Hindawi
Computational and Mathematical Methods
Volume 2022, Article ID 4266576, 7 pages
https://doi.org/10.1155/2022/4266576

Research Article

The Solution of Absolute Value Equations Using Two New Generalized Gauss-Seidel Iteration Methods

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Received 23 September 2021; Accepted 7 January 2022; Published 6 May 2022

Academic Editor: Jianming Shi

In this paper, we provide two new generalized Gauss-Seidel (NGGS) iteration methods for solving absolute value equations $Ax - |x| = b$, where $A \in R^{n\times n}$, $b \in R^n$, and $x \in R^n$ are unknown solution vectors. Also, convergence results are established under mild assumptions. Eventually, numerical results prove the credibility of our approaches.

1. Introduction

Consider the absolute value equation (AVE):

$$Ax - |x| = b, \tag{1}$$

where $A \in R^{n\times n}$, $|x| = (|x_1|, |x_2|, \cdots, |x_n|)^T$, and $b \in R^n$. A more general structure of the AVE is

$$Ax + B|x| = b, \tag{2}$$

where $B \in R^{n\times n}$, $B \neq 0$. When $B = -I$, where I denotes the identity matrix, then Eq. (2) reduces to the special form (1). The AVEs are significant nondifferentiable and nonlinear problems that appear in optimization, e.g., linear programming, journal bearings, convex quadratic programming, linear complementarity problems (LCPs), and network prices [1–13].

The numerical techniques for AVEs have received a lot of attention in recent years, and several approaches have been suggested, such as Li [14] proposed the preconditioned AOR iterative technique to determine AVE (1) and established the novel convergence results of the suggested scheme. To solve the AVE (1), Ke and Ma [15] introduced an SOR-like approach. Chen et al. [16] studied the concept of [15] extensively and presented an optimal parameter SOR-like approach. Huang and Hu [17] reformulated the AVE system as a standard LCP without any premise and showed some convexity and existence outcomes for determining AVE (1). Fakharzadeh and Shams [18] recommended the mixed-type splitting (MTS) iterative scheme for determining AVE (1) and established the novel convergence properties. Zhang et al. [19] developed a novel algorithm that transformed the AVE problem into an optimization problem associated with convexity. Caccetta et al. [20] examined a smoothing Newton technique for determining (1) and showed that the technique is globally convergent when $\|A^{-1}\| < 1$. Saheya et al. [11] investigated smoothing type techniques for determining (1) and showed that their techniques have global and local quadratic convergence. Gu et al. [21] proposed the nonlinear CSCS-like approach as well as the Picard-CSCS approach in order to determine (1), which concerns the Toeplitz matrix. Wu and Li [22] developed a special shift splitting technique to determine AVE (1) and demonstrated the novel convergence outcomes for the approach. Edalatpour et al. [23] established the generalized Gauss-Seidel (GGS) techniques for determining (1) and analyzed its convergence properties and others; see [24–35] and the references therein.

This article describes two new iterative approaches to determine AVEs. The main contributions we made to the article are as follows:

Table 1: Numerical results for Example 4 with $\Psi = 0.3$ and $\lambda = 0.95$.

Methods	n	1000	2000	3000	4000
SLM	Itr	18	18	18	18
	Time	3.0156	13.1249	33.9104	65.1345
	RSV	6.92e–07	6.93e–07	6.93e–07	6.94e–07
SM	Iter	14	14	14	14
	Time	2.8128	9.0954	17.3028	29.1644
	RSV	8.91e-07	8.92e-07	8.93e-07	8.93e-07
NGGS method I	Itr	10	10	10	10
	Time	2.4630	7.6228	14.6325	23.0959
	RSV	9.70e-07	6.85e–07	5.60e–07	4.85e–07
NGGS method II	Itr	6	6	6	6
	Time	1.1322	2.7342	4.0845	7.7877
	RSV	3.70e-06	3.68e-07	3.69e-07	3.69e-07

(i) We extend the GGS technique [23] to the general case. To achieve this goal, we impose two additional parameters (λ and Ω) that can accelerate the convergence procedure.

(ii) A variety of novel conditions are used to investigate the convergence properties of the newly developed methods.

The remainder of this paper is organized in the following manner. In Section 2, we present some notations and a lemma that will be used throughout the remainder of this study. In Section 3, we propose the NGGS procedures and discuss their convergence. We demonstrate the efficiency of these algorithms in Section 4 by providing numerical examples. In the last section, we make concluding remarks.

