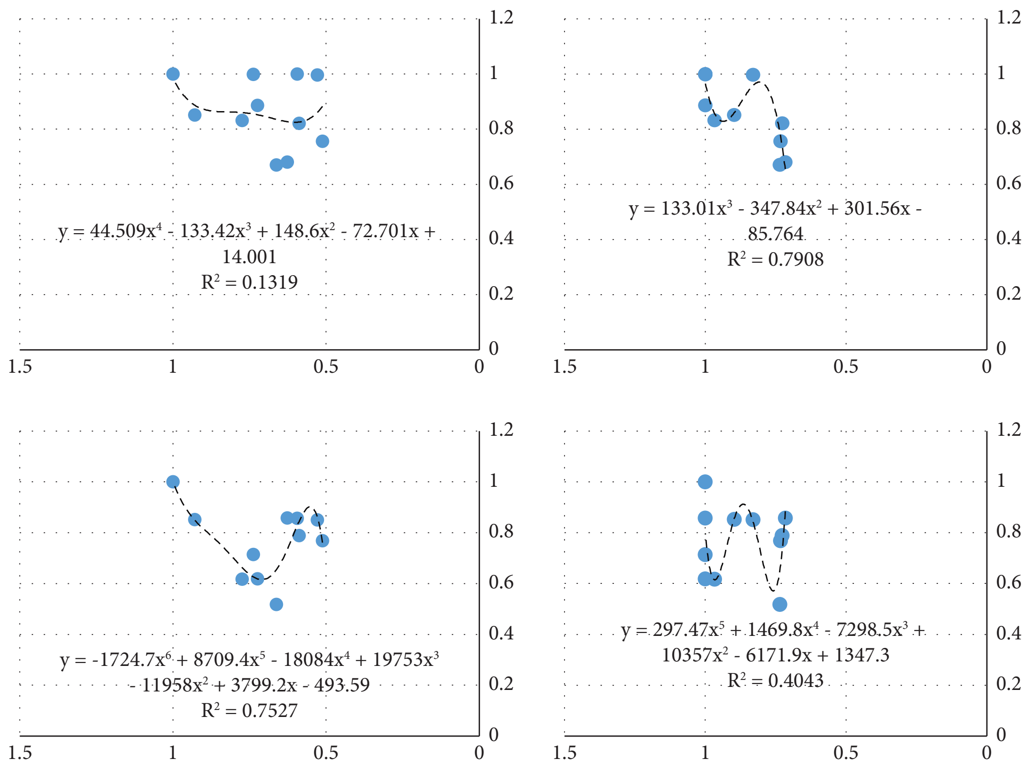


Figure 3: Connection between the overall efficiency score and the efficiency scores of the first stage.

Further, Table 12 displays that the average growth rate of the total productivity of DMUs for the overall process, the first, and the second stages are, respectively, 1.191, 1.084, and 2.652 representing an average increase of total productivity over the two periods of 2020 and 2021. Such an increase rises from the increased mean of TC (Technological Change). In Table 12, four petroleum wells (DMU1, DMU5, DMU7, and DMU11), out of 11 petroleum wells, sustained a regress in the total productivity index between 2020 and 2021, and the other seven petroleum wells have improved productivity. It is noted that TC values less than one indicate the regress of the technology, and TC values more than one indicate the progress of the technology. Also, EC (Efficiency Change) values of more than one indicate an increase in performance, and EC values of less than one show a decrease in performance. Now, we consider EC and TC values gained in both substages in the last two sets of columns. The values gained for the first stage show that DMU1, DMU 2, DMU 9, and DMU11 have improved their MPI over the 2020 and 2021 periods, while the other DMUs have experienced a reduction. Similar to the overall process, DMUs differ in their relevant EC and TC values from the first stage over the two periods of 2020 and 2021, with DMU6 experiencing the worst efficiency change. The last set of columns represents that the MPI of the second stage has improved over the years 2020 and 2021 periods in all DMUs except for DMU3, DMU5, DMU8, and DMU11. Once again, DMUs differ in their relevant EC and TC values, with DMU1 and DMU4 performing the opposite behavior in both efficiencies and DMU8 showing the worst technology change. Finally, for most DMUs, the EC value was less than one, which is why their efficiency decreased from 2020 to 2021, and their total productivity index for some of them is bigger than one. The TC values for all DMUs except for DMU3 were bigger than one; however, their EC values were less than one. This means that, even though they applied fewer new technologies, they made the optimal use of the new technologies and there was no negative impact on their progress. DMU7 had a TC value of less than one and was inefficient; however, it progressed compared to the previous year. In other words, DMU7 used new technologies more optimally and more frequently, leading to its progress.

