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INFLUENCE OF ASYMMETRIC PULSES IN SPREAD SPECTRUM SYSTEMS

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This paper examines the impact of asymmetric pulses in NRZ waveforms generated by maximum-length sequences and used in spread spectrum systems. The data asymmetry phenomenon is produced by differential propagation delays through logic circuits in the payloads. A model of three elementary pulse shapes is employed to characterize the signal source and the occurrence probabilities for each is calculated. The autocorrelation function of the waveform is eventually obtained taking into account the asymmetry, and some numerical evaluations are finally presented to show the deviation from the theoretic case where no asymmetry is considered.

Keywords: Asymmetric pulses; Spread spectrum systems; M-sequences; Autocorrelation function

I. INTRODUCTION

The properties of PCM-NRZ waveforms have been widely discussed in the literature (*e.g.* [1, 2]), as this particular waveform is often used in different transmission techniques. The phenomenon of data asymmetry is a very important characteristic of this waveform, as an asymmetric NRZ data stream may be responsible for various problems in a communication link. The cause of this phenomenon lies in the differential propagation delays through logic circuits in the payloads. The unequal rise and falling times of these logic gating circuits produce data transitions at other than the nominal time instants in the stream of -1 and $+1$ pulses.

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In order to estimate and analyze the effects of the phenomenon on the parameters of a communication system, suitable asymmetry models have been developed. The NRZ asymmetric waveform with chip duration T_c has been considered to be formed by an M-ary Markov source, so that one of a set of M elementary signals is transmitted in each T_c interval with given *a priori* and transition probabilities. The asymmetry is taken into account through the definition of those elementary signals. These models have been successfully used to quantitatively determine the degradation of the error probability performance, and therefore the SNR of various types of data detectors, due to the asymmetry [3]. They have also been used to derive the spectral density of an asymmetric NRZ data stream generated by a purely random source [4, 5]. The results observed have been used to create a model to predict the threshold levels of undesired spectral components that fall into the carrier tracking loop bandwidth of a space telemetry system [4].

In the cases mentioned above, the source of the NRZ signal is considered to be purely random, where the signal transmitted in a given signaling interval is independent of those transmitted in previous signaling intervals, and its probability of occurrence can be calculated by using the probabilities of transmitting a positive or negative pulse. In this paper the same asymmetry models are employed to determine the autocorrelation function of a spreading waveform, denoted as c(t), which takes on values ± 1 (NRZ signal) and therefore the phenomenon of chip asymmetry is also present. This waveform is used as the input to a spreading or despreading modulator and it is often generated by a maximal-length sequence, a sequence of ones and zeros, whose properties are given in the literature [6–8]. M-sequences are a special category of pseudorandom codes (PN codes) used as spreading codes in spread spectrum systems. Although they are periodic codes with a specific number of ones and zeros in each period, and therefore cannot be called random, they have a well-defined statistical distribution for the runs of ones and zeros, through which the frequency of occurrence for each of the elementary signals that form the waveform can be determined and used as "pseudo-probability" in the analysis that takes place in the paper. After the probabilities of the elementary signals have been calculated, they can be used to define certain other parameters eventually combined to give the desired expression of the autocorrelation function.

II. DEFINITION OF THE ASYMMETRY MODEL

Consider a spreading waveform c(t) generated by a maximal-length sequence, in which a certain chip asymmetry is observed in the -1s and $+1$s stream. In this paper, we use the data asymmetry model proposed in [3], which assumes that $+1$ NRZ symbols are elongated by $\Delta T_c/2$ (relative to their nominal value of T_c) when a negative-going data transition occurs and -1 bits are shortened by the same amount when a positive-going transition occurs. When no transitions occur, the symbols maintain their nominal T_c value. Thus, ΔT_c represents the relative difference in length observed between the elongated $+1$ and shortened -1 symbols, and the parameter Δ is referred to as the *relative fractional asymmetry*.

