In this paper, we provide two new generalized Gauss-Seidel (NGGS) iteration methods for solving absolute value equations $Ax - |x| = b$, where $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$, and $x \in \mathbb{R}^n$ are unknown solution vectors. Also, convergence results are established under mild assumptions. Eventually, numerical results prove the credibility of our approaches.
Recalling that the AVE (1) has
\[ \text{NGGS Method I for AVE.} \]
and NGGS method II) for determining AVE (1).

3. NGGS Iteration Methods

Here, we examine the suggested methods (NGGS method I and NGGS method II) for determining AVE (1).

3.1. NGGS Method I for AVE. Recalling that the AVE (1) has the following form,
\[ Ax - \lambda |x| = b. \]
Multiplying \( \lambda \), then we get
\[ \lambda Ax - \lambda |x| = \lambda b. \] Let
\[ A = D_A - L_A - U_A = (\Omega + D_A - L_A) - (\Omega + U_A), \]
where \( D_A = \text{diag}(A) \), \( L_A \), and \( U_A \) are strictly lower and upper triangular parts of \( A \), respectively. Furthermore, \( \Omega = \Psi I \), where \( 0 \leq \Psi \leq 1 \) and \( I \) denote the identity matrix. Using (3) and (4), the NGGS Method 1 is suggested as
\[ (\Omega + D_A - \lambda L_A)x - \lambda |x| = [(1 - \lambda)(\Omega + D_A) + \lambda(\Omega + U_A)]x + \lambda b. \]
Using the iterative scheme, so (5) can be written as
\[ (\Omega + D_A - \lambda L_A)x^{i+1} - \lambda |x^{i+1}| = [(1 - \lambda)(\Omega + D_A) + \lambda(\Omega + U_A)]x^i + \lambda b, \]
where \( i = 0, 1, 2, \ldots \), and \( 0 < \lambda \leq 1 \) (see Appendix). Note that if \( \lambda = 1 \) and \( \Omega = 0 \), then Eq. (6) reduces to the GGS method [23].

The next step in the analysis is to verify the convergence of NGGS method I by using the following theorem.

**Theorem 2.** Suppose that AVE (1) is solvable, let the diagonal values of \( A > I \) and \( D_A - L_A - I \) matrix be the strictly row wise diagonally dominant. If
\[ \| (\Omega + D_A - \lambda L_A)^{-1} [(1 - \lambda)(\Omega + D_A) + \lambda(\Omega + U_A)] \|_\infty < 1 - \lambda \| (\Omega + D_A - \lambda L_A)^{-1} \|_\infty, \]
then the sequence \( \{ x^i \} \) of the NGGS method I converges to the unique solution \( x^* \) of AVE (1).

**Proof.** We will prove first \( \| (\Omega + D_A - \lambda L_A)^{-1} \|_\infty < 1 \). Clearly, if we put \( L_A = 0 \), then \( \| (\Omega + D_A - \lambda L_A)^{-1} \|_\infty = \| (\Omega + D_A)^{-1} \|_\infty < 1 \). If we assume that \( L_A \neq 0 \), we get
\[ 0 \leq \| L_A \| \| t < (\Omega + D_A - I) t, \] if we take
\[ \| L_A \| t < (\Omega + D_A - I) t. \]
Taking both side by \( (\Omega + D_A)^{-1} \), we get
\[ (\Omega + D_A)^{-1} L_A t < (\Omega + D_A)^{-1} [(\Omega + D_A) - I] t, \]
\[ \lambda (\Omega + D_A)^{-1} L_A t < (\Omega + D_A)^{-1} (\Omega + D_A - I) t, \]
\[ (\Omega + D_A)^{-1} \lambda L_A t < (\Omega + D_A)^{-1} (\Omega + D_A - I) t, \]
\[ (\Omega + D_A)^{-1} \lambda L_A t < (\Omega + D_A)^{-1} L_A t, \]
\[ (\Omega + D_A)^{-1} L_A t < (\Omega + D_A)^{-1} L_A t, \]
where \( Q = \lambda (\Omega + D_A)^{-1} L_A \) and \( t = (1, 1, \cdots, 1)^T \). Also, we have