2. Preliminaries

Here we briefly examine some of the notations and concepts used in this article.

Let $A = (a_{ij}) \in R^{n\times n}$, we indicate the norm and absolute value as $\|A\|_\infty$ and $|A| = (|a_{ij}|)$, respectively. The matrix $A \in R^{n\times n}$ is called an Z-matrix if $a_{ij} \le 0$ for $i \neq j$ and an M-matrix if it is a nonsingular Z-matrix and with $A^{-1} \ge 0$.

Lemma 1. [36]. *Suppose x and $\in R^n$, then* $\|x - z\|_\infty \ge \||x| - |z|\|_\infty$.

3. NGGS Iteration Methods

Here, we examine the suggested methods (NGGS method I and NGGS method II) for determining AVE (1).

3.1. NGGS Method I for AVE. Recalling that the AVE (1) has the following form,

$Ax - |x| = b.$

Multiplying λ, then we get

$$\lambda Ax - \lambda|x| = \lambda b. \quad (3)$$

Let

$$A = D_A - L_A - U_A = (\Omega + D_A - L_A) - (\Omega + U_A), \quad (4)$$

where $D_A = \text{diag}\,(A)$,L_A, and U_A are strictly lower and upper triangular parts of A, respectively. Furthermore, $\Omega = \Psi I$, where $0 \le \Psi \le 1$ and I denote the identity matrix. Using (3) and (4), the NGGS Method I is suggested as

$$(\Omega + D_A - \lambda L_A)x - \lambda|x| = [(1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A)]x + \lambda b. \quad (5)$$

Using the iterative scheme, so (5) can be written as

$$(\Omega + D_A - \lambda L_A)x^{i+1} - \lambda\left|x^{i+1}\right| = [(1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A)]x^i + \lambda b, \quad (6)$$

where $i = 0, 1, 2, \cdots$, and $0 < \lambda \le 1$ (see Appendix). Note that if $\lambda = 1$ and $\Omega = 0$, then Eq. (6) reduces to the GGS method [23].

The next step in the analysis is to verify the convergence of NGGS method I by using the following theorem.

Theorem 2. *Suppose that AVE (1) is solvable, let the diagonal values of $A > 1$ and $D_A - L_A - I$ matrix be the strictly row wise diagonally dominant. If*

$$\|(\Omega + D_A - \lambda L_A)^{-1}[(1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A)]\|_\infty < 1 - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty, \quad (7)$$

then the sequence $\{x^i\}$ of the NGGS method I converges to the unique solution x^* of AVE (1).

Proof. We will prove first $\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty < 1$. Clearly, if we put $L_A = 0$, then $\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty = \|(\Omega + D_A)^{-1}\|_\infty < 1$. If we assume that $L_A \neq 0$, we get

$0 \le |\lambda L_A|\, t < (\Omega + D_A - I)t$,if we take

$|\lambda L_A|\, t < (\Omega + D_A - I)t$.

Taking both side by $(\Omega + D_A)^{-1}$, we get

$(\Omega + D_A)^{-1}\,|\lambda L_A|\, t < (\Omega + D_A)^{-1}((\Omega + D_A) - I)t$,

$|\lambda(\Omega + D_A)^{-1}L_A|\, t < (I - (\Omega + D_A)^{-1})t$,

$|\lambda(\Omega + D_A)^{-1}L_A|\, t < t - (\Omega + D_A)^{-1}t$,

$(\Omega + D_A)^{-1}t < t - |\lambda(\Omega + D_A)^{-1}L_A|\, t$,

$$(\Omega + D_A)^{-1}t < (1 - |Q|)t, \quad (8)$$

where $Q = \lambda(\Omega + D_A)^{-1}L_A$ and $t = (1, 1, \cdots, 1)^T$. Also, we have

Table 2: Numerical results for Example 5 with $\Psi = 0.2$ and $\lambda = 0.98$.

Methods	n	64	256	1024	4096
	Itr	14	14	15	35
AOR	Time	0.3483	1.9788	2.3871	5.8097
	RSV	5.215e-07	6.293e-07	6.548e-07	8.741e-07
	Itr	14	14	15	25
MTS	Time	0.3168	1.0952	1.9647	2.2194
	RSV	4.310e-07	5.468e-07	5.069e-07	9.384e-07
	Itr	12	13	13	14
NGGS method I	Time	0.2131	0.5285	1.8553	2.0033
	RSV	8.80e-07	7.32e-07	9.49e-07	4.11e-07
	Itr	5	5	5	5
NGGS method II	Time	0.1475	0.4187	1.3582	1.9283
	RSV	1.26e-07	1.40e-07	1.45e-07	1.47e-07

Table 3: Numerical results for Example 6 with $\Psi = 0.2$ and $\lambda = 0.98$.