In this study, regression analysis has been performed to assess the relationship between the overall efficiency score and the efficiency scores of stages. Some curves examined to recognize the relationship between the overall efficiency scores and those of the first stage for the year 2020 are exhibited in Figure 3. According to Figure 3, we can see that the polynomial equation of order six shows the highest R-squared values defining the correlation between the overall efficiency score and that of the first stage in 2020. This shows a complicated relation within the DMU such as the one illustrated in Figure 1. The same process is utilized to analyze the relation existing between the overall efficiency score of the DMU and that of the second stage in 2021. The corresponding results are exhibited in Figure 4.

These regression analyses reflect complicated relations between the stages and their internal transactions. Therefore, dynamic interactions between periods are excessive since the structure remains nonlinear through both the first and second stages when taking into account either the year 2020 or 2021. Thus, even though Figures 3 and 4 reflect the

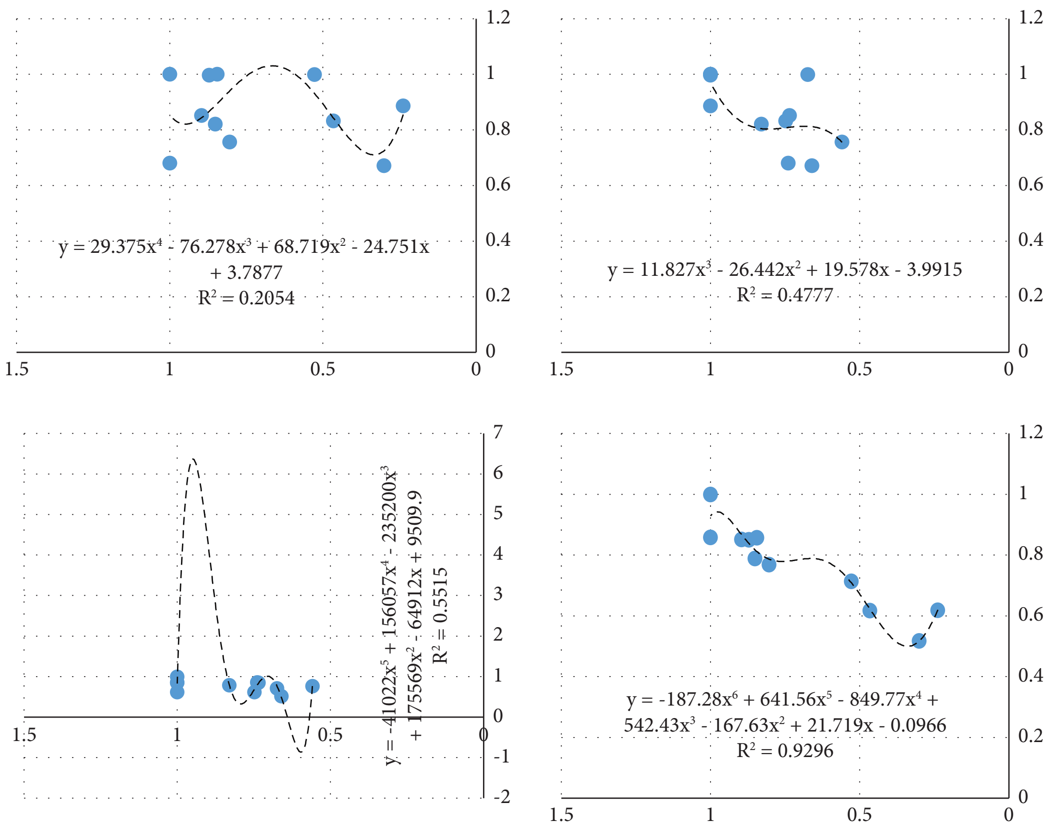


Figure 4: Connection between the overall efficiency score and the efficiency scores of the second stage.

network structure of the well in 2020, very similar polynomial results are gained when considering both stages in 2021.

