

Research Article

Predefined Time Synchronization Control for Uncertain Chaotic Systems

Yun Liu¹ and Fang Zhu² 

¹School of Finance and Mathematics, Huainan Normal University, Huainan 232038, China

²College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

Correspondence should be addressed to Fang Zhu; zhufang198305@126.com

Received 14 December 2021; Accepted 14 February 2022; Published 3 March 2022

Academic Editor: Ahmed Mostafa Khalil

Copyright © 2022 Yun Liu and Fang Zhu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this study, the predefined time synchronization problem of a class of uncertain chaotic systems with unknown control gain function is considered. Based on the fuzzy logic system and varying-time terminal sliding mode control technology, the predefined time synchronization between the master system and the slave system can be realized by the proposed control method in this study. The simulation results confirm the theoretical analysis.

1. Introduction

In recent decades, chaotic synchronization has been a research hotspot. The main reason is its wide application, such as in the fields of secure communication, biological systems, and so on [1–6]. Up to now, there are many synchronization methods between two chaotic systems, such as adaptive control [7–11], active control [12–14], impulsive control [15–17], and sliding mode control [18–23]. Among them, sliding mode control is deeply concerned by scholars because of its simple control principle and good robustness. Under the influence of unknown parameters and disturbances, two kinds of sliding mode synchronization methods were studied in [19]. Subsequently, to realize the state transient performance of the controlled system, many terminal sliding mode control methods were proposed. For example, a terminal sliding mode control method was employed in [22] and the synchronization of coronary artery system was realized. For fractional-order chaotic systems, [23] proposed a fractional-order terminal sliding mode control method, which synchronized two uncertain fractional-order systems. It should be pointed out that the initial value of the system should not be too far from the sliding mode; otherwise, the control performance will be affected.

It should also be considered that the gain of the discontinuous controller should not be large, which will increase the serious chattering problem. In order to solve the above problems, a varying-time terminal sliding mode control method will be used to realize the predefined time synchronization of two uncertain chaotic systems.

In this study, the predefined time synchronization of the main system and the slave system is considered. The main highlights are as follows: the synchronization of two uncertain chaotic systems is realized by the varying-time sliding mode control method, and the case where the controller gain is unknown is considered. The rest of this study is organized as follows. Some preliminaries are given in Section 2. A preset time terminal sliding mode is proposed and main results are investigated in Section 3. A synchronization example is shown in Section 4. Finally, Section 5 gives a brief conclusion.

2. Preliminaries

The master system is described as

$$\begin{cases} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = f_1(t, \xi_1, \xi_2), \end{cases} \quad (1)$$

where $\xi_1, \xi_2 \in \mathbb{R}$ are the states of system (1), and $f_1(t, \xi_1, \xi_2) \in \mathbb{R}$ is a nonlinear function.

The slave system is described as

$$\begin{cases} \dot{\eta}_1 = \eta_2, \\ \dot{\eta}_2 = f_2(t, \eta_1, \eta_2) + g(t, \eta_1, \eta_2)u, \end{cases} \quad (2)$$

where $\eta_1, \eta_2 \in \mathbb{R}$ are the states of system (2), $f_2(t, \eta_1, \eta_2) \in \mathbb{R}$ is a nonlinear function, $u \in \mathbb{R}$ is the control input, and $g(t, \eta_1, \eta_2)$ is a control gain function.

Define synchronization errors $e_1 = \eta_1 - \xi_1$, $e_2 = \eta_2 - \xi_2$. The aim of this study is to design a varying-time terminal sliding mode control method, so that the synchronization error e_1 reaches a small neighborhood of zero in the pre-defined time. According to (1) and (2), one gets the synchronization error system as

$$\begin{cases} \dot{e}_1 = e_2, \\ \dot{e}_2 = f_2(t, \eta_1, \eta_2) - f_1(t, \xi_1, \xi_2) + g(t, \eta_1, \eta_2)u. \end{cases} \quad (3)$$

In order to design the controller in this study, the following assumptions need to be made.

Assumption 1. States $\xi_1, \xi_2, \eta_1, \eta_2$ are measurable, and initial values $\xi_2(0) = \eta_2(0)$.

Assumption 2. $f_1(t, \xi_1, \xi_2)$ and $f_2(t, \eta_1, \eta_2)$ are unknown but bounded.

Assumption 3. $g(t, \eta_1, \eta_2)$ is unknown strictly positive and there exists a positive constant χ , such that $g(t, \eta_1, \eta_2) > \chi$.

Remark 1. $\xi_2(0) = \eta_2(0)$ in Assumption 1 is to ensure that the initial value of error system (3) belongs to the sliding mode, which will be designed later. Assumption 2 ensures that the fuzzy logic system can be used to estimate the unknown function.