Methods	n	100	400	900	1600	4900
	Itr	97	190	336	706	384
AOR	Time	0.4721	2.8203	3.2174	6.3887	9.2344
	RSV	9.80e-07	9.61e-07	9.73e-07	9.84e-07	9.36e-07
	Itr	88	157	250	386	342
MTS	Time	0.4041	1.7953	3.0219	5.7626	8.8965
	RSV	8.91e-07	9.65e-07	9.18e-07	9.56e-07	9.89e-07
	Itr	39	59	76	92	112
NGGS method I	Time	0.2309	0.4250	1.9633	2.5413	3.4387
	RSV	8.39e-07	8.90e-07	9.32e-07	9.31e-07	7.42e-07
	Itr	22	33	43	52	88
NGGS method II	Time	0.1486	0.2537	0.9255	1.3671	1.7898
	RSV	5.09e-07	7.45e-07	7.63e-07	9.17e-07	8.90e-07

Table 4: Numerical results for Example 7 with $\Psi = 0.3$ and $\lambda = 0.95$.

Methods	n	1000	2000	3000	4000	5000
	Itr	13	13	14	14	14
SA	Time	3.9928	8.8680	24.4031	51.3946	73.3394
	RSV	6.04e-07	8.54e-07	2.33e-07	2.69e-07	3.01e-07
	Itr	12	13	13	13	13
SOR	Time	1.5136	3.3817	6.1262	7.1715	9.5261
	RSV	9.45e-08	2.69e-08	3.29e-08	3.80e-08	4.25e-07
	Itr	10	10	10	10	10
NGGS method I	Time	1.3911	2.9736	3.6003	5.9112	7.7228
	RSV	5.77e−07	5.78e−07	5.78e−07	5.78e−07	5.78e−07
	Itr	5	6	6	6	6
NGGS method II	Time	0.2753	0.9910	1.3985	2.3515	2.9527
	RSV	7.15e−07	1.67e−08	2.04e−08	2.62e−08	3.81e−07

$$0 \le |(I-Q)^{-1}| = |I + Q + Q^2 + Q^3 + \cdots + Q^{n-1}|,$$

$$\le (I + |Q| + |Q|^2 + |Q|^3 + \cdots + |Q|^{n-1}) = (I - |Q|)^{-1}. \quad (9)$$

Thus, from (8) and (9), we get

$|(\Omega + D_A - \lambda L_A)^{-1}|t = |(I-Q)^{-1}(\Omega + D_A)^{-1}|t \le |(I-Q)^{-1}||(\Omega + D_A)^{-1}|t,$

$< (I - |Q|)^{-1}(I - |Q|)t = t.$

So, we obtain

$\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty < 1.$

Uniqueness: Let $x^\star$ and $z^\star$ be two different solutions of the AVE (1). Using (5), we get

$$x^\star = \lambda(\Omega + D_A - \lambda L_A)^{-1}|x^\star| + (\Omega + D_A - \lambda L_A)^{-1}[((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))x^\star + \lambda b], \quad (10)$$

$$z^\star = \lambda(\Omega + D_A - \lambda L_A)^{-1}|z^\star| + (\Omega + D_A - \lambda L_A)^{-1}[((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))z^\star + \lambda b]. \quad (11)$$

From (10) and (11), we get

$x^\star - z^\star = \lambda(\Omega + D_A - \lambda L_A)^{-1}(|x^\star| - |z^\star|) + (D_A - \lambda L_A)^{-1}((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))(x^\star - z^\star).$

Based on Lemma 1 and Eq. (7), the above equation can be expressed as follows:

$\|x^\star - z^\star\|_\infty \le \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty \||x^\star| - |z^\star|\|_\infty + \|(\Omega + D_A - \lambda L_A)^{-1}((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))\|_\infty \|x^\star - z^\star\|_\infty,$

$< \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty \|x^\star - z^\star\|_\infty + (1 - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty)\|x^\star - z^\star\|_\infty,$

$\|x^\star - z^\star\|_\infty - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty \|x^\star - z^\star\|_\infty < (1 - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty)\|x^\star - z^\star\|_\infty,$

