Research Article

Elasticity Solutions for Sandwich Arches considering Permeation Effect of Adhesive

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Received 21 October 2019; Accepted 7 March 2020; Published 28 March 2020

Guest Editor: Guangming Chen

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In this work, analytical solution of simply supported sandwich arches considering permeation effect of adhesives is presented. The permeation layer is described by the functionally graded material, exponentially graded in the radial direction. The stresses and deformations of each layer are based on the two-dimensional (2D) elasticity theory in the polar coordinate. The governing equations of the arch are solved by the layer-wise method, which turns the differential equations with variable coefficients into constant coefficients. The solution can be obtained efficiently by means of the recursive matrix method, especially for the arch with many layers. The present solution agrees well with the finite element solution with a fine mesh, while the finite element method is time consuming in mesh division and calculation. The one-dimensional (1D) solution based on the Euler–Bernoulli theory is close to the present one; however, the error increases as the arch becomes thick. The effect of permeation layer thickness on the stresses is studied. It is indicated that the stress distributions tend to be smooth along the radial direction as the permeation layer thickness increases.

1. Introduction

Owing to various advantages such as corrosion resistance, antifatigue, and high specific strength and stiffness, composite structures are increasingly used in different branches of engineering. In addition, their mechanical behavior can be optimized by ingeniously choosing the material type and composite pattern. A typical application is the sandwich structures [1–5], usually composed of two stiff face layers and a relatively soft core layer, bonded together by the adhesive. It is known that the interfacial damage exists widely in sandwich structures due to the discontinuity of material property at the interface. In practice, the adhesive will permeate into adjacent macrovoid materials such as glass fiber, balsa wood, and aluminum honeycomb, which leads to a refined sandwich model, as shown in Figure 1. The permeation layer is the mixture of the face (or core) layer and the adhesive. This makes the material property smoothly vary along the radial direction, like of functionally graded material (FGM). Such a problem deserves to be deeply

investigated. For sandwich plates considering permeation effect, analytical solution has been proposed by Huo et al. [6]. In addition to sandwich plates, sandwich arches have also received a lot of attention due to their advantage of artistic appearance and excellent load-carrying capacity. The present study aims at extending their work to investigate the sandwich arch with permeation effect.

A review of the literature indicates that many analytical models have been proposed for the mechanical behaviors of sandwich structures. Galuppi and Royer-Carfagni [7] studied the bending behavior of the sandwich arch formed by two elastic facial layers and a thin polymeric interlayer. In their report, the deformation of the arch was based on the Euler–Bernoulli assumption, which was suitable for thin arches with moderate curvature. The stress distribution in a sandwich plate with the FG face was studied by Raissi et al. [8] by the use of the layer-wise method based on the first-order shear deformation theory. The vibration of sandwich microplates with nanocomposite face layers under magnetic and electric fields was studied by Mohammadimehr et al. [9]

by virtue of the sinusoidal shear deformation theory. Livani et al. [10] investigated the supersonic flutter of doubly curved sandwich panels with variable thickness subjected to thermal load. In their study, the governing equations were based on a new higher-order shear deformation theory, in which the vertical and in-plane displacement components were assumed as quadratic and cubic functions, respectively. Bouchafa et al. [11] presented a new refined hyperbolic shear deformation theory with only four unknown functions to analyze the thermoelastic bending of FG sandwich plates. Bending, vibration, and buckling analysis of FG sandwich plates were analyzed by Nguyen et al. [12] according to the refined higher-order shear deformation theory. By using a higher-order sandwich theory, Rahmani et al. [13] tackled the free vibration of the composite sandwich shell with a flexible core.

In addition to the aforementioned analytical models, some numerical methods are efficient for the analysis of sandwich structures. By means of the differential quadrature finite element method, Liu et al. [14] investigated bending of the sandwich shell consisting of two homogenous face sheets and a FG core. Natarajan and Manickam [15] presented a QUAD-8 shear flexible element method based on higher-order shear deformation theory to study the bending and free flexural vibration of FG sandwich plates. Dynamic behavior of corrugated-core sandwich plates was analyzed by Peng et al. [16] via a mesh-free Galerkin method, in which the domain of the plate was discretized by scattered nodes, and no elements were required. By virtue of the isogeometric method, Thai et al. [17] studied the free vibration and buckling analysis of sandwich plates. This method has the advantage to exactly represent domains and achieve approximation with arbitrarily high smoothness. Mantari et al. [18] developed a layer-wise finite element formulation-based trigonometric layer-wise shear deformation theory for the bending of thick sandwich plates.

The analytical solution based on the elasticity theory is important because it can be used as a benchmark to assess other simplified solutions [19]. Based on the elasticity theory, Pagano [20] proposed an analytical solution for the bending of simply supported composite and sandwich plates. The elasticity solution of the cylindrical sandwich shell subjected to radial and axial loads was presented by Kardomateas [21]. Zenkour [22] studied the bending problem of cross-ply laminated plates by the use of the elasticity equations linked with the state space method. Kashtalyan and Menshykova [23] developed elasticity solution for sandwich plates with a FG core.