<table>
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<th>Methods</th>
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<th>2000</th>
<th>3000</th>
<th>4000</th>
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<td>18</td>
<td>18</td>
<td>18</td>
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<td>SLM</td>
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</tr>
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<td></td>
<td>RSV</td>
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<td>6.93e-7</td>
<td>6.93e-7</td>
<td>6.94e-7</td>
</tr>
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<td></td>
<td>Iter</td>
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<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>SM</td>
<td>Time</td>
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<td>9.0954</td>
<td>17.3028</td>
<td>29.1644</td>
</tr>
<tr>
<td></td>
<td>RSV</td>
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<td>8.92e-7</td>
<td>8.93e-7</td>
<td>8.93e-7</td>
</tr>
<tr>
<td></td>
<td>Itr</td>
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<td>10</td>
<td>10</td>
<td></td>
</tr>
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</tr>
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<td>4.85e-7</td>
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<td></td>
<td>Itr</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>NGGS method II</td>
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<td>3.69e-7</td>
<td>3.69e-7</td>
</tr>
</tbody>
</table>

(i) We extend the GGS technique [23] to the general case. To achieve this goal, we impose two additional parameters (\( \lambda \) and \( \Omega \)) that can accelerate the convergence procedure.

(ii) A variety of novel conditions are used to investigate the convergence properties of the newly developed methods.

The remainder of this paper is organized in the following manner. In Section 2, we present some notations and a lemma that will be used throughout the remainder of this study. In Section 3, we propose the NGGS procedures and discuss their convergence. We demonstrate the efficiency of these algorithms in Section 4 by providing numerical examples. In the last section, we make concluding remarks.

2. Preliminaries

Here we briefly examine some of the notations and concepts used in this article.

Let \( A = (a_{ij}) \in R^{n \times n} \), we indicate the norm and absolute value as \( ||A||_\infty \) and \( |A| = (|a_{ij}|) \), respectively. The matrix \( A \in R^{n \times n} \) is called an Z-matrix if \( a_{ij} \leq 0 \) for \( i \neq j \) and an M -matrix if it is a nonsingular Z-matrix and with \( A^{-1} \geq 0 \).

**Lemma 1.** [36]. Suppose \( x \) and \( \epsilon \in R^n \), then \( ||x - \epsilon||_\infty \geq ||x||_\infty - ||\epsilon||_\infty \).

3. NGGS Iteration Methods

Here, we examine the suggested methods (NGGS method I and NGGS method II) for determining AVE (1).
Table 2: Numerical results for Example 5 with $\Psi = 0.2$ and $\lambda = 0.98$.

<table>
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<th>1024</th>
<th>4096</th>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AOR</td>
<td></td>
<td>14</td>
<td>14</td>
<td>15</td>
<td>35</td>
</tr>
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<td></td>
<td>Time</td>
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<td>5.8097</td>
</tr>
<tr>
<td></td>
<td>Itr</td>
<td></td>
<td>14</td>
<td>14</td>
<td>15</td>
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<td>5.069e-07</td>
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</tr>
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<td>Itr</td>
<td></td>
<td>12</td>
<td>13</td>
<td>13</td>
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<tr>
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<td>0.2131</td>
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<td>2.0033</td>
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<td>RSV</td>
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<td>Itr</td>
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Table 3: Numerical results for Example 6 with $\Psi = 0.2$ and $\lambda = 0.98$.

<table>
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<td>AOR</td>
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<td>7.63e-07</td>
<td>9.17e-07</td>
<td>8.90e-07</td>
</tr>
</tbody>
</table>

Table 4: Numerical results for Example 7 with $\Psi = 0.3$ and $\lambda = 0.95$.

