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Research Article

Modeling Nile Tilapia Heterogeneous Growth under Different Stocking Densities during Pre-Grow-Out Stage

V. H. Fuentes-Andraca,[1] **M. E. Araneda-Padilla**,[2] **R. Domínguez-May**,[3] **M. Gullian-Klanian**,[4] **E. J. Marin-Coria**,[5] **J. C. Quintana-Casares**,[6] **D. Peñalosa-Martinell**,[7] **and G. Ponce-Díaz**[1]

[1]*Instituto Politécnico Nacional-Centro Interdisciplinario de Ciencias Marinas, Avenue Instituto Politécnico Nacional s/n Col, Playa Palo de Santa Rita, C.P. 23096, La Paz, B.C.S., Mexico*
[2]*Benchmark Genetics Chile SPA, Unidad de Bioeconomía y Control Santa Rosa 560 Oficina 25 B, Puerto Varas 5550000, Chile*
[3]*Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Unidad Mérida, Carretera Mérida–Progreso Km. 6, Mérida C.P. 97310, Yucatán, Mexico*
[4]*Universidad Marista de Mérida, Periférico Norte Tablaje Catastral, 13941, Carretera Mérida–Progreso, Mérida C.P. 97300, Yucatán, Mexico*
[5]*Instituto de Ecología A.C. (INECOL), Xalapa 91073, Veracruz, Mexico*
[6]*Maricultura del Pacífico S.A. de C.V., Avenue Dr. Carlos Canseco 5994 Colonia El Cid, Mazatlán C.P. 82110, Sinaloa, Mexico*
[7]*Centro de Investigaciones Biológicas del Noroeste (CIBNOR), P.O. Box, 128, La Paz, C.P. 23000, Baja CA Sur, Mexico*

Correspondence should be addressed to G. Ponce-Díaz; gponced@ipn.mx

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The heterogeneous and homogeneous growth (HmG) assumptions in an intensive tilapia *Oreochromis niloticus* pre-grow-out production were evaluated under the different stocking scenarios. Four growth models (von Bertalanffy, Logistic, Pütter, and Gompertz) were tested and modified to include the effect of stocking density and growth difference. Models with homogeneous growth were fitted using a nonlinear regression, while heterogeneous growth (HtG) models were parameterized using quantile (0.05, 0.15, 0.25, 0.50, 0.75, 0.85, and 0.95) regression. According to goodness of fit and validation, the models that best fit homogeneous and HtG were Logistic and von Bertalanffy, respectively, which confirm the existence of the dense dependency effect on growth performance. Density and growth have an inversely proportional relationship. Quantile regression provided greater efficiency in predicting growth of the different groups of individuals in the population. The results obtained can be used by the aquaculture farmer to select stocking management strategies and optimal transfer time for tilapia juveniles.

1. Introduction

The tilapia *Oreochromis niloticus* is the third most farmed freshwater fish in the world and represents an important food source in the different countries [1]. Tilapia production includes three stages: nursery (1–30 g body weight), pre-grow-out (30–220 g body weight), and grow-out (>220 g body weight) [2]. Pre-grow-out is considered a crucial stage in tilapia culture, especially in a full cycle or vertically integrated companies because it reduces mortality, grow-out time, disease risk, high implementation, and operation costs, exposure to extreme environmental variables and increases per unit area productivity in the intensive systems [3–12].

Intensive systems are characterized by high-stocking densities (25–300 fish m^{-3}), which affect growth performance [5, 11, 13–18]. Therefore, an important management factor is to select the optimal stocking density in the three tilapia (nursery, pre-grow-out, and grow-out) culture stages, because this factor determines growth and other productive variables of economic interest [19–21]. However, high-stocking densities cause high levels of stress and trigger disease outbreaks due to impaired immunity and higher vulnerability to pathogens [22, 23].

Growth in aquaculture has been evaluated through different nonlinear growth models that represent the balance

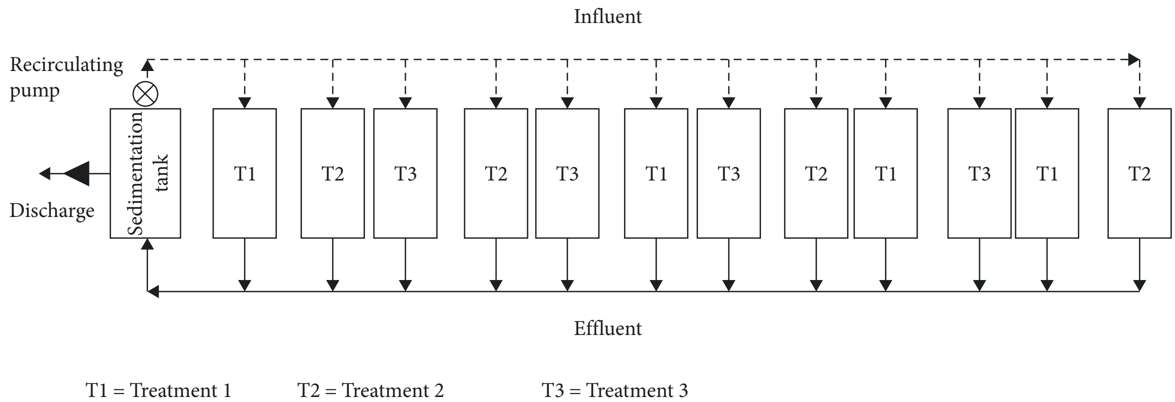

Figure 1: Layout of experimental tanks in the pre-grow-out area of tilapia *Oreochromis niloticus* in a biofloc system.

between anabolism and catabolism and incorporate the principle of Pütter's equation [24–28]. Some of these models have been modified to include the effect of important management or environmental variables [28–35].

Most growth models used in aquaculture consider the mean weight and assume size homogeneity [18, 36]. Parameters for these models are usually obtained using nonlinear regression methods [37, 38]. However, there is evidence that organisms on farms grow heterogeneously under different rearing conditions. Since slight changes in final weights could cause significant changes during economic valuation, some studies have evaluated heterogeneous growth (HtG) for determining productive performance and economic benefit [21, 39–46].

Models with HtG can be fitted using quantile regression, which is suitable for nonparametric analyses, offers a closer idea of weight distribution, and is not sensitive to the presence of outliers [42, 47–55].

Determining HtG of tilapia in the pre-grow-out stage is fundamental, mainly when the producer is required to carry out transfer strategies without overestimating or underestimating growth [56]. Overestimating growth leads to revenue and earnings expectations that would likely not materialize; on the contrary, underestimating may lead to a delay in harvest time and nonoptimal feeding [38].

Tilapia growth has been previously modeled in the pre-grow-out stage considering size homogeneity [2]. However, models with HtG are still scarce [31, 43, 44, 57]. To date, to our knowledge, works regarding the HtG of Nile tilapia in pre-grow-out stage have not been found. Therefore, the objective of the present research study is to evaluate the effect of stocking density on the HtG of tilapia reared in pre-grow-out stage under the different stocking densities.

2. Materials and Methods

2.1. Experimental Design and Culture Conditions. The experiment was developed in a tilapia enterprise located in Sinaloa, Mexico using different stocking densities (D_0) in an intensive pre-grow-out facility under a Biofloc system. The stocking densities used were chosen based on the previous studies of tilapia cultures [58–63] with three treatments: $T1 = 59$ fish m^{-3}, $T2 = 89$ fish m^{-3}, and $T3 = 117$ fish m^{-3} with four replicates per treatment. The three treatments were randomly assigned in 12-indoor rectangular tanks of 40 m^{-3} with brackish water (4 PPT) and constant aeration using a diffuser hose Aero-tube® (Aero-Tube, Sparks, NV, USA). The experiment used 42,400 tilapias with an average initial individual weight of 27.00 ± 6.63 g and lasted 69 days (Figure 1).