We consider the asymmetric NRZ waveform to be formed by an M-ary source, characterized by one of M signals, referred to as *elementary signals*. These signals represent the waveforms which are possible to occur between adjacent integer multiples of the bit time T_c, and are therefore characterized by a T_c duration. As proposed in [5], the source model can be described in terms of three elementary signals, namely:

$$
\begin{aligned}
s_1(t) &= A \quad 0 \le t \le T_c \\
s_2(t) &= \begin{cases} A & 0 \le t \le \eta T_c \\ -A & \eta T_c \le t \le T_c \end{cases} \\
s_3(t) &= -A \quad 0 \le t \le T_c
\end{aligned}
\tag{1}
$$

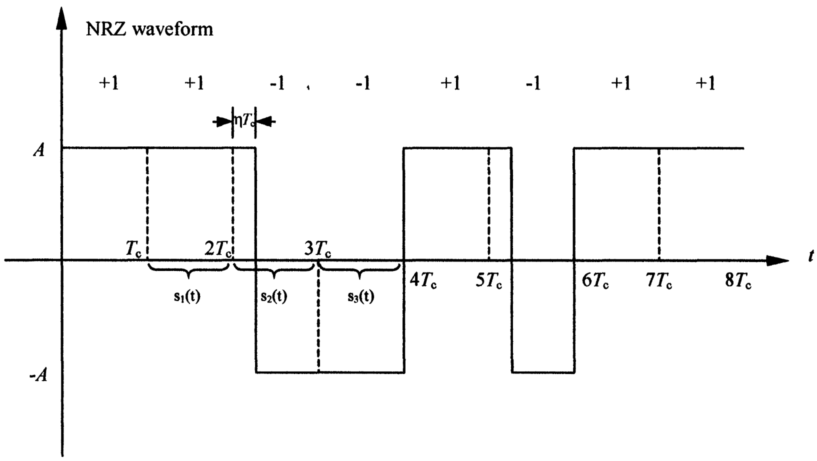

FIGURE 1 Unbalanced asymmetric waveform for a typical data sequence.

where A is the signal amplitude and $\eta = \Delta/2$. In an NRZ waveform, $s_1(t)$ occurs whenever the sequences $(1, 1)$ or $(-1, 1)$ occur, $s_2(t)$ whenever a $(1, -1)$ sequence occurs, and $s_3(t)$ whenever a $(-1, -1)$ sequence occurs. An NRZ waveform corresponding to this asymmetry model is illustrated in Figure 1. For the case of a spreading waveform we let $A = 1$.

III. CALCULATION OF THE PROBABILITIES OF THE ELEMENTARY SIGNALS

In order to express the autocorrelation function of a spreading waveform which is created by an M-sequence, in the presence of chip asymmetry, we must first determine the probability of occurrence for each of the elementary signals defined above. To do this we use the properties of M-sequences to calculate the probabilities of occurrence of each of the sequences $(1, 1)$, $(-1, 1)$, $(1, -1)$ and $(-1, -1)$ in the spreading waveform.

Consider an M-sequence generated by a PN sequence generator of length r. The period of the M-sequence will be $N = 2^r - 1$. If we define a *run* as a subsequence of identical symbols (ones or zeros) within the M-sequence, and the length of the run as the length of the subsequence, then based on the properties of these sequences [6–8] there is in any period N:

(1) 1 run of ones of length r.
(2) 1 run of zeros of length $r-1$.
(3) 1 run of ones and 1 run of zeros of length $r-2$.
(4) 2 runs of ones and 2 runs of zeros of length $r-3$.
(5) 4 runs of ones and 4 runs of zeros of length $r-4$.

$$\vdots$$

$r \cdot 2^{r-3}$ runs of ones and 2^{r-3} runs of zeros of length 1.

The sequence with the above properties is used to form a spreading waveform (NRZ signal) of period NT_c, where the ones and zeros in the M-sequence correspond to $+1$ and -1 pulses in the waveform, respectively.