However, in the above investigations, the permeation effect of the adhesive between layers is neglected. In this paper, 2D elasticity solution is presented for a refined sandwich arch model considering the permeation effect of the adhesive. In the present work, the permeation layer is described as FGM, exponentially graded along the radial

FIGURE 1: Permeation effect in the sandwich arch.

direction. The general solution of stresses and deformations with unknown coefficients is obtained by means of the layer-wise method. The coefficients are determined via the recursive matrix method, which is efficient for arches with many layers. The solution obtained is compared with the finite element solution and 1D solution based on the Euler–Bernoulli theory, respectively.

2. Analytical Model

2.1. Basic Equations. Without loss of generality, we consider a sandwich arch with internal radius R_1 , external radius R_2 , thickness H , and angle β , composed of p FG layers with each thickness h_i , as shown in Figure 2. The adjacent layers are perfectly bonded, i.e., the deformations are continuous along the radial direction. A polar coordinate system $r-\theta$ is established. The arch is simply supported at two edges and is subjected to radial load $q(\theta)$ acting on the external surface. The permeation layer is the material consisting in the mixture of face (or core) layer material and the adhesive material. Here, we assume its elastic modulus to be exponentially graded along the radial direction, i.e., $E_i(r) = E_0^i e^{k_i r}$, in which E_0^i represents the elastic modulus at the internal surface of the i -th layer and k_i denotes the gradient index. Poisson's ratio μ_i is constant in each layer. The homogeneous layers, i.e., face, core, and adhesive layers, can be included in the model by letting $k_i = 0$ only.

According to the 2D elasticity theory in the polar coordinate [24], the constitutive relations are given by

$$\begin{aligned}\sigma_r^i &= \frac{E_i(r)}{1 - \mu_i^2} (\epsilon_r^i + \mu_i \epsilon_\theta^i), \\ \sigma_\theta^i &= \frac{E_i(r)}{1 - \mu_i^2} (\mu_i \epsilon_r^i + \epsilon_\theta^i), \\ \tau_{r\theta}^i &= \frac{E_i(r)}{2(1 + \mu_i)} \gamma_{r\theta}^i,\end{aligned}\tag{1}$$

$$i = 1, 2, \dots, p,$$

FIGURE 2: Simply supported sandwich FGM arch.

in which σ_θ^i and σ_r^i are the circumferential and radial normal stresses, respectively; $\tau_{r\theta}^i$ is the shear stress; ϵ_θ^i and ϵ_r^i are the circumferential and radial normal strains, respectively; and $\gamma_{r\theta}^i$ is the shear strain. The geometrical equations are

$$\begin{aligned}\epsilon_r^i &= \frac{\partial u_r^i}{\partial r}, \\ \epsilon_\theta^i &= \frac{u_r^i}{r} + \frac{1}{r} \frac{\partial u_\theta^i}{\partial \theta}, \\ \gamma_{r\theta}^i &= \frac{1}{r} \frac{\partial u_r^i}{\partial \theta} + \frac{\partial u_\theta^i}{\partial r} - \frac{u_\theta^i}{r},\end{aligned}\quad (2)$$

$i = 1, 2, \dots, p,$

in which u_θ^i and u_r^i are the deformation components in the circumferential and radial directions, respectively. The equilibrium equations are given by

$$\begin{aligned}\frac{\partial \sigma_r^i}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}^i}{\partial \theta} + \frac{\sigma_r^i - \sigma_\theta^i}{r} &= 0, \\ \frac{1}{r} \frac{\partial \sigma_\theta^i}{\partial r} + \frac{\partial \tau_{r\theta}^i}{\partial r} + \frac{2\tau_{r\theta}^i}{r} &= 0,\end{aligned}\quad (3)$$

$i = 1, 2, \dots, p.$

The simply supported boundary conditions of the arch are

$$\sigma_\theta^i = u_r^i = 0, \quad \text{at } \theta = 0, \beta, i = 1, 2, \dots, p. \quad (4)$$

The load condition on the external and internal surfaces of the arch can be expressed by

$$\begin{aligned}\sigma_r^p &= -q(\theta), \\ \tau_{r\theta}^p &= 0, \\ \sigma_r^1 &= 0, \\ \tau_{r\theta}^1 &= 0,\end{aligned}\quad \begin{aligned} &\text{at } r = R_2, \\ &\text{at } r = R_1.\end{aligned}\quad (5)$$

2.2. General Solution for a Sublayer. By combining equations (1)–(3), two partial differential equations with respect to u_θ^i and u_r^i are derived:

$$\begin{aligned}\frac{\partial^2 u_r^i}{\partial r^2} + \frac{1}{r} \frac{\partial u_r^i}{\partial r} - \frac{u_r^i}{r^2} + \frac{1 + \mu_i}{2} \frac{1}{r} \frac{\partial^2 u_\theta^i}{\partial r \partial \theta} - \frac{3 - \mu_i}{2} \frac{1}{r^2} \frac{\partial u_\theta^i}{\partial \theta} \\ + \frac{1 - \mu_i}{2r^2} \frac{\partial^2 u_r^i}{\partial \theta^2} + k_i \frac{\partial u_r^i}{\partial r} + k_i \mu_i \frac{u_r^i}{r} + k_i \mu_i \frac{1}{r} \frac{\partial u_\theta^i}{\partial \theta} = 0, \\ \frac{\partial^2 u_\theta^i}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta^i}{\partial r} - \frac{u_\theta^i}{r^2} + \frac{1 + \mu_i}{1 - \mu_i} \frac{1}{r} \frac{\partial^2 u_r^i}{\partial r \partial \theta} + \frac{3 - \mu_i}{1 - \mu_i} \frac{1}{r^2} \frac{\partial u_r^i}{\partial \theta} \\ + \frac{2}{1 - \mu_i} \frac{1}{r^2} \frac{\partial^2 u_\theta^i}{\partial \theta^2} + k_i \frac{1}{r} \frac{\partial u_r^i}{\partial \theta} + k_i \frac{\partial u_\theta^i}{\partial r} - k_i \frac{u_\theta^i}{r} = 0,\end{aligned}\quad (6)$$

$i = 1, 2, \dots, p.$

Here, the above equations are solved by the layer-wise method [25]. According to this method, each arch layer is divided into λ sublayers each with very small thickness so that the variable r in equation (6) can be taken as a constant. The radius, the middle plane of the j -th ($j = 1, 2, \dots, p\lambda$) sublayer, is denoted by $\hat{r}_j$. The distances from the arch center O to the internal and external surfaces of the j -th sublayer are, respectively, represented by d_j^0 and d_j^1 . Then, we expand the deformation components of the j -th sublayer into Fourier series:

$$\begin{aligned}u_r^j(r, \theta) &= \sum_{m=1}^{\infty} R_m^j(r) \sin(\alpha_m \theta), \\ u_\theta^j(r, \theta) &= \sum_{m=1}^{\infty} \Theta_m^j(r) \cos(\alpha_m \theta),\end{aligned}\quad (7)$$

$j = 1, 2, \dots, p\lambda,$

where $\alpha_m = (m\pi/\beta)$. Substituting equation (7) into equation (6), we obtain two ordinary differential equations:

$$\begin{aligned}\frac{d^2 R_m^j(r)}{dr^2} + \frac{1}{\hat{r}_j} \frac{dR_m^j(r)}{dr} - \left[1 + \frac{(1 - \mu_i)(\alpha_m)^2}{2} \right] \frac{R_m^j(r)}{(\hat{r}_j)^2} \\ - \frac{(1 + \mu_i)\alpha_m}{2} \frac{1}{\hat{r}_j} \frac{d\Theta_m^j(r)}{dr} + \frac{(3 - \mu_i)\alpha_m}{2} \frac{\Theta_m^j(r)}{(\hat{r}_j)^2} \\ + k_i \frac{dR_m^j(r)}{dr} + k_i \mu_i \frac{R_m^j(r)}{\hat{r}_j} - k_i \alpha_m \mu_i \frac{\Theta_m^j(r)}{\hat{r}_j} = 0, \\ \frac{d^2 \Theta_m^j(r)}{dr^2} + \frac{1}{\hat{r}_j} \frac{d\Theta_m^j(r)}{dr} - \left[1 + \frac{2(\alpha_m)^2}{1 - \mu_i} \right] \frac{\Theta_m^j(r)}{(\hat{r}_j)^2} \\ + \frac{(1 + \mu_i)\alpha_m}{1 - \mu_i} \frac{1}{\hat{r}_j} \frac{dR_m^j(r)}{dr} + \frac{(3 - \mu_i)\alpha_m}{1 - \mu_i} \frac{R_m^j(r)}{(\hat{r}_j)^2} \\ + k_i \alpha_m \frac{R_m^j(r)}{\hat{r}_j} + k_i \frac{d\Theta_m^j(r)}{dr} - k_i \frac{\Theta_m^j(r)}{\hat{r}_j} = 0,\end{aligned}$$

$m = 1, 2, 3, \dots, j = 1, 2, \dots, p\lambda, i = \left[\frac{j}{\lambda} \right] + 1,$

(8)

where $[j/\lambda]$ means the integer part of j/λ . The solution of equation (8) is

$$\begin{aligned} R_m^j(r) &= \sum_{n=1}^4 a_{mn}^j e^{s_{mn}^j r}, \\ \Theta_m^j(r) &= \sum_{n=1}^4 \xi_{mn}^j a_{mn}^j e^{s_{mn}^j r}, \end{aligned} \quad (9)$$

$$m = 1, 2, 3, \dots, j = 1, 2, \dots, p\lambda,$$

in which a_{mn}^j are unknown coefficients, and s_{mn}^j are the four roots of the following quartic equation:

$$\begin{aligned} &\left[(\widehat{r}_j)^2 (s_{mn}^j)^2 + \widehat{r}_j (k_i \widehat{r}_j + 1) s_{mn}^j - k_i \widehat{r}_j - 1 + \frac{2(\alpha_m)^2}{1 - \mu_i} \right] \\ &\cdot \left[(\widehat{r}_j)^2 (s_{mn}^j)^2 + \widehat{r}_j (k_i \widehat{r}_j + 1) s_{mn}^j + k_i \mu_i \widehat{r}_j - 1 + \frac{(1 - \mu_i)(\alpha_m)^2}{2} \right] \\ &+ \left[\frac{(1 + \mu_i)\alpha_m}{1 - \mu_i} \widehat{r}_j s_{mn}^j + k_i \alpha_m \widehat{r}_j + \frac{(3 - \mu_i)\alpha_m}{1 - \mu_i} \right] \\ &\times \left[\frac{(1 + \mu_i)\alpha_m}{2} \widehat{r}_j s_{mn}^j + k_i \alpha_m \mu_i \widehat{r}_j - \frac{(3 - \mu_i)\alpha_m}{2} \right] = 0, \end{aligned} \quad (10)$$

$$m = 1, 2, 3, \dots, n = 1, 2, 3, 4, j = 1, 2, \dots, p\lambda, i = \left[\frac{j}{\lambda} \right] + 1,$$

and ξ_{mn}^j can be expressed by s_{mn}^j as follows:

$$\xi_{mn}^j = \frac{(\widehat{r}_j)^2 (s_{mn}^j)^2 + k_i (\widehat{r}_j)^2 s_{mn}^j + k_i \mu_i \widehat{r}_j + s_{mn}^j \widehat{r}_j + ((1 - \mu_i)(\alpha_m)^2/2) - 1}{((1 + \mu_i)\alpha_m/2) r s_{mn}^j + k_i \alpha_m \mu_i \widehat{r}_j - ((3 - \mu_i)\alpha_m/2)}, \quad (11)$$

$$m = 1, 2, 3, \dots, n = 1, 2, 3, 4, j = 1, 2, \dots, p\lambda, i = \left[\frac{j}{\lambda} \right] + 1.$$

Substituting equation (9) into equation (7), the general solutions of the deformation components with unknown coefficients are obtained:

$$\begin{aligned} u_r^j(r, \theta) &= \sum_{m=1}^{\infty} \sum_{n=1}^4 a_{mn}^j e^{s_{mn}^j r} \sin(\alpha_m \theta), \\ u_{\theta}^j(r, \theta) &= \sum_{m=1}^{\infty} \sum_{n=1}^4 \xi_{mn}^j a_{mn}^j e^{s_{mn}^j r} \cos(\alpha_m \theta), \end{aligned} \quad (12)$$

$$j = 1, 2, \dots, p\lambda.$$

Substitution of equation (12) into equations (1) and (2) gives the general solutions of the stress components

$$\begin{aligned} \sigma_r^j(r, \theta) &= \frac{E_i(r)}{1 - \mu_i^2} \sum_{m=1}^{\infty} \sum_{n=1}^4 \left(s_{mn}^j + \frac{\mu_i}{r} - \frac{\alpha_m \mu_i \xi_{mn}^j}{r} \right) a_{mn}^j e^{s_{mn}^j r} \sin(\alpha_m \theta), \\ \sigma_{\theta}^j(r, \theta) &= \frac{E_i(r)}{1 - \mu_i^2} \sum_{m=1}^{\infty} \sum_{n=1}^4 \left(\mu_i s_{mn}^j + \frac{1}{r} - \frac{\alpha_m \xi_{mn}^j}{r} \right) a_{mn}^j e^{s_{mn}^j r} \sin(\alpha_m \theta), \\ \tau_{r\theta}^j &= \frac{E_i(r)}{2(1 + \mu_i)} \sum_{m=1}^{\infty} \sum_{n=1}^4 \left(\frac{\alpha_m}{r} + \xi_{mn}^j s_{mn}^j - \frac{\xi_{mn}^j}{r} \right) a_{mn}^j e^{s_{mn}^j r} \cos(\alpha_m \theta), \end{aligned}$$

$$j = 1, 2, \dots, p\lambda, i = \left[\frac{j}{\lambda} \right] + 1. \quad (13)$$

2.3. Recursive Matrix Method. The out-of-plane variables, i.e., u_{θ}^j , u_r^j , σ_r^j , and $\tau_{r\theta}^j$, of each sublayer can be rearranged into the vector form

$$\begin{bmatrix} u_{\theta}^j(r, \theta) \\ u_r^j(r, \theta) \\ \sigma_r^j(r, \theta) \\ \tau_{r\theta}^j(r, \theta) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} \Theta_m^j(r) \cos(\alpha_m \theta) \\ R_m^j(r) \sin(\alpha_m \theta) \\ Y_m^j(r) \sin(\alpha_m \theta) \\ Z_m^j(r) \cos(\alpha_m \theta) \end{bmatrix}, \quad j = 1, 2, \dots, p\lambda. \quad (14)$$