$(1 - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty)\|x^\star - z^\star\|_\infty < (1 - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty)\|x^\star - z^\star\|_\infty,$

$\|x^\star - z^\star\|_\infty < \|x^\star - z^\star\|_\infty$, which is a contradiction. Thus, $x^\star = z^\star$. □

Convergence: We will consider $x^\star$ as the unique solution to AVE (1). Consequently, from (10) and

$x^{i+1} = \lambda(\Omega + D_A - \lambda L_A)^{-1}|x^{i+1}| + (\Omega + D - \lambda L_A)^{-1}[((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))x^i + \lambda b]$, we deduce

$x^{i+1} - x^\star = \lambda(\Omega + D_A - \lambda L_A)^{-1}(|x^{i+1}| - |x^\star|) + (\Omega + D_A - \lambda L_A)^{-1}[((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))(x^i - x^å)].$

By taking infinity norm and Lemma 1, we have

$\|x^{i+1} - x^\star\|_\infty - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty \|x^{i+1} - x^\star\|_\infty \le \|(\Omega + D_A - \lambda L_A)^{-1}((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))\|_\infty \|x^i - x^\star\|_\infty$, and since $\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty < 1$, it follows that

$\|x^{i+1} - x^\star\|_\infty \le \|(\Omega + D_A - \lambda L_A)^{-1}((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))\|_\infty / 1 - \lambda\|(\Omega + D_A - \lambda L_A)^{-1}\|_\infty \|x^i - x^\star\|_\infty.$

According to the inequality above, the presented approach converges to the solution when condition (7) is met.

3.2. NGGS Method II for AVE. In this section, we describe the NGGS method II. Based on (3) and (4), we can express the suggested method for determining AVE (1) as follows (see Appendix):

$(\Omega + D_A - \lambda L_A)x^i + 1 - \lambda|x^{i+1}| = [(1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A)]x^{i+1} + \lambda b, i = 0, 1, 2, \cdots.$

In the following, we will examine the convergence results for NGGS method II.

Theorem 3. *Suppose that AVE (1) is solvable, let the diagonal values of $A > 1$ and $D_A - L_A - I$ be row diagonally dominant, and then the sequence of the NGGS method II converges to the unique solution $x^\star$ of AVE (1).*

Proof. The uniqueness can be inferred directly from Theorem 2. For convergence, consider □

$x^{i+1} - x^\star = \lambda(\Omega + D_A - \lambda L_A)^{-1}|x^{i+1}| + (\Omega + D_A - \lambda L_A)^{-1}[((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))x^{i+1} + \lambda b] - (\lambda(\Omega + D_A - \lambda L_A)^{-1}|x^\star| + (\Omega + D_A - \lambda L_A)^{-1}[((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))x^\star + \lambda b]),$

$(\Omega + D_A - \lambda L_A)(x^{i+1} - x^\star) = \lambda(|x^{i+1}| - |x^\star|) + ((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))(x^{i+1} - x^\star),$

$\lambda(D_A - L_A - U_A)x^{i+1} - \lambda|x^{i+1}| = \lambda(D_A - L_A - U_A)x^\star - \lambda|x^\star|,$

$$(D_A - L_A - U_A)x^{i+1} - |x^{i+1}| = (D_A - L_A - U_A)x^\star - |x^\star|. \quad (12)$$

From (4) and (12), we have

$Ax^{i+1} - |x^{i+1}| = Ax^\star - |x^\star|$

$Ax^{i+1} - |x^{i+1}| = b.$

Therefore, x^{i+1} solves the system of AVE (1).

4. Numerical Tests

Here, four examples are provided to illustrate the performance of the novel approaches from three different perspectives:

(i) The number of iterations (indicated by "Itr")

(ii) The computational time (s) (exposed by "Time")

(iii) The residual error (represented by "RSV")

Here, "RSV" is defined by

$\mathrm{RSV} := \|Ax^i - |x^i| - b\|_2 / \|b\|_2 \le 10^{-6}.$

All numerical tests were conducted on a personal computer with 1.80 GHz CPU (Intel(R) Core (TM) i5-3337U) and 4 GB of memory using MATLAB (2016a). In addition, the zero vector is the initial vector for Example 4

Example 4. Let

$$A = \mathrm{tridiag}(-1, 4, -1) \in R^{n\times n}.$$

Calculate $b = Ax^\star - |x^\star| \in R^n$ with $x^\star = (x_1, x_2, x_3, \cdots, x_n)^T \in R^n$ such that $x_i = (-1)^i$. Here, the proposed methods are compared to two existing methods: the SOR-like optimal parameters technique shown in [16] (expressed by SLM using $\omega = 0.825$) and the shift splitting iteration approach described in [22] (represented by SM). The results are provided in Table 1.