We need to calculate the following probabilities:

p_{s1}: probability of occurrence of $(1,1)$ sequence
p_{s2}: probability of occurrence of $(-1,1)$ sequence
p_{s3}: probability of occurrence of $(1,-1)$ sequence
p_{s4}: probability of occurrence of $(-1,-1)$ sequence

Calculation of p_{s1}. We must first define the number of $(1,1)$ sequences appearing in one period of the spreading waveform, which is the same as the number of $(1,1)$ sequences in the M-sequence.
In the (sole) run of ones of length r, there will be $r-1$ $(1,1)$ sequences.
In the (sole) run of ones of length $r-2$, there will be $r-3$ $(1,1)$ sequences.
In the two runs of ones of length $r-3$, there will be $2\cdot(r-4)$ such sequences *etc.*
In total: $(r-1)+(r-3)+2\cdot(r-4)+2^2\cdot(r-5)+\cdots+2^{r-4}\cdot 1=2^{r-2}$

The number of sequences of length 2–regardless of type–contained in one period of the spreading waveform is $N=2^r-1$. In these we have also considered the sequence of length 2 formed by the last symbol of each period and the first one of the next period, assuming a very large number of repetitions. Therefore:

$$p_{s1}=\frac{2^{r-2}}{2^r-1}=\frac{N+1}{4N} \tag{2}$$

Calculation of p_{s2}. We must define the number of $(-1,1)$ sequences appearing in one period of the spreading waveform. A transition from -1 to 1 will occur 2^{r-3} times in each period because of the runs of zeros of length 1 in each period of the M-sequence (at the end of each run), 2^{r-4} times because of the runs of zeros of length 2, *etc.*
In total: $2^{r-3}+2^{r-4}+2^{r-5}+\cdots+2^{r-r}+1=2^{r-2}$. Therefore:

$$p_{s2}=\frac{2^{r-2}}{2^r-1}=\frac{N+1}{4N} \tag{3}$$

Calculation of p_{s3}. The transitions from 1 to -1 are calculated as for p_{s2} and the probability is the same as in (3):

$$p_{s3}=\frac{2^{r-2}}{2^r-1}=\frac{N+1}{4N} \tag{4}$$

Calculation of p_{s4}. The procedure is similar to the one for p_{s1}, with the difference that the maximum length of the runs of zeros is now $r-1$, therefore:

$$p_{s4} = \frac{2^{r-2} - 1}{2^r - 1} = \frac{N-3}{4N} \tag{5}$$

Based on (2)–(5) and on the observations made in the earlier section, we can now define the probabilities of occurrence p_1, p_2, p_3 for each of the elementary signals $s_1(t), s_2(t)$ and $s_3(t)$ as:

$$\begin{aligned} p_1 &= p_{s1} + p_{s2} = \frac{N+1}{2N} \\ p_2 &= p_{s3} = \frac{N+1}{4N} \\ p_3 &= p_{s4} = \frac{N-3}{4N}. \end{aligned} \tag{6}$$

IV. THE AUTOCORRELATION FUNCTION

Consider the following signals:

$$\begin{aligned} u(t) &= \begin{cases} 1 & 0 \le t < \eta T_c \\ 0 & \text{elsewhere} \end{cases} \\ \nu(t) &= \begin{cases} 1 & \eta T_c \le t \le T_c \\ 0 & \text{elsewhere} \end{cases} \end{aligned} \tag{7}$$

From (1), (7) we can now write (assuming $A = 1$):

$$\begin{aligned} s_1(t) &= u(t) + v(t) \\ s_2(t) &= u(t) - v(t) \\ s_3(t) &= -u(t) - v(t) \end{aligned} \tag{8}$$

An appropriate representation of the spreading waveform, assuming that this is formed by an infinite number of repetitions of the same period, can therefore be:

$$c(t) = \sum_{n=-\infty}^{\infty} a_n u(t - nT_c) + \sum_{n=-\infty}^{\infty} b_n v(t - nT_c) \tag{9}$$

The parameters (a_n, b_n) in (9) take on the following values, in accordance with the definitions in (8):

$$(a_n, b_n) = \begin{cases} (1, 1) & \text{with probability } p_1 \\ (1, -1) & \text{with probability } p_2 \\ (-1, -1) & \text{with probability } p_3 \end{cases} \tag{10}$$

The probabilities p_1, p_2, and p_3 refer to the probability of occurrence of each of the elementary signals $s_1(t), s_2(t)$ and $s_3(t)$ and are given by (6).

