## SHORT COMMUNICATION Comments on Size Effects in Metallic Films

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In a recent paper<sup>1</sup> a new approximate expression for polycrystalline metal film resistivity has been compared to previously proposed expressions.<sup>1-4</sup> We attempt in this paper to discuss the validity of the theoretical formulation.

The well-known Fuchs–Sondheimer (FS) conduction model<sup>5</sup> is applicable to thin films in which background and film surface scatterings occur. The film resistivity  $\rho_F^{FS}$  is given by:<sup>5</sup>

$$\rho_F^{FS} = \rho_B \cdot [F(k,p)]^{-1}$$
 (1)

where  $\rho_B$  is the bulk resistivity (i.e. the resistivity of an infinitely thick film), k the reduced thickness, p is the reflection coefficient at the film surfaces and F the usual size effect function or FS function. (Refer to Warkusz<sup>1</sup> (Eq. 2) and Sondheimer<sup>5</sup>).

Eq. (1) cannot be used to express the relation between the resistivity of a polycrystalline film  $\rho_F^{MS}$  and the resistivity  $\rho_g$  of an infinitely thick polycrystalline film in the Mayadas-Shatzkes model<sup>6</sup> (when background, grain-boundary and surfaces scatterings occur simultaneously) since the background relaxation time  $\tau$  of the F-S model is replaced by a relaxation time  $\tau^*$  representing the combined effect of background and grain-boundary scatterings (Refer to Mayadas and Shatzkes,<sup>6</sup> Eq. (76)). To obtain the total film resistivity in presence of both background, grain-boundary and surface scatterings, the relaxation time  $\tau^*$  is then substituted for  $\tau$  in the Boltzmann equation and in relations deduced from boundary conditions at film surfaces; hence, the obtained relation between  $\rho_E^{MS}$  and  $\rho_g$ (Mayadas and Shatzkes<sup>6</sup> Eq. (15) and Mola and Heras<sup>7</sup>) cannot be reduced to Eq. (1).

Consequently, all relations related to polycrystalline films which could be deduced from the following incorrect relation:1

$$\rho_F^{MS} = \rho_g \cdot [F(k,p)]^{-1}$$
 (2)

are questionable. This is the case for Eq. (6) of Warkusz<sup>1</sup> and all derived equations. For large ranges of k, p and the physical parameter  $\alpha^6$  (specially in the usual thin films ranges); it is observed,<sup>1</sup> as expected, that this formulation markedly deviates from the theoretical Mayadas—Shatzkes (MS) relation. However no marked discrepancy is observed when  $k \ge 1$  since the physical effect arising because of the geometrical limitation of the mean free path imposed by the film surfaces vanishes, i.e.

$$\rho_F^{MS}/\rho_g \approx 1.$$

It has been shown<sup>8-15</sup> that several methods exist to reduce MS expressions to analytical expressions and specially to Fuchs—Sondheimer expressions.<sup>10,11,14</sup> For instance an effective mean free path may be introduced, leading to a substitution of k by  $k_g$  where  $k_g$  is defined by:

$$k_g = k \cdot [G(\alpha)]^{-1} \tag{3}$$

with  $G(\alpha) = \rho_0 / \rho_g$  where  $\rho_0$  is the resistivity of a single crystal, i.e. the resistivity that refers to background scattering. Hence, convenient expressions for  $\rho_F^{MS}$  could be<sup>10,11,14</sup>

$$\rho_F^{MS} / \rho_g = [F(k_g, p)]^{-1};$$
  

$$\rho_F^{MS} / \rho_0 = [F(k_g, p)]^{-1} \cdot [G(\alpha)]^{-1}$$
(4)

In the limit when  $k_g$  becomes large;<sup>12</sup>

$$\rho_F^{MS} / \rho_g \approx 1 + (1 - p)k^{-1} \cdot \frac{3}{8} G(\alpha)$$
(5)

It has been previously shown<sup>4</sup> that an approximate expression for  $\rho_F^{MS}$  could be:

$$\rho_F^{MS}/\rho_g \approx 1 + (1-p)k^{-1} \cdot h(\alpha) \tag{6}$$

In the limiting  $k_g$ -range  $(k_g \ge 1)$  it appears that:

$$h(\alpha) \approx \frac{3}{8} G(\alpha) \tag{7}$$

Numerical data<sup>4</sup> have shown that the following relation

$$h(\alpha) = \frac{3}{8} \cdot G(\alpha) \tag{8}$$

proposed by Warkusz<sup>1</sup> leads to more significant deviation when we consider large  $\alpha$ , p and k ranges than the model previously derived by Tellier and Tosser.<sup>4</sup>

Hence, we may conclude from the above discussions that the conduction model proposed by Warkusz is questionable.

## REFERENCES

- 1. F. Warkusz, Electrocomp. Sci & Technol., 5, 99 (1978).
- 2. P. Wissman, Thin Solid Films, 5, 329 (1970).
- 3. F. Thieme and W. Kirstein, *Thin Solid Films*, **30**, 371 (1975).
- 4. C. R. Tellier and A. J. Tosser, *Thin Solid Films*, 33, L 19 (1976).
- 5. E. H. Sondheimer, Adv. Phys., 1, 1 (1952).
- 6. A. F. Mayadas and M. Shatzkes, *Phys. Rev. B*, 1, 1382 (1970).
- 7. E. E. Mola and J. M. Heras, *Thin Solid Films*, 18, 137 (1973).
- C. R. Tellier, Electrocomp. Sci & Technol., 5, 127 (1978).
- 9. C. Tellier and A. Tosser, Le Vide, Suppl., 189, 25 (1978).
- 10. C. Tellier and A. Tosser, Le Vide, 189, 129 (1977).
- 11. C. R. Tellier, A. J. Tosser, and C. Boutrit, *Thin Solid Films*, 44, 201 (1977).
- 12. C. R. Tellier, Thèse Doctorat ès Sciences, Nancy (1977).
- C. R. Tellier and A. J. Tosser, *Thin Solid Films*, 43, 261 (1977).
- 14. C. R. Tellier, Thin Solid Films, 51, 311 (1978).
- 15. C. R. Tellier and A. J. Tosser, Int. Conf. Thin Films, Loughborough G.B. (1978) *Thin Solid Films*, 57, 163 (1979).





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