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TIME DOMAIN RESPONSE OF UNIFORM RC LINES WITH RC TERMINATION AT BOTH ENDS

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The time domain response of uniform RC lines with RC termination at both ends is obtained from the approximate poles of the transfer function. A simple model is developed for the URC line. The model consists of three passive elements only and can be easily implemented for computer-aided analysis of URC lines with RC termination at both ends.

INTRODUCTION

Uniformly distributed (URC) lines are widely used in many diverse fields, for example for modelling of VLSI interconnects. The analytical solutions for time-domain response of open-circuit, short-circuit, capacitively loaded, infinitely long, and capacitively-resistively loaded URC lines have received considerable attention in the literature [1–12]. In all these cases, it is assumed that the URC line is driven by an ideal voltage source with zero source resistance. In an attempt to include the effect of the source resistance, recently Abuelma'atti [13] presented an analytical solution for the time-domain response of uniform RC lines with resistive termination at both ends. However, in most practical cases, especially in VLSI interconnects, the source and load terminations are not purely resistive. A more realistic situation should take into consideration the capacitive effects at both ends. This would result in a mathematically untractable transfer function for the voltage driven URC line. Probably this is the reason why this problem has not yet been considered in the available literature. It is the purpose of this paper to present an analytical solution based on finding the approximate poles of the transfer function of the URC line with RC termination at both ends, over a wide range of load, source, and line parameters. Based on this analytical solution, a simple equivalent circuit, formed by three passive elements only, is derived for the URC line with RC termination at both ends. This model is compatible with previously published models and would be more appropriate for computer-aided analysis of circuits and systems comprising URC lines with RC termination at both ends.

ANALYSIS

The transfer function of a voltage driven (Fig. 1) URC line with RC termination at both ends can be expressed by

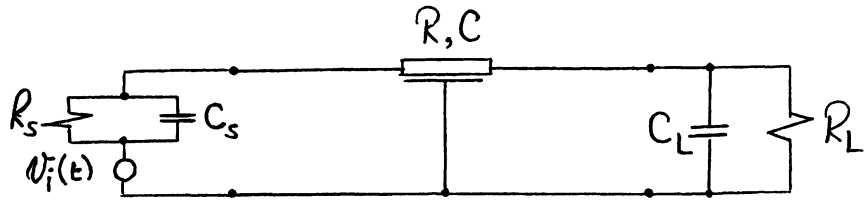

FIGURE 1 Voltage-driven URC line with RC termination at both ends.

$$\frac{V_L(s)}{V_i(s)} = \frac{1}{((1 + Z_S/Z_L)\cosh\sqrt{sCR} + ((R/Z_L)/\sqrt{sCR} + (Z_S/R)\sqrt{sCR})\sinh\sqrt{sCR})} \tag{1}$$

Where Z_S is the source impedance, Z_L is the load impedance, R is the total resistance, and C is the total capacitance of the line. Substituting $Z_S = R_S/(1 + sC_SR_S)$ and $Z_L = R_L/(1 + sC_LR_L)$ and denoting $sCR = \psi$, $\alpha = R_S/R$, $\beta = R/R_L$, $\gamma = C_L/C$, and $\delta = C_S/C$, eqn. (1) reduces to

$$\frac{V_L(\psi)}{V_i(\psi)} = \frac{1}{\left(1 + \frac{\alpha\beta + \alpha\gamma\psi}{1 + \alpha\delta\psi}\right)\cosh\sqrt{\psi} + ((\beta + \gamma\psi)/\sqrt{\psi})\sinh\sqrt{\psi} + \alpha/(1 + \alpha\delta\psi)\sqrt{\psi}\sinh\sqrt{\psi}}$$
$$= \frac{1}{Q(\alpha, \beta, \gamma, \delta, \psi)} \tag{2}$$

The transfer function of an open-circuit URC line can be obtained from (2) by setting $\beta = \gamma = 0$. The transfer function of a URC with resistive termination at both ends can be obtained from (2) by setting $\gamma = \delta = 0$.