$$\Psi_m^j(r) = [\Theta_m^j(r) \ R_m^j(r) \ Y_m^j(r) \ Z_m^j(r)]^T,$$

$$\Gamma_m^j = [a_{m1}^j \ a_{m2}^j \ a_{m3}^j \ a_{m4}^j]^T,$$

$$\Omega_m^j(r) = \begin{bmatrix} \xi_{m1}^j a_{m1}^j e^{s_{m1}^j r} & \xi_{m2}^j a_{m2}^j e^{s_{m2}^j r} & \xi_{m3}^j a_{m3}^j e^{s_{m3}^j r} & \xi_{m4}^j a_{m4}^j e^{s_{m4}^j r} \\ a_{m1}^j e^{s_{m1}^j r} & a_{m2}^j e^{s_{m2}^j r} & a_{m3}^j e^{s_{m3}^j r} & a_{m4}^j e^{s_{m4}^j r} \\ \frac{E_i(r)}{1-\mu_i^2} \left(s_{m1}^j + \frac{\mu_i - \mu_i \alpha_m \xi_{m1}^j}{r} \right) a_{m1}^j e^{s_{m1}^j r} & \frac{E_i(r)}{1-\mu_i^2} \left(s_{m1}^j + \frac{\mu_i - \mu_i \alpha_m \xi_{m1}^j}{r} \right) a_{m2}^j e^{s_{m2}^j r} & \frac{E_i(r)}{1-\mu_i^2} \left(s_{m1}^j + \frac{\mu_i - \mu_i \alpha_m \xi_{m1}^j}{r} \right) a_{m3}^j e^{s_{m3}^j r} & \frac{E_i(r)}{1-\mu_i^2} \left(s_{m1}^j + \frac{\mu_i - \mu_i \alpha_m \xi_{m1}^j}{r} \right) a_{m4}^j e^{s_{m4}^j r} \\ \frac{E_i(r)}{2(1+\mu_i)} \left(\frac{\alpha_m - \xi_{m1}^j}{r} + \xi_{m1}^j s_{m1}^j \right) a_{m1}^j e^{s_{m1}^j r} & \frac{E_i(r)}{2(1+\mu_i)} \left(\frac{\alpha_m - \xi_{m2}^j}{r} + \xi_{m2}^j s_{m2}^j \right) a_{m2}^j e^{s_{m2}^j r} & \frac{E_i(r)}{2(1+\mu_i)} \left(\frac{\alpha_m - \xi_{m3}^j}{r} + \xi_{m3}^j s_{m3}^j \right) a_{m3}^j e^{s_{m3}^j r} & \frac{E_i(r)}{2(1+\mu_i)} \left(\frac{\alpha_m - \xi_{m4}^j}{r} + \xi_{m4}^j s_{m4}^j \right) a_{m4}^j e^{s_{m4}^j r} \end{bmatrix}. \quad (16)$$

By replacing r in the above equation with d_j^0 and d_j^1 , respectively, one has

$$\begin{aligned} \Psi_m^j(d_j^0) &= \Omega_m^j(d_j^0) \Gamma_m^j, \\ \Psi_m^j(d_j^1) &= \Omega_m^j(d_j^1) \Gamma_m^j, \end{aligned} \quad (17)$$

$$m = 1, 2, 3, \dots, j = 1, 2, \dots, p\lambda,$$

By eliminating Γ_m^j in the two equations of equation (17), we obtain

$$\Psi_m^j(d_j^1) = \Omega_m^j(d_j^1) \Omega_m^j(d_j^0)^{-1} \Psi_m^j(d_j^0), \quad m = 1, 2, 3, \dots, j = 1, 2, \dots, p\lambda. \quad (18)$$

The out-of-plane variables are continuous between the adjacent sublayers, i.e.,

$$\Psi_m^j(d_j^1) = \Psi_m^j(d_{j+1}^0), \quad m = 1, 2, 3, \dots, j = 1, 2, \dots, p\lambda. \quad (19)$$

By combining equation (18) with equation (19), the relation between the internal and external surfaces of the arch is obtained:

$$\begin{aligned} \Psi_m^p(d_p^1) &= \left[\prod_{j=p\lambda}^1 \Omega_m^j(d_j^1) \Omega_m^j(d_j^0)^{-1} \right] \Psi_m^1(d_1^0), \\ m &= 1, 2, 3, \dots, j = 1, 2, \dots, p\lambda. \end{aligned} \quad (20)$$

We define

By substituting equations (12) and (13) into equation (14), $\Theta_m^j(r)$, $R_m^j(r)$, $Y_m^j(r)$, and $Z_m^j(r)$ can be further rearranged into the matrix form as follows:

$$\Psi_m^j(r) = \Omega_m^j(r) \Gamma_m^j(t), \quad m = 1, 2, 3, \dots, j = 1, 2, \dots, p\lambda, i = \left\lceil \frac{j}{\lambda} \right\rceil + 1, \quad (15)$$

in which

$$\begin{bmatrix} S_m^{11}(t) & S_m^{12}(t) \\ S_m^{21}(t) & S_m^{22}(t) \end{bmatrix} = \left[\prod_{j=p\lambda}^1 \Omega_m^j(d_j^1) \Omega_m^j(d_j^0)^{-1} \right], \quad m = 1, 2, 3, \dots, \quad (21)$$

in which $S_m^{11}(t)$, $S_m^{12}(t)$, $S_m^{21}(t)$, and $S_m^{22}(t)$ are the 2×2 submatrices. Equation (20) can be written as

$$\begin{bmatrix} \Theta_m^p(d_p^1) \\ R_m^p(d_p^1) \\ Y_m^p(d_p^1) \\ Z_m^p(d_p^1) \end{bmatrix} = \begin{bmatrix} S_m^{11}(t) & S_m^{12}(t) \\ S_m^{21}(t) & S_m^{22}(t) \end{bmatrix} \begin{bmatrix} \Theta_m^1(d_1^0) \\ R_m^1(d_1^0) \\ Y_m^1(d_1^0) \\ Z_m^1(d_1^0) \end{bmatrix}, \quad m = 1, 2, 3, \dots \quad (22)$$