The Biofloc system was initiated four weeks before the introduction of fish in the three tanks. This system was prepared by adding Winfish® (Belenes, Zapopan, Jal., México) commercial feed with 55% crude protein, Epicin-hatcheries® (Megasupply, Eastampton, NJ, USA) commercial probiotic (*Bacillus subtilis*, *Bacillus licheniformis*, *Bacillus coagulans*, *Lactobacillus acidophilus*, and *Saccharomyces cerevisiae*) with a concentration of 4.0×10^{-9} colony forming units (CFU)/g, microalgae of the genus *Thalassiosira* sp. with a concentration of 5×10^{-4} mL^{-1}, molasses and corn flour. A carbon–nitrogen ratio of 12 : 1 was maintained following the methodology proposed by De Schryver et al. [64]. At the end of 4 weeks, 10 m^{-3} of the water were added to each culture tank and the tilapias were stocked.

Fish were fed using a commercial feed Winfish® (Belenes, Zapopan, Jal., México) for tilapia with 35%–45% crude protein on a daily ratio between 3% and 4% of the biomass, distributed from six to seven rations. Weekly biometrics were conducted to evaluate tilapia growth to adjust the feed ration and the carbon sources to be administered in each tank according to the biomass (kg) obtained. Monitoring was carried out by weighing individually 30 fish per tank using a digital scale (± 0.01 g).

Feed was applied directly on the surface of the tanks, and the carbon sources were weighed and mixed individually in a clean bucket with tank water, followed by a uniform and direct application to the tank surface after the first feeding. Water temperature and dissolved oxygen content were measured in each tank with an oximeter YSI 55® (YSI Inc., Yellow Springs, OH, USA) four times a day (06:00, 12:00, 18:00, and 24:00 hr). Water pH was measured with a potentiometer Waterproof pHTestr 10® (Cole-Parmer, East Bunker Court Vernon Hills, IL, USA). Weekly measurements of ionized and nonionized ammonium, nitrite and alkalinity

Table 1: Water quality of intensive pre-grow-out of Nile tilapia *Oreochromis niloticus* in a biofloc system with three stocking densities.

Variable	Treatment 1	Treatment 2	Treatment 3
Temperature (°C)	31.78 ± 0.54	31.58 ± 0.84	31.85 ± 0.51
Disolved oxygen (mg L^{-1})	4.99 ± 0.27	4.75 ± 0.35	4.93 ± 0.33
pH	7.98 ± 0.11	7.95 ± 0.09	7.95 ± 0.10
Ionized ammonium (mg L^{-1})	1.25 ± 0.63	0.94 ± 0.57	0.53 ± 0.41
Nonionized ammonium (mg L^{-1})	1.26 ± 0.92	1.09 ± 0.67	0.64 ± 0.53
Nitrite (mg L^{-1})	1.15 ± 0.56	1.16 ± 0.39	1.13 ± 0.72
Alkalinity (mg L^{-1})	315.17 ± 27.53	299.16 ± 10.20	291.60 ± 11.25

were performed with a photometer YSI 9300® (YSI Inc., Yellow Springs, OH, USA) at 08:00 hr (Table 1). These water parameters were maintained in the optimal conditions for tilapia culture [65, 66].

Settling solids were determined using Imhoff® (DWK Life Sciences, Millville, NJ, USA) cones with a settling time of 20 min and taken daily alternating each tank once per treatment at 8:00 hr. An optimal level for juvenile tilapia from 20 to 50 mL L^{-1} was maintained throughout the culture; when the allowed level exceeded, it was controlled with limited water exchanges (10%) and a sedimentation tank connected to the culture tanks with recirculating water [66–71].

The water exchange rate was 10% every other day from Days 7 to 33 and 30% 1 day a week from Days 55 to 69; water recirculation through the sedimentation tank was 8 hr/day from Days 34 to 69. To ensure that water management was adequate, the volume of flocs was measured before and after these processes [70].

2.2. Modeling Homogeneous and Heterogeneous Growth Scenarios. Two growth scenarios were studied in the present investigation. The first one assumes homogeneous growth (HmG) where all the initially stocked fish ($t = 0$) show a similar growth pattern; size variability is ignored, which means that from stocking to transfer time ($t = T$), the population is represented by the average weight $x(t)$ of the individuals, where t is time in days. This process has been the most common way of representing growth of aquaculture populations via a model. The second scenario using quantile regression assumes the HtG in which the fish of the same population are considered to show different growth patterns. This process starts with the stocking of fish ($t = 0$) of the same age and origin (cohort) which is distributed in the different sizes over time until transfer time ($t = T$).

2.2.1. Homogeneous Growth (HmG). To represent tilapia growth in HmG, mathematical models were tested based on bioenergetic principles and biologically interpretable parameters [24, 72, 73]. For this purpose, von Bertalanffy, Logistic, Pütter, and Gompertz models were selected because they are widely used in aquaculture. Such growth models were modified in the anabolic component with the objective of including the effect of initial density D_0. The inclusion of this effect (initial density) on the growth models has been similar to that described by Araneda et al. [31]. Thus, the mathematical expressions used to model pre-grow-out of Nile tilapia in a biofloc system under the different stocking densities were:

(a) von Bertalanffy

$$g(x; D_0) = \alpha_0 f(D_0) x^{\frac{2}{3}} - \alpha_2 x, \tag{1}$$

(b) Logistic 1

$$g(x; D_0) = \alpha_0 f(D_0) x - \alpha_2 x^2, \tag{2}$$

(c) Pütter

$$g(x; D_0) = \alpha_0 f(D_0) x^{\beta_1} - \alpha_2 x, \tag{3}$$

(d) Logistic 2

$$g(x; D_0) = \alpha_0 f(D_0) x^{\beta_1} - \alpha_2 x^2, \tag{4}$$

(e) Gompertz

$$g(x; D_0) = \alpha_0 f(D_0) x^{\beta_1} - \alpha_2 x ln(x), \tag{5}$$

where x is the weight (g) of each tilapia; D_0 represents the initial density; α_0, α_2, and β_1 are parameters within each growth function; $f(D_0)$, represents the effect of initial density, which is given by [31]:

$$f(D_0) = e^{-\alpha_1 (\ln D_0)^2}, \tag{6}$$

where α_1 is a parameter.

2.2.2. Heterogeneous Growth (HtG). Under the heterogeneous scenario, the same models described by HmG were considered (Equations (1)–(5)) but $\beta = 1$ was assumed in Equations (3)–(5). Quantile regression was used to parameterize growth. In this study, the quantiles 0.05, 0.15, 0.25, 0.50, 0.75, 0.85, and 0.95 were selected to estimate the HtG of tilapia. This selection was made to consider central (quantiles 0.25, 0.50, and 0.75) and extreme (quantiles 0.05, 0.15, 0.85, and 0.95) values [51]. According to the empirical data, these models admitted a modification with the effect of the initial density (D_0) in the parameter that represents catabolism (α_2) as indicated below.

A new perspective of the effect of the initial density in tilapia intensive pre-grow-out emerges. However, the modified Equations (3) and (4) did not show convergence in the estimation of the parameters. Therefore, the three equations used to model the growth of the organisms with size heterogeneity were:

(a) von Bertalanffy:

$$g(x; D_0) = \alpha_0 f(D_0) x^{\frac{2}{3}} - \alpha_2 f(D_0) x, \quad (7)$$

(b) Logistic 1:

$$g(x; D_0) = \alpha_0 f(D_0) x - \alpha_2 f(D_0) x^2, \quad (8)$$

(c) Gompertz:

$$g(x; D_0) = \alpha_0 f(D_0) x - \alpha_2 f(D_0) x ln(x), \quad (9)$$

where x is the size in weight (g) of each tilapia; D_0 represents the initial density; α_0, and α_2 are coefficients within each growth function; and $f(D_0)$ represents the effect of initial density, which was defined in Equation (6).

To obtain the weight of the organisms at time t from Equations (1)–(5) and (7)–(9), the following equation was used:

$$X_t = X_0 + \int_0^T g(x; D_0) dt, \quad (10)$$

where x_0 is the initial tilapia weight and T represents the final integration time.