As the spreading waveform is periodic the following property also applies:

$$(a_n, b_n) = (a_{n+iN}, b_{n+iN})$$

for all the integers i. The parameter N refers to the period of the M-sequence generating the spreading waveform.

In order to calculate the autocorrelation function we need to obtain the expectation of the product $c(t) \cdot c(t+\tau)$ [9]:

$$\begin{aligned} \langle c(t) \cdot c(t+\tau) \rangle = & \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \langle a_n a_m \rangle \cdot u(t - nT_c) \cdot u(t + \tau - mT_c) \\ & + \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \langle a_n b_m \rangle \cdot u(t - nT_c) \cdot v(t + \tau - mT_c) \\ & + \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \langle b_n a_m \rangle \cdot v(t - nT_c) \cdot u(t + \tau - mT_c) \\ & + \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \langle b_n b_m \rangle \cdot v(t - nT_c) \cdot v(t + \tau - mT_c) \end{aligned} \tag{11}$$

where the expectations of the products $a_n\, a_m$, $a_n b_m$, $b_n a_m$ and $b_n b_m$ need to be defined. From (10) it is possible to obtain:

$$a_n = \begin{cases} 1 & \text{with probability } p_1 + p_2 \\ -1 & \text{with probability } p_3 \end{cases} \tag{12}$$

$$b_n = \begin{cases} 1 & \text{with probability } p_1 \\ -1 & \text{with probability } p_2 + p_3 \end{cases} \tag{13}$$

Therefore:

$$a_n a_m = \begin{cases} 1 & \text{with probability } (p_1+p_2)^2+p_3^2 \\ -1 & \text{with probability } 2p_3(p_1+p_2) \end{cases} \tag{14}$$

In (14) the product takes on the value 1 whenever $a_n = a_m = 1$ or $a_n = a_m = -1$ and the probability is obtained by multiplying the relative probabilities as given in (12), (13). The value -1 appears whenever $a_n = 1$ and $a_m = -1$, or $a_n = -1$ and $a_m = 1$, justifying the factor 2 at the estimated probability.

Similarly, we obtain:

$$a_n b_m = b_n a_m = \begin{cases} 1 & \text{with probability } (p_1+p_2)p_1+p_3(p_2+p_3) \\ -1 & \text{with probability } (p_1+p_2)(p_2+p_3)+p_3p_1 \end{cases} \tag{15}$$

$$b_n b_m = \begin{cases} 1 & \text{with probability } p_1^2+(p_2+p_3)^2 \\ -1 & \text{with probability } 2p_1(p_2+p_3) \end{cases} \tag{16}$$

From (14)–(16) the expectations are given by:

$$\begin{aligned} \langle a_n a_m \rangle_{\text{total}} &= (p_1+p_2)^2+p_3^2-2p_3(p_1+p_2) \\ \langle a_n b_m \rangle_{\text{total}} &= \langle b_n a_m \rangle_{\text{total}} = (p_1-p_3)^2-p_2^2 \\ \langle b_n b_m \rangle_{\text{total}} &= p_1^2+(p_2+p_3)^2-2p_1(p_2+p_3) \end{aligned} \tag{17}$$

The notation "total" has been used for the above expectations because these apply irrespective of the values of m and n. In the case, though, that $m-n=iN$ (with i any integer), the products give specific results which must be separately taken into consideration, in accordance with the definition of the autocorrelation function of a PN sequence. For $m-n=iN$ we have $a_n = a_m$, $b_n = b_m$ and the products are therefore expressed as:

$$a_n^2 = \begin{cases} 1^2 & \text{with probability } p_1+p_2 \\ & \quad \text{(probability of occurrence of } a_n) \\ (-1)^2 & \text{with probability } p_3 \end{cases} \tag{18}$$