Then, equation (22) is further decomposed into two matrix equations as follows:

$$\begin{aligned} \begin{bmatrix} \Theta_m^p(d_1^0) \\ R_m^p(d_1^0) \end{bmatrix} &= S_m^{11}(t) \begin{bmatrix} \Theta_m^1(d_1^0) \\ R_m^1(d_1^0) \end{bmatrix} + S_m^{12}(t) \begin{bmatrix} Y_m^1(d_1^0) \\ Z_m^1(d_1^0) \end{bmatrix}, \\ \begin{bmatrix} Y_m^p(d_1^0) \\ Z_m^p(d_1^0) \end{bmatrix} &= S_m^{21}(t) \begin{bmatrix} \Theta_m^1(d_1^0) \\ R_m^1(d_1^0) \end{bmatrix} + S_m^{22}(t) \begin{bmatrix} Y_m^1(d_1^0) \\ Z_m^1(d_1^0) \end{bmatrix}, \end{aligned} \quad m = 1, 2, 3, \dots \quad (23)$$

The load function $q(\theta)$ in equation (5) can be expanded into Fourier series

$$q(\theta) = \sum_{m=1}^{\infty} q_m \sin(\alpha_m \theta), \quad (24)$$

$$q_m = \frac{2}{\beta} \int_0^{\beta} q(\theta) \sin(\alpha_m \theta) d\theta.$$

Substituting the load condition into equation (23), one has

$$\begin{bmatrix} \Theta_m^1(d_1^0) \\ R_m^1(d_1^0) \end{bmatrix} = S_m^{21}(t)^{-1} \begin{bmatrix} -q_m \\ 0 \end{bmatrix}, \quad m = 1, 2, 3, \dots \quad (25)$$

By reusing equations (18) and (19), $\Psi_m^j(d_j^1)$ is obtained:

$$\Psi_m^j(d_j^1) = \left[\prod_{k=j}^1 \Omega_m^k(d_k^1) \Omega_m^k(d_k^0)^{-1} \right] \Psi_m^1(d_1^0), \quad (26)$$

$$j = 1, 2, \dots, p\lambda, m = 1, 2, 3, \dots$$

Finally, the coefficients are determined:

$$\Gamma_m^j = \Omega_m^j(d_j^1)^{-1} \Psi_m^j(d_j^1), \quad j = 1, 2, \dots, p\lambda, m = 1, 2, 3, \dots \quad (27)$$

Substituting the coefficients back into equations (12) and (13), the solution of stress and deformation components is obtained.

It can be observed that, as the layer number increases, only the computation effort in equation (23) slightly increases. Therefore, the present method is highly efficient for arches with many layers. Moreover, other boundary conditions can also be applied in the present model. For example, the clamped condition can be equivalent to the simply supported one acted by the unknown horizontal reaction, which can be further determined by the zero displacement condition at the clamped end [26].

3. Example and Discussion

In the following examples, the infinite series in the solution are truncated into the finite term, i.e., $m = 1, 2, \dots, N$.

3.1. Convergence Study. Consider a simply supported sandwich arch with $R_1 = 1000$ mm, $R_2 = 1240$ mm, and $\beta = \pi/4$. The face, core, and adhesive layers are made of carbon fiber, polyurethane foam, and epoxy, respectively. The thickness, elastic modulus, and Poisson's ratio of each arch layer are listed in Table 1.

Since the governing equations of equation (6) are solved by the layer-wise method, the effect of the divided number on the solution accuracy should be analyzed first. We assume the arch is subjected to radial load $q(\theta) = \sin(\pi/\beta)$ N/mm. For sinusoidal load, one series term is sufficient, i.e., $N = 1$. Table 2 gives the present solution of stresses and deformations in the face layer and permeation layer with different divided number λ , respectively. It can be found that the solution tends to be convergent with the increase of λ . Moreover, the solution in the face layer converges faster than that in the permeation layer. The solution is accurate with at

TABLE 1: The thickness and material property for each layer in the sandwich arch with nine layers.

i	Layer type	h_i (mm)	E_i (MPa)	μ_i
1	Face	18	120000	0.3
2	Permeation	20	$E_1 e^{[(r-18)\ln(E_3/E_1)]/h_2}$	0.3
3	Adhesive	2	11000	0.3
4	Permeation	20	$E_3 e^{[(r-40)\ln(E_5/E_3)]/h_4}$	0.3
5	Core	120	70	0.3
6	Permeation	20	$E_5 e^{[(r-180)\ln(E_7/E_5)]/h_6}$	0.3
7	Adhesive	2	11000	0.3
8	Permeation	20	$E_7 e^{[(r-222)\ln(E_9/E_7)]/h_8}$	0.3
9	Face	18	120000	0.3