Table 1 presents the solution $x^\star$ for various values of n. The result of this comparison shows that our proposed techniques are more efficient than SLM and SM approaches in terms of "Itr" and "Time."

Example 5. Consider $A = M + 4I \in R^{n\times n}$ and $b = Ax^\star - |x^\star| \in R^n$ with

$M = \text{tridiag}(-I_n, H_n, -I_n) \in R^{n\times n}, x^\star = (-1, 1, -1, 1, \cdots, -1, 1)^T \in R^n$, where $H_n = \text{tridiag}(-1, 4, -1) \in R^{v\times v}$, $I \in R^{v\times v}$, being a unit matrix and $n = v^2$. For Examples 5 and 6, use the same stopping criterion and initial guess mentioned in [18]. The recommended methods are compared with the AOR approach [14] and the mixed-type splitting (MTS) iterative technique [18]. The outcomes are summarized in Table 2.

In Table 2, we present the numeric outcomes of the AOR method, MTS method, NGGS method I, and NGGS method II, respectively. Our results indicate that the proposed methods are more effective than both AOR and MTS approaches.

Example 6. Consider $A = M + I \in R^{n\times n}$ and $b = Ax^\star - |x^\star| \in R^n$ with

$M = \text{tridiag}(-1.5I_n, H_n, -0.5I_n) \in R^{n\times n}, x^\star = (1, 2, 1, 2, \cdots)^T \in R^n$, where $H_n = \text{tridiag}(-1.5, 4, -0.5) \in R^{v\times v}$ and $n = v^2$. The findings are summarized in Table 3.

Table 3 presents the solution $x^\star$ for various values of n. The result of this comparison shows that our proposed techniques are more efficient than AOR and MTS approaches in terms of "Itr" and "Time."

Example 7. Let

$A = \text{tridiag}(-1, 8, -1) \in R^{n\times n}, x^\star = ((-1)^h, h = 1, 2, \cdots, n)^T \in R^n$ and $b = Ax^\star - |x^\star| \in R^n$. Applying the same stopping criteria and initial guess as given in [37], we compare the novel approaches with the technique shown in [37] (expressed by SA using $\omega = 1.0455$) and the SOR-like technique presented in [15] (denoted by SOR).

Table 4 shows that all tested techniques can quickly compute AVE (1). However, we see that the "Itr" and "Time" of the proposed approaches are less than the other known approaches. In conclusion, we find that the proposed approaches are feasible and useful for AVEs.

5. Conclusions

In this work, two novel NGGS approaches are presented for the purpose of determining AVEs, and their convergence properties are discussed in detail. Then, numerical experiments are used to demonstrate their effectiveness. Ultimately, the numerical tests show that the recommended procedures are more efficient in iteration steps and computing time than the existing methods.

Appendix

Here, we describe the implementation of the novel methods. From $Ax - |x| = b$, we have

$$x = A^{-1}(|x| + b). \tag{A.1}$$

Thus, we can approximate x^{i+1} as follows:

$$x^{i+1} \approx A^{-1}\left(\left|x^i\right| + b\right). \tag{A.2}$$

This approach is known as the Picard approach [9]. Our next discussion concerns the algorithm for NGGS Method I.

Algorithm for the NGGS Method I is as follows:

(1) Select the parameters Ψ and λ, an initial guess $x^0 \in R^n$, and put $i = 0$

(2) Compute

$$y^i = x^{i+1} \approx A^{-1}\left(\left|x^i\right| + b\right). \tag{A.3}$$

(3) Calculate

$$x^{i+1} = \lambda(\Omega + D_A - \lambda L_A)^{-1}\left|y^i\right| + (\Omega + D_A - \lambda L_A)^{-1}[((1-\lambda)(\Omega + D_A) + \lambda(\Omega + U_A))x^i + \lambda b]. \tag{A.4}$$

(4) If $x^{i+1} = x^i$, then end. Otherwise, put $i = i + 1$ and go to step 2

Similar considerations apply to the NGGS Method II.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

Author has no conflict of interest for this submission.

Acknowledgments

The author would like to thank the anonymous referees for their significant comments and suggestions.

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