## 6. Conclusions

One of the main objectives of management in an organization is to improve productivity over a long period. Productivity is assumed to be a function of effectiveness and efficiency over time. However, measuring productivity growth under uncertainty is important for identifying the development patterns followed by different economies. Since its inception, MPI has been widely used as an acceptable index for analyzing performance changes. We proposed a model under DEA and MPI to evaluate the efficiency and productivity of a two-stage NDEA system that minimizes the maximum value of the objective function with the optimization of the worst-case scenario. By refusing a conservatism level, the feasibility of optimal solutions is warranted. The presented approach not only specifies patterns of productivity change and gives a new interpretation along with the managerial implication of each Malmquist ingredient but also identifies strategic orientations of DMUs in past periods for suitable choices in future periods. Moreover, by adjusting the conservatism degree in the proposed model, we get better results. This approach handled to evaluate the proficiency of 11 wells in the Persian Gulf over the 2020 and 2021 periods. The results gained are attended helpful to better the perception of Iranian wells and their internal structures. These results show the changes in productivity and efficiency production in detail showing that the management of resource consumption is inefficient, and the investments are inadequate to raise the growth of the relative technology levels. It is noted that the rate of degradation of capital facilities in the petroleum land is extremely high, confirming the necessity to increase investment to substitute the underestimated assets. The idea of uncontrollable inputs also has pervasive applications, so including it in the presented model would be an absorbing future study direction. In addition, we can consider the addition of more criteria, particularly qualitative ones, to evaluate efficiency and productivity in a more precise process. Also, focusing on various external parameters affecting the proficiency of the petroleum wells would form similar research.

## Data Availability

The dataset analyzed during the current study is confidential.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References

[1] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," *European Journal of Operational Research*, vol. 2, no. 6, pp. 429–444, 1978.

[2] W. D. Cook and J. Zhu, *A Handbook on the Modeling of Internal Structures and Networks*, Springer, New York, NY, USA, 2nd edition, 2014.

[3] S. Malmquist, “Index numbers and indifference surfaces,” *Trabajos de Estadística*, vol. 4, no. 2, pp. 209–242, 1953.

[4] M. M. Yu and L. H. Chen, “Productivity change of airlines: a global total factor productivity index with network structure,” *Journal of Air Transport Management*, vol. 109, pp. 403–102403, 2023.

[5] M. Li, X. Wang, F. O. Agyeman, Y. Gao, and M. Sarfraz, “Efficiency evaluation and the impact factors of sustainable forestry development in China: adoption of super-efficiency data envelopment analysis and malmquist index methods,” *Forests*, vol. 14, no. 5, p. 909, 2023.

[6] L. Liang, W. D. Cook, and J. Zhu, “DEA models for two-stage processes: game approach and efficiency decomposition,” *Naval Research Logistics*, vol. 55, no. 7, pp. 643–653, 2008.

[7] P. Wanke, M. Abul Kalam Azad, A. Emrouznejad, and J. Antunes, “A dynamic network DEA model for accounting and financial indicators: a case of efficiency in MENA banking,” *International Review of Economics & Finance*, vol. 61, pp. 52–68, 2019.

[8] L. Yang and X. Zhang, “Assessing regional eco-efficiency from the perspective of resource, environmental and economic performance in China: a bootstrapping approach in global data envelopment analysis,” *Journal of Cleaner Production*, vol. 173, pp. 100–111, 2018.

[9] H. Nedaei, G. J. Naini, and S. A. Makui, “A dynamic DEA model to measure the learning rates of efficient Frontier and DMUs: an application to oil and gas wells drilling,” *Computers & Industrial Engineering*, vol. 144, pp. 106–434, 2020.

[10] R. Mahmoudi and A. Emrouznejad, “A multi-period performance analysis of airlines: a game-SBM-NDEA and Malmquist Index approach,” *Research in Transportation Business & Management*, vol. 46, Article ID 100801, 2023.

[11] K. Tone, Q. L. Kweh, W. M. Lu, and I. W. K. Ting, “Modeling investments in the dynamic network performance of insurance companies,” *Omega*, vol. 88, pp. 237–247, 2019.