Fig. 2 shows a plot of $Q(\alpha, \beta, \gamma, \delta, \psi)$ for different values of α, β, γ, δ and ψ. Inspection of Fig. 2 reveals that, in general, $Q(\alpha, \beta, \gamma, \delta, \psi)$ can be represented by polynomials of the Nth order of the form

$$Q(\alpha, \beta, \gamma, \delta, \psi) = A_o + \sum_{n=1}^{N} A_n \psi^n \tag{3}$$

where $A_o = 1 + \beta(1 + \alpha)$. In general, the parameters A_n can be obtained by using standard curve-fitting subroutines available in most mainframe computers. However, for a second-order polynomial, with N = 2, the parameters A_1 and A_2 can be obtained by hand calculations using the Lagrange interpolating polynomial [14]. This procedure yields a family of parameters A_1 and A_2 that depend on α, β, γ, and δ. These parameters are fitted to simple closed-form analytical expressions, giving

$$A_1 = 0.4885(1 + 1.92756\alpha(-0.33 + 1.33/(1 - 1.195\alpha\delta + 1.725(\alpha\delta)^2)))$$
$$(1 + 0.39918\beta)(1 + 1.94575\gamma) \tag{4}$$

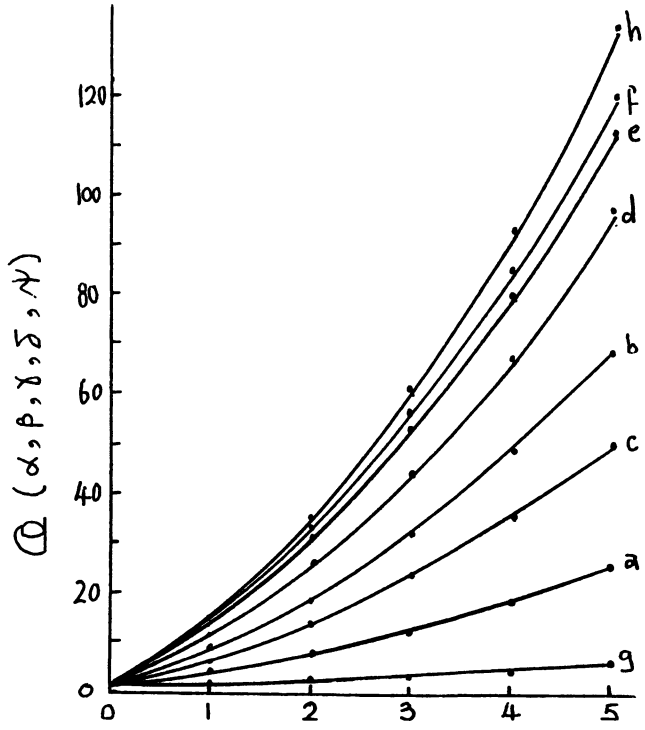

FIGURE 2 Variation of Q(α, β, γ, δ, ψ) with the parameters α, β, γ, δ. a. α = 0, β = 0, γ = 2, δ = 8 b. α = 0.02, β = 0.06, γ = 6, δ = 4 c. α = 0.04, β = 0.04, γ = 4, δ = 0 d. α = 0.04, β = 0.01, γ = 8, δ = 0 e. α = 0.06, β = 0.1, γ = 10, δ = 10 f. α = 0.08, β = 0.06, γ = 10, δ = 2 g. α = 0.1, β = 0.0, γ = 0, δ = 0 h. α = 0.1, β = 0.1, γ = 10, δ = 0 ----Calculated using (2). Calculated using (3)-(5).

$$A_2 = 0.0515(1 + 4.36893\alpha(0.13 + 0.87/(1 + 16.576\alpha\delta)))(1 + 0.097087\beta)(1 + 4.42574\gamma) \tag{5}$$

Using (3)–(5), calculations were made with N = 2, and are shown in Fig. 2, from which it is seen that the proposed second-order polynomial of (3) accurately represents the denominator of (1) with an average root-mean-square error contained within 0.95% for $0 < \alpha < 1.0$, $0 < \beta < 1.0$, $0 < \gamma < 10$ and $0 < \delta < 10$.

RESULTS

By combining (3)–(5) and substituting into (2), assuming N=2, the unit step response due to unput $V_i(t) = u(t)$ is

$$V_L(s) = \frac{1}{s}\frac{1}{\gamma_o + \gamma_1\psi + \gamma_2\psi^2}$$

or

$$\frac{V_L(\psi)}{RC} = \frac{1}{\gamma_2}\frac{1}{\psi^2 + 2b\psi + \omega^2} \tag{6}$$

where $2b = A_1/A_2$ and $\omega_0^2 = A_0/A_2$. Since $\omega_0^2 < b^2$, the inverse Laplace transform of (6) is obtained as [14]

$$V_L(t) = (1 - \frac{\omega_o^2}{p - m}(\frac{e^{mt/RC}}{m} - \frac{e^{pt/RC}}{p})) \tag{7}$$

where m and p are the roots of the equation

$$\psi^2 + 2b\,\psi + \omega_o^2 = 0$$

Fig. 3 depicts the unit-step response for the URC line with RC termination at both ends for different values of source and load resistances and capacitances. From Fig. 3 it is obvious that the approximations of (3)–(5) accurately predicts the performance of the URC line terminated at both ends by a parallel RC combination.