This means that $a_n^2 = 1$ always. Similarly:

$$b_n^2 = 1 \quad \text{always} \tag{19}$$

$$a_n b_n = \begin{cases} 1 & \text{with probability } p_1 + p_3 \\ -1 & \text{with probability } p_2 \end{cases} \tag{20}$$

based on the three possible pairs (a_n, b_n) as seen in (10). The expressions (18)–(20) yield:

$$\begin{aligned} \langle a_n^2 \rangle &= 1 \\ \langle b_n^2 \rangle &= 1 \\ \langle a_n b_n \rangle &= p_1 - p_2 + p_3 \end{aligned} \tag{21}$$

Finally, we need to define the expectations of the products $a_n a_m$, $a_n b_m$, $b_n a_m$ and $b_n b_m$ after the above cases have been excluded, *i.e.*, when $m - n \neq iN$. For this purpose, we must take into consideration the fact that the total number of different combinations of m and n are N^2 for each product (where N is the period of the PN sequence), from which N refer to the case $m - n = iN$, and $N^2 - N$ are the combinations that refer to the case $m - n \neq iN$. Thus, having already calculated the expressions (19) and (21), we have:

$$\langle a_n a_m \rangle_{\text{total}} = \frac{N}{N^2} \langle a_n^2 \rangle + \frac{N^2 - N}{N^2} \langle a_n a_m \rangle_{m-n \neq iN}$$

which gives:

$$\langle a_n a_m \rangle_{m-n \neq iN} = \frac{N}{N-1} \left[\langle a_n a_m \rangle_{\text{total}} - \frac{1}{N} \langle a_n^2 \rangle \right] \tag{22}$$

Similarly:

$$\langle b_n b_m \rangle_{m-n \neq iN} = \frac{N}{N-1} \left[\langle b_n b_m \rangle_{\text{total}} - \frac{1}{N} \langle b_n^2 \rangle \right] \tag{23}$$

$$\langle a_n b_m \rangle_{m-n \neq iN} = \langle b_n a_m \rangle_{m-n \neq iN} = \frac{N}{N-1} \left[\langle a_n b_m \rangle_{\text{total}} - \frac{1}{N} \langle a_n b_n \rangle \right] \tag{24}$$

Expression (11) and the results obtained in (21)–(24), after a lengthy but straightforward algebra, where integration over period T_c and the replacement $m-n=l+iN$ for the cases where $m-n\neq iN$ (with $l=1, 2,\ldots,N-1$) have taken place, lead to:

$$
\begin{aligned}
R_c(\tau) = & \frac{1}{T_c}\langle a_n^2\rangle \sum_{i=-\infty}^{\infty}\int_0^{\eta T_c} u(t)\cdot u(t+\tau-iNT_c)dt+ \\
& +\frac{1}{T_c}\langle a_n b_n\rangle \sum_{i=-\infty}^{\infty}\int_0^{\eta T_c} u(t)\cdot v(t+\tau-iNT_c)dt+ \\
& +\frac{1}{T_c}\langle a_n b_n\rangle \sum_{i=-\infty}^{\infty}\int_{\eta T_c}^{T_c} v(t)\cdot u(t+\tau-iNT_c)dt+ \\
& +\frac{1}{T_c}\langle b_n^2\rangle \sum_{i=-\infty}^{\infty}\int_{\eta T_c}^{T_c} v(t)\cdot u(t+\tau-iNT_c)dt+ \\
& +\frac{1}{T_c}\langle a_n a_m\rangle_{m-n\neq iN} \sum_{i=-\infty}^{\infty}\sum_{l=1}^{N-1}\int_0^{\eta T_c} u(t)\cdot u(t+\tau-(l+iN)T_c)dt+ \\
& +\frac{1}{T_c}\langle a_n b_m\rangle_{m-n\neq iN} \sum_{i=-\infty}^{\infty}\sum_{l=1}^{N-1}\int_0^{\eta T_c} u(t)\cdot v(t+\tau-(l+iN)T_c)dt+ \\
& +\frac{1}{T_c}\langle b_n a_m\rangle_{m-n\neq iN} \sum_{i=-\infty}^{\infty}\sum_{l=1}^{N-1}\int_{\eta T_c}^{T_c} v(t)\cdot u(t+\tau-(l+iN)T_c)dt+ \\
& +\frac{1}{T_c}\langle b_n b_m\rangle_{m-n\neq iN} \sum_{i=-\infty}^{\infty}\sum_{l=1}^{N-1}\int_{\eta T_c}^{T_c} v(t)\cdot v(t+\tau-(l+iN)T_c)dt
\end{aligned}
\tag{25}
$$