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<th>3000</th>
<th>4000</th>
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<td>14</td>
</tr>
<tr>
<td></td>
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</tr>
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<td>2.69e-07</td>
<td>3.01e-07</td>
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<td>Itr</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>SOR</td>
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<td></td>
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</tr>
<tr>
<td>NGGS method I</td>
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<td>2.04e-08</td>
<td>2.62e-08</td>
<td>3.81e-07</td>
</tr>
</tbody>
</table>
Thus, from (8) and (9), we get
\[
0 \leq |(I - Q)^{-1}| = |I + Q + Q^2 + Q^3 + \cdots + Q^{n-1}|,
\]
\[
\leq (I + |Q||Q|^2 + |Q|^3 + \cdots + |Q|^{n-1}) = (I - |Q|)^{-1}.
\]

(9)

Thus, from (8) and (9), we get
\[
||Ω + D_A - LAΩ||^{-1}t = ||(I - Q)^{-1}(Ω + D_A)||^{-1}t \leq ||(I - Q)^{-1}||||Ω + D_A||^{-1}t < (I - Q)^{-1}t = t.
\]

So, we obtain
\[
(Ω + D_A - LAΩ)^{-1} ||x||_∞ < 1.
\]

Uniqueness: Let \(x^*\) and \(z^*\) be two different solutions of the AVE (1). Using (5), we get
\[
x^* = λ(Ω + D_A - LAΩ)^{-1}|x|^1 + (Ω + D_A - LAΩ)^{-1}[(1 - λ)(Ω + D_A) + λ(Ω + U_A)]x^* + λb,
\]
\[
z^* = λ(Ω + D_A - LAΩ)^{-1}|z|^1 + (Ω + D_A - LAΩ)^{-1}[(1 - λ)(Ω + D_A) + λ(Ω + U_A)]z^* + λb,
\]
\[(10, 11)\]

From (10) and (11), we get
\[
x^* - z^* = λ[(Ω + D_A - LAΩ)^{-1}|x|^1 + (Ω + D_A - LAΩ)^{-1}[(1 - λ)(Ω + D_A) + λ(Ω + U_A)](x^* - z^*)
\]
\[
\text{Based on Lemma 1 and Eq. (7), the above equation can be expressed as follows:}
\]
\[
||x^* - z^*||_∞ \leq λ ||(Ω + D_A - LAΩ)^{-1}||_∞ |||x|^1|| - ||z^*||_∞ + ||(Ω + D_A - LAΩ)^{-1}[(1 - λ)(Ω + D_A) + λ(Ω + U_A)]||_∞ ||x^* - z^*||_∞
\]
\[
< λ ||(Ω + D_A - LAΩ)^{-1}||_∞ ||x^* - z^*||_∞ + (1 - λ)(Ω + D_A - LAΩ)^{-1} ||x^* - z^*||_∞ < 1
\]
\[
(1 - λ)(Ω + D_A - LAΩ)^{-1} ||x^* - z^*||_∞ < 1 - λ(Ω + D_A - LAΩ)^{-1} ||x^* - z^*||_∞ < 1 - λ(Ω + U_A)\]
\[
||x^* - z^*||_∞ < ||x^* - z^*||_∞\text{which is a contradiction. Thus, } x^* = z^*.
\]

Convergence: We will consider \(x^*\) as the unique solution to AVE (1). Consequently, from (10) and
\[
x^* + 1 = λ[(Ω + D_A - LAΩ)^{-1}||x^* + 1|| + (Ω + D_A - LAΩ)^{-1}[(1 - λ)(Ω + D_A) + λ(Ω + U_A)]x^* + λb\]
\]
\[
we deduce
\]
\[
x^* + 1 = λ[(Ω + D_A - LAΩ)^{-1}||x^* + 1|| + (Ω + D_A - LAΩ)^{-1}[(1 - λ)(Ω + D_A) + λ(Ω + U_A)]x^* + λb.
\]
\[
\]
By taking infinity norm and Lemma 1, we have
\[
||x^* + 1 - x^*||_∞ \leq ||(Ω + D_A - LAΩ)^{-1}||_∞ ||x^* + 1 - x^*||_∞ ≤ \frac{1}{(1 - λ)(Ω + U_A)^{-1} + λ(Ω + D_A) - LAΩ^{-1}} ||x^* - x^*||_∞
\]
\[
and hence, since \((Ω + D_A - LAΩ)^{-1}||_∞ < 1\), it follows that
\]
\[
||x^* + 1 - x^*||_∞ ≤ \frac{1}{(1 - λ)(Ω + D_A) + λ(Ω + U_A)}||x^* - x^*||_∞.
\]
\[
According to the inequality above, the presented approach converges to the solution when condition (7) is met.