In order to achieve the aim of this study, the time-varying terminal sliding mode is considered:

$$z = \begin{cases} e_2 + \beta e_1 + 2\lambda_1 t + \lambda_2 + \lambda_3(e_1 + \lambda_1 t^2 + \lambda_2 t + \alpha)^{q/p}, & t \leq T, \\ e_2 + \beta e_1 + \lambda_3 e_1^{q/p}, & t > T, \end{cases} \quad (4)$$

where T is a preset time, $0 < q/p < 1$, q and p are the odds, α, β are the design positive constant, and λ_1, λ_2 , and λ_3 satisfy the following conditions:

(1) In order to ensure that the initial value of system (3) belongs to the sliding mode (4), i.e.,

$$\beta e_1(0) + \lambda_2 + \lambda_3(e_1(0) + \alpha)^{q/p} = 0. \quad (5)$$

(2) The sliding mode (4) is continuous at $t = T$, i.e.,

$$\begin{cases} 2\lambda_1 T + \lambda_2 = 0, \\ \lambda_1 T^2 + \lambda_2 T + \alpha = 0. \end{cases} \quad (6)$$

(3) In order to ensure that sliding mode (4) can quickly approach the origin, i.e.,

$$\lambda_3 > 0. \quad (7)$$

Let

$$\Delta = \begin{cases} e_1 + \lambda_1 t^2 + \lambda_2 t + \alpha, & t \leq T, \\ e_1, & t > T. \end{cases} \quad (8)$$

Remark 2. The derivation of $\Delta^{q/p}$ with respect to time t may appear singular problem, and we modify $\Delta^{q/p-1} \dot{\Delta}$ as

$$\Delta^{q/p-1} \dot{\Delta} = \begin{cases} \Delta^{q/p-1} (e_2 + 2\lambda_1 t + \lambda_2) & , \text{ for } t \leq T \text{ and } \Delta \neq 0, \\ 0 & , \text{ for } t \leq T \text{ and } \Delta = 0, \\ \Delta^{q/p-1} e_2 & , \text{ for } t > T \text{ and } \Delta \neq 0, \\ 0 & , \text{ for } t > T \text{ and } \Delta = 0. \end{cases} \quad (9)$$

3. Main Result

Since $\xi_1, \xi_2, \eta_1, \eta_2$ are measurable, unknown functions $f_1(t, \xi_1, \xi_2)$, $f_2(t, \eta_1, \eta_2)$, and $g(t, \eta_1, \eta_2)$ can be estimated by fuzzy logic systems [24, 25]. For $f_1(t, \xi_1, \xi_2)$, $f_2(t, \eta_1, \eta_2)$, and $g(t, \eta_1, \eta_2)$, there exist $\theta_{f_1}^{*T} \varphi_{f_1}(\xi_1, \xi_2)$, $\theta_{f_2}^{*T} \varphi_{f_2}(\eta_1, \eta_2)$, and $\theta_g^{*T} \varphi_g(\eta_1, \eta_2)$, such that

$$\begin{cases} f_1(t, \xi_1, \xi_2) = \theta_{f_1}^{*T} \varphi_{f_1}(\xi_1, \xi_2) + \varepsilon_{f_1}(\xi_1, \xi_2), \\ f_2(t, \eta_1, \eta_2) = \theta_{f_2}^{*T} \varphi_{f_2}(\eta_1, \eta_2) + \varepsilon_{f_2}(\eta_1, \eta_2), \\ g(t, \eta_1, \eta_2) = \theta_g^{*T} \varphi_g(\eta_1, \eta_2) + \varepsilon_g(\eta_1, \eta_2), \end{cases} \quad (10)$$

where $\varepsilon_{f_1}(\xi_1, \xi_2)$, $\varepsilon_{f_2}(\eta_1, \eta_2)$, and $\varepsilon_g(\eta_1, \eta_2)$ are the bounded fuzzy estimation errors, $\theta_{f_1}^{*T}$, $\theta_{f_2}^{*T}$, and θ_g^{*T} are the ideal weight vectors, and $\varphi_{f_1}(\xi_1, \xi_2)$, $\varphi_{f_2}(\eta_1, \eta_2)$, and $\varphi_g(\eta_1, \eta_2)$ are the Gaussian functions.