2.3. Fit, Parameterization, and Validation of the Models. In the HmG, the model coefficients were estimated through fitting the average observed weight from each tank per treatment of initial density. The parameterization of the growth models was performed using nonlinear regression methods [74] in Statistica v12 (StatSoft Inc., Tulsa, OK, USA).

In HtG, coefficients were estimated through the fit of all the observed weights from each tank per treatment of initial density. The parameterization of the growth models was performed using quantile regression methods [75] and quantreg package for quantile regression available in R statistical software (R Development Core Team, 2014 and RStudio v2.15.2).

The parameterized Equations (1)–(5) and (7)–(9) were solved by numerical integration (Equation (10)) and simulated in Excel MS© (Microsoft Corp., Redmond, WA, USA) using the Euler method with a step size of 1 day (*ds = 1*).

Validation was made using indicators for system dynamics [76]. The indicators used include the coefficient of determination (R^2), root-mean-square error (RMSE), and Theil's inequality coefficient (U) with a critical value of 0.2 ($U \leq 0.2$) [77] and three proportions of inequality in mean (U^m), variance (U^s), and covariance (U^c) [31, 78–80].

Table 2: Parameters, estimation, standard error, and t statistic for the modified models of homogeneous growth of tilapia *Oreochromis niloticus*.

Models	Parameters	Estimation	Standard error	t Statistic
von bertalanffy	α_0	0.4439	0.10	4.44
	α_1	−0.0309	0.09	−3.65
	α_2	0.0307	0.01	3.33
Logistic 1	α_0	0.1120	0.02	4.63
	α_1	−0.0461	0.01	−4.72
	α_2	0.0002	0.00	6.87
Pütter	α_0	1.9181	0.32	5.91
	α_1	−0.0009	0.00	−3.39
	β_1	0.9891	0.00	535.03
	α_2	1.7676	0.30	5.90
Logistic 2	α_0	0.0081	0.00	11.58
	α_1	−0.0060	0.00	−4.94
	β_1	1.8837	0.01	163.11
	α_2	0.0040	0.00	9.83
Gompertz	α_0	0.1711	0.02	7.09
	α_1	−0.0078	0.00	−2.59
	β_1	1.1119	0.03	36.87
	α_2	0.0486	0.01	3.41

Note: α_0, α_1, α_2, and β_1 are parameters within each growth function.

3. Results

3.1. Modeling Homogeneous Growth (HmG)

3.1.1. Parameter Estimation. The nonlinear regression analyses proposed in Equations (1)–(5) showed statistical significance for the estimation of growth rates ($p < 0.1$). The parameter α_1 of each function (Equations (1)–(5)) includes the effect of the initial density D_0 on growth rate g (x; D_0) of each size x (Table 2). All parameters were significant ($p < 0.1$) in the modified HmG models (von Bertalanffy, Logistic 1, Pütter, Logistic 2, and Gompertz).

The predictions of each function showed a decrease in the growth rates per size as initial density increased (Figure 2). It is important to highlight that depending on density, there are different growth trends, and every trend has a maximum growth rate.

At the end of the experiment, the von Bertalanffy model estimated differences in growth of 53.9% between T1 (2.91 g day^{-1}) and T3 (1.67 g day^{-1}). On the other hand, the Logistic 1, Pütter, Logistic 2, and Gompertz models estimated differences in growth of 55.6, 53.5, 57.9, and 55.3%, respectively, between T1 and T3.

3.1.2. Validation and Simulation. In the HmG, all modified growth functions adequately represented the increase in weight of tilapia in the pre-grow-out stage. However, the fit and validation statistical indicators showed that the Logistic model 1 (Equation (2)) was the one that best represents the empirical data from the period $t = 0$ (stocking) to $t = 69$ (transfer).

In each initial culture density (D_0), the coefficient of determination (R^2) explained from 97% to 98% the variation

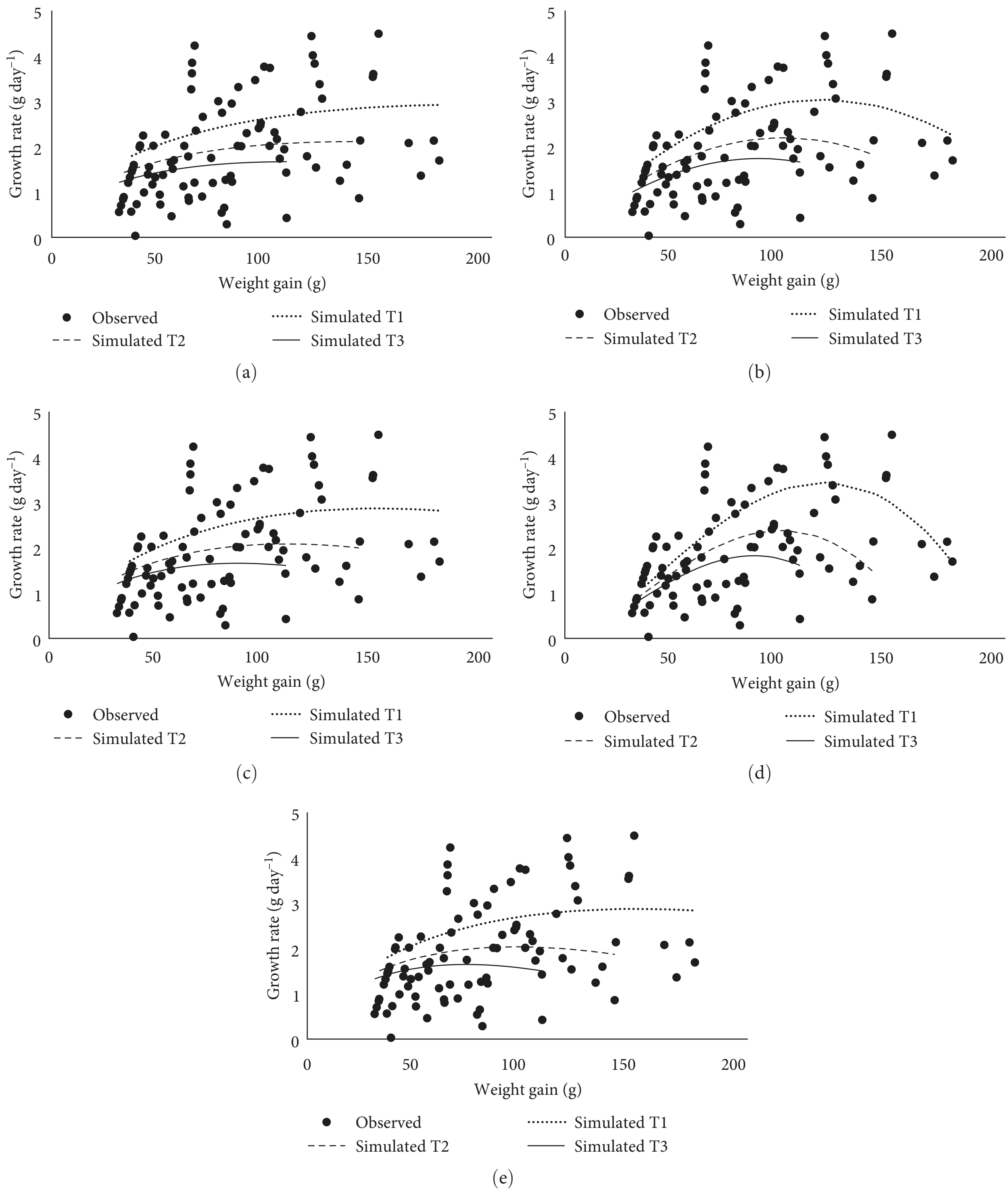

Figure 2: Observed (points) and simulated tilapia *Oreochromis niloticus* growth rates (g day^{-1}) (lines) for the modified growth models in homogeneous scenario: (a) von Bertalanffy; (b) Logistic 1; (c) Pütter; (d) Logistic 2; and (e) Gompertz. Each curve represents the simulated growth rate for a given initial density (D_0).

of the model concerning the observed values of weight and the RMSE estimated values of 5.9–6.5 g. This result was confirmed by Theil's inequality coefficient (U), showing that the Logistic model 1 generated the observed data more precisely. In each initial density, the values of U were lower than the critical value of 0.2, with low error in the mean and variance (Table 3).