3.2. NGGS Method II for AVE. In this section, we describe the NGGS method II. Based on (3) and (4), we can express the suggested method for determining AVE (1) as follows (see Appendix):
\[
(Ω + D_A - LAΩ)x^* + 1 - λ|x^*|| = [(1 - λ)(Ω + D_A) + λ(Ω + U_A)]x^* + λb, i = 0, 1, 2, \ldots
\]

In the following, we will examine the convergence results for NGGS method II.

Theorem 3. Suppose that AVE (1) is solvable, let the diagonal values of A > 1 and D_A - L_A to be row diagonally dominant, and then the sequence of the NGGS method II converges to the unique solution \(x^*\) of AVE (1).

Proof. The uniqueness can be inferred directly from Theorem 2. For convergence, consider
\[
x^* + 1 - x^* = λ[(Ω + D_A - LAΩ)^{-1}||x^* + 1|| + (Ω + D_A - LAΩ)^{-1}[(1 - λ)(Ω + D_A) + λ(Ω + U_A)]x^* + λb.
\]

\[
(Ω + D_A - LAΩ)[x^* + 1 - x^*] = λ[(|x^*| - |x^*|) + ((1 - λ)(Ω + D_A) + λ(Ω + U_A))x^* - λ|x^*|,
\]
\[
(D_A - L_A - U_A)x^* = (D_A - L_A - U_A)x^* - x^*.
\]

(12)

From (4) and (12), we have
\[
Ax^* + |x^*| - Ax^* = |x^*|
\]
\[
Ax^* - |x^*| = b.
\]

Therefore, \(x^*\) solves the system of AVE (1).

4. Numerical Tests

Here, four examples are provided to illustrate the performance of the novel approaches from three different perspectives:

(i) The number of iterations (indicated by “Itr”)

(ii) The computational time (s) (exposed by “Time”)

(iii) The residual error (represented by “RSV”)

Here, “RSV” is defined by
\[
\text{RSV} = \|Ax^* - |x^*| - b\|/\|b\| \leq 10^{-6}.
\]

All numerical tests were performed on a personal computer with 1.80 GHz CPU (Intel(R) Core (TM) i5-3337U) and 4 GB of memory using MATLAB (2016a). In addition, the zero vector is the initial vector for Example 4

Example 4. Let
\[
A = \text{tridiag}(-1, 4, -1) \in R^{nxn}.
\]

Calculate \(b = Ax^* - |x^*| \in R^n\) with \(x^* = (x_1, x_2, x_3, \ldots, x_n)^T \in R^n\) such that \(x_i = (-1)^i\). Here, the
proposed methods are compared to two existing methods: the SOR-like optimal parameters technique shown in [16] (expressed by SLM using $\omega = 0.825$) and the shift splitting iteration approach described in [22] (represented by SM). The results are provided in Table 1.

Table 1 presents the solution $x^*$ for various values of $n$. The result of this comparison shows that our proposed techniques are more efficient than SLM and SM approaches in terms of “Itr” and “Time.”