From (3) and (4), the derivative of z with respect to t can be obtained as

$$\begin{aligned} \dot{z} &= f_2(t, \eta_1, \eta_2) - f_1(t, \xi_1, \xi_2) + \beta e_2 + g(t, \eta_1, \eta_2)u \\ &+ \begin{cases} 2\lambda_1 + \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, & t \leq T \\ \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, & t > T \end{cases} \\ &= \theta_{f_2}^{*T} \varphi_{f_2}(\eta_1, \eta_2) - \theta_{f_1}^{*T} \varphi_{f_1}(\xi_1, \xi_2) + \beta e_2 + \varepsilon_{f_2}(\eta_1, \eta_2) \\ &\quad - \varepsilon_{f_1}(\xi_1, \xi_2) + \theta_g^{*T} \varphi_g(\eta_1, \eta_2)u_e + g(t, \eta_1, \eta_2)u \\ &+ \begin{cases} 2\lambda_1 + \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, & t \leq T, \\ \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, & t > T. \end{cases} \end{aligned} \quad (11)$$

Now, design the controller as

$$\begin{aligned}
 u &= u_e + u_s, \\
 u_e &= \frac{\tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2)}{\left(\tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2)\right)^2 + 1} \\
 &\quad \left[\begin{array}{l} -\tilde{\theta}_{f_2}^T \varphi_{f_2}(\eta_1, \eta_2) + \tilde{\theta}_{f_1}^T \varphi_{f_1}(\xi_1, \xi_2) - k_1 z - \beta e_2 \\ + \left\{ \begin{array}{l} -2\lambda_1 - \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, t \leq T \\ -\lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, t > T \end{array} \right. \end{array} \right] \quad (12) \\
 &\triangleq \frac{\tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2) \Pi}{\left(\tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2)\right)^2 + 1}, \\
 u_s &= -\frac{|\Pi| \text{sign}(z)}{\left(\left(\tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2)\right)^2 + 1\right) \chi}
 \end{aligned}$$

where k_1 is a design positive constant, and $\hat{\theta}_{f_1}$, $\hat{\theta}_{f_2}$, and $\hat{\theta}_g$ are the estimations of $\theta_{f_1}^*$, $\theta_{f_2}^*$, and θ_g^* . The parameter adaptation laws of $\hat{\theta}_{f_1}$ and $\hat{\theta}_{f_2}$ are given by

$$\begin{aligned}
 \dot{\hat{\theta}}_{f_1} &= \gamma_{f_1} (-z \varphi_{f_1}(\xi_1, \xi_2) - \delta_{f_1} \hat{\theta}_{f_1}), \\
 \dot{\hat{\theta}}_{f_2} &= \gamma_{f_2} (z \varphi_{f_2}(\eta_1, \eta_2) - \delta_{f_2} \hat{\theta}_{f_2}), \\
 \dot{\hat{\theta}}_g &= \gamma_g (z u_e \varphi_g(\eta_1, \eta_2) - \delta_g \hat{\theta}_g),
 \end{aligned} \quad (13)$$

where γ_{f_1} , γ_{f_2} , γ_g , δ_{f_1} , δ_{f_2} , and δ_g are the design positive constants. Let $\varepsilon(t) = \varepsilon_{f_1}(\xi_1, \xi_2) + \varepsilon_{f_2}(\eta_1, \eta_2) + \varepsilon_g(\eta_1, \eta_2) u_e$. Obviously, $\varepsilon(t)$ is bounded, i.e., there exists a positive constant ε^* , such that $|\varepsilon(t)| \leq \varepsilon^*$.

Theorem 1. *Under Assumptions 1–3, if the time-varying terminal sliding mode (4), controller (12), and parameter adaptive laws (13) are employed, then all signals in (14) are bounded.*

Proof. Consider the following Lyapunov function:

$$V_1 = \frac{1}{2} z^2 + \frac{1}{\gamma_{f_1}} \tilde{\theta}_{f_1}^T \tilde{\theta}_{f_1} + \frac{1}{\gamma_{f_2}} \tilde{\theta}_{f_2}^T \tilde{\theta}_{f_2} + \frac{1}{\gamma_g} \tilde{\theta}_g^T \tilde{\theta}_g, \quad (14)$$

where $\tilde{\theta}_{f_1} = \theta_{f_1}^* - \hat{\theta}_{f_1}$, $\tilde{\theta}_{f_2} = \theta_{f_2}^* - \hat{\theta}_{f_2}$, and $\tilde{\theta}_g = \theta_g^* - \hat{\theta}_g$. From (11), derivation of \dot{V}_1 with respect to t yields