[12] R. C. Pittman, T. E. Carew, C. K. Glass, S. R. Green, C. A. Taylor Jr., and A. D. Attie, “A radioiodinated, intracellularly trapped ligand for determining the sites of plasma protein degradation in vivo,” *Biochemical Journal*, vol. 212, no. 3, pp. 791–800, 1983.

[13] D. W. Caves, L. R. Christensen, and W. E. Diewert, “Multilateral comparisons of output, input, and productivity using superlative index numbers,” *The Economic Journal*, vol. 92, no. 365, pp. 73–86, 1982.

[14] K. Tone, “Dealing with undesirable outputs in DEA: a slacks-based measure (sbm) approach,” in *Proceedings of the North American Productivity Workshop*, Toronto, Canada, June 2004.

[15] G. Halkos and K. N. Petrou, “Treating undesirable outputs in DEA: a critical review,” *Economic Analysis and Policy*, vol. 62, pp. 97–104, 2019.

[16] M. Wang and C. Feng, “Regional total-factor productivity and environmental governance efficiency of China’s industrial sectors: a two-stage network-based super DEA approach,” *Journal of Cleaner Production*, vol. 273, pp. 110–123110, 2020.

[17] J. Zhu and B. Lin, “Economic growth pressure and energy efficiency improvement: empirical evidence from Chinese cities,” *Applied Energy*, vol. 307, no. C, Article ID 118275, 2022.

[18] Y. Li, J. Wang, B. Liu, H. Li, Y. Guo, and X. Guo, “Regional green total factor performance analysis of China’s construction industry based on a united framework combining static and dynamic indexes,” *Research Square*, 2022.

[19] M. Li, X. Wang, F. O. Agyeman, Y. Gao, and M. Sarfraz, “Efficiency evaluation and the impact factors of sustainable forestry development in China: adoption of super-efficiency data envelopment analysis and malmquist index methods,” *Forests*, vol. 14, no. 5, p. 909, 2023.

[20] N. Aghayi, M. Tavana, and B. A. Maleki, “A malmquist productivity index with directional distance function and uncertain data,” *Scientia Iranica*, 2018.

[21] F. Shirazi and E. Mohammadi, “Evaluating efficiency of airlines: a new robust DEA approach with undesirable output,” *Research in Transportation Business & Management*, vol. 33, pp. 467–100467, 2019.

[22] M. Toloo and J. Hanclova, “Multi-valued measures in DEA in the presence of undesirable outputs,” *Omega*, vol. 94, Article ID 102041, 2020.

[23] Q. X. An, Q. F. Wu, J. L. Li, B. B. Xiong, and X. H. Chen, “Environmental efficiency evaluation for Xiangjiang River basin cities based on an improved SBM model and Global Malmquist index,” *Energy Economics*, vol. 81, pp. 95–103, 2019.

[24] K. Asanimoghadam, M. Salahi, A. Jamalian, and R. Shakouri, “A Two-stage structure with undesirable outputs: slacks-based and additive slacks-based measures DEA models,” *RAIRO-Operations Research*, vol. 56, no. 4, pp. 2513–2534, 2022.

[25] M. Salahi, A. Jamalian, R. Shakouri, and K. Asanimoghadam, “Additive slack-based measure for a two-stage structure with shared inputs and undesirable feedback,” *Advances in Operations Research*, vol. 2022, Article ID 7596736, 14 pages, 2022.

[26] R. Shakouri and M. Salahi, “Stochastic P-robust approach to a centralized two-stage DEA system,” *Advances in Operations Research with Resource Waste*, vol. 2022, Article ID 1133882, 14 pages, 2022.

[27] X. Zhang, R. Li, and J. Zhang, “Understanding the green total factor productivity of manufacturing industry in China: analysis based on the super-SBM model with undesirable outputs,” *Sustainability*, vol. 14, no. 15, p. 9310, 2022.

[28] L. Chen, Y. M. Wang, and Y. Huang, “Cross-efficiency aggregation method based on prospect consensus process,” *Annals of Operations Research*, vol. 288, no. 1, pp. 115–135, 2019.

[29] B. Arabi, S. M. Doraisamy, A. Emrouznejad, and A. Khoshroo, “Eco-efficiency measurement and material balance principle: an application in power plants Malmquist Luenberger Index,” *Annals of Operations Research*, vol. 255, no. 1-2, pp. 221–239, 2017.