Example 5. Consider $A = M + 4I \in \mathbb{R}^{n \times n}$ and $b = Ax^* - |x^*| \in \mathbb{R}^n$ with

$$M = \text{tridiag}(-I_n, H_n, -I_n) \in \mathbb{R}^{n \times n}, x^* = (-1, 1, -1, 1, \ldots, -1, 1)^T \in \mathbb{R}^n, \quad H_n = \text{tridiag}(-1, 4, -1) \in \mathbb{R}^{n \times n}, \quad I \in \mathbb{R}^{n \times n},$$

being a unit matrix and $n = v^2$. For Examples 5 and 6, use the same stopping criterion and initial guess mentioned in [18]. The recommended methods are compared with the AOR approach [14] and the mixed-type splitting (MTS) iterative technique [18]. The outcomes are summarized in Table 2.

In Table 2, we present the numeric outcomes of the AOR method, MTS method, NGGS method I, and NGGS method II, respectively. Our results indicate that the proposed techniques are more effective than both AOR and MTS approaches.

Example 6. Consider $A = M + I \in \mathbb{R}^{n \times n}$ and $b = Ax^* - |x^*| \in \mathbb{R}^n$ with

$$M = \text{tridiag}(-1.5I_n, H_n, -0.5I_n) \in \mathbb{R}^{n \times n}, x^* = (1, 2, 1, 2, \ldots)^T \in \mathbb{R}^n, \quad H_n = \text{tridiag}(-1.5, 4, -1) \in \mathbb{R}^{n \times n}, \quad n = v^2.$$

The findings are summarized in Table 3. Table 3 presents the solution $x^*$ for various values of $n$. The result of this comparison shows that our proposed techniques are more efficient than AOR and MTS approaches in terms of “Itr” and “Time.”

Example 7. Let

$$A = \text{tridiag}(-1, 8, -1) \in \mathbb{R}^{n \times n}, \quad x^* = ((-1)^{h}, h = 1, 2, \ldots, n)^T \in \mathbb{R}^n \text{and } b = Ax^* - |x^*| \in \mathbb{R}^n.$$ Applying the same stopping criteria and initial guess as given in [37], we compare the novel approaches with the technique shown in [37] (expressed by SA using $\omega = 1.0455$) and the SOR-like technique presented in [15] (denoted by SOR).

Table 4 shows that all tested techniques can quickly compute AVE (1). However, we see that the “Itr” and “Time” of the proposed approaches are less than the other known approaches. In conclusion, we find that the proposed approaches are feasible and useful for AVEs.

5. Conclusions

In this work, two novel NGGS approaches are presented for the purpose of determining AVEs, and their convergence properties are discussed in detail. Then, numerical experiments are used to demonstrate their effectiveness. Ultimately, the numerical tests show that the recommended procedures are more efficient in iteration steps and computing time than the existing methods.

Appendix

Here, we describe the implementation of the novel methods. From $Ax - |x| = b$, we have

$$x = A^{-1}(|x| + b). \quad (A.1)$$

Thus, we can approximate $x^{i+1}$ as follows:

$$x^{i+1} \approx A^{-1}(|x^i| + b). \quad (A.2)$$

This approach is known as the Picard approach [9]. Our next discussion concerns the algorithm for NGGS Method I. Algorithm for the NGGS Method I is as follows:

1. Select the parameters $\Psi$ and $\lambda$, an initial guess $x^0 \in \mathbb{R}^n$, and put $i = 0$

2. Compute

$$y^i = x^{i+1} \approx A^{-1}(|x^i| + b). \quad (A.3)$$

3. Calculate

$$x^{i+1} = \lambda(\Omega + D_A - \lambda L_A)^{-1}|y^i| + (\Omega + D_A - \lambda L_A)^{-1}[(1 - \lambda)(\Omega + D_A)

$$+ \lambda(\Omega + U_A)x^i + \lambda b]. \quad (A.4)$$

4. If $x^{i+1} = x^i$, then end. Otherwise, put $i = i + 1$ and go to step 2.

Similar considerations apply to the NGGS Method II.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

Author has no conflict of interest for this submission.

Acknowledgments

The author would like to thank the anonymous referees for their significant comments and suggestions.

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