$$\begin{aligned}
 \dot{V}_1 &= z \dot{z} - \frac{1}{\gamma_{f_1}} \tilde{\theta}_{f_1}^T \dot{\tilde{\theta}}_{f_1} - \frac{1}{\gamma_{f_2}} \tilde{\theta}_{f_2}^T \dot{\tilde{\theta}}_{f_2} \\
 &= z \left[\begin{array}{l} \theta_{f_2}^{*T} \varphi_{f_2}(\eta_1, \eta_2) - \theta_{f_1}^{*T} \varphi_{f_1}(\xi_1, \xi_2) + \beta e_2 + \varepsilon(t) \\ + \theta_g^{*T} \varphi_g(\eta_1, \eta_2) u_e + g(t, \eta_1, \eta_2) u_s \\ + \left\{ \begin{array}{l} 2\lambda_1 + \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, t \leq T \\ \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, t > T \end{array} \right. \end{array} \right] \\
 &\quad - \frac{1}{\gamma_{f_1}} \tilde{\theta}_{f_1}^T \dot{\tilde{\theta}}_{f_1} - \frac{1}{\gamma_{f_2}} \tilde{\theta}_{f_2}^T \dot{\tilde{\theta}}_{f_2} - \frac{1}{\gamma_g} \tilde{\theta}_g^T \dot{\tilde{\theta}}_g \\
 &= z \left[\begin{array}{l} \tilde{\theta}_{f_2}^T \varphi_{f_2}(\eta_1, \eta_2) - \tilde{\theta}_{f_1}^T \varphi_{f_1}(\xi_1, \xi_2) + \beta e_2 + \varepsilon(t) \\ \tilde{\theta}_{f_2}^T \varphi_{f_2}(\eta_1, \eta_2) - \tilde{\theta}_{f_1}^T \varphi_{f_1}(\xi_1, \xi_2) + \tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2) u_e \\ + \tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2) u_e + g(t, \eta_1, \eta_2) u_s \\ + \left\{ \begin{array}{l} 2\lambda_1 + \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, t \leq T \\ \lambda_3 \frac{q}{p} \Delta^{p/q-1} \dot{\Delta}, t > T \end{array} \right. \end{array} \right] \\
 &\quad - \frac{1}{\gamma_{f_1}} \tilde{\theta}_{f_1}^T \dot{\tilde{\theta}}_{f_1} - \frac{1}{\gamma_{f_2}} \tilde{\theta}_{f_2}^T \dot{\tilde{\theta}}_{f_2} - \frac{1}{\gamma_g} \tilde{\theta}_g^T \dot{\tilde{\theta}}_g.
 \end{aligned} \quad (15)$$

Substituting (12) and (13) to (15) yields

$$\begin{aligned}
 \dot{V}_1 &= -k_2 z^2 - \frac{z \Pi}{\left(\tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2)\right)^2 + 1} - \frac{g(t, \eta_1, \eta_2) |z| |\pi|}{\left(\left(\tilde{\theta}_g^T \varphi_g(\eta_1, \eta_2)\right)^2 + 1\right) \chi} \\
 &\quad + z \varepsilon(t) + \delta_{f_1} \tilde{\theta}_{f_1}^T \tilde{\theta}_{f_1} + \delta_{f_2} \tilde{\theta}_{f_2}^T \tilde{\theta}_{f_2} + \delta_g \tilde{\theta}_g^T \tilde{\theta}_g \\
 &\leq -k_2 z^2 + z \varepsilon(t) + \delta_{f_1} \tilde{\theta}_{f_1}^T \tilde{\theta}_{f_1} + \delta_{f_2} \tilde{\theta}_{f_2}^T \tilde{\theta}_{f_2} + \delta_g \tilde{\theta}_g^T \tilde{\theta}_g.
 \end{aligned} \quad (16)$$

Since the following inequalities hold:

$$z\varepsilon(t) \leq \frac{1}{4}z^2 + \varepsilon^{*2},$$

$$\delta_{f_1} \tilde{\theta}_{f_1}^T \tilde{\theta}_{f_1} \leq \frac{\delta_{f_1}}{2} \left(-\tilde{\theta}_{f_1}^T \tilde{\theta}_{f_1} + \theta_{f_1}^{*T} \theta_{f_1}^* \right), \quad (17)$$

$$\delta_{f_2} \tilde{\theta}_{f_2}^T \tilde{\theta}_{f_2} \leq \frac{\delta_{f_2}}{2} \left(-\tilde{\theta}_{f_2}^T \tilde{\theta}_{f_2} + \theta_{f_2}^{*T} \theta_{f_2}^* \right),$$

substituting (12) into \dot{V}_1 yields

$$\dot{V}_1 \leq -\left(k_1 - \frac{1}{4}\right)z^2 - \frac{\delta_{f_1}}{2} \tilde{\theta}_{f_1}^T \tilde{\theta}_{f_1} - \frac{\delta_{f_2}}{2} \tilde{\theta}_{f_2}^T \tilde{\theta}_{f_2} + R^*, \quad (18)$$

where $R^* = \varepsilon^{*2} + \delta_{f_1}/2\theta_{f_1}^{*T}\theta_{f_1}^* + \delta_{f_2}/2\theta_{f_2}^{*T}\theta_{f_2}^*$. Selecting k_1 , δ_1 , and δ_2 , such that $\iota_1 \triangleq \min\{2k_1 - 1/2, \gamma_{f_1}\delta_{f_1}, \gamma_{f_2}\delta_{f_2}\} > 0$, then

$$\dot{V}_1 \leq -\iota_1 V_1 + R^*. \quad (19)$$

According to (19), we can conclude that all signals in (14) are bounded. This completes the proof. \square