TABLE 2: The present solution of stresses and deformations in the face layer ($i = 1$) and permeation layer ($i = 2$) with different sublayer divided number λ , respectively.

r (mm)	λ	σ_{θ}^i (MPa) $\theta = \pi/8$	σ_r^i (MPa) $\theta = \pi/8$	$\tau_{r\theta}^i$ (MPa) $\theta = 0$	u_{θ}^i (mm) $\theta = 0$	u_r^i (mm) $\theta = \pi/8$
1018 ($i = 1$)	2	-24.52	0.8247	-4.114	-1.271	-5.295
	4	-25.24	0.8295	-4.141	-1.287	-5.366
	6	-25.40	0.8303	-4.145	-1.290	-5.379
	8	-25.44	0.8306	-4.147	-1.291	-5.384
	10	-25.47	0.8307	-4.147	-1.292	-5.386
1038 ($i = 2$)	2	-33.75	-0.02910	-1.485	-0.8844	-5.290
	4	-25.42	-0.01452	-1.541	-0.8952	-5.360
	6	-23.07	-0.01168	-1.553	-0.8973	-5.373
	8	-21.98	-0.01068	-1.557	-0.8980	-5.376
	10	-21.90	-0.01061	-1.558	-0.8984	-5.378

least three significant figures when $\lambda = 10$. Therefore, the divided number is taken as $\lambda = 10$ for the following calculations.

Then, the convergence property associated with the series terms for the solution is studied. The arch is acted by uniform load $q(\theta) = 1$ N/mm. The present solution of stresses and deformations with different series number N is given in Table 3. It can be found that the present solution is rapidly convergent and has high convergence precision with four significant digits when $N = 9$. Thus, the number of series terms is fixed at $N = 9$ unless stated.

3.2. Comparison Study. We still consider the simply supported sandwich arch in Section 3.1; however, the acting load is $q(\theta) = 1$ N/mm and β is the variable. The present solution is compared with the finite element (FE) solution obtained by ANSYS. In the FE modeling, the PLANE-182 element is used to simulate all the layers. Due to the symmetry of the arch, only half part of the arch is modeled, as shown in Figure 3. The FE mesh is created by dividing the arch length into 50 elements, while the thicknesses of face, adhesive, core, and permeation layers is divided into 4, 1, 20, and 10 elements, respectively. Figure 4 shows the comparison of stresses and deformations between the present solution and FE solution with different arch angle β . It can be found that the FE solution is in good agreement with the present one; however, the FE solution with the fine mesh is time

TABLE 3: The convergence property of the present solution in the face layer ($i = 1$) and permeation layer ($i = 2$) with different series number N , respectively.

r (mm)	N	σ_θ^i (MPa) $\theta = \pi/8$	σ_r^i (MPa) $\theta = \pi/8$	$\tau_{r\theta}^i$ (MPa) $\theta = 0$	u_θ^i (mm) $\theta = 0$	u_r^i (mm) $\theta = \pi/8$
1018 ($i = 1$)	1	-32.43	1.058	-5.280	-1.645	-6.858
	3	-32.13	1.068	-5.424	-1.645	-6.851
	5	-32.13	1.068	-5.427	-1.645	-6.851
	7	-32.13	1.068	-5.428	-1.645	-6.851
	9	-32.13	1.068	-5.428	-1.645	-6.851
1038 ($i = 2$)	1	-27.17	-0.01300	-1.984	-1.144	-6.850
	3	-26.92	0.02640	-2.037	-1.143	-6.843
	5	-26.93	0.02445	-2.039	-1.143	-6.844
	7	-26.93	0.02459	-2.039	-1.143	-6.844
	9	-26.93	0.02459	-2.039	-1.143	-6.844

FIGURE 3: Schematic of the half part in the FE modeling.

FIGURE 4: Continued.

FIGURE 4: The comparison of stresses and deformations between the present solution and FE solution with different arch angles. (a) σ_θ^i at $\theta = \beta/2$. (b) σ_r^i at $\theta = \beta/2$. (c) $\tau_{r\theta}^i$ at $\theta = 0$. (d) u_r^1 at $r = R_1$.

TABLE 4: Comparisons of FE results with the present results for different length-to-thickness ratios a/H when $t = 104$ (s), respectively.

h_1 (mm)	Solutions	Present	FE	1D	R_{FE} (%)	R_{1D} (%)
25	σ_m (MPa)	204.8	204.7	204.3	0.0805	0.199
	τ_m (MPa)	-0.7713	-0.7698	-0.7645	0.198	0.691
	u_{rm} (mm)	-15.58	-15.55	-15.44	0.163	0.710
40	σ_m (MPa)	87.21	86.72	86.30	0.563	0.487
	τ_m (MPa)	-0.3445	-0.3441	-0.3413	0.105	0.799
	u_{rm} (mm)	-4.268	-4.246	-4.134	0.516	2.64
50	σ_m (MPa)	57.73	57.28	55.97	0.778	2.28
	τ_m (MPa)	-0.2404	-0.2400	-0.2381	0.165	0.820
	u_{rm} (mm)	-2.326	-2.304	-2.108	0.956	8.53
75	σ_m (MPa)	26.40	26.06	23.44	1.27	10.1
	τ_m (MPa)	-0.1326	-0.1321	-0.1309	0.408	0.878
	u_{rm} (mm)	-0.7969	-0.7856	-0.6401	1.41	18.5

Note: R_{FE} means $|(FE-Present)/Present|$; R_{1D} means $|(1D-Present)/Present|$.