Equation (25) expresses the autocorrelation function in a way that considers the asymmetry of the NRZ waveform. The first four summations represent the cases where $m-n=iN$ and therefore the expressions in (21) are used; the rest refer to the cases where $m-n\neq iN$. As it will be shown below, in the absence of asymmetry it results in the typical autocorrelation function given in the literature for spreading waveforms created by M-sequences.

Expression (25) represents the case when the spreading waveform is correlated with a delayed version of itself having the same asymmetry factor η. However, it can also be used to yield the autocorrelation of the waveform when its version at the transmitter is combined with a delayed version at the receiver of a spread spectrum system. In such a case, the chip asymmetry introduced by the NRZ generator may differ, introducing a form of self-interference to the system. For the purposes of the analysis, two different sets of signals $\{u_t(t), v_t(t)\}$ and $\{u_r(t), v_r(t)\}$ may be considered as in (7), with asymmetry factors η_t and η_r, to define the spreading waveform for the transmitter and the receiver, respectively, as in (9). The final expression will be similar to (25); the asymmetry factor η must be replaced, though, by η_t, in all the integration intervals above.

V. NUMERICAL EVALUATIONS

Illustrated in Figures 2 and 3 are evaluations of the analytical autocorrelation results as given by Eq. (25), where a common asymmetry is considered for the waveform and its delayed version, as noted in the previous section. As can be seen, the results obtained for $\eta = 0$, *i.e.*, when no asymmetry is observed, are in agreement with the autocorrelation function of a spreading waveform generated by a maximal-length sequence as described in the literature. In the presence

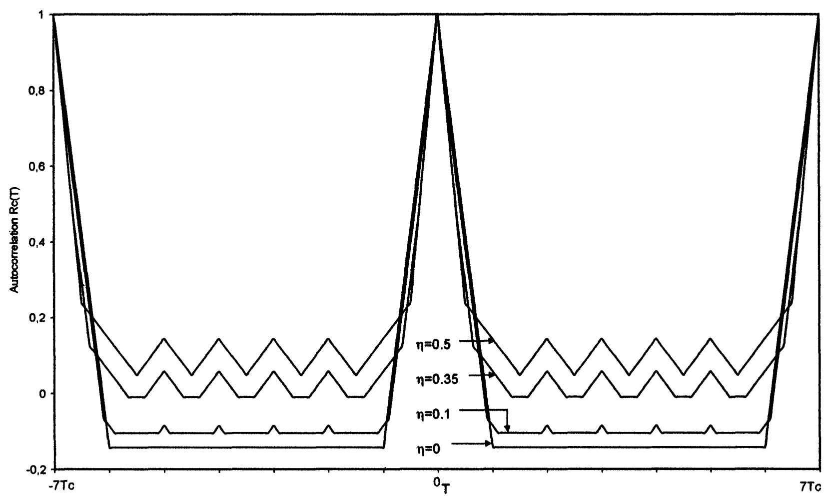

FIGURE 2 Autocorrelation function for maximal-length sequences with chip duration T_c and $N = 7$ in the presence of chip asymmetry.