All HmG models had a good fit regarding the observed data. However, R^2 and U were the indicators considered to select the best model. Logistic 1 was chosen as the best model for the three

Table 3: Validation of the modified growth models in the homogeneous scenario for tilapia *Oreochromis niloticus*.

Models	Statistics	Treatment 1	Treatment 2	Treatment 3
von Bertalanffy	R^2	0.985.79	0.97	0.98
	RMSE	0.06	6.56	7.39
	U	0.09	0.09	0.09
	U^m	0.00	0.20	0.64
	U^s	0.91	0.00	0.12
	U^c		0.80	0.24
Logistic 1	R^2	0.99	0.97	0.98
	RMSE	6.58	5.97	6.45
	U	0.06	0.09	0.09
	U^m	0.31	0.11	0.51
	U^s	0.15	0.00	0.19
	U^c	0.55	0.88	0.30
Pütter	R^2	0.99	0.97	0.98
	RMSE	5.62	6.60	8.21
	U	0.06	0.09	0.09
	U^m	0.08	0.24	0.66
	U^s	0.01	0.00	0.15
	U^c	0.92	0.76	0.20
Logistic 2	R^2	0.99	0.97	0.98
	RMSE	6.54	5.84	4.90
	U	0.07	0.09	0.09
	U^m	0.03	0.05	0.17
	U^s	0.35	0.00	0.25
	U^c	0.64	0.95	0.59
Gompertz	R^2	0.99	0.97	0.98
	RMSE	6.29	7.35	9.99
	U	0.06	0.09	0.10
	U^m	0.24	0.39	0.71
	U^s	0.03	0.00	0.15
	U^c	0.74	0.61	0.14

Note: R^2 = coefficient of determination; RMSE = root- mean- square error; U = Theil's coefficient of inequality; U^m = proportion of mean; U^s = proportion of variance; U^c = proportion of covariance.

treatments. Theil's inequality coefficient (U) showed that Logistic 1 was the most appropriate model with values of 0.062, 0.086, and 0.090 for T1, T2, and T3, respectively.

Over time, the model adequately simulated a decrease in weight (g) as the initial density increased (Figure 3). At the end of the culture, T1 obtained a simulated average weight of 184.01 g, T2 of 142.72 g, and T3 of 118.27 g. Growth trajectories with the best model (Logistic 1) differed according to initial density. The initial density negatively affected the final average weight of the tilapia culture in the biofloc system.

3.2. Modeling Heterogeneous Growth (HtG). The objective of the HtG modeling was to select the mathematical function that best simulates the observed quantile growth trajectories. The growth model was selected using the same criteria as the HmG, through the statistical significance of its parameters and the simulation validation of each quantile growth curves.

3.2.1. Parameter Estimation. The nonlinear fitted models (Equations (7)–(9)) applying quantile regression showed significant parameters ($p < 0.1$ for each initial density (D_0)) in Logistic 1 and Gompertz model. Parameters α_0 and α_2 of the von Bertalanffy model were not significant in 0.05, 0.15, and 0.25 quantiles. Parameter α_1 of equation $f(D_0)$ (Equation (6)) included the effect of initial density (D_0) on growth rate in both model components (anabolic and catabolic). These parameters explain the changes that happen in the growth rate due to initial density (D_0) (Table 4).

The highest growth rates occur in T1 and the lowest ones are in T3. For example, for the von Bertalanffy model (Equation (7)) T1 had a maximum growth rate of 3.15 g day^{-1}, T2 of 2.66 g day^{-1}, and T3 of 2.32 g day^{-1} in the 0.50 quantile. For the Logistic model 1 (Equation (8)) T1 had a maximum growth rate of 3.43 g day^{-1}, T2 of 2.07 g day^{-1}, and T3 of 1.41 g day^{-1} in the same quantile. For the Gompertz model (Equation (9)), T1 had a maximum growth rate of 3.70 g day^{-1}, T2 of 1.91 g day^{-1}, and T3 of 1.39 g day^{-1} in the same quantile.

3.2.2. Validation and Simulation. In the HtG, the modified growth models represented the increase in weight of tilapias in the pregrowth stage. However, the statistical comparison showed that the von Bertalanffy model was the one that best represented the HtG data of tilapia from the period $t = 0$ (stocking) to $t = 69$ (transfer). In each initial culture density (D_0), the coefficient of determination (R^2) explained from

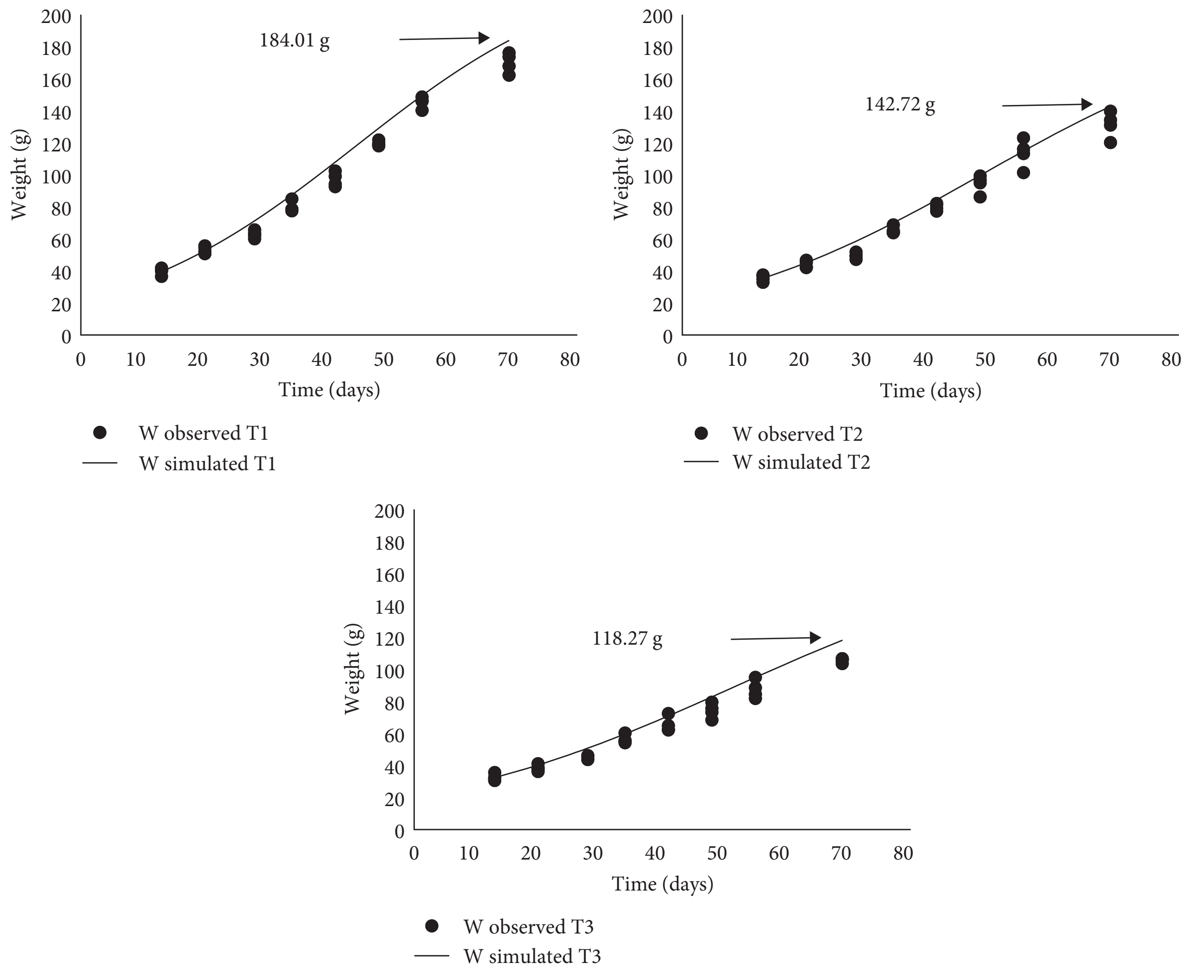

Figure 3: Results of modeling tilapia *Oreochromis niloticus* growth in the homogeneous scenario for each of the initial culture densities (D_0) with the Logistic 1 modified model. The numbers indicate the final simulated size.