From (2)–(5) it can be shown that the transfer function of the URC line terminated at both ends by a parallel RC combination corresponds to the equivalent circuit shown in Fig. 4. This equivalent circuit consists of three passive

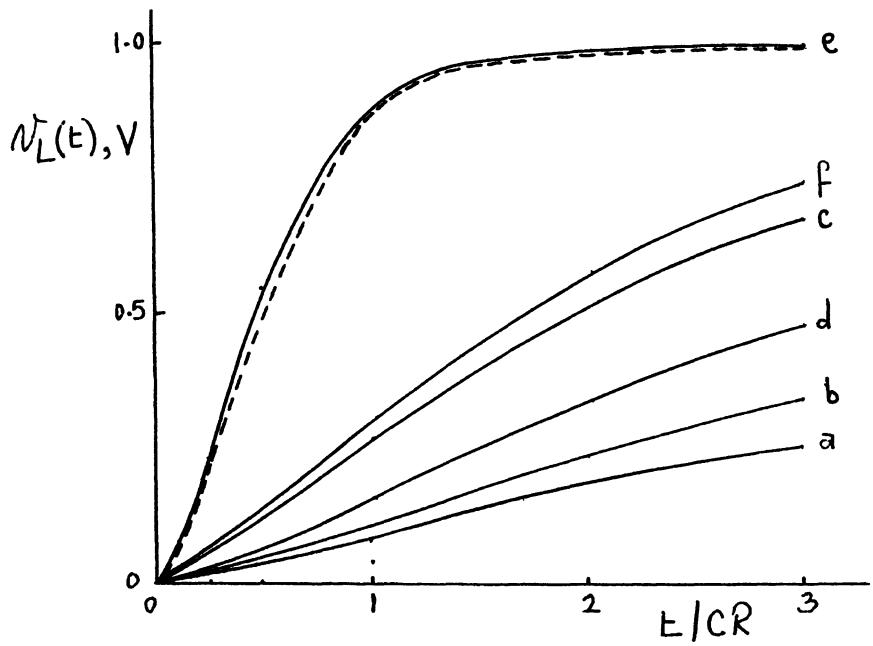

FIGURE 3 Unit-step response for URC line with RC termination at both ends. ----: Calculated (7) ---: Exact a. $\alpha = 0.02$, $\beta = 0.1$, $\gamma = 10$, $\delta = 10$ b. $\alpha = 0.04$, $\beta = 0$, $\gamma = 6$, $\delta = 4$ c. $\alpha = 0.06$, $\beta = 0.02$, $\gamma = 2$, $\delta = 2$ d. $\alpha = 0.1$, $\beta = 0.1$, $\gamma = 4$, $\delta = 10$ e. $\alpha = 0$, $\beta = 0$, $\gamma = 0$, $\delta = 0$ f. $\alpha = 0$, $\beta = 0.1$, $\gamma = 2$, $\delta = 2$.

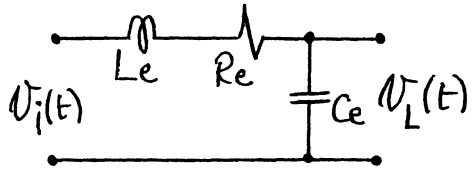

FIGURE 4 Proposed model for URC line with RC termination at both ends. Re = R, Ce = $A_1 / A_0 C$, $Le = A_2 / A_0$, $A_o = 1 + \beta(1 + \alpha)$, A_1 and A_2 are given by eqns (4) and (5) respectively.

elements only and is, therefore, appropriate for computer-aided design of circuits and systems comprising URC lines with RC termination at both ends.

CONCLUSION

In this paper, a second-order polynomial representation has been proposed for the transfer function of a URC line terminated at both ends by a parallel RC combination. Based on this representation, a lumped network with three passive elements only can be used for modeling the performance of URC lines with RC termination at both ends. This model can be easily implemented for computer-aided analysis of circuits comprising URC lines with RC termination at both ends.

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