Remark 3. For $t > T$, $e_2 = \dot{e}_1 = z - \beta e_1 - \lambda_3 e_1^{q/p}$, with the boundedness of z ; there exists unknown constant b^* , such that $|z| \leq b^*$. Let $V_2 = 1/2e_1^2$, and one has

$$\begin{aligned} \dot{V}_2 &= e_1 \dot{e}_1 \\ &= e_1 (z - \beta e_1 - \lambda_3 e_1^{q/p}) \\ &\leq -\left(\beta - \frac{1}{4}\right)e_1^2 - \lambda_3 e_1^{q/p-1} + b^{*2}. \end{aligned} \quad (20)$$

Let $\beta > 1/4$ and define

$$\Omega_e = \left\{ e_1 \mid \lambda_3 e_1^{q/p-1} \leq \frac{b^{*2}}{1-\nu} \right\}, \quad (21)$$

where $\nu \in (0, 1)$. Obviously, if $e_1 \in \bar{\Omega}_e$, $\dot{V}_2 \leq -\lambda_3 e_1^{q/p-1} + b^{*2} < -\lambda_3 \nu e_1^{q/p-1} < 0$, V_2 will monotonically decrease only to enter Ω_e . Therefore, we obtain the convergence range of the tracking error e_1 .

4. Numerical Simulations

In this section, the chaotic gyroscope system [26] is taken as an example to show the effectiveness of the proposed method (12). For the master system (1), define $f_1(t, \xi_1, \xi_2) = -10^2(1 - \cos \xi_1)^2 / \sin^3 \xi_1 + \sin \xi_1 - 0.5\xi_2 - 0.05\xi_2^3 + 35.7 \sin(2t) \sin \xi_1$. For the slave system (2), define $f_2(t, \eta_1, \eta_2) = -10^2(1 - \cos \eta_1)^2 / \sin^3 \eta_1 + \sin \eta_1 - 0.5\eta_2 - 0.05\eta_2^3 + 35.5 \sin(2t) \sin \eta_1$, $g(t, \eta_1, \eta_2) = 5 + \sin \eta_2$. Obviously, $g(t, \eta_1, \eta_2) > \chi \triangleq 3$. The initial values $\xi_1(0) = -1$, $\xi_2(0) = 1$, $\eta_1(0) = 2$, and $\eta_2(0) = 1$. The fuzzy membership functions are selected as

$$\varphi(\rho) = \exp \left[-\frac{1}{2} \left(\frac{\rho + 7.5 - 2.5j}{1.2} \right)^2 \right], \quad (22)$$

where $\rho = \xi_1, \xi_2, \eta_1, \eta_2$; $j = 1, 2, 3, 4, 5$. First, select a group of parameters as $T = 2, k_1 = 3, q = 3, p = 5, \beta = 3, \alpha = -5, \lambda_1 =$

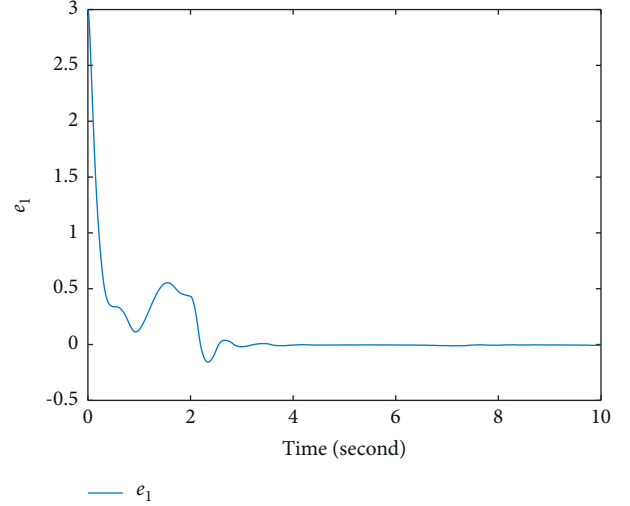


FIGURE 1: Time response trajectory of e_1 by using the proposed method (12) with $T = 2s$.