Table 4: Quantiles, parameters, estimate, standard error, and *t* statistic of modified models in heterogeneous scenario for tilapia *Oreochromis niloticus* growth.

Models	Quantiles	Parameters	Estimate	Standard error	*t* Statistic
		α_0	0.1749	6897.75	0.42
	0.05	α_1	0.0254	0.01	5.39
		α_2	0.0123	0.01	0.84
		α_0	0.1806	11922.30	0.65
	0.15	α_1	0.0247	0.00	11.46
		α_2	0.0091	0.00	1.46
		α_0	0.2000	7745.88	0.86
	0.25	α_1	0.0269	0.00	16.01
		α_2	0.0106	0.00	1.89
		α_0	0.3270	315.83	3.49
von Bertalanffy	0.50	α_1	0.0326	0.00	17.98
		α_2	0.0316	0.00	5.05
		α_0	0.2971	1160.61	1.81
	0.75	α_1	0.0328	0.00	14.47
		α_2	0.0232	0.00	3.06
		α_0	0.3716	315.34	3.70
	0.85	α_1	0.0367	0.00	18.40
		α_2	0.0353	0.00	5.14
		α_0	0.5021	280.58	2.79
	0.95	α_1	0.0430	0.01	8.50
		α_2	0.0545	0.01	2.95

Table 4: Continued.

Models	Quantiles	Parameters	Estimate	Standard error	t Statistic
		α_0	1.0510	3.63	28.85
	0.05	α_1	0.1404	0.02	6.52
		α_2	0.0100	0.52	2.04
		α_0	0.6866	2.83	41.84
	0.15	α_1	0.1204	0.02	7.29
		α_2	0.0058	0.25	2.70
		α_0	0.6600	2.50	50.38
	0.25	α_1	0.1192	0.02	7.82
		α_2	0.0052	0.22	2.99
		α_0	0.7363	3.35	44.02
Logistic 1	0.50	α_1	0.1303	0.02	7.85
		α_2	0.0049	0.24	3.01
		α_0	0.9016	5.14	34.90
	0.75	α_1	0.1521	0.01	10.82
		α_2	0.0050	0.23	3.85
		α_0	1.6741	4.56	43.05
	0.85	α_1	0.1909	0.01	16.55
		α_2	0.0085	0.33	5.02
		α_0	4.2535	6.21	36.38
	0.95	α_1	0.2449	0.02	10.31
		α_2	0.0188	1.86	2.29
		α_0	5.2550	4.07	30.76
	0.05	α_1	0.1725	0.02	10.47
		α_2	1.0877	0.37	2.94
		α_0	6.3795	2.47	54.90
	0.15	α_1	0.1782	0.01	17.40
		α_2	1.2999	0.28	4.69
		α_0	5.3211	2.86	50.86
	0.25	α_1	0.1699	0.01	18.68
		α_2	1.0682	0.21	5.19
		α_0	6.8542	3.29	51.25
Gompertz	0.50	α_1	0.1871	0.01	17.78
		α_2	1.3369	0.29	4.66
		α_0	5.3795	5.22	38.52
	0.75	α_1	0.1833	0.01	18.16
		α_2	1.0143	0.22	4.61
		α_0	5.3080	8.07	27.11
	0.85	α_1	0.1868	0.01	15.19
		α_2	0.9851	0.22	4.40
		α_0	8.6631	8.75	28.33
	0.95	α_1	0.2134	0.01	15.86
		α_2	1.5713	0.38	4.17

Note: α_0, α_1, α_2, and β_1 are parameters within each growth function.

95% to 99% of the variation of the model concerning the observed weight values and RMSE estimated values from 3.1 to 24.3 g. This result was confirmed by Theil's inequality coefficient (U), showing that the modified von Bertalanffy model was the best one for weight observations in the heterogeneous case.

In each treatment, the values of U were lower than the critical value of 0.2, with low error in the mean and variance (Table 5). The best model was chosen considering R^2 and U values. In this case, von Bertalanffy is shown to have the best fit with observed data for the three densities and showed higher values in R^2, and lower ones in U. Although as mentioned earlier, some parameters were not significant.

In tilapia culture at 59 fish m^{-3} the quantile 0.25 indicates that 25% of the population has a body weight lower than 141.80 g while the other 25% has a body weight higher than 183.55 g in the quantile 0.75. Quantile 0.50 indicates a final body weight of 171.79 g in the median (Figure 4). Following this example in the other densities (89 and 117 fish m^{-3}), initial density has a negative effect on body weight. Arrows in Figure 4 show the final weight of central quantile 0.50 and extreme quantiles 0.05 and 0.95.

Table 5: Validation of the modified models in heterogeneous scenario for tilapia *Oreochromis niloticus* growth.

Models	Statistics	Quantiles at treatment 1							Quantiles at treatment 2							Quantiles at treatment 3						
		0.05	0.15	0.25	0.5	0.75	0.85	0.95	0.05	0.15	0.25	0.5	0.75	0.85	0.95	0.05	0.15	0.25	0.5	0.75	0.85	0.95
von Bertalanffy	R^2	0.98	0.98	0.98	0.99	0.98	0.98	0.99	0.99	0.98	0.98	0.98	0.98	0.99	0.96	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	RMSE	10.55	9.78	10.31	8.48	15.83	16.46	24.37	6.34	4.46	4.48	4.32	5.54	6.14	10.22	3.20	5.08	4.99	7.71	5.98	6.60	7.18
	U	0.16	0.12	0.11	0.09	0.11	0.11	0.12	0.13	0.08	0.08	0.07	0.07	0.07	0.11	0.06	0.07	0.07	0.10	0.08	0.08	0.10
	U^m	0.40	0.41	0.49	0.33	0.63	0.61	0.74	0.29	0.17	0.18	0.01	0.17	0.25	0.19	0.64	0.66	0.64	0.61	0.50	0.55	0.33
	U^s	0.54	0.48	0.38	0.44	0.28	0.25	0.22	0.63	0.35	0.31	0.00	0.16	0.27	0.13	0.13	0.25	0.26	0.37	0.47	0.39	0.62
	U^c	0.13	0.17	0.17	0.29	0.13	0.17	0.07	0.16	0.52	0.55	0.99	0.69	0.52	0.70	0.25	0.12	0.13	0.07	0.08	0.11	0.13
Logistic 1	R^2	0.72	0.78	0.81	0.86	0.95	0.96	0.98	0.94	0.95	0.96	0.98	0.98	0.99	0.96	0.98	0.98	0.98	0.99	0.99	0.99	0.99
	RMSE	19.88	18.59	18.11	18.18	11.45	11.59	9.50	9.71	9.50	8.94	5.89	7.80	15.61	26.04	7.02	7.72	7.05	2.13	8.90	17.97	28.44
	U	0.28	0.26	0.24	0.21	0.12	0.11	0.08	0.12	0.11	0.10	0.08	0.10	0.16	0.24	0.10	0.09	0.09	0.04	0.12	0.26	0.39
	U^m	0.40	0.25	0.20	0.19	0.11	0.12	0.04	0.65	0.66	0.59	0.39	0.25	0.54	0.60	0.77	0.77	0.73	0.36	0.63	0.63	0.70
	U^s	0.03	0.11	0.12	0.15	0.20	0.12	0.09	0.00	0.00	0.01	0.04	0.51	0.47	0.38	0.09	0.09	0.07	0.01	0.40	0.41	0.34
	U^c	0.57	0.65	0.69	0.69	0.72	0.78	0.89	0.35	0.34	0.41	0.57	0.31	0.05	0.07	0.15	0.15	0.21	0.64	0.02	0.01	0.00
Gompertz	R^2	0.80	0.78	0.82	0.85	0.94	0.95	0.97	0.95	0.95	0.96	0.97	0.97	0.98	0.96	0.97	0.97	0.97	0.99	0.99	0.99	0.99
	RMSE	23.68	24.51	23.62	24.39	16.54	28.18	11.40	9.80	11.01	10.42	7.37	7.35	10.34	15.91	5.06	5.29	5.23	3.74	7.63	11.12	17.10
	U	0.23	0.24	0.22	0.20	0.12	0.12	0.09	0.11	0.11	0.11	0.09	0.11	0.13	0.16	0.08	0.08	0.08	0.08	0.12	0.17	0.20
	U^m	0.66	0.57	0.56	0.55	0.52	0.79	0.28	0.72	0.72	0.68	0.47	0.01	0.25	0.41	0.72	0.67	0.58	0.09	0.50	0.55	0.70
	U^s	0.00	0.01	0.00	0.01	0.02	0.04	0.05	0.00	0.00	0.00	0.04	0.49	0.61	0.44	0.01	0.00	0.01	0.66	0.53	0.48	0.33
	U^c	0.34	0.43	0.44	0.44	0.47	0.18	0.69	0.28	0.28	0.32	0.50	0.56	0.22	0.21	0.27	0.33	0.42	0.33	0.04	0.03	0.01