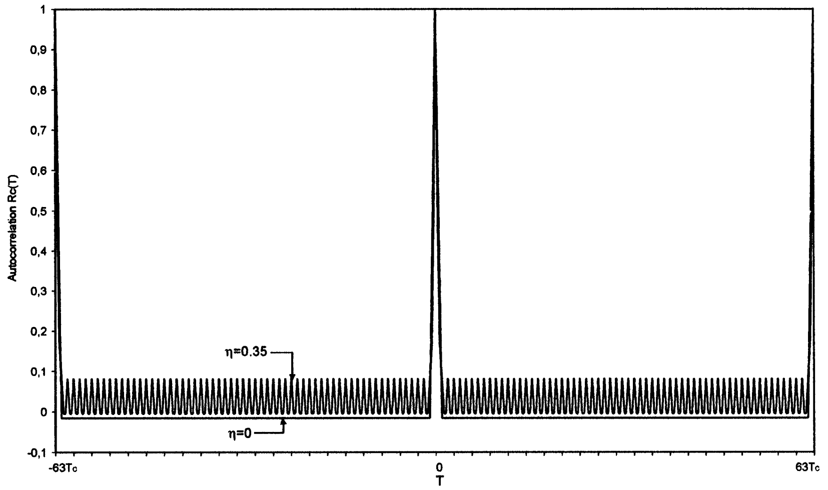

FIGURE 3 Autocorrelation function for maximal-length sequences with chip duration T_c and $N=63$.

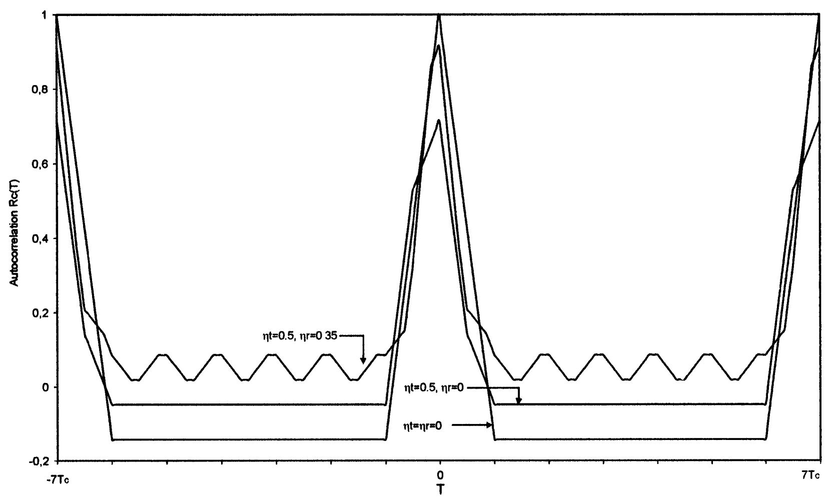

FIGURE 4 Autocorrelation in a system of unequal asymmetry values at the transmitter and receiver.

of chip asymmetry, though, the autocorrelation appears to be different from the theoretical, and the disagreement increases with η. The number of extra peaks appearing due to the asymmetry is equal to $N-3$.

In the case when the chip asymmetry of the waveform takes on different values at the transmitter and the receiver of a spread spectrum system, the numerical results vary according to the specific values of the parameters η_t and η_r, which were previously described. Two examples of the form of the autocorrelation function are illustrated in Figure 4, where the cases of single asymmetry (only at the transmitter) and double asymmetry (transmitter and receiver) are both presented.

VI. CONCLUSION

The impact of chip asymmetry on the autocorrelation function of an M-sequence waveform used as spreading code in spread spectrum systems has been investigated. This phenomenon is caused by a non-ideal response of the logic gating circuits producing the NRZ waveform which is used as spreading code in the system. The analysis was based on a model of three elementary pulse shapes forming the spreading waveform. For this purpose, the occurrence probabilities of the elementary signals were calculated by considering the properties of M-sequences, and they were eventually combined to yield an analytical expression of the autocorrelation function where the asymmetry appears as a parameter. The numerical evaluation which followed for different values of the asymmetry showed a significant deviation from the theoretical waveform, introducing a form of interference to the spread spectrum systems using M-sequences.

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