[30] P. Peykani, E. Mohammadi, and A. Emrouznejad, “An adjustable fuzzy chance-constrained network DEA approach with application to ranking investment firms,” *Expert Systems with Applications*, vol. 166, no. 15, pp. 938–113938, 2021.

[31] A. H. Dar, S. K. Mathur, and S. Mishra, “The efficiency of Indian banks: a DEA, malmquist and sfa analysis with bad output,” *Journal of Quantitative Economics*, vol. 19, no. 4, pp. 653–701, 2021.

[32] M. Salahi, N. Torabi, and A. Amiri, “An optimistic robust optimization approach to common set of weights in DEA,” *Measurement*, vol. 93, pp. 67–73, 2016.

[33] M. Salahi, M. Toloo, and Z. Hesabirad, “Robust russell and enhanced russell measures in DEA,” *Journal of the Operational Research Society*, vol. 70, pp. 1–9, 2019.

[34] M. Salahi, M. Toloo, and N. Torabi, “A new robust optimization approach to common weights formulation in DEA,” *Journal of the Operational Research Society*, vol. 72, pp. 1–13, 2020.

[35] E. Soltanzadeh and H. Omrani, "Dynamic network data envelopment analysis model with fuzzy inputs and outputs: an application for Iranian airlines," *Applied Soft Computing*, vol. 63, pp. 268–288, 2018.
[36] D. Akbarian, "Overall profit Malmquist productivity index under data uncertainty," *Financial Innovation*, vol. 6, no. 1, p. 6, 2020.
[37] R. Shakouri, M. Salahi, and S. Kordrostami, "Stochastic p-robust DEA efficiency scores approach to banking sector," *Journal of Modelling in Management*, vol. 15, no. 3, pp. 893–917, 2020.
[38] R. Shakouri, M. Salahi, and S. Kordrostami, "Stochastic p-robust approach to two-stage network DEA model," *Quantitative Finance and Economics*, vol. 3, no. 2, pp. 315–346, 2019.
[39] S. Mehdizadeh, A. R. Amirteimoori, M. H. Behzadi, and S. Kordrostami, "Stochastic two-stage network-structures under P-models: a DEA based approach," *International Journal of Industrial Mathematics*, vol. 12, no. 3, p. 1332, 2020.
[40] M. Tavana, K. Khalili-Damghani, F. J. Santos Arteaga, and A. Hashemi, "A Malmquist productivity index for network production systems in the energy sector," *Annals of Operations Research*, vol. 284, no. 1, pp. 415–445, 2020.
[41] T. Kuosmanen, "Weak disposability in non-parametric production analysis with undesirable outputs," *American Journal of Agricultural Economics*, vol. 87, no. 4, pp. 1077–1082, 2005.
[42] J. P. Chavas and T. L. Cox, "Production analysis: a non-parametric time series application to U.S. agriculture," *Journal of Agricultural Economics*, vol. 48, no. 1–3, pp. 330–348, 1997.
[43] A. Hailu and T. S. Veeman, "Non-parametric productivity analysis with undesirable outputs: an application to the Canadian pulp and paper industry," *American Journal of Agricultural Economics*, vol. 83, no. 3, pp. 605–616, 2001.
[44] T. Kuosmanen and R. Kazemi Matin, "Duality of weakly disposable technology," *Omega*, vol. 39, no. 5, pp. 504–512, 2011.
[45] A. Charnes and W. W. Cooper, "Programming with linear fractional functional," *Naval Research Logistics*, vol. 9, pp. 181–185, 1962.
[46] A. Fakhru Lrazi, A. Pendashteh, L. C. Abdullah, D. R. A. Biak, S. S. Madaeni, and Z. Z. Abidin, "Review of technologies for oil and gas produced water treatment," *Journal of Hazardous Materials*, vol. 170, no. 2-3, pp. 530–551, 2009.
[47] Z. Zhou, L. Lin, H. Xiao, C. Ma, and S. Wu, "Stochastic network DEA models for two-stage systems under the centralized control organization mechanism," *Computers & Industrial Engineering*, vol. 110, pp. 404–412, 2017.
[48] L. V. Snyder and M. S. Daskin, "Stochastic p-robust location problems," *IIE Transactions*, vol. 38, no. 11, pp. 971–985, 2006.