Note: R^2 = coefficient of determination; RMSE = root-mean-square error; U = Theil's coefficient of inequality; U^m = proportion of mean; U^s = proportion of variance; U^c = proportion of covariance.

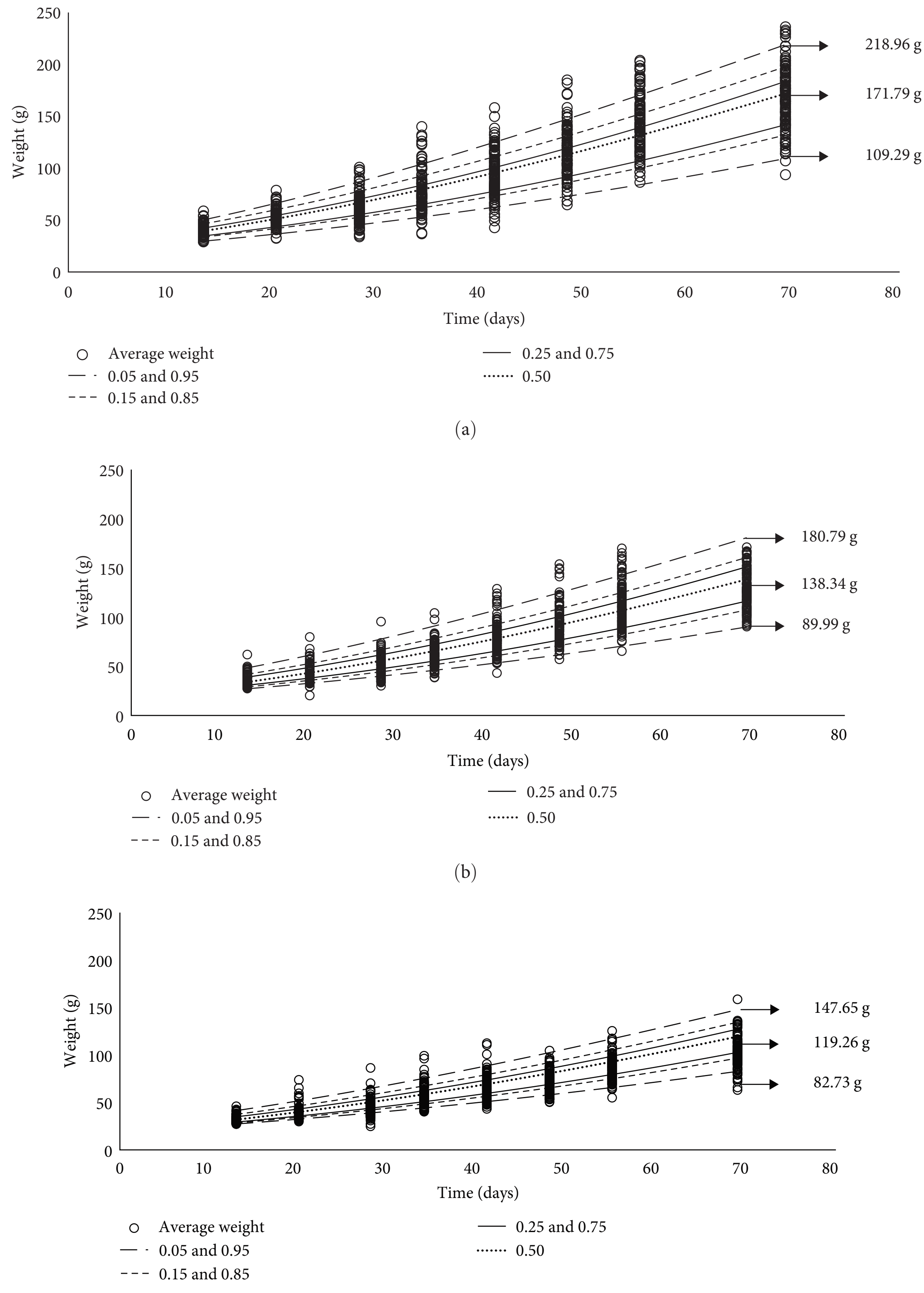

Figure 4: Results of the heterogeneous growth modeling of tilapia *Oreochromis niloticus* for each of the initial culture densities (D_0) with the modified models. Treatments (a) T1, (b) T2, and (c) T3.

TABLE 6: Simulated final weights in the three stocking densities with heterogeneous growth of tilapia *Oreochromis niloticus*.

Quantiles	Treatment 1	Treatment 2	Treatment 3
0.05	109.29 g	89.99 g	82.73 g
0.15	132.02 g	107.32 g	96.63 g
0.25	141.80 g	116.35 g	102.62 g
0.50	171.79 g	138.34 g	119.26 g
0.75	183.55 g	151.19 g	127.06 g
0.85	197.77 g	160.62 g	134.48 g
0.95	218.96 g	180.79 g	147.65 g

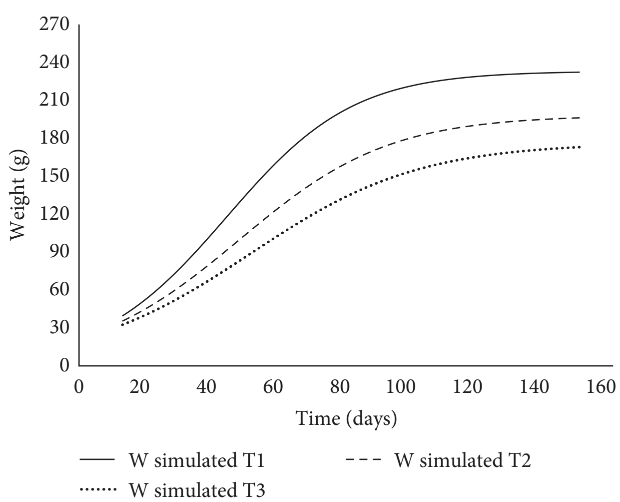

FIGURE 5: Simulation growth trajectories with sizes homogeneity in tilapia *Oreochromis niloticus* culture for 150 days.

The model adequately simulated a decrease in weight (g) in all quantiles as the initial density (D_0) increased (Table 6).

3.3. Forecasting Scenarios in Tilapia Pre-Grow-Out. With the best models of both scenarios (HmG and HtG), tilapia growth was projected in time. Weight prediction is important, especially in vertically integrated or complete cycle farms, because the aquaculture farmer needs to organize management plans and make the best decisions. For this reason, growth patterns in HmG and HtG were simulated with the best models (Logistic 1 and von Bertalanffy) during a pre-grow-out stage of 150 days with the same initial densities (T1, T2, and T3). HmG model shows evidence of the effect of initial density on growth (Figure 5). All densities have an initial increase in weight, which tends to stabilize through the culture of 150 days. Growth patterns of tilapia in pre-grow-out stage with HmG show logistic growth.