TABLE 5: The thickness and material properties for each layer in the sandwich arch with seven layers.

i	Layer type	h_i (mm)	E_i (MPa)	μ_i
1	Face	16	40000	0.3
2	Adhesive	4	9000	0.3
3	Permeation	h_3	$E_2 e^{[(z-20)\ln(E_4/E_2)]/h_3}$	0.3
4	Core	$60 - 2h_3$	70	0.3
5	Permeation	h_3	$E_4 e^{[(z-80+h_3)\ln(E_6/E_4)]/h_5}$	0.3
6	Adhesive	4	9000	0.3
7	Face	16	40000	0.3

consuming, especially in mesh division and calculation. Besides, the stresses and deformations, in absolute values, increase with the increase of β .

By letting $k_i = 0$ in the basic equations, the present solution can be used to predict stresses and deformations for the fully homogeneous layered arch. Consider a sandwich arch composed two face layers sandwiching a relatively soft thin interlayer, which was studied by Galuppi

and Royer-Carfagni [7] based on the 1D Euler-Bernoulli theory. The geometric and material parameters are fixed at $q(\theta) = 0.5 \text{ N/mm}^2$, $E_1 = E_3 = 70000 \text{ MPa}$, $E_2 = 2.6 \text{ MPa}$, $\mu_1 = \mu_2 = \mu_3 = 0.3$, $R_1 = 1000 \text{ mm}$, $\beta = \pi/4$, $h_1 = h_3$, and $h_2 = 2 \text{ mm}$. We define three variables: $\sigma_m = \sigma_\theta^1$ at $\theta = 0.5\beta$, $r = R_1$; $\tau_m = \tau_{r\theta}^1$ at $\theta = 0$, $r = R_1 + h_1$; and $u_{rm} = u_r^1$ at $\theta = 0.5\beta$, $r = R_1$. The comparison of σ_m , τ_m , and u_{rm} between the present, FE, and 1D solutions with different layer

FIGURE 5: Stress distribution along the radial direction with different permeation thicknesses $h_3 = 0, 10, 15$, and 20 mm, respectively. (a) σ_θ^i at $\theta = \beta/2$. (b) Enlarged view of σ_θ^i . (c) $\tau_{r\theta}^i$ at $\theta = 0$. (d) Enlarged view of $\tau_{r\theta}^i$. (e) σ_r^i at $\theta = 0$.

thicknesses is shown in Table 4. It can be found that the FE solution agrees well with the present solution for all cases. The 1D solution is close to the present one for the thin arch, while the error of 1D solution increases when the arch becomes thick. The errors of σ_m and u_{rm} in 1D solution reach 10.1% and 18.5%, respectively.

3.3. Parametric Study. Consider a simply supported sandwich arch with five layers with $R_1 = 2000$ mm, $R_2 = 2100$ mm, and $\beta = \pi/8$ acted by distributed load $q(\theta) = \sin(\pi/\beta)$ N/mm. Here, only the permeation effect between the adhesive and core layers is considered. The thickness and the material property of each layer are given in Table 5. The distributions of

σ_{θ}^i , $\tau_{r\theta}^i$, and σ_r^i along the radial direction with different permeation thicknesses $h_3 = 0, 10, 15$, and 20 mm are shown in Figure 5, in which $h_3 = 0$ means the special case of the sandwich arch without the permeation layer. It can be found that (i) the absolute value of σ_{θ}^i at the internal and external surfaces in the face layers decreases with the increase of h_3 ; (ii) the absolute peak value of $\tau_{r\theta}^i$ in the face layers decreases with the increase of h_3 , while that in the core layer increases with the increase of h_3 ; (iii) the distributions of σ_{θ}^i and $\tau_{r\theta}^i$ tend to be smooth with the increase of h_3 ; and (iv) the distribution of σ_r^i is less affected by h_3 .

4. Conclusions

Based on the elasticity theory in the polar coordinate, a refined model of the simply supported sandwich arch considering the permeation effect of the adhesive is presented. By means of the recursive matrix method, the solution can be efficiently obtained for the arch with many layers. Conclusions are summarized as follows:

- (1) The present solution is rapidly convergent with the increase of series term and sublayer number.
- (2) The finite element solution with the fine mesh is in agreement with the present solution, while the finite element method is time consuming in mesh division and calculation.
- (3) The 1D solution, based on the Euler–Bernoulli theory, is close to the present solution for thin arches; however, the error increases with the increase of arch thickness.
- (4) The permeation thickness has considerable effect on the stress and deformation distributions. The distributions of circumferential normal stress and the shear stress tend to be smooth in the radial direction as the permeation thickness increases.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was financially supported by the National Natural Science Foundation of China (Grant nos. 51608264, 51978336, and 51778285) and the Natural Science Foundation for distinguished Young Scholars of Jiangsu Province (Grant no. BK20190034).

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