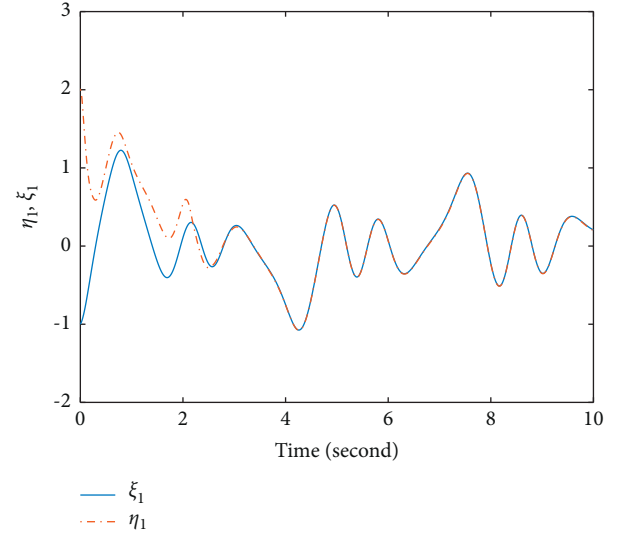


FIGURE 2: Time response trajectories of ξ_1 and η_1 by using the proposed method (12) with $T = 2s$.

$-5/4$, $\lambda_2 = 5, \lambda_3 = 14/2^{3/5}$, and the simulation results are shown in Figures 1–3. Figures 1 and 2 show that the state ξ_1 of master system (1) and the state η_1 of slave system (2) are synchronized after $T = 2s$. In order to overcome the influence of unknown gain $g(t, \eta_1, \eta_2)$, Figure 3 shows that the controller u fluctuates at $T = 2s$, and then, the controller has a small chattering phenomenon.

Extend the predefined time to $T = 5s$, and parameters modify as $\lambda_1 = -1/5, \lambda_2 = 2, \lambda_3 = 11/2^{3/5}$; other parameters remain unchanged. The simulation results are shown in Figures 4–6. Figures 4 and 5 show that states ξ_1 and η_1 are synchronized after $T = 5s$ (Figure 6). The controller u also has a small fluctuation at $t = 5s$, and the chattering phenomenon is very small.

Obviously, the proposed control method (12) in this study can ensure the synchronization between master

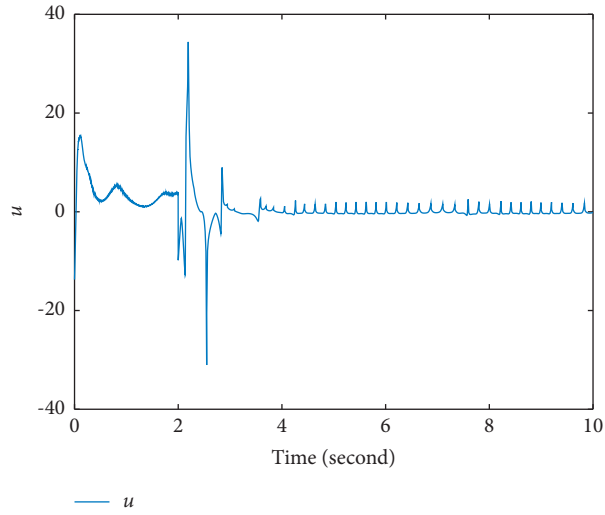


FIGURE 3: Time response trajectory of controller u by using the proposed method (12) with $T = 2s$.

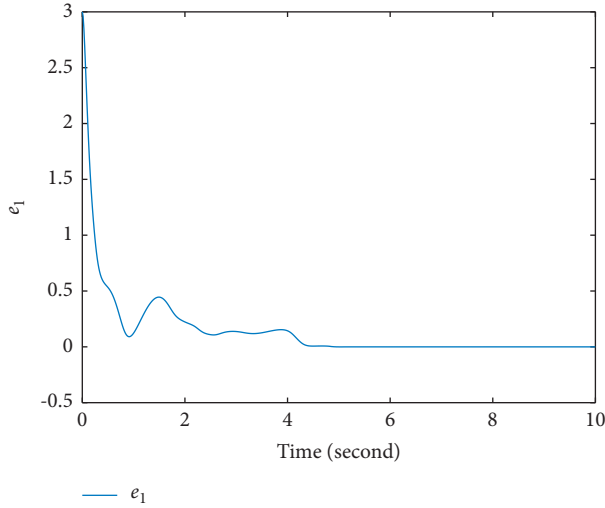


FIGURE 4: Time response trajectory of e_1 by using the proposed method (12) with $T = 5s$.