In the HtG model, simulation mainly considered growth curves corresponding to 0.25, 0.50, and 0.75, because other quantiles represent extreme variability [42]. The HtG model also shows the evident effect of initial density (Figure 6). Under the same quantile, organisms grow less at high densities and vice versa. Growth patterns of tilapia in pre-grow-out stage with HtG showed exponential growth.

The daily growth rates of quantiles 0.25, 0.5, and 0.75 at each initial density were also simulated (Figure 7). Depending on size, the organisms with the same initial density have different growth patterns. The effect of the initial density is evident since the low density (T1) shows a higher growth rate compared to the medium (T2) and high densities (T3). Instantaneous growth rates show a positive increase.

4. Discussion

The present research study shows homogeneous (HmG) and heterogeneous (HtG) growth models in intensive fattening of Nile tilapia with different initial densities. As recommended by several authors, both models contain the least possible number of parameters [31, 81], so they comply with the parsimony principle [82]. The modified Logistic model 1 was the most effective equation for projecting HmG growth, and the modified von Bertalanffy model was the most appropriate for casting HtG.

Simulated growth rates in HmG decreased with increasing initial stocking density, which is consistent with other studies of tilapia reared at different stocking densities [5, 20, 60, 71, 83–85]. The results confirm that stocking density affects the growth rate of tilapia raised in the pre-grow-out stage and with a biofloc system at the intensive level.

The Logistic model 1 described the continuous evolution of tilapia growth in the pre-grow-out stage at the intensive level considering the average body weight and showed the best fit homogeneous data, given its high-R^2 value and low RSME and $U < 0.2$. The results in this study are consistent with the findings of other authors who tested different growth models in tilapia and rainbow trout cultures considering size homogeneity [38, 86–88].

The modified Logistic model 1 of this study is the first evidence of a growth model expressed as the growth rate (differential equation) and altered in the anabolic component with the effect of population density, like the growth rate models studied by Araneda et al. [31]. The authors of the present research tested three modified models (von Bertalanffy, Gompertz, and Pütter) in intensive cultures of white shrimp in the growth stage.

Although the knowledge of suitable models to describe the HmG of tilapia in the pre-grow-out stage is still scarce, the Logistic model 1 can be implemented in intensive tilapia farms to model the HmG in this stage. The growth patterns of this study confirm that increased tilapia densities in the pre-grow-out stage can lead to biomass reduction due to crowding stress [13, 89, 90]. As a result, management decisions should be considered at high-population densities because brooding time is longer, organisms are less efficient, and economic performance is likely to suffer.

Even though Logistic 1 was the best model overall, it slightly overestimates average growth. Most of the growth models used in the aquaculture consider a single average value, which is a limiting factor to show various growth patterns of different batches or sizes in the same population. Logistic 1 was observed to adequately represented the

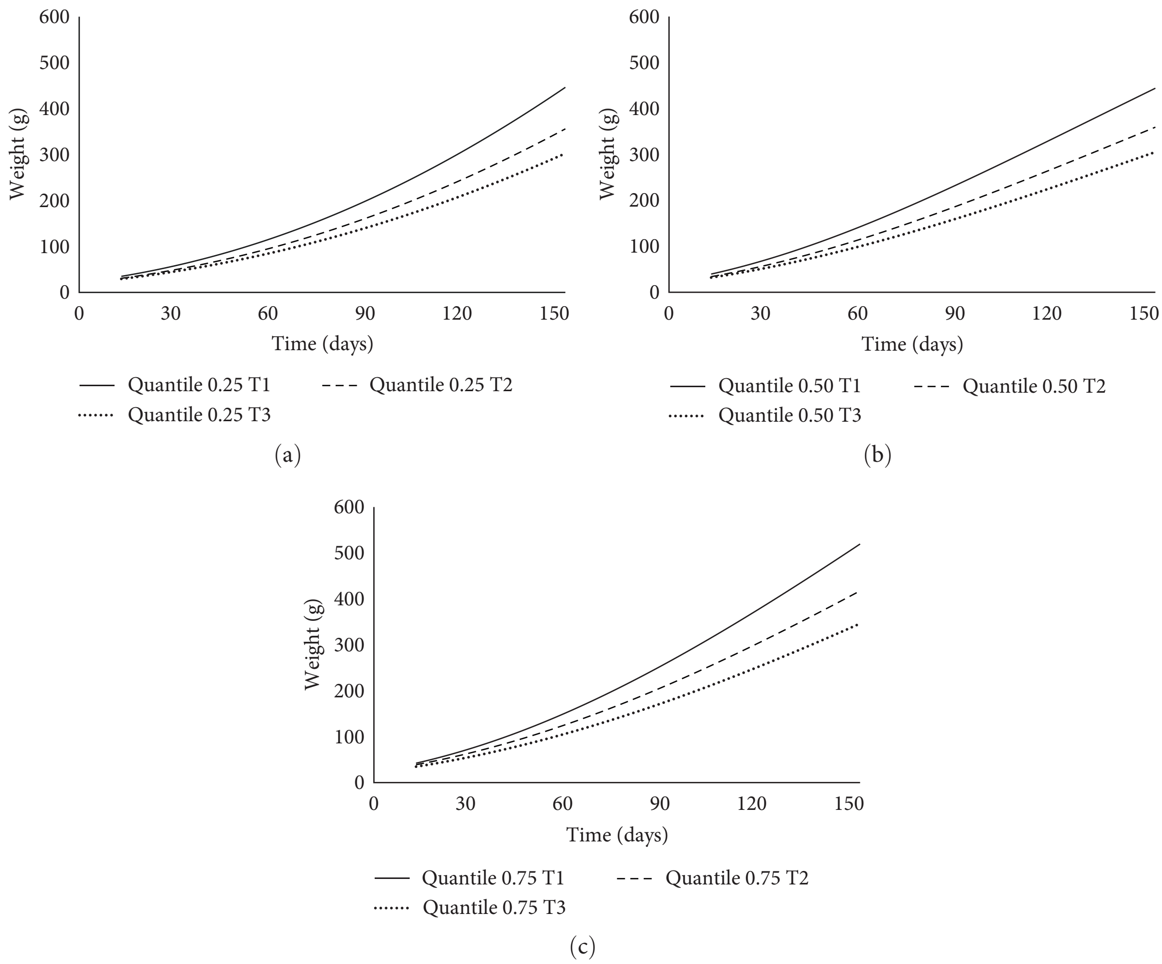

Figure 6: Simulation growth trajectories with size heterogeneity in tilapia *Oreochromis niloticus* culture for 150 days.

homogeneous tilapia growth at the intensive level in the pre-grow-out stage, which agrees with previous authors who used the Logistic function to model the HmG of tilapia in the grow-out stage [38, 86].

Furthermore, different growth models have been evaluated in the present study applying a quantile regression. Other authors have made these modifications considering, for example, bioenergetics growth models [34, 35, 91–93]. Thus, this contribution is important for the aquaculture industry in the production of organisms in the pre-grow-out stage.

The modification of the von Bertalanffy model shown in this research is, to our knowledge, the first evidence of a model modified in both components (anabolic and catabolic) with the effect of stocking density and fitted with quantile regression and the purpose of modeling HtG (Tables 4 and 5). These results are consistent with other authors, who have used a quantile regression mixed-TGC model to predict the HtG of gilthead sea bream considering the effect of water temperature [42, 49]. Other authors used a model size–structure to predict the HtG of tilapia [29, 40, 43, 94] and shrimp considering the effect of stocking density [31, 33].

Despite von Bertalanffy was the best growth model in HtG scenario, some parameters were not significant ($p < 0.1$) in lower quantiles (0.05, 0.15, and 0.25). This problem could be due to the small amount of data in the first quantiles, which affects the parameterization and could be solved with more data on each sample. Nevertheless, it is not a limiting factor for the contribution of this research study to the approximation of size heterogeneity in pre-grow-out stage. Martínez and Seijo [95] modeled the effect of water temperature on shrimp growth and some parameters of the growth model were not significant but subsequently, the growth model was successfully used in a bioeconomic analysis. Additionally, choosing the best model is complicated; in some cases, the statistics results contrast with the biological interpretation [96].

The quantile regression can serve as a predictive and decision-making support tool in intensive systems of tilapia or other farmed species, mainly in the pre-grow-out stage since it describes different growth patterns through time and gives relevant information to find optimal transfer times (from pre-grow-out to grow-out stages). Additionally, size heterogeneity is significant in economic terms because small fish with slow growth increases the culture costs, and different sizes have different market prices that directly affect revenue, thus net profit [21, 29, 38]. Therefore, the quantile regression could provide beneficial results in tilapia

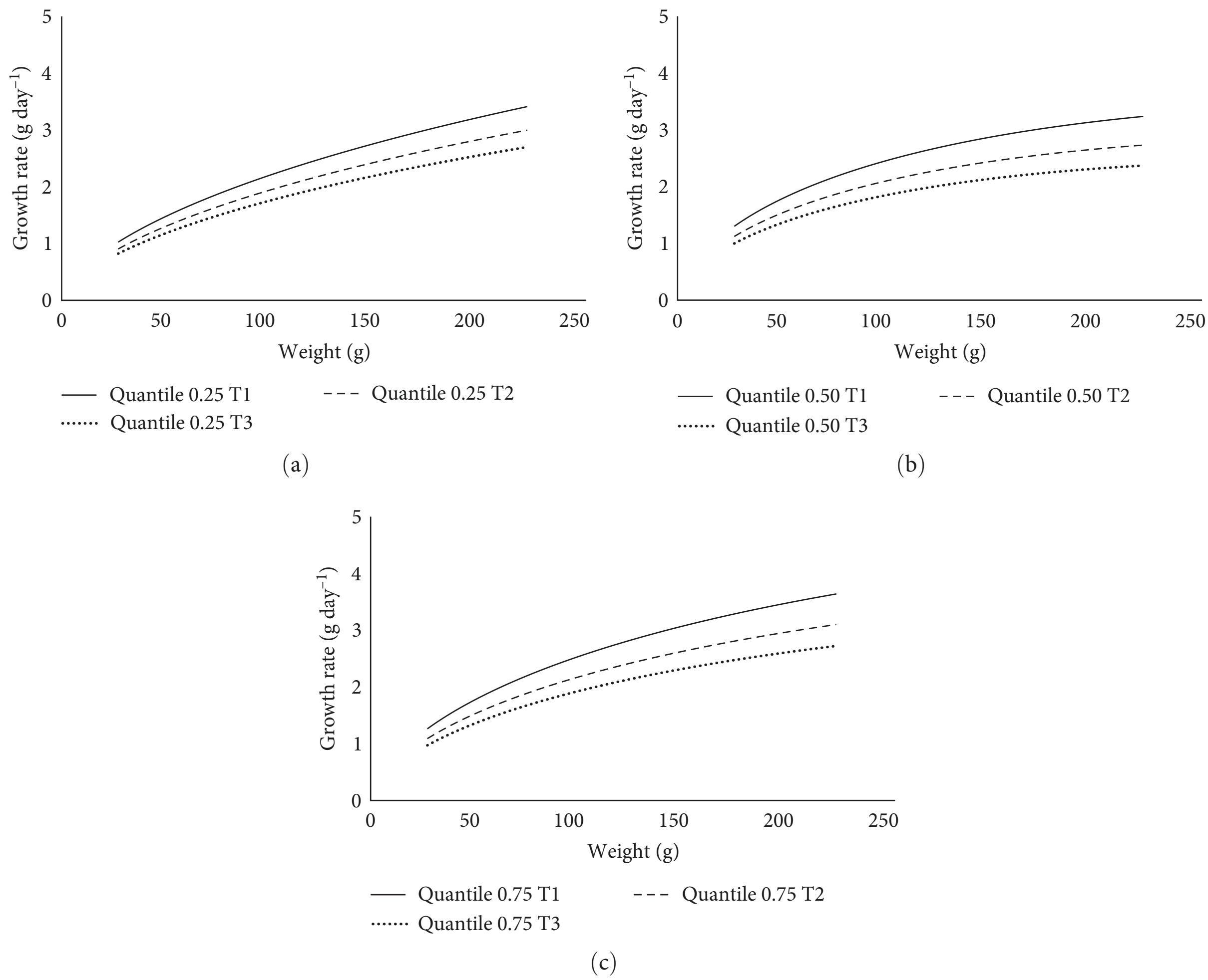

Figure 7: Simulated daily growth rates of each initial density treatments (T1, T2, and T3) considering three quantiles (0.25, 0.50, and 0.75).

aquaculture, especially when the stocking density is a crucial management point that determines the final yield and strengthens the estimates of final fish size at harvest or transfer.

Based on longer-time simulation, the von Bertalanffy model shows higher asymptotic growth than the Logistic model 1 [37, 72, 97, 98]. This result is also in line with the those obtained by Júnior et al. [86] during 1,000 days of tilapia rearing in net cages and Musa et al. [99] during 400 days of African catfish rearing in a recirculation aquaculture system. Despite the Logistic 1 and von Bertalanffy models developed showed to be suitable for tilapia pre-grow-out in the biofloc system, rearing data beyond 69 days would be needed to validate the projections of these models. Quantile regression successfully modeled HtG; in the future, the model would be improved by including other growth drivers, such as water temperature and nutrient dynamics. Additionally, in the case of von Bertalanffy model, the growth rate continues to increase, indicating that the tilapia before pre-grow-out has not yet reached the maturity stage.

Decision-making in intensive systems, such as optimal population densities, is essential when these decisions affect production costs, income, and net profit [11, 32, 40]. Optimizing a tilapia farm at different growth stages is crucial in aquaculture, mainly since the performance of one stage goes hand in hand with the next one until the organisms reach commercial size [5, 21, 45, 100]. In this case, stocking density affected tilapia growth in the pre-grow-out stage showing an inverse relationship, i.e., growth rates decreased as the initial density increased [101]. The determination of intrapopulation heterogeneity is important since it provides information closer to reality and helps decision-making in the different production stages [42–44], especially in vertically integrated farms where the objective of pre-grow-out stage is to optimize scarce resources and reduce production costs determining the optimal transfer time to the grow-out stage [102].

The predictive application that is here in this study proposed could be used as a tool based on the business analytics and applied to real production conditions as a tool for production control and surveillance, validation for production plans and comparison between key performance indicators versus observed results. Based on the latter and the results, designing through hypotheses the prescriptive analysis would allow an aquaculture company to estimate the optimal values of management variables, such as ration or seeding density, that would allow minimizing the production costs of the company and determine the optimal transfer time [103–105]. Finally, the growth models analyzed in this study using quantile regression can be easily applied to other species grown under the similar conditions and incorporate the effects of other variables that influence size dispersion. For example, future research on tilapia should consider dense population dependency and its dynamic effect on growth and survival [33].

5. Conclusion

Quantile regression proved to be a valuable and accurate tool to describe the HtG of tilapia in the pre-grow-out stage at different stocking densities. Unlike other models that use the average value of the population, the quantile regression does not penalize size variation in growth and decreases uncertainty in its estimates. Through quantile regression, the modified von Bertalanffy model allowed different growth patterns to be predicted in intensive tilapia culture, successfully evaluating size heterogeneity throughout the production cycle. In contrast, the modified Logistic model 1 was the most effective in predicting HmG. The findings in the present study on HmG and HtG at intensive level in the pre-grow-out stage may be the basis for future studies aimed at determining the culture time that minimizes production costs (optimal transfer time) during the pregrowing stage in commercial aquaculture of tilapia and other aquatic species in vertically integrated farms.

Data Availability

Information about the data supporting this research article is available from the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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