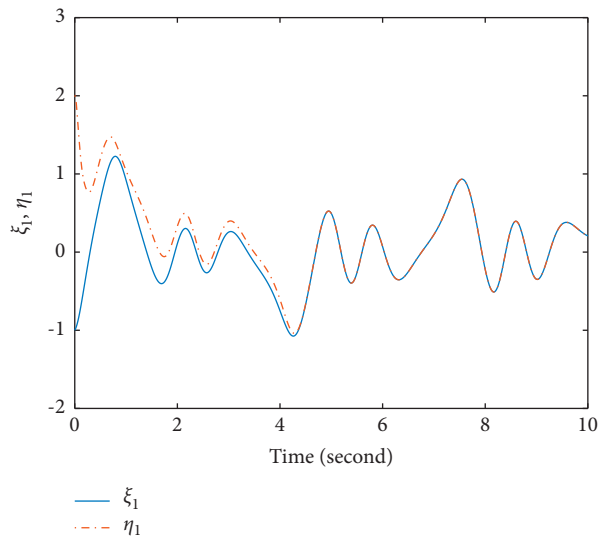


FIGURE 5: Time response trajectories of ξ_1 and η_1 by using the proposed method (12) with $T = 5s$.

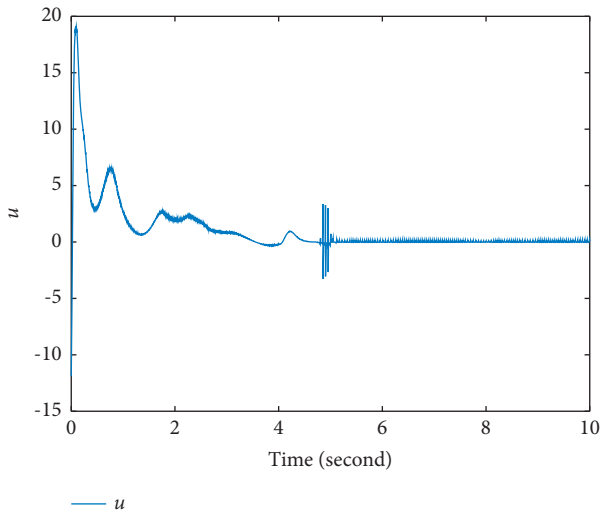


FIGURE 6: Time response trajectory of controller u by using the proposed method (12) with $T = 5s$.

system (1) and slave system (2) at a predefined time and can also overcome the influence of unknown gain $g(t, \eta_1, \eta_2)$.

5. Conclusion

In this study, the predefined time synchronization problem of uncertain chaotic systems was investigated. The fuzzy logic system was used to estimate the unknown function. A time-varying sliding mode was constructed. The proposed varying-time terminal sliding mode control method in this study made all signals bounded and the synchronization error entered a small neighborhood of zero after the predefined time. Simulation results show the effectiveness of the method.

Data Availability

The datasets generated for this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

The authors gratefully acknowledge the support of Anhui Province University Excellent Talents Funding Project (gxbjZD2021075).

References

- [1] B. Wang, X. Zhang, and X. Dong, "Novel secure communication based on chaos synchronization," *IEICE-Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol. 101, no. 7, pp. 1132–1135, 2018.
- [2] P. Wang, G. Wen, X. Yu, W. Yu, and T. Huang, "Synchronization of multi-layer networks: from node-to-node synchronization to complete synchronization," *IEEE*

- Transactions on Circuits and Systems I: Regular Papers*, vol. 66, no. 3, pp. 1141–1152, 2018.
- [3] B. Vaseghi, M. A. Pourmina, and S. Mobayen, "Secure communication in wireless sensor networks based on chaos synchronization using adaptive sliding mode control," *Nonlinear Dynamics*, vol. 89, no. 3, pp. 1689–1704, 2017.
- [4] G. Wen, P. Wang, X. Yu, W. Yu, and J. Cao, "Pinning synchronization of complex switching networks with a leader of nonzero control inputs," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 66, no. 8, pp. 3100–3112, 2019.
- [5] M. F. Hassan and M. Hammuda, "A new approach for constrained chaos synchronization with application to secure data communication," *Journal of the Franklin Institute*, vol. 356, no. 12, pp. 6697–6723, 2019.
- [6] S. Khorashadizadeh and M.-H. Majidi, "Chaos synchronization using the fourier series expansion with application to secure communications," *AEU - International Journal of Electronics and Communications*, vol. 82, pp. 37–44, 2017.
- [7] H. Liu, S.-G. Li, H.-X. Wang, and G.-J. Li, "Adaptive fuzzy synchronization for a class of fractional-order neural networks," *Chinese Physics B*, vol. 26, no. 3, Article ID 030504, 2017.
- [8] S. Vaidyanathan and A. T. Azar, "Adaptive Control and Synchronization of Halvorsen Circulant Chaotic Systems," in *Advances in Chaos Theory and Intelligent Control*, pp. 225–247, Springer, Berlin, Germany, 2016.
- [9] S. Vaidyanathan and A. T. Azar, "Generalized Projective Synchronization of a Novel Hyperchaotic Four-wing System via Adaptive Control Method," in *Advances in Chaos Theory and Intelligent Control*, pp. 275–296, Springer, Berlin, Germany, 2016.
- [10] S. Vaidyanathan, O. A. Abba, G. Betchewe, and M. Alidou, "A new three-dimensional chaotic system: its adaptive control and circuit design," *International Journal of Automation and Control*, vol. 13, no. 1, pp. 101–121, 2019.
- [11] S. Kumar, A. E. Matouk, H. Chaudhary, and S. Kant, "Control and synchronization of fractional-order chaotic satellite systems using feedback and adaptive control techniques," *International Journal of Adaptive Control and Signal Processing*, vol. 35, no. 4, pp. 484–497, 2021.
- [12] C. Huang and J. Cao, "Active control strategy for synchronization and anti-synchronization of a fractional chaotic financial system," *Physica A: Statistical Mechanics and Its Applications*, vol. 473, pp. 262–275, 2017.
- [13] I. Ahmad, A. B. Saaban, A. B. Ibrahim, M. Shahzad, and N. Naveed, "The synchronization of chaotic systems with different dimensions by a robust generalized active control," *Optik*, vol. 127, no. 11, pp. 4859–4871, 2016.
- [14] S. Çiçek, A. Ferikoğlu, and İ. Pehlivan, "A new 3d chaotic system: dynamical analysis, electronic circuit design, active control synchronization and chaotic masking communication application," *Optik*, vol. 127, no. 8, pp. 4024–4030, 2016.
- [15] B. Liu, Z. Sun, Y. Luo, and Y. Zhong, "Uniform synchronization for chaotic dynamical systems via event-triggered impulsive control," *Physica A: Statistical Mechanics and Its Applications*, vol. 531, Article ID 121725, 2019.
- [16] Z. Xu, D. Peng, and X. Li, "Synchronization of chaotic neural networks with time delay via distributed delayed impulsive control," *Neural Networks*, vol. 118, pp. 332–337, 2019.
- [17] K. Tian, H.-P. Ren, and C. Bai, "Synchronization of hyperchaos with time delay using impulse control," *IEEE Access*, vol. 8, pp. 72570–72576, 2020.
- [18] S. Mobayen, "Chaos synchronization of uncertain chaotic systems using composite nonlinear feedback based integral

- sliding mode control,” *ISA Transactions*, vol. 77, pp. 100–111, 2018.
- [19] X. Chen, J. H. Park, J. Cao, and J. Qiu, “Adaptive synchronization of multiple uncertain coupled chaotic systems via sliding mode control,” *Neurocomputing*, vol. 273, pp. 9–21, 2018.
- [20] U. E. Kocamaz, B. Cevher, and Y. Uyaroglu, “Control and synchronization of chaos with sliding mode control based on cubic reaching rule,” *Chaos, Solitons & Fractals*, vol. 105, pp. 92–98, 2017.
- [21] J. Sun, Y. Wang, Y. Wang, and Y. Shen, “Finite-time synchronization between two complex-variable chaotic systems with unknown parameters via nonsingular terminal sliding mode control,” *Nonlinear Dynamics*, vol. 85, no. 2, pp. 1105–1117, 2016.
- [22] Z. Zhao, X. Li, J. Zhang, and Y. Pei, “Terminal sliding mode control with self-tuning for coronary artery system synchronization,” *International Journal of Biomathematics*, vol. 10, no. 03, Article ID 1750041, 2017.
- [23] J. Ni, L. Liu, C. Liu, and X. Hu, “Fractional order fixed-time nonsingular terminal sliding mode synchronization and control of fractional order chaotic systems,” *Nonlinear Dynamics*, vol. 89, no. 3, pp. 2065–2083, 2017.
- [24] Y. Lu, “Adaptive-fuzzy control compensation design for direct adaptive fuzzy control,” *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3222–3231, 2018.
- [25] L. Wang, H. Li, Q. Zhou, and R. Lu, “Adaptive fuzzy control for nonstrict feedback systems with unmodeled dynamics and fuzzy dead zone via output feedback,” *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2400–2412, 2017.
- [26] S. Vaidyanathan, “Global chaos regulation of a symmetric nonlinear gyro system via integral sliding mode control,” *International Journal of Chemical Research*, vol. 9, no. 5, pp. 462